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Fuzzy reliability analysis based on support vector machine

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Abstract

For a structure with implicit performance function structure and less sample data, it is difficult to obtain accurate probability distribution parameters by traditional statistical analysis methods. To address the issue, the probability distribution parameters of samples are often regarded as fuzzy numbers. In this paper, a novel fuzzy reliability analysis method based on support vector machine is proposed. Firstly, the fuzzy variable is converted into an equivalent random variable, and the equivalent mean and equivalent standard deviation are calculated. Secondly, the support vector regression machine with excellent small sample learning ability is used to train the sample data. Subsequently, and the performance function is approximated. Finally, the Monte Carlo method is used to obtain fuzzy reliability. Numerical examples are investigated to demonstrate the effectiveness of the proposed method, which provides a feasible way for fuzzy reliability analysis problems of small sample data.

Keywords: Fuzzy variable, Support vector machine, Monte Carlo method

1 Introduction

With the development of science and technology, reliability technique has been developed quickly [1-3]. In the case of small sample size, the traditional statistical analysis methods cannot accurately obtain the probability distribution parameters. To address the problem, the probability distribution parameters can often be considered as fuzzy numbers [4]. Therefore, it is very important to study fuzzy reliability problems under small samples [5,6].

When structural performance function is an implicit function, some surrogate model-based-methods can be used, such as response surface method [7], Kriging method [8], neural network method [9], and radial basis method [10]. For response surface method, numerous test points are often needed to determine the coefficients; Kriging is a local interpolation method, which is an unbiased optimal estimation of the variables in a finite region; neural network method exhibits strong nonlinear ability. In general, above-mentioned methods require more sample data to achieve a better fitting effect. However, for reliability problems with small sample size, above-mentioned methods are not suitable. Therefore, a novel fuzzy reliability analysis method based on support vector machine is proposed in this paper.

Support vector machine [11-12] (SVM) is a machine learning algorithm based on statistical learning theory that was proposed by Professor Vapnik. It adopts the principle of minimizing structural risk and has good generalization performance. It is applicable for small samples, nonlinear and high dimensional problems, and is mainly used for pattern recognition and function fitting. The earliest use of support vector machines for reliability calculations was studied by Rocco and Moreno to evaluate the reliability of network systems [13]. Hurtado [14] proved that the support vector machine classification algorithm is suitable for structural reliability analysis with small samples. Li [15] proposed a second-order moment

reliability analysis method based on support vector machine and Monte Carlo simulation. Yuan [16] proposed a reliability analysis method by combining Markov chain and support vector machine, the samples were generated using Markov chain to explore the limit-state regions, and then support vector machine was used to obtain approximate limit state function. Lin [17] integrated the cat swarm optimization (CSO) into the support vector machine classifier, and constructed the CSO + SVM data classification model to reduce the number of features. Suykens [18] proposed a least squares support vector machine. Yang [19] proposed a multi-least squares recursive projection dual support vector machine (MLSPTSVM) for multi-classification problems. Tehrany [20] used support vector machines with different kernel functions for spatial prediction of flood occurrence. Mohammadpour [21] predicted the water quality index of free constructed wetland using support vector machine and two artificial neural network methods. Ceperic [22] proposed a general strategy for short-term load forecasting based on support vector regression.

Nowadays, fuzzy support vector machines have been received considerable attention [23-25]. The fuzzy support vector machine is an improved support vector machine. It is based on the support vector machine and the fuzzy membership function. Lin [26] proposed the concept of fuzzy support vector machine. Based on support vector machine and fuzzy set theory, the fuzzy membership degree was introduced into the input samples of support vector machine, and the support vector machine was reconstructed to form fuzzy support method based on sample weighting. The key issue of this method is how to determine the degree of membership, i.e., the weights of the sample. Duan [27] proposed a construction algorithm of fuzzy membership degree to realize the solution of fuzzy support vector machine. Hao [28] introduced the concept of fuzzy theory into the model of support vector machine, and used different types of kernel functions to blur the weights and deviations of the regression model. Hong [29] used support vector machines for multivariate fuzzy linear and nonlinear regressions. Abe [30] proposed a fuzzy support vector machine for multi-label classification. Baser [31] proposed a fuzzy regression function with support vector machines for solar radiation estimation. Zhou [32] proposed a novel method of CCPR by integrating fuzzy support vector machine with hybrid kernel function and genetic algorithm (GA).

In this paper, for fuzzy reliability problems with small sample data and implicit functions, fuzzy reliability problem is firstly transformed into random reliability problem, and fuzzy variables are converted into equivalent random variables, then equivalent mean and standard deviation are calculated. The samples are generated by the Latin hyper-cube sampling. The support vector machine is used to train the samples and fit the performance function. Finally, the Monte Carlo method is used to calculate the fuzzy reliability.

In Section 2, fuzzy variables are converted into equivalent random variables. In Section 3, performance function is fitted by support vector machine. In Section 4, fuzzy reliability is solved. Section 5 is numerical examples. Section 6 is conclusions.

2. Fuzzy variable transformations

2.1. Fuzzy variables transformation

According to the fuzzy decomposition principle, a fuzzy variable can be decomposed into interval variables under different cut sets.

For a fuzzy variable $\tilde{x}$, its probability distribution function is $\pi_x(x)$ [33]. For an arbitrary confidence level λ , the fuzzy variable $\tilde{x}$ can be converted into an interval variable $x_\lambda = [c_\lambda, d_\lambda]$, and its upper and lower bounds are:

$$c_\lambda = x^r(\lambda) = (\pi_x^r)^{-1}(\lambda) \quad (1)$$

$$d_\lambda = x^l(\lambda) = (\pi_x^l)^{-1}(\lambda) \quad (2)$$

where π_x^l, π_x^r are the left and right branches of probability distribution function.

Let the strength of a machine part be a random variable r and the stress be a fuzzy variable $\tilde{s}$. The failure probability can be expressed as follows:

$$F = P(\tilde{s} - r) \quad (3)$$

For an arbitrary confidence value λ , the fuzzy variable $\tilde{s}$ can be transformed into an interval number $s_\lambda = [c_\lambda, d_\lambda]$. The failure probability F can be calculated as:

$$\begin{aligned}
F &= P(\tilde{s} - r) = \int_0^1 P(s_\lambda - r) d\lambda \\
&= \int_{-\infty}^m f_r(r) \left[\int_0^{L(r)} \frac{d_\lambda - r}{d_\lambda - c_\lambda} d\lambda + 1 - L(r) \right] dr + \int_m^{+\infty} f_r(r) \left[\int_0^{R(r)} \frac{d_\lambda - r}{d_\lambda - c_\lambda} d\lambda \right] ds \quad (4)
\end{aligned}$$

where $f_r(r)$ is the probability density function of random strength r , m , $L(x)$ and $R(x)$ are the mean, left reference function, and right reference function of the fuzzy variable $\tilde{s}$, respectively.

According to the stress intensity interference model, the failure probability F can be rewritten as:

$$F = \int_{-\infty}^{+\infty} \left[f_r(r) \int_r^{+\infty} f_s^T(s) ds \right] dr = \int_{-\infty}^m \left\{ f_r(r) [1 - F_s^T(r)] \right\} dr + \int_m^{+\infty} \left\{ f_r(r) [1 - F_s^T(r)] \right\} dr \quad (5)$$

where s^T is the equivalent random stress of fuzzy stress $\tilde{s}$, $f_s^T(s)$ is the probability density function of equivalent random stress s^T , $F_s^T(s)$ is the cumulative density function (CDF).

Comparing Equations (4) and (5), the CDF $F_s^T(s)$ can be expressed as:

$$F_s^T(s) = \begin{cases} \int_0^{L(s)} \frac{s - c_\lambda}{d_\lambda - c_\lambda} d\lambda & (s \leq m) \\ \int_0^{R(s)} \frac{s - c_\lambda}{d_\lambda - c_\lambda} d\lambda + 1 - R(s) & (s > m) \end{cases} \quad (6)$$

Subsequently, the probability density function $f_s^T(s)$ can be obtained as:

$$f_s^T(s) = \begin{cases} \int_0^{L(s)} \frac{1}{d_\lambda - c_\lambda} d\lambda & (s \leq m) \\ \int_0^{R(s)} \frac{1}{d_\lambda - c_\lambda} d\lambda & (s > m) \end{cases} \quad (7)$$

Based on Equation (7), a fuzzy variable $\tilde{x}$ can be transformed into an equivalent random variable x^T , and its equivalent probability density function [33] is:

$$f_{x^T}(x) = \int_0^\lambda \frac{1}{d_\lambda - c_\lambda} d\lambda \quad (8)$$

where $\lambda = \begin{cases} L(x), x \leq m \\ R(x), x > m \end{cases}$, m , $L(x)$ and $R(x)$ are the mean, left reference function, and right reference function of the fuzzy variable $\tilde{x}$, respectively.

2.2 Mean and standard deviation of equivalent random variables

When fuzzy variable is transformed into an equivalent random variable, the equivalent mean and equivalent standard deviation can be, respectively, calculated as follows.

$$\mu_{x^T} = E_{x^T}(x) = \int_{-\infty}^{+\infty} x f_{x^T}(x) dx \quad (9)$$

$$\sigma_{x^T} = \sqrt{\int_{-\infty}^{+\infty} (x - \mu_{x^T})^2 f_{x^T}(x) dx} = \sqrt{E(x^2) - \mu_{x^T}^2} \quad (10)$$

For some commonly used membership functions, the equivalent mean and equivalent standard deviation are calculated as follows:

(1) Linear membership function

The linear membership functions of fuzzy variables are expressed as follows:

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$$\mu_{\tilde{x}}(x) = \begin{cases} L(x) = 1 - \frac{m-x}{\alpha}, m-\alpha < x \leq m \\ R(x) = 1 - \frac{x-m}{\beta}, m < x \leq m+\beta \\ 0, \text{others} \end{cases} \quad (11)$$

Then, for a given threshold, the fuzzy variables are transformed into interval numbers as follows.

$$x_{\lambda} = [c_{\lambda}, d_{\lambda}] = [m - \alpha(1 - \lambda), m + \beta(1 - \lambda)] \quad (12)$$

The equivalent probability density function can be expressed as:

$$f_{x^r}(x) = \begin{cases} \int_0^{m-\alpha(1-\lambda)} \frac{1}{(\alpha+\beta)(1-\lambda)} d\lambda, m-\alpha < x \leq m \\ \int_0^{m+\beta(1-\lambda)} \frac{1}{(\alpha+\beta)(1-\lambda)} d\lambda, m < x \leq m+\beta \end{cases} = -\frac{\ln[1-\lambda]}{\alpha+\beta} \quad (13)$$

The equivalent probability density function can be substituted into equations (9) and (10) to calculate the equivalent mean and equivalent standard deviation when fuzzy variables are linear membership functions as follows:

$$\mu_{x^r} = m + \frac{\beta - \alpha}{4} \quad (14)$$

$$\sigma_{x^r} = \sqrt{\frac{\alpha\beta}{9} + \frac{7(\beta - \alpha)^2}{144}} \quad (15)$$

(2) Normal membership function.

The normal membership function is given as follows.

$$\mu_{\tilde{x}}(x) = \begin{cases} L(x) = \exp[-\frac{(x-m)^2}{\alpha^2}], x \leq m \\ R(x) = \exp[-\frac{(x-m)^2}{\beta^2}], x > m \end{cases} \quad (16)$$

Then, for a given threshold, the fuzzy variables are transformed into interval numbers as follows.

$$\tilde{x}_{\lambda} = [c_{\lambda}, d_{\lambda}] = [m - \alpha\sqrt{-\ln \lambda}, m + \beta\sqrt{-\ln \lambda}] \quad (17)$$

The equivalent mean and equivalent standard deviation for fuzzy variables with linear membership functions can be, respectively, calculated as:

$$a = 1, \quad \mu_{x^r} = m + \frac{\sqrt{\pi}(\beta - \alpha)}{4} \quad (18)$$

$$\sigma_{x^r} = \sqrt{\frac{\alpha\beta}{3} + \frac{(16 - 3\pi)(\beta - \alpha)^2}{48}} \quad (19)$$

(3) Parabolic membership function.

The parabolic membership function is expressed as follows.

$$\mu_{\tilde{x}}(x) = \begin{cases} \left(\frac{x-\alpha}{m-\alpha}\right)^k, & \alpha \leq x < m \\ \left(\frac{c-x}{c-m}\right)^k, & m \leq x < c \\ 0, & \text{others} \end{cases} \quad (20)$$

Based on Equation (20), the equivalent mean and equivalent standard deviation are, respectively, calculated as:

$$\mu_{x^r} = \frac{1}{2(k+1)}(a+c+2mk) \quad (21)$$

$$\sigma_{x^r} = \frac{1}{12(k+1)^3} \left\{ k \left[5(a+c)^2 - 12m(a+c) - 8ac + 12m^2 \right] + 2(a-c)^2 \right\} \quad (22)$$

(4) Cauchy membership function.

The Cauchy membership function is given by

$$\mu_{\tilde{x}}(x) = \begin{cases} \frac{1}{1+\alpha(x-m)^k}, & x \leq m \\ \frac{1}{1+\beta(x-m)^k}, & x > m \end{cases} \quad (23)$$

where $\alpha > 0$, $\beta > 0$, and k is an even number that is greater than 0.

The equivalent mean and equivalent standard deviation can be, respectively, calculated as follows:

$$\mu_{x^r} = m + \frac{\pi}{2k \sin \frac{\pi}{k}} \left[\left(\frac{1}{\beta} \right)^{\frac{1}{k}} - \left(\frac{1}{\alpha} \right)^{\frac{1}{k}} \right] \quad (24)$$

$$\sigma_{x^r} = \frac{2\pi}{k \sin \frac{\pi}{k}} \left[\beta^{-\frac{2}{k}} - \alpha \beta^{-\frac{1}{k}} + \alpha^{-\frac{2}{k}} \right] - \left(\frac{\pi}{2k \sin \frac{\pi}{k}} \right)^2 \left[\beta^{-\frac{1}{k}} - \alpha^{-\frac{1}{k}} \right] \quad (25)$$

3 Fitting structure Performance function using support vector machine

The support vector machine adopts the principle of minimizing structural risk and has good generalization performance. It can resolve small sample size, nonlinear and high dimensional problems, and is mainly used for pattern recognition and function fitting. The support vector regression machine maps the input space and output space by nonlinear transformation [13,14].

3.1 Sampling strategy

For a structure with implicit performance functions, the support vector machine achieves structural reliability analysis by fitting the performance function. Sampling methods, including direct sampling method, importance sampling method, uniform direction sampling, and Latin hyper-cube sampling (LHS), can be used to generate samples. The Latin hyper-cube sampling method is a stratified sampling technique that can effectively estimate the small probability of failure.

According to the 5σ principle, Latin hyper-cube sampling is performed in the feasible interval $[\mu-5\sigma, \mu+5\sigma]$ of random variable to ensure that the sampling contains enough failure sample points.

3.2 Data preprocessing

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Based on Latin hyper-cube sampling method, N samples $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T$ ($i = 1, 2, \dots, N$) are generated, and corresponding response values of these samples are calculated by numerical analysis methods such as finite element. All collected data is divide into two groups, i.e., training set and test set. The normalized pre-processing of sample data is then performed as follows:

$$f : x_{in} \rightarrow y_{in} = \frac{x_{in} - x_{\min}}{x_{\max} - x_{\min}} \quad (26)$$

where $x_{in}, y_{in} \in R^n$, $x_{\min} = \min(x_i)$ and $x_{\max} = \max(x_i)$.

The result of data normalization is to normalize the original data to interval $[0,1]$, $y_{in} \in [0,1], i = 1, 2, \dots, n$.

3.3 Selection of kernel function

The basic idea of support vector machine is to map the input space and output space by nonlinear transformation. Based on experiences, the selection of kernel function is very important in structural reliability analysis.

One of the key issues for using SVM to solve structural reliability is selecting a proper kernel function. The linear kernel function is suitable for separable cases, and is usually used in the support vector classifier. The polynomial kernel function is a global kernel function, which can map low-dimensional input space to high-dimensional feature space. However, the polynomial kernel function has many parameters and is complicated. When the polynomial index q is large, the learning complexity is increasing and easy to cause "over-fitting" problem. The Gaussian kernel function is one of the most widely used kernel functions. Compared with polynomial kernel functions, it has fewer parameters. Moreover, it has good anti-interference ability to the noise existing in the data. When using sigmoid kernel function, the support vector machine implements a multi-layer perceptron neural network, which has good generalization ability for unknown samples.

Based on aforementioned discussion, it is necessary to combine the characteristics of various kernel functions according to the actual situation. In general, prior knowledge can be used to select kernel functions that match the data distribution. Furthermore, cross-validation methods can also be used to try different kernel functions that to select kernel function with the smallest error.

3.4 Parameter Optimization

The penalty parameter C and kernel function parameter $\mathcal{E}$ is essential for support vector machines. In this paper, the K-fold Cross Validation (K-CV) method is used to perform parameter optimization.

4 Solution of Fuzzy Reliability

4.1 Reliability Solution Based on Support Vector Machine

A certain amount of sample data is extracted according to sampling strategy, and the estimated values of sample points are calculated by a numerical analysis method such as finite element. These data are divided into training set and test set, which are used to train the support vector regression machine model and to evaluate the performance of established support vector machine model. Finally, the fitted SVM model is available by preprocessing the data, selecting the appropriate kernel function, and optimizing the parameters. The Monte-Carlo method (Monte-Carlo) and fitted SVM are combined to calculate structural reliability. The estimated failure probability can be expressed as follows.

$$P_f = P(g(x) < 0) \approx P(g^*(x) < 0) \approx \frac{N_r}{N} \quad (27)$$

where $g(x)$ is the real structural performance function; $g^*(x)$ is the fitted SVM; N is the total sample data such as 10^6 ; N_r is the number of samples residing in failure domain.

4.2 Calculation Process

Firstly, the fuzzy variable is transformed into the equivalent random variable, and the equivalent mean and standard deviation are calculated. Subsequently, the fuzzy reliability problem is transformed into

random reliability problem. Secondly, based on Latin hyper-cube sampling method, building SVM based on data pre-processing, kernel function selection and support vector machine parameters optimization. Finally, the Monte Carlo method is used to calculate fuzzy reliability. The detailed calculation processes are shown in Fig.1.

Fig. 1 Flow chart of reliability calculation

5 Numerical Examples

In the following examples, the proposed method, i.e., fuzzy support vector machine (FSVM), is used to calculate fuzzy reliability. The fuzzy decomposition theorem combined with Monte Carlo Simulation (FMCS) and response surface method (FRSM) are performed for comparison. The result from fuzzy Monte Carlo method (FMCS) is viewed as benchmark solution. For system with small sample size, 30 samples are taken in this paper. All examples are performed with MATLAB.

5.1 Example1

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Assume that the geometry size and pressure of spherical pressure vessels are normal random variables [34] with the following distributions: wall thickness $t \sim N(70, 0.35^2)$ mm, Inner diameter $D \sim N(1000, 5^2)$ mm, and pressure $p \sim N(28, 4^2)$ MPa.

Container material is 18MnMoNb and its yielded strength $\tilde{r}$ is a fuzzy variable, $\tilde{r} \sim N(175, 15^2)$ MPa. The membership function is shown in Fig. 2.

Fig. 2 Membership function of $\tilde{r}$

This example is to solve the fuzzy reliability, and the performance function is defined as follows.

$$g(\tilde{r}, q) = \tilde{r} - q = 0 \quad (28)$$

According to reference [35], mean and standard deviation of stress are $\bar{\sigma}_q = 129.2$ and $s_{\sigma_q} = 18.5$.

According to the proposed method and the membership function given in the literature [34], the interval number of fuzzy strengths is obtained as $\tilde{r}_\lambda = [a_\lambda, b_\lambda] = [150 + 25\lambda, 225 - 50\lambda]$, $\alpha = 25, \beta = 50, m = 175$. The equivalent mean and equivalent standard deviation of fuzzy strength can be computed as follows.

$$\mu_{r^*} = m + \frac{\beta - \alpha}{4} = 181.25 \quad (29)$$

$$\sigma_{r^*} = \sqrt{\frac{\alpha\beta}{9} + \frac{7(\beta - \alpha)^2}{144}} = 13.0104 \quad (30)$$

According to the calculated mean and standard deviation of stress and fuzzy strength, Latin hypercube sampling is performed to fit support vector machine.

The different kernel functions are selected to show the effects. For selection of polynomial kernel function is selected, the reliability is $R_r = 1$; For Gaussian kernel function, the reliability is $R_r = 0.985535$; For sigmoid kernel function, the reliability is $R_r = 0.990477$. Compared with the result from FMCS, the reliability error of using sigmoid kernel function is smaller than others, thus, the sigmoid kernel function is selected. The parameters are optimized by cross-validation method, and the optimal parameters $C = 8$, $\varepsilon = 0.03125$ are obtained. The structural performance function is fitted by the support vector machine, and the structural fuzzy reliability is calculated. The results of all compared methods are listed in Tab.1.

Tab.1 Reliability probabilities of pressure vessel structure

method	FMCS	FSVM	FRSM
reliability	0.989345	0.990477	0.983463
absolute error	0	1.13E-03	5.90E-03
relative error	0	0.11%	0.60%

From Tab.1, it can be seen that structural reliability calculated by the proposed method is 0.990477 with using 30 samples, the absolute error is 0.00113 compared to the FMCS, and the relative error is 0.11%. The error is very small and can meet the engineering requirements. Compared to FRSM with 0.983463, the reliability obtained by FSVM has higher accuracy. Therefore, the method proposed is more suitable for solving fuzzy reliability problem with small sample data.

5.2 Example2

A free beam with uniform load is shown in Fig. 3[36].

Fig.3 Force diagram of a free beam

The length l , section width b , section height h and load are basic variables, $l = 4000\text{mm}$, $b = 105$, $h = 210\text{mm}$. The load $\tilde{q}$ is a fuzzy variable with $\tilde{q} \sim N(210, 9^2)\text{N/mm}^2$, and its membership function is shown in Fig. 4. The material of the beam is 45-steel whose strength $\tilde{R}$ is a fuzzy random variable with $\tilde{R} \sim N(550, 20^2)\text{MPa}$, and the membership function $\mu_{\tilde{R}}(r)$ is shown in Fig. 5.

Fig. 4 Membership function of $\tilde{q}$

Fig. 5 Membership function of $\tilde{R}$

In this example, the structural performance function is defined as

$$g(x) = \tilde{R} - \frac{0.75\tilde{q}l^2}{bh^2} = x - \frac{0.75yl^2}{bh^2} \quad (31)$$

Using the proposed method, the fuzzy variables are transformed into equivalent random variables. Based on the membership function given in the literature [35], the equivalent mean value and equivalent standard deviation are calculated as $\mu_{q^*} = 211.5$, $\sigma_{q^*} = 3.1225$, $\mu_{R^*} = 545$, $\sigma_{R^*} = 10.4083$.

The samples are selected by the Latin hypercube sampling method. When 30 samples are generated, the reliability is 0.973649, the absolute error is 0.0195 and the relative error is 2.05%.

After comparisons, the sigmoid kernel function is selected, and the cross-validation method is used to optimize the parameters to obtain the optimal parameters $C=11.3137$, $\varepsilon=0.03125$. The structural performance function is fitted by support vector machine to calculate fuzzy reliability. The results of all compared methods are listed in Tab. 2.

From Tab. 2, it can be seen that the reliability calculated by the proposed method is 0.973649. Compared with the reliability by FMCS of 0.954121, the absolute error is 0.0195, and the relative error is 2.05%. The reliability from FRSM method is 0.753529, the absolute error is 0.2006, and the relative

error is 21.02%. Thus, the proposed method is more accurate than FRSM. In this example, the fuzzy support vector machine method still uses Latin hypercube sampling method to generate 30 samples for training and testing, and the result shows that it is feasible.

Tab.2 Reliability probabilities of beam structure

method	FMCS	FSVM	FRSM
reliability	0.954121	0.973649	0.753529
absolute error	0	1.95E-02	2.006E-01
relative error	0	2.05%	21.02%

5.3 Example 3

A ten-bar truss structure [37], as shown in Fig. 6, is considered in example 3.

Fig. 6 The diagram of 10-bar truss structure

The length of horizontal and vertical rods are the same, i.e., L . Assuming that the cross-sectional area of horizontal rod is $\tilde{A}_1$, the cross-sectional area $\tilde{A}_2$ of vertical rod and the cross-sectional area $\tilde{A}_3$ of inclined rod are mutually independent fuzzy variables, which obey normal distributions as $\tilde{A}_1 \sim N(13, 1.3^2) \text{ inch}^2$, $\tilde{A}_2 \sim N(2, 0.2^2) \text{ inch}^2$, $\tilde{A}_3 \sim N(9, 0.9^2) \text{ inch}^2$, and the membership functions are triangular membership functions as follows.

$$\mu_{\tilde{A}_1}(A_1) = \begin{cases} 0, & A_1 < 7.8 \\ \frac{A_1 - 7.8}{5.2}, & 7.8 \leq A_1 < 13 \\ \frac{15.6 - A_1}{2.6}, & 13 \leq A_1 < 15.6 \\ 0, & A_1 \geq 15.6 \end{cases}$$

$$\mu_{\tilde{A}_2}(A_2) = \begin{cases} 0, & A_2 < 1.2 \\ \frac{A_2 - 1.2}{0.8}, & 1.2 \leq A_2 < 2 \\ \frac{2.4 - A_2}{0.4}, & 2 \leq A_2 < 2.4 \\ 0, & A_2 \geq 2.4 \end{cases}$$

$$\mu_{\tilde{A}_3}(A_3) = \begin{cases} 0, & A_3 < 5.4 \\ \frac{A_3 - 5.4}{3.6}, & 5.4 \leq A_3 < 9 \\ \frac{10.8 - A_3}{1.8}, & 9 \leq A_3 < 10.8 \\ 0, & A_3 \geq 10.8 \end{cases}$$

In this example, the other parameters such as elastic modulus, material density, rod length, and acting load are all deterministic variables. Load is $P = 100000 \text{ bang}$, length of rod is $L = 360 \text{ inch}$, modulus of elasticity is $E = 10^7 \text{ psi}$, allowable displacement of vertex 2 is $d_{allow} = 4.5 \text{ inch}$.

The fuzzy variables are first transformed into equivalent random variables, based on the membership function given in the literature [36], the equivalent mean value and equivalent standard deviation of the fuzzy variables $\tilde{A}_1, \tilde{A}_2, \tilde{A}_3$ are $\mu_{A_1^r} = 12.35, \sigma_{A_1^r} = 1.3531, \mu_{A_2^r} = 1.9, \sigma_{A_2^r} = 0.2082$,

$\mu_{A_3^T} = 8.55, \sigma_{A_3^T} = 0.9367$. The 30 sample points are generated by Latin hypercube sampling method, and the estimated values of generated samples are obtained by Finite element simulation, as shown in Tab. 3.

Tab.3 Sample data based on LHS.

No.	x	y	z	g
1	7.428023	2.766266	5.590229	-1.598989
2	10.902587	3.050309	15.7732	1.236507
3	21.199204	2.472248	8.091026	1.31142
4	5.725285	3.729313	11.517416	-1.067865
5	22.970024	1.101878	16.781107	2.357362
6	19.663881	3.114384	9.807808	1.591681
7	13.937386	1.229696	13.523065	1.450781
8	16.840877	1.044695	14.174617	1.786675
9	9.599554	2.318746	10.427449	0.397492
10	13.741842	1.833099	12.62959	1.383104
11	24.043356	1.897704	5.146359	0.288537
12	15.942505	2.873745	16.596233	1.967842
13	15.460827	3.273971	6.880098	0.590776
14	9.101802	1.568418	7.308865	-0.442973
15	14.565776	2.954179	16.077721	1.798512
16	17.302386	1.66562	11.225901	1.567491
17	17.9346	3.414449	14.798446	2.029047
18	12.683078	3.312506	9.049161	0.803901
19	12.080272	1.330341	7.442667	0.211491
20	24.332523	2.413919	3.997413	-0.532183
21	6.742312	2.163209	12.884805	-0.36256
22	21.998386	2.124236	9.51966	1.629572
23	22.223579	3.552426	10.830836	1.894122
24	20.858629	1.395209	12.213598	1.900804
25	7.821076	1.460813	4.345641	-2.35968
26	10.574966	0.909502	8.407039	0.121558
27	11.332734	3.663395	15.089483	1.288464
28	18.511233	2.655811	13.873925	1.977553
29	8.451524	1.962732	5.998768	-1.079353
30	19.302364	2.572618	6.238606	0.629067

In the example, the RBF kernel function is selected, and the cross-validation method is used to optimize the parameters. Then, the optimal parameters are $C=724.0773$, $\varepsilon=0.0221$. The results of all compared methods are list in Tab.4.

Tab.4 Reliability probabilities of beam structure

method	FMCS	FSVM	FRSM
reliability	-	0.906700	0.634800
absolute error	-	-	-
relative error	-	-	-

From Tab.4, the reliability results from the proposed methods and FRSM are 0.906700 and 0.634800, respectively. Note that the result from FMCS is not reported because performance functions are implicit functions. Compared to the FRSM, the proposed method has higher accuracy. This example has shown that the proposed method is suitable for implicit reliability problem.

6 Conclusions

In this paper, a fuzzy reliability analysis method based on support vector machine is proposed for a structure with implicit functions and small size of sample. Firstly, the fuzzy decomposition theorem is used to convert fuzzy variables into equivalent random variables, subsequently, the fuzzy reliability problem can be converted into random reliability problem, and the equivalent mean value and equivalent

standard deviation are calculated. Based on the equivalent mean and equivalent standard deviation, Latin hyper-cube sampling is performed to obtain training samples for support vector machine. For reliability problem, a proper kernel function should be selected and the cross-validation method is used to optimize the parameters. Finally, the MCS is used to estimate fuzzy reliability. The calculation accuracy is achieved and the feasibility of the proposed method is verified by numerical examples.

Based on numerical examples, it can be seen that the proposed is generally more accurate than FRSM. Thus, the proposed method is applicable for a structure with implicit functions and small sample size. In order to facilitate the comparison with FMCS, the first two examples with explicit performance functions, and the third example with implicit performance function. All examples have shown the effectiveness of the proposed method. Support vector machines have the good small sample learning capabilities, it combines Monte Carlo simulation providing a useful way for fuzzy reliability problems.

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Declarations

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