Research On Adaptive Optimal Iterative Learning Control Based on Least Squares

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Title page

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ORIGINAL ARTICLE

Research on adaptive optimal iterative learning control based on least squares

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Abstract: Traditional Optimal Iterative Learning Control (TOILC) can effectively improve the tracking performance of the servo system. However, there may be parameter perturbation in the running process of the servo system, and its parameters are constantly changing slowly. As a result, the convergence of TOILC becomes worse, and the tracking performance of the system deteriorates seriously. Therefore, in view of the time-varying characteristics of the system, a least squares optimal iterative learning control (LSAOILC) algorithm is proposed. In the process of iteration, the nominal model of the system is identified according to the input and output signals so as to update the optimal iterative learning controller, which does not need to obtain the exact system model information in advance, making up for the shortage of TOILC. The simulations and experiments prove the effectiveness of the proposed strategy for the servo system.

Keywords: Optimal iterative learning, Servo system, System identification, Adaptive control, Data-driven.

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1 Introduction

Iterative learning control is generally used to eliminate the repetitive error of a system when performing a repetitive task. The initial iterative learning can obtain good control performance without considering the precise system model in the design process and is widely used in motion control occasions such as manipulator. Arimoto et al. [1-4] adopted P-type or PD-type iterative learning control. The design of its controller is similar to PID feedback control. The controller is designed to meet performance requirements through tuning. In the design process, only a rough understanding of the system, the hysteresis and the size of the first Markov parameters are used to ensure the stability of the system without requiring an accurate system model. With the in-depth research of scientific researchers, it is found that the accuracy of the model of the controlled object has a certain impact on the performance of the iterative learning controller. Literature [5] applied P-type iterative learning control and H_∞ iterative learning control and optimized iterative learning control to ABB manipulators, and compared their performance. The results show that H_∞ iterative learning control and optimized iterative learning control are due to the more accurate system model has better tracking performance. At present, the main research direction of the researchers is to combine optimal control, H_∞, filter design and other methods with iterative learning, and fully consider the model information of the system during the design process for controller design [6-10], among which the most the optimal iterative learning control has good transient behavior and monotonic convergence along the iterative direction. Therefore, the optimal iterative learning control has been widely used and researched. Gunnarsson [11-14] et al. obtained the Toeplitz matrix of the system through identification methods, and then used the optimization theory to design the iterative learning controller based on the Toeplitz matrix. Obtain good tracking performance. O.H et al. [15-17] designed an iterative learning controller based on an optimization method and introduced a window matrix to suppress the residual vibration of the lithography machine motion control system. All of the above methods rely on the Toeplitz matrix of the system, and the parameter perturbation is not considered which is caused by the long-term operation of the system. This may cause the performance deterioration of the iterative learning algorithm.

At present, iterative learning control is mainly designed for linear time-invariant systems, but there will be certain parameter perturbations during the long-term operation of the system and the system parameters will change slowly. Aiming at the time-varying characteristics of the system, literature [18] proposed a data-driven recursive least squares identification algorithm. The algorithm is only a research of system parameter identification research and does not mention the combination of iterative learning control in the iterative process. Literature [19] designed a model predictive iterative learning control algorithm based on quadratic criteria. Based on model identification, it combined iterative learning control with model predictive control, and designed an iterative learning controller based on model prediction. It only conducts theoretical and simulation analysis, and has no practical application. Yu et al. [20] proposed an improved iterative learning recursive least-squares identification algorithm for the unknown time-varying of high-speed train systems, which combined with traditional iterative learning control to continuously identify nonlinear system model parameters in the iterative process. And good results have been achieved in the simple model experiment of high-speed trains. These methods require prior knowledge of the system to construct the system model structure and perform parameter identification on this basis, but it is actually difficult to construct an accurate system model structure. Literature [21-23] proposed a data-driven optimal iterative learning control framework and applied it to nonlinear system control. The algorithm does not require precise system model information during the controller design process, but only needs to be bsed on system input-output data is designed for the controller, but this paper only carries on the theoretical simulation and does not apply to the actual physical object.

Therefore, according to the time-varying characteristics of the servo system, this paper proposes a LSAOILC algorithm that combines non-parametric model identification with TOILC; In the process of the system running, the nominal model of the system is constantly identified, which makes up for the shortage that TOILC cannot cope with the time-varying of the system; in addition, the performance of TOILC depends on the accuracy of the system model obtained in advance, while LSAOILC only needs system input and output data in the implementation process without obtaining an accurate system model in advance; and the algorithm is applied to the motion control of the servo system. The system still has good tracking performance when the system has time-varying characteristics.

The structure of this paper is as follows. Section II introduces the mathematical model of the servo system. Section III is the design of TOILC controller and analysis of its convergence. Section IV proposes a LSAOILC based on the influence of system time-variant. The simulation and experimental verification of the algorithm are presented in Section V. Finally, Section VI summarizes the conclusions.

2 Preliminaries

The goal of this article is to design a controller for a servo system with repetitive motion characteristics, so that it can achieve good trajectory tracking performance. This section gives a system description, and the block diagram of the control system is shown in Figure 1.

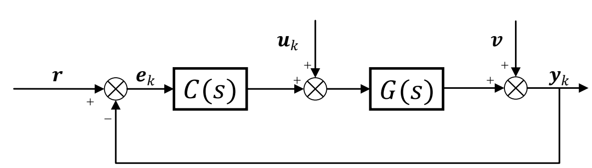

Figure 1 Control system block diagram

Here, $G(s)$ is the plant, $C(s)$ is the feedback controller, v is the noise, $\boldsymbol{r}$ is the reference trajectory, $\boldsymbol{e}_k$, $\boldsymbol{u}_k$ and $\boldsymbol{y}_k$ represent the error signal, feedforward signal and output signal of the k^{th} iteration, respectively. All signals are defined at finite time intervals $t = 0,\ldots,N$. The output of the k^{th} iteration is

$$\boldsymbol{y}_k = \left[y_k(0), y_k(1), y_k(2) \ldots y_k(N-1) \right]^T \tag{1}$$

N denotes the sampling points, $\boldsymbol{r}$, $\boldsymbol{e}_k$ and $\boldsymbol{y}_k$ are represented by vectors in the same form as $\boldsymbol{y}_k$. Regardless of the influence of noise, the system output equation is

$$\boldsymbol{y}_k = \boldsymbol{T}_r \boldsymbol{r} + \boldsymbol{T}_u \boldsymbol{u}_k \tag{2}$$

Toeplitz matrix $\boldsymbol{T}_u$ is formed by the impulse response coefficients of the transfer operator $G(s)/(1+G(s)C(s))$, i.e.

$$\boldsymbol{T}_u = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(N-1) & h(N-2) & \cdots & h(0) \end{bmatrix} \tag{3}$$

and $\boldsymbol{T}_r$ is defined analogously. When the change of system parameters is not considered, according to Eq. (2) the difference between the k^{th} and $(k-1)^{th}$ outputs is

$$\Delta \boldsymbol{y}_k = \boldsymbol{T}_u \Delta \boldsymbol{u}_k \tag{4}$$

where $\Delta \boldsymbol{y}_k = \boldsymbol{y}_k - \boldsymbol{y}_{k-1}$, $\Delta \boldsymbol{u}_k = \boldsymbol{u}_k - \boldsymbol{u}_{k-1}$. With the basic mathematical model of the servo system, the controller is designed and analyzed.

3 Problem formulation

This section first introduces the framework of TOILC, and then analyzes the convergence of the algorithm. Finally, the influence of system time-varying factors on TOILC is analyzed, and the solution is put forward.

3.1 TOILC scheme

TOILC usually uses the control structure shown in Figure 2. A general updating equation is given by

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \boldsymbol{L}\boldsymbol{e}_k \tag{5}$$

where

$$\boldsymbol{e}_{k+1} = \boldsymbol{r} - \boldsymbol{T}_r \boldsymbol{r} - \boldsymbol{T}_u \boldsymbol{u}_{k+1} \tag{6}$$

The idea is to determine $\boldsymbol{u}_{k+1}$ such that the error $\boldsymbol{e}_{k+1}$ becomes as small as possible by minimizing the criterion

$$J_{k+1} = \boldsymbol{e}_{k+1}^T \boldsymbol{W}_e \boldsymbol{e}_{k+1} + \Delta \boldsymbol{u}_{k+1}^T \boldsymbol{W}_{du} \Delta \boldsymbol{u}_{k+1} \tag{7}$$

where $\boldsymbol{W}_e$ and $\boldsymbol{W}_{du}$ are positive definite weight matrices such that the objective J_{k+1} is strictly convex. Usually, $\boldsymbol{W}_e$ is the identity matrix $\boldsymbol{I}$, $\boldsymbol{W}_{du}=\rho \cdot \boldsymbol{I}$ and $\rho \in (0,1]$ is the controller design parameter.

Based on Eq. (6) and Eq. (7), differentiating J_{k+1} with respect to $\boldsymbol{u}_{k+1}$ and setting this derivative equal to zero yields ILC updating Eq. (5) with

$$\boldsymbol{L} = (\rho \cdot \boldsymbol{I} + \boldsymbol{T}_u^T \boldsymbol{W}_e \boldsymbol{T}_u)^{-1} \boldsymbol{T}_u^T \boldsymbol{W}_e \tag{8}$$

Remark 1. The weight matrices $\boldsymbol{W}_{du}$ regulates the effect of noise and time-varying disturbance on the next iteration feedforward signal, thus affecting the asymptotic error. Introducing into $\boldsymbol{W}_{du}$ and the objective function J_{k+1} can improve the robustness of the ILC algorithm such that the tracking performance of the system is satisfied when the system parameters change little.

Whereas TOILC uses a parametric plant model to obtain the Toeplitz matrix $\boldsymbol{T}_u$ before iteration. This has two shortcomings. Firstly, it is impossible to obtain accurate $\boldsymbol{T}_u$ due to some non-linear factors and noise. Secondly, the parameters of the servo system may change slowly during operation, which makes the TOILC algorithm unable to converge and deteriorates the tracking performance. A method to identify the Toeplitz matrix $\boldsymbol{T}_u$ using only input and output data is proposed [21],[23], and the estimated convolution matrix is then used in TOILC.

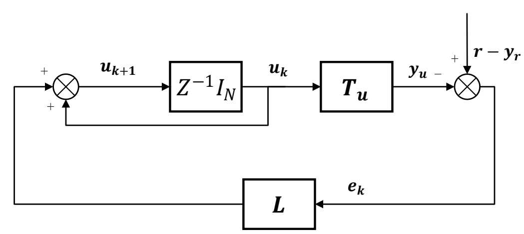

Figure 2 ILC block diagram

3.2 Convergence analysis

There is a common understanding of the convergence condition of ILC, that is,

$$\bar{\sigma}(\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_u) < 1 \tag{9}$$

where $\bar{\sigma}(\cdot)$ represents the largest singular value of the matrix.

When the system parameters do not change, $\boldsymbol{T}_u$ is a constant and the singular value decomposition of $\boldsymbol{T}_u$ can be obtained:

$$\boldsymbol{T}_u = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T \tag{10}$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices, $\boldsymbol{\Sigma}$ is the diagonal matrix.

Take $\boldsymbol{W}_e = \boldsymbol{I}$ and combine Eq. (8) and Eq. (10) to get the equation:

$$\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_u = \boldsymbol{V}(\rho \cdot \boldsymbol{I} + \boldsymbol{\Sigma}^2)^{-1}(\rho \cdot \boldsymbol{I})\boldsymbol{V}^T \tag{11}$$

Therefore, the singular value of $\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_u$ can be obtained from the knowledge of matrix theory, that is:

$$\sigma(\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_u) = \rho / (\rho + \sigma_i^2) \tag{12}$$

where $\sigma_i > 0$ is the diagonal of $\boldsymbol{\Sigma}$, so Eq. (12) satisfies the inequality:

$$\rho / (\rho + \sigma_i^2) < 1 \tag{13}$$

From inequality (13), when the system is not affected by time-varying factors, the maximum singular value of matrix $\boldsymbol{I} - \boldsymbol{L}\boldsymbol{T}_u$ is less than 1, which satisfies the convergence condition (9).

Remark 2. The introduction of parameter ρ can ensure that the TOILC algorithm meets the convergence conditions, rather than relying solely on the characteristics of the system itself. So that the system has a certain degree of robustness.

3.3 The impact of system time-variant

Although TOILC has a certain degree of robustness, its performance will be severely affected when the system parameters change to a certain degree. The ILC block diagram with system time-varying characteristics is shown in the Figure 3.

As shown in Figure 3, $\tilde{\boldsymbol{T}}_u$ represents the nominal model of the system with time variation. Since $\boldsymbol{T}_u \neq \tilde{\boldsymbol{T}}_u$, $\sigma(\boldsymbol{I} - \boldsymbol{L}\tilde{\boldsymbol{T}}_u)$ cannot be derived to Eq. (12). Therefore, when the system has time-varying characteristics, TOILC cannot ensure that the system meets the convergence condition (9).

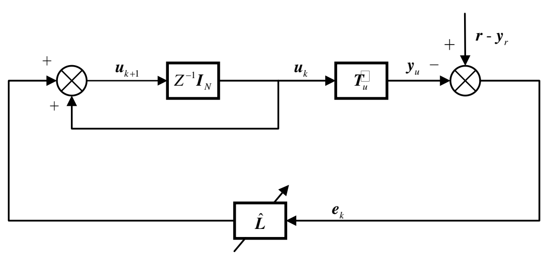

Figure 3 ILC block diagram with system time-variant

So, the controller $\boldsymbol{L}$ needs to be updated in the iterative process, as show in Figure 3, that is:

$$\hat{\boldsymbol{L}} = (\rho \cdot \boldsymbol{I} + \hat{\boldsymbol{T}}_u^T \boldsymbol{W}_e \hat{\boldsymbol{T}}_u)^{-1} \hat{\boldsymbol{T}}_u^T \boldsymbol{W}_e \tag{14}$$

where $\hat{\boldsymbol{T}}_u$ represents the identification value of $\tilde{\boldsymbol{T}}_u$, the convergence condition can be rewritten as:

$$\bar{\sigma}(I - \hat{\boldsymbol{L}}\tilde{\boldsymbol{T}}_u) < 1 \tag{15}$$

When the estimation error is small, this means $\hat{\boldsymbol{T}}_u \approx \tilde{\boldsymbol{T}}_u$, the system can still meet the convergence condition. In this way, the influence of system parameters on tracking performance can be effectively improved.

The main contribution of this paper is to propose an adaptive optimal iterative learning control algorithm based on least squares. It mainly includes:

1) System parametric model does not need to be constructed, and the controller is only designed based on the input and output data of the system.

2) A unit impulse response identification algorithm based on least squares is proposed, which is identified according to the input and output changes of the system.

3) The controller $\boldsymbol{L}$ is updated in the iterative process, so as to effectively deal with the influence of system time-variant.

4 Adaptive ILC based on least squares

In this section, the system impulse response identification method based on least squares is proposed and combined with TOILC to form an adaptive ILC, namely LSAOILC. The proposed LSAOILC can effectively deal with the impact of system parameter changes on tracking performance.

4.1 Impulse response identification

The impulse response is identified by input-output data to form the Toeplitz matrix of the system. When the system parameters remain unchanged, Eq. (4) can be reformulated as:

$$\Delta \boldsymbol{y}_k = \Delta \boldsymbol{U}_k \boldsymbol{h}_k \tag{16}$$

where $\boldsymbol{h}_k = \begin{bmatrix} h(0) & h(1) & \cdots & h(N-1) \end{bmatrix}^T$ denotes impulse response,

$$\Delta \boldsymbol{U}_k = \begin{bmatrix} \Delta u_k(0) & 0 & \cdots & 0 \\ \Delta u_k(1) & \Delta u_k(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Delta u_k(N-1) & \Delta u_k(N-2) & \cdots & \Delta u_k(0) \end{bmatrix} \tag{17}$$

denotes the lower-triangular Toeplitz matric of vector $\Delta \boldsymbol{u}_k$. The objective function can be obtained according to the least square estimation algorithm:

$$J = \left(\Delta \boldsymbol{y}_k - \Delta \boldsymbol{U}_k \hat{\boldsymbol{h}}_k\right)^T \left(\Delta \boldsymbol{y}_k - \Delta \boldsymbol{U}_k \hat{\boldsymbol{h}}_k\right) \tag{18}$$

Where $\hat{\boldsymbol{h}}_k$ represents the estimated value of impulse response $\boldsymbol{h}_k$, differentiating J with respect to $\hat{\boldsymbol{h}}_k$ and setting this derivative equal to zero:

$$\frac{\partial J}{\partial \hat{\boldsymbol{h}}_k} = -2\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{y}_k + 2\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k \hat{\boldsymbol{h}}_k = 0 \tag{19}$$

Therefore, the impulse response estimate can be obtained:

$$\hat{\boldsymbol{h}}_k = \left(\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k\right)^{-1} \Delta \boldsymbol{U}_k^T \Delta \boldsymbol{y}_k \tag{20}$$

In many practical situations, the inverse of the matrix $\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k$ does not exist, resulting in no solution to Eq. (5). In this case, the algorithm will be unstable. Therefore, the Moore-Penrose generalized inverse matrix is substituted into

Eq. (20), that is:

$$\hat{\boldsymbol{h}}_k = \left(\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k\right)^+ \Delta \boldsymbol{U}_k^T \Delta \boldsymbol{y}_k \tag{21}$$

So, the estimated value $\hat{\boldsymbol{T}}_{\boldsymbol{u}}$ of the system Toeplitz matrix can be generated by Eq. (21).

Remark 3. Moore-Penrose generalized inverse matrix can be obtained by matrix singular value decomposition. Using the generalized inverse matrix to solve can avoid the influence of singular values and make the estimation algorithm more stable. On the other hand, this identification algorithm is only applicable for the case that the system parameters remain unchanged or remain unchanged within a certain period of time.

4.2 LSAOILC procedure

In some practical cases, the parameters of the servo system do not change all the time but a slow changing process. In other words, from the perspective of the iteration domain, system parameters change slowly in the iteration domain, but not every iteration. On the other hand, the change of system parameters will lead to poor convergence of the algorithm and result in larger tracking error. So, the performance index function is introduced:

$$\|\boldsymbol{e}_k\| < \xi \tag{22}$$

Where ξ is a positive constant, this inequality is used to monitor system tracking error in the iteration process. When the error does not meet the performance requirements, the identification algorithm and TOILC can be combined to form LSAOILC. Therefore, the LSAOILC procedure is proposed.

LSAOILC Procedure
(1) Choose the appropriate weight matrices $\boldsymbol{W}_{\boldsymbol{e}}$ and $\boldsymbol{W}_{\boldsymbol{du}}$.
(2) Obtain the impulse response of the system through simple experiments to obtain the imprecise system Toeplitz matrix, the initial value of the control signal $\boldsymbol{u}_0$ is set to 0.
(3) In the iterative process, judge whether the tracking error satisfies inequality (22), if not, use Eq. (21) for identification to obtain the system model estimate $\hat{\boldsymbol{T}}_{\boldsymbol{u}}$, otherwise no identification.
(4) Substitute $\hat{\boldsymbol{T}}_{\boldsymbol{u}}$ into Eq. (14) to calculate the controller $\hat{\boldsymbol{L}}$, if there is no identification, the controller $\hat{\boldsymbol{L}}$ remains unchanged.
(5) The controller $\hat{\boldsymbol{L}}$ is substituted into Eq. (5) to calculate the next control signal $\boldsymbol{u}_{k+1}$.
(6) Repeat process (3)-(5) continuously to keep the servo system meeting the tracking performance.

4.3 Analysis of LSAOILC

In practice, however, the measured output is always subject to noise[24]. Therefore, measured out change $\Delta \boldsymbol{y}_k^n = \Delta \boldsymbol{y}_k + \boldsymbol{n}_k$, the impulse response estimate is rewritten as:

$$\begin{aligned}\hat{\boldsymbol{h}}_k &= \left(\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k\right)^{-1} \Delta \boldsymbol{U}_k^T \Delta \boldsymbol{y}_k^n \\ &= \left(\Delta \boldsymbol{U}_k^T \Delta \boldsymbol{U}_k\right)^{-1} \Delta \boldsymbol{U}_k^T (\Delta \boldsymbol{y}_k + \boldsymbol{n}_k)\end{aligned} \tag{23}$$

On the other hand, combining Eq. (4) and Eq. (5) are available, larger $\boldsymbol{e}_{k-1}$ will inevitably lead to larger $\Delta \boldsymbol{u}_k$ and $\Delta \boldsymbol{y}_k$. From Eq. (23), the accuracy of the LSAOILC identification algorithm is not only related to $\Delta \boldsymbol{y}_k$, but also affected by noise. Therefore, inequality (22) can effectively suppress the influence of noise on identification accuracy.

Secondly, according to Remark 1, TOILC has certain robustness, and the performance of the algorithm will not be affected when the system parameters change small. For the servo system with time-varying characteristics, the parameters change slowly. Only when the parameter changes to a certain extent, the convergence of TOILC will deteriorate, which will seriously deteriorate the tracking performance of the system. Therefore, the introduction of inequality (22) can effectively avoid the running the identification algorithm in each iteration and improve the running speed of the algorithm.

At last, from Eq. (21) and inequality (22), it can be seen that the identification algorithm of LSAOILC does not always run in the iterative domain, so the convergence of the identification algorithm can be guaranteed. Therefore, as long as the identification accuracy is high, the convergence of LSAOILC can be guaranteed. The most important point is that the identification algorithm of LSAOILC only needs to be run once to get an accurate $\hat{\boldsymbol{T}}_{\boldsymbol{u}}$. Therefore, the identification algorithm of LSAOILC converges fast.

Remark 4. The identification of impulse response does not require the construction of a parameterized model of the system. LSAOILC can effectively deal with the impact of time-varying system, and make up for the shortcomings of TOILC.

5 Simulation analysis and experiment test

In this section, the LSAOILC is verified through simulation and experiment. The designed simulation experiments verify the performance of LSAOILC in dealing with time-varying system, and verify it on the experimental platform of Brushless DC servo motor.

5.1 Simulations

In the simulation, TOILC and LSAOILC are used to control the plant respectively. The effect of noise is not considered in the simulation. The third-order reference trajectory shown in Figure 4 is adopted and its trajectory parameters are: $r_{\max} = 3600$ deg , $v_{\max} = 8000$ deg $\cdot$ s^{-1} , $a_{\max} = 50000$ deg $\cdot$ s^{-2} , $j_{\max} = 1 \times 10^8$ deg $\cdot$ s^{-3}, $t = 1.0235$ s .

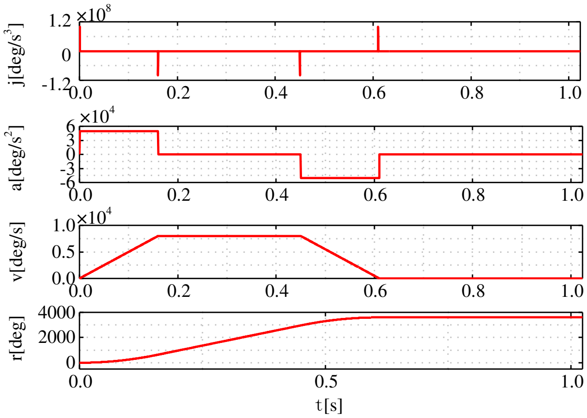

Figure 4 Third-order reference trajectory

The simulation object consists of the second-order system and the oscillation and anti-oscillation links, and its transfer function is:

$$G(s)=\frac{1}{24s^2+10000s}\times\frac{s^2+0.14\times100s+100^2}{s^2+0.16\times110s+110^2} \quad (24)$$

The simulation uses the control structure shown in Figure 1, and its PID parameters are: $K_p=503957$, $K_i=65807879$, $K_d=0$. Divide the simulation into two parts: 1) system parameters remain unchanged, 2) system parameters change.

5.1.1 System Parameters Remain Unchanged

Use TOILC for control when the system parameters remain unchanged and $\boldsymbol{W}_e=\boldsymbol{I}$, $\rho=10^{-14}$.

When the system parameters remain unchanged, the tracking error two-norm of tool TOILC is shown in Figure 5. It can be seen from Figure 5 that TOILC has better convergence when the system parameters remain unchanged.

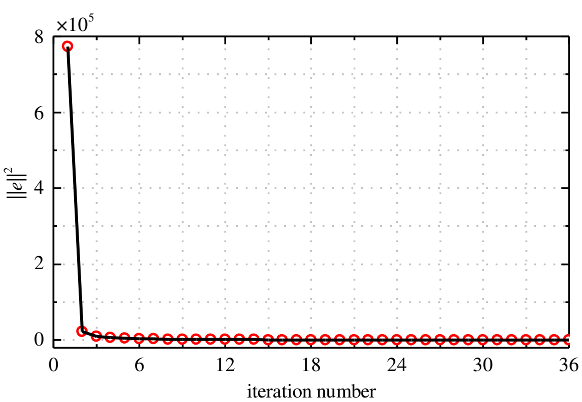

Figure 5 Two-norm tracking error of TOILC without parameter change

The error comparison curve between different iterations of TOILC is shown in Figure 6. It is obvious that the maximum error order of magnitude before iteration is 10^1 deg, and the maximum error order of magnitude of the 10^{th} iteration is reduced to 10^{-1} deg. Therefore, the system tracking error can be effectively reduced by TOILC when the system parameters remain unchanged.

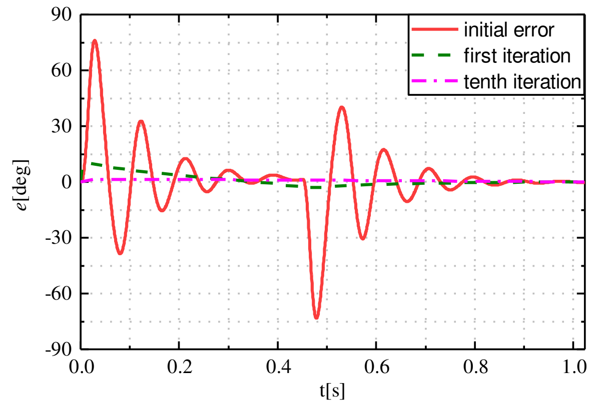

Figure 6 Tracking error of TOILC without parameter change

5.1.2 System Parameters Change

It can be seen from Remark1 that the performance of TOILC will deteriorate when the system parameters change to a certain extend, and the servo system parameters change slowly. Therefore, the simulation deteriorates TOILC performance by changing system parameters at the 18^{th} iteration. The new transfer function after changing the system parameters is as follows:

$$G(s)=\frac{1.7}{24s^2+10000s}\times\frac{s^2+0.14\times100s+100^2}{s^2+0.16\times110s+110^2} \quad (25)$$

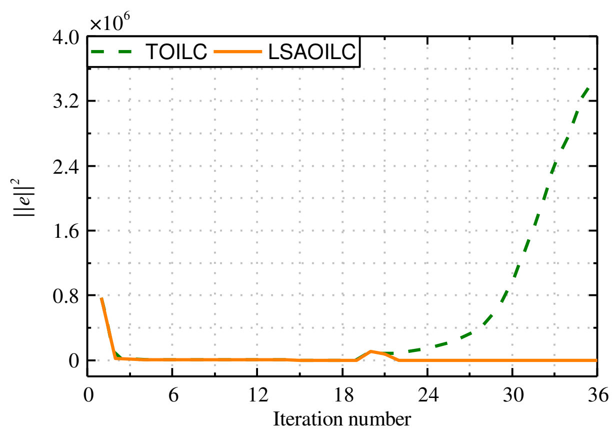

Figure 7 Two-norm of the tracking error

TOILC and LSAOILC are used in the control system, respectively. Figure 7 shows the objective function of the two algorithms when the system parameters change in the 18^{th} iteration. It can be seen from the Figure 7 that TOILC diverges rapidly after the system parameters change and the LSAOILC converges after several iterations. The LSAOILC algorithm is shown more clearly in Figure 8. It is found that LSAOILC only needs three iterations to converge, which shows that it has good convergence performance.

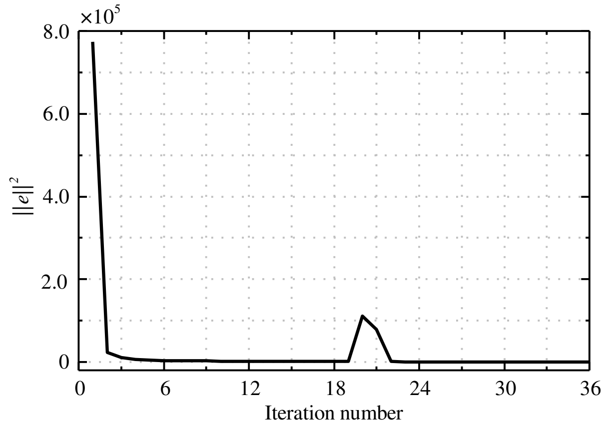

Figure 8 Two-norm of the tracking error for LSAOILC

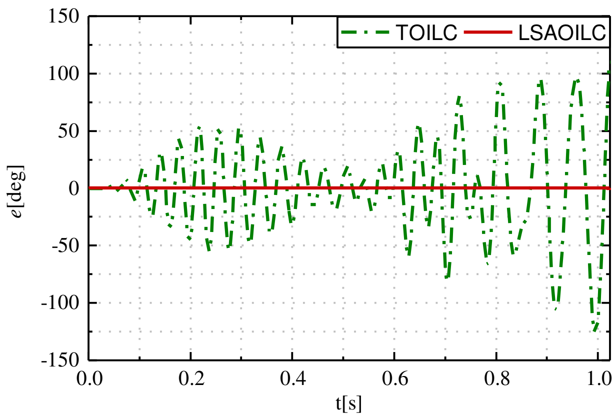

Figure 9 The 36^{th} iteration tracking error

The tracking error of the 36^{th} iteration of the two algorithms is shown in Figure 9 and the maximum error of TOILC is deteriorated severely, reaching 10^2 deg orders of magnitude. On the other hand, the maximum error of LSAOILC is reduced to the order of 10^{-1} deg .

Through simulation, it can be obtained that TOILC cannot cope with the time-varying characteristics of the system, but LSAOILC can effectively cope with it.

5.2 Experiments

The experimental platform is shown in Figure 10. Motion control platform composed of DC brushless motors. The torque output range of the actuator is -30mN·m~30mN·m. The rated voltage of the motor is 24V, the rated current is 1.17A, the rated speed is 6000r/min, the locked-rotor torque is 104 mN·m, the torque constant is 27.8mN·m /A, and the encoder is 2000 lines. The master computer is a PC, and the slave computer is a four-axis drive-control integrated control card designed with an ARM chip.

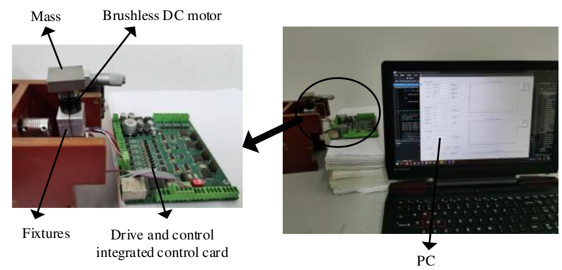

Figure 10 Servo motor experiment platform

The control structure shown in Figure 11 was used in the experiment. Among them, $G(s)$ represents a mechanical system composed of DC brushless motors and loads, and $P(s)$ represents a parameter perturbation simulator designed inside the motion control card for algorithm verification. $P(s)$ and $G(s)$ together constitute the controlled object of the servo system. The transfer function of $P(s)$ is:

$$P(s)=\frac{s^2+2\cdot\beta_{zero}\cdot\omega_{zero}\cdot s+\omega_{zero}^2}{s^2+2\cdot\beta_{pole}\cdot\omega_{pole}\cdot s+\omega_{pole}^2}\cdot\frac{\omega_{pole}^2}{\omega_{zero}^2} \tag{26}$$

where $\omega_{zero}=2\cdot\pi\cdot f_{zero}$, $\omega_{pole}=2\cdot\pi\cdot f_{pole}$ and the values are shown in Table I.

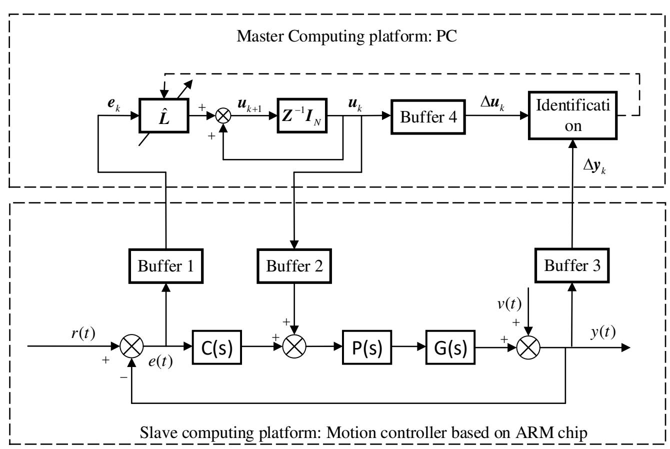

Figure 11 Experimental control plan structure diagram

Table 1 Parameters of $P(s)$

Parameter	Value
f_{zero}	12
f_{pole}	12
β_{zero}	0.25
β_{pole}	0.25

In the iterative process, the parameter changes caused by long-term operation of the system are simulated by changing $P(s)$. The parameters of the PID feedback controller are shown in Table II.

Table 2 Parameters of PID feedback controller

Parameters	Values
K_p	18.75
K_i	0.39
K_d	1250

According to Figure 11, during the experiment, LSAOILC have an identification process but TOILC does not.

5.2.1 System parameters remain unchanged

The experiment continues to use the third-order trajectory as shown in Figure 4 and $\boldsymbol{W_e} = \boldsymbol{I}$, $\rho = 10^{-6}$. When the parameter of $P(s)$ remains unchanged, use TOILC and LSAOILC to control respectively.

From Figure 12, it can be seen that TOILC and LSAOILC have good convergence when $P(s)$ remains unchanged. Therefore, the identification algorithm of LSOILC does not affect the original performance of its own ILC. The error curve of LSAOILC is shown in Figure 13. After four iterations, the maximum error is reduced from 10^0 deg to 10^{-1} deg . This also shows that LSAOILC does not affect the original performance of TOILC.

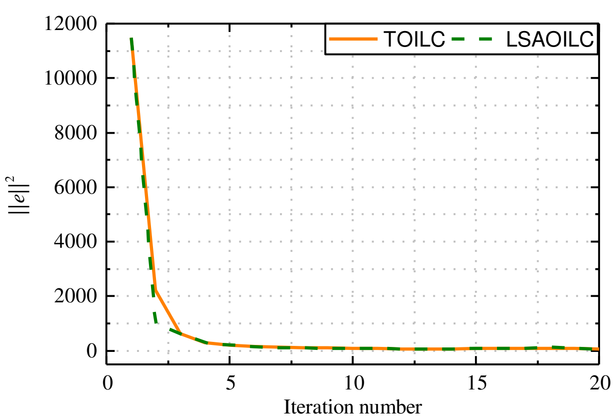

Figure 12 Two-norm of the tracking error when $P(s)$ does not change

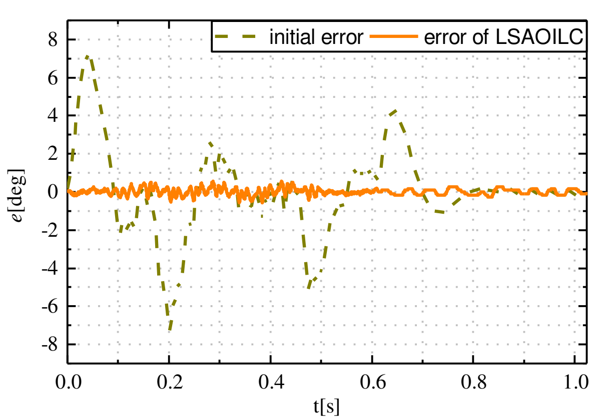

Figure 13 The tracking error of LSAOILC and initial error when $P(s)$ does not change

5.2.2 System parameters change

On the other hand, in the 5^{th} iteration, the parameter β_{zero} of $P(s)$ is changed to 0.15. In this case, the two-norm error of TOILC is shown in Figure 14. It is obvious that the convergence of TOILC becomes very poor when $P(s)$ changes.

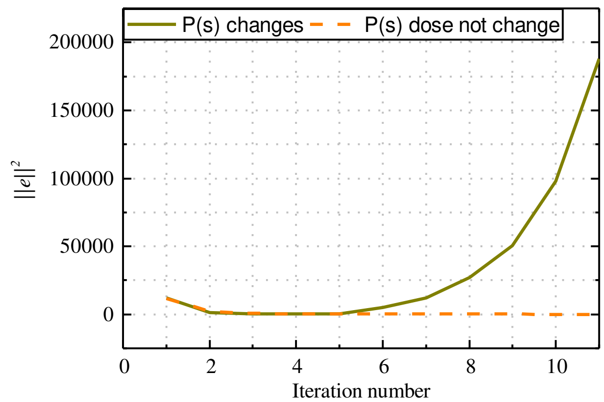

Figure 14 The two-norm error of TOILC when $P(s)$ does not change and $P(s)$ changes

The two-norm error of LSAOILC is shown in Figure 15. When the parameters of $P(s)$ change, LSAOILC tends to converge after several iterations. And it can be seen from the error curve in Figure 16. After 26 iterations, the maximum of LSAOILC was also reduced to the order of 10^{-1} deg , but the maximum error of TOILC has been deteriorated to the order of 10^1 deg when the $P(s)$ changes. Therefore, the impact of system time-varying on TOILC is very serious, but LSAOILC can deal with the change of system parameters effectively.

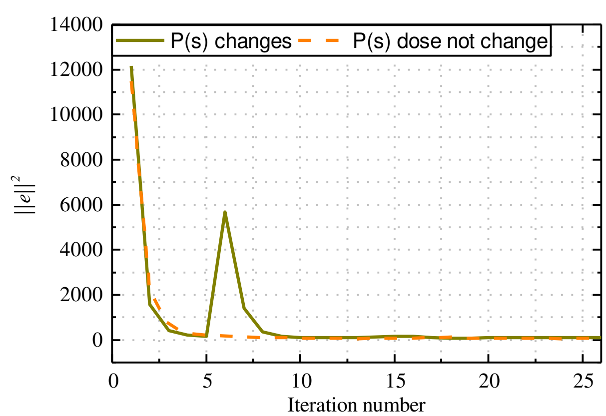

Figure 15 The two-norm error of LSAOILC when $P(s)$ does not change and $P(s)$ changes

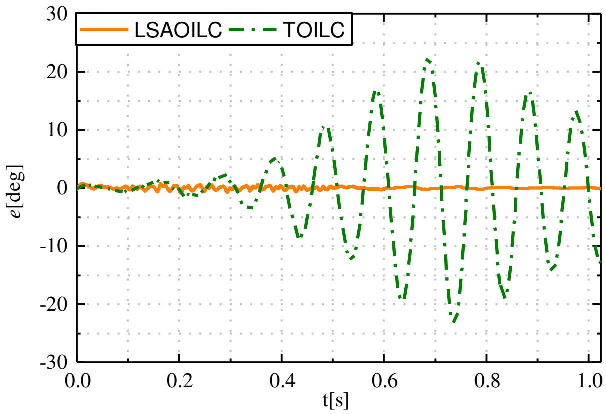

Figure 16 The tracking error of LSAOILC and TOILC when $P(s)$ changes

According to the error curve shown in Figure 17, LSAOILC reduces the maximum error to 10^{-1} deg order of magnitude when system parameters change. So LSAOILC can effectively cope with system time changes and make up for the shortcomings of TOILC.

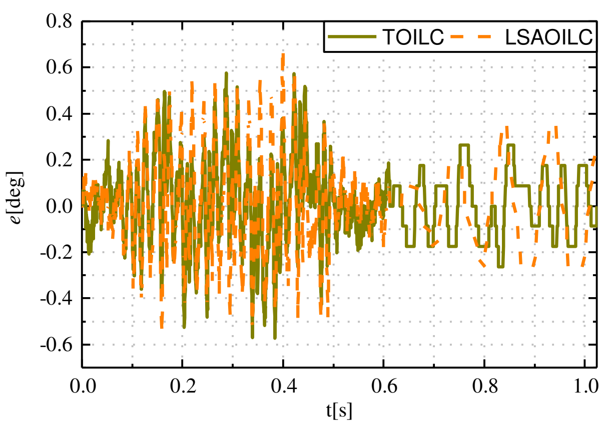

Figure 17 The tracking error of TOILC when $P(s)$ does not change and the tracking error of LSAOILC when $P(s)$ changes

It is worth mentioning that both experiment and simulation adopt the method of systems with abrupt parameter changes. The actual system changes very slowly, but it has been proved that LSAOILC can also have good performance in this extreme sudden change. Therefore, when the system changes slowly, LSAOILC can also have good performance.

6 Conclusions

This paper conducts a comprehensive analysis of the TOILC algorithm. According to the time-varying characteristics of the system, a method to estimate the impulse response of the system by using the input and output data of the previous iteration is proposed, and LSAOILC is formed by combining it with TOILC. In the iterative process, the controller is updated to effectively deal with the influence of time-varying system, improve the trajectory tracking performance of time-varying system, and make up for the deficiency of TOILC.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

The author' contributions are as follows: Liang-Liang Yang was in charge of the whole trial; Rui Yuan wrote the manuscript; Xiang Luo and Hui Zhang assisted with sampling and laboratory analyses.

Competing interests

The authors declare no competing financial interests.

Consent for publication

All the authors agreed to publish this paper

Ethics approval and consent to participate

Not applicable

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