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T.L. Yookesh (✉ renu_yookesh@yahoo.co.in)

Vignan University

T.P. Latchoumi

Vignan University

K.Balamurugan

Vignan University

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An Iterative Technique to Solve Fuzzy Delay Integro-Differential Equations

^{1*}T.L. Yookesh, ²T.P. Latchoumi, ³K.Balamurugan

¹Department of Science and Humanities, VFSTR (Deemed to be University), Guntur, India

²Department of Computer Science and Engineering, SRM Institute of Science and Technology,
Ramapuram Campus, Chennai, Tamilnadu, India.

³Department of Mechanical Engineering, VFSTR (Deemed to be University), Guntur, India

E. Mail: ¹renu_yookesh@yahoo.co.in

Abstract— In this exploration work, we make an assessment on the get together of nonlinear Volterra fuzzy delay integro-differential conditions, with the help of an iterative technique. The outcome is contrasted with the precise outcome with verify the legitimacy and proficiency of the method to deal with nonlinear Volterra fuzzy delay integro-differential condition. To display the suitable highlights of this proposed system, a numerical worldview is represented.

Keywords— *Volterra integro-differential equations, convergence, iterative technique*

I. INTRODUCTION

In mathematics, an integro-differential equation is a condition that includes commonly to integrals and subsidiaries of functions. Delay, which uncovers the past estimations of the capacity, makes modern, pay its enthusiasm on delay integro-differential equations. Researchers and scientists are exploring the point of delay integro-differential equations through their exertion in science applications, for example, heat transfer, dispersion process, neutron dissemination organic breeds concurrent together with expanding and diminishing percentage of producing. Additionally, these sorts of equations start in the regions of material science, science and building applications. A portion of the applications are uneven streamlined features and air versatile wonders, numerous models of populace development, polymer technology, scientific displaying of the dissemination of discrete particles in a violent liquid, populace elements, compartmental frameworks, atomic reactors and numerical demonstrating of genetic marvels. For instance, Volterra [25] was explored the populace development, focusing his investigation on the inherited impacts and a few creators [5, 7, 17, 18, 19, 29] talked about the integro-differential frameworks.

Recently because of the modern enthusiasm for blur makes their normal results even more unreasonably reasonable. The uses of the assumption of integro-differential conditions of fuzzy delay have been extended [30-32]. Fuzzy delay integro-differential equations assume significant job in demonstrating vulnerability of dynamical frameworks. Fuzzy was presented by Zadeh [28]. Fuzzy integro-differential conditions are in practice of eagerness, because of their significance to the assessment of marvels with memory where inexactness is the trademark. K. Balachandran [8] dealt with controllability of nonlinear Volterra integro-differential frameworks and K.

Balachandran et al. [9] considered the current outcomes for fuzzy delay integro-differential conditions. Toshiyuko Koto [23] discover the dependability of Θ -technique and Runge-Kutta strategies to explain the delay integro-differential conditions. Discover the answer for the n^{th} request fuzzy integro-differential conditions utilizing the iterative strategy is by A. A. Hemeda [14]. H. M. El-Hawary et al. [11] found the numerical answer for delay integro-differential equations with spline and ghostly techniques. Park and Han in [21] examined the truth and peculiarity hypothesis for an answer of fuzzy Volterra fundamental conditions. Allahviranloo et al. in [6] introduced another technique for tackling fuzzy integro-differential equation under complete differentiability. After that various experts discussed the procedure for courses of action of these sorts of conditions, in view of different quick to fuzzy numbers, there are a couple of research systems to look at fuzzy conditions. [9,14,22, 24].

This paper proposed a strategy dependent on iterative system called an Adomian decomposition technique to illuminate the Volterra delay integro-differential equations with fuzzy. Center our exploration is to discover how this proposed strategy chips away at Volterra delay integro-differential conditions under fuzzy foundation. These procedures join rapidly towards the precise plan and this technique does exclude any discretization of factors and consequently, it is exempted from changing off slip-ups. We embody a numerical manual to explain the proposed system.

II. PRELIMERIES

A. Membership Function

Give δ a chance to be a nonempty set. A fuzzy set $\bar{A}$ in δ is characterized by its membership function $\mu_{\bar{A}} : \delta \rightarrow [0,1]$ and $\mu_{\bar{A}(x)}$ is explained as the degree of membership of element x in fuzzy set $\bar{A}$ for each $x \in \delta$. It is clear that $\bar{A}$ is determined by the set of tuples $\bar{A} = \{(x, \mu_{\bar{A}(x)}) \mid x \in \delta\}$.

B. Triangular Fuzzy Number

Let the triangular fuzzy number of fuzzy set $\bar{A}$ with mid as c , left span $m > 0$ and right span $n > 0$, then

$$\mu(x) = \begin{cases} 1 - \frac{c-x}{m}, & \text{if } c-m < x < c \\ 1 - \frac{x-c}{n}, & \text{if } c < x < c+n \\ 0 & \text{or else} \end{cases}$$

Fig. 1. Triangular Fuzzy Number

III. NONLINEAR VOLTERRA FUZZY DELAY INTEGRO-DIFFERENTIAL EQUATIONS

The delay Volterra integro-differential equation of the second kind is stated by,

$$\psi'(q) = p(q) + \int_a^q k(q, \nu) P(\psi(\nu), \psi(\nu - \tau)) d\nu \quad (3.1)$$

with initial condition $\psi(a) = c_1$, where 'a' is a real constant, $k(q, \nu)$ and $p(q)$ are real valued functions. The functions $P(\psi(\nu))$ and $P(\psi(\nu - \tau))$ are nonlinear function and nonlinear delay function of $\psi(\nu)$ respectively. If $p(q)$ is a crisp function then the solutions of equation (3.1) are also crisp. However, if $p(q)$ is a fuzzy function this equation may posses only fuzzy solutions. Let us introduce α -level set of ψ , p and P to the above equation and we obtain,

$$\begin{aligned} \psi(q, \alpha) &= [\underline{\psi}(q, \alpha), \bar{\psi}(q, \alpha)] \\ \psi'(q, \alpha) &= [\underline{\psi}'(q, \alpha), \bar{\psi}'(q, \alpha)] \\ p(q, \alpha) &= [\underline{p}(q, \alpha), \bar{p}(q, \alpha)] \\ P(\underline{\psi}(\nu, \alpha), \bar{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \bar{\psi}(\nu - \tau, \alpha)) &= [G(\underline{\psi}(\nu, \alpha), \bar{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \bar{\psi}(\nu - \tau, \alpha)), \\ &\quad H(\underline{\psi}(\nu, \alpha), \bar{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \bar{\psi}(\nu - \tau, \alpha))] \end{aligned} \quad (3.2)$$

where $0 \leq \alpha \leq 1$ and $q \in [a, b]$

Moreover all the derivatives with respect to q , are fuzzy functions. Now, the equation of (3.1) can be written as follows

$$\begin{cases} \underline{\psi}'(q) = \underline{p}(q) + \int_a^q k(q, \nu) \underline{P}(\underline{\psi}(\nu), \underline{\psi}(\nu - \tau)) d\nu \\ \bar{\psi}'(q) = \bar{p}(q) + \int_a^q k(q, \nu) \bar{P}(\bar{\psi}(\nu), \bar{\psi}(\nu - \tau)) d\nu \end{cases} \quad (3.3)$$

for $0 \leq \alpha \leq 1$. Suppose $k(q, \nu)$ be continuous in $a \leq \nu \leq b$ and for fix q , the sign of $k(q, \nu)$ does not change over $[a, q]$, therefore we have

$$\begin{aligned}
\underline{\psi}'(q) &= \underline{p}(q) + \int_a^q k(q, \nu) G(\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)) d\nu \\
\overline{\psi}'(q) &= \overline{p}(q) + \int_a^q k(q, \nu) H(\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)) d\nu
\end{aligned} \tag{3.4}$$

with subject to initial condition $[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha)] = [\underline{b}_1(\alpha), \overline{b}_1(\alpha)]$ for each $0 \leq \alpha \leq 1$ and $\nu \in [a, b]$. We can see that equation (3.4) are arrangement of Volterra fuzzy delay integro-differential equations in the crisp case for each $0 \leq \alpha \leq 1$.

IV. DESCRIPTION OF THE METHOD

Here, we clarify the principle calculation of Adomian decomposition technique [2, 3] that applies to a Volterra fuzzy delay integro-differential equations[1, 27] of the structure

$$\begin{aligned}
\underline{\psi}'(q, \alpha) &= \underline{f}(q, \alpha) + \int_a^q k(q, \nu) G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu \\
\overline{\psi}'(q, \alpha) &= \overline{f}(q, \alpha) + \int_a^q k(q, \nu) H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu
\end{aligned} \tag{4.1}$$

The Volterra fuzzy delay integro-differential equations can be written as follows

$$\begin{aligned}
L\underline{\psi}'(q, \alpha) &= \underline{f}(q, \alpha) + \int_a^q k(q, \nu) G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu \\
L\overline{\psi}'(q, \alpha) &= \overline{f}(q, \alpha) + \int_a^q k(q, \nu) H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu
\end{aligned} \tag{4.2}$$

where L be the derivative with respect to q and $G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)]$ and $H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)]$ are nonlinear terms. Now handle the inverse operator L^{-1} to two hands of (4.2), then we have the form

$$\begin{aligned}
\underline{\psi}'(q, \alpha) &= \underline{g}(q, \alpha) + L^{-1} \left(\int_a^q k(q, \nu) G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu \right) \\
\overline{\psi}'(q, \alpha) &= \overline{g}(q, \alpha) + L^{-1} \left(\int_a^q k(q, \nu) H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] d\nu \right)
\end{aligned} \tag{4.3}$$

where the functions $L^{-1}\underline{f}(q, \alpha) = \underline{g}(q, \alpha)$ and $L^{-1}\overline{f}(q, \alpha) = \overline{g}(q, \alpha)$. The ADM expect an unbounded arrangement answer for the obscure capacities $[\underline{\psi}(q, \alpha), \overline{\psi}(q, \alpha)]$, given by

$$\begin{aligned}
\underline{\psi}(\nu, \alpha) &= \sum_{i=0}^{\infty} \underline{\psi}_i(q, \alpha) \text{ and} \\
\overline{\psi}(\nu, \alpha) &= \sum_{i=0}^{\infty} \overline{\psi}_i(q, \alpha)
\end{aligned} \tag{4.4}$$

Then nonlinear operators $G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)]$, $H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha), \underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)]$ into an unending arrangement of polynomials given by

$$\begin{aligned}
G[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha)] &= \sum_{i=0}^{\infty} \underline{A}_n; G[\underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] = \sum_{i=0}^{\infty} \overline{A}_n \text{ and} \\
H[\underline{\psi}(\nu, \alpha), \overline{\psi}(\nu, \alpha)] &= \sum_{i=0}^{\infty} \underline{B}_n; H[\underline{\psi}(\nu - \tau, \alpha), \overline{\psi}(\nu - \tau, \alpha)] = \sum_{i=0}^{\infty} \overline{B}_n
\end{aligned} \tag{4.5}$$

Where $\square A_n = [\underline{A}_n; \overline{A}_n]$, $\square B_n = [\underline{B}_n; \overline{B}_n]$, $n \geq 0$ are the Adomian polynomial characterize by,

$$\begin{aligned}
\underline{A}_n &= \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} G \left(\sum_{i=0}^n \lambda^i \underline{\psi}_i, \sum_{i=0}^n \lambda^i \overline{\psi}_i \right) \right]_{\lambda=0} \\
\overline{A}_n &= \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} G \left(\sum_{i=0}^n \lambda^i \underline{\psi}_i, \sum_{i=0}^n \lambda^i \overline{\psi}_i \right) \right]_{\lambda=0} \\
\underline{B}_n &= \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} H \left(\sum_{i=0}^n \lambda^i \underline{\psi}_i, \sum_{i=0}^n \lambda^i \overline{\psi}_i \right) \right]_{\lambda=0} \\
\overline{B}_n &= \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} H \left(\sum_{i=0}^n \lambda^i \underline{\psi}_i, \sum_{i=0}^n \lambda^i \overline{\psi}_i \right) \right]_{\lambda=0}
\end{aligned} \tag{4.6}$$

On substituting (4.4) and (4.5) into (4.3), we get

$$\begin{aligned}
\underline{\psi}_0 &= \underline{g}(q, \alpha), \\
\underline{\psi}_{n+1} &= L^{-1} \int_a^q k(q, \nu) \underline{A}_n d\nu, n \geq 0
\end{aligned} \tag{4.7}$$

and

$$\begin{aligned}
\overline{\psi}_0 &= \overline{g}(q, \alpha), \\
\overline{\psi}_{n+1} &= L^{-1} \int_a^q k(q, \nu) \overline{B}_n d\nu, n \geq 0
\end{aligned} \tag{4.8}$$

□
 We approximate $\underline{\psi} = [\underline{\psi}(q, \alpha), \overline{\psi}(q, \alpha)]$ by $\underline{\varphi} = \sum_{i=0}^{n-1} \underline{\psi}_i(q, \alpha); \overline{\varphi} = \sum_{i=0}^{n-1} \overline{\psi}_i(q, \alpha)$, where $\lim_{n \rightarrow \infty} \underline{\varphi} = \underline{\psi}_i(q, \alpha)$ and $\lim_{n \rightarrow \infty} \overline{\varphi} = \overline{\psi}_i(q, \alpha)$.

V. UNIQUENESS AND CONVERGENCE OF THE METHOD

A. Uniqueness theorem

If $\underline{\psi}(q) = \underline{p}(q) + \int_0^{q-\tau} \underline{P}(\psi(q)) d\nu; \overline{\psi}(q) = \overline{p}(q) + \int_0^{q-\tau} \overline{P}(\psi(q)) d\nu$ has a unique solutions whenever $0 < \alpha < 1$,

where $\alpha = LMT$.

Proof: Let $\underline{\psi}, \underline{\psi}^*; \overline{\psi}, \overline{\psi}^*$ are the two different solutions to $\underline{\psi}(u); \overline{\psi}(u)$, then

$$\begin{aligned} |\underline{\psi} - \underline{\psi}^*| &= \left| \int_0^{q-\tau} k(u, \nu) [\underline{E}(\psi) - \underline{E}^*(\psi)] d\nu \right|; \\ |\overline{\psi} - \overline{\psi}^*| &= \left| \int_0^{q-\tau} k(u, \nu) [\overline{F}(\psi) - \overline{F}^*(\psi)] d\nu \right| \\ &\leq \int_0^{q-\tau} |k(u, \nu)| \left| [\underline{E}(\psi) - \underline{E}^*(\psi)] \right| d\nu; \\ &\quad \int_0^{q-\tau} |k(u, \nu)| \left| [\overline{F}(\psi) - \overline{F}^*(\psi)] \right| d\nu \\ &\leq LM |\underline{\psi} - \underline{\psi}^*| \int_0^{q-\tau} d\nu; LM |\overline{\psi} - \overline{\psi}^*| \int_0^{q-\tau} d\nu \\ &\leq \alpha |\underline{\psi} - \underline{\psi}^*|; \alpha |\overline{\psi} - \overline{\psi}^*| \end{aligned} \tag{5.1}$$

from which we get $(1 - \alpha) |\underline{\psi} - \underline{\psi}^*| \leq 0$ and $(1 - \alpha) |\overline{\psi} - \overline{\psi}^*| \leq 0$ implies $\underline{\psi} = \underline{\psi}^*; \overline{\psi} = \overline{\psi}^*$.

B. Convergence theorem

The arrangement $\overline{\psi}(w, \alpha) = \sum_{i=0}^{\infty} \overline{\psi}_i(w, \alpha)$ of the problem (4.2) using ADM convergence when

$$0 < \alpha < 1, |\underline{\psi}_1(w, \alpha)| < \infty.$$

Proof: Confine the series of partial sums $\overline{\varphi}_n$, let $\overline{\varphi}_n$ and $\overline{\varphi}_m$ be arbitrary partial sums with $n \geq m$. We have to show that $\overline{\varphi}_n$ is a Cauchy alignment in this Banach space.

Now,

$$\begin{aligned}
\|\overline{\varphi_n} - \overline{\varphi_m}\| &= \max_{\forall x \in [a, b]} |\overline{\varphi_n} - \overline{\varphi_m}| \\
&= \max_{\forall x \in [a, b]} \left| \sum_{i=m+1}^n \overline{\psi_i}(w, \alpha) \right| \\
&= \max_{\forall x \in [a, b]} \left| \sum_{i=m+1}^n \left(\int_a^r k(w, \nu) \overline{\overline{A_i}} d\nu \right) \right| \\
&= \max_{\forall x \in [a, b]} \left| \int_a^r k(w, \nu) \left(\sum_{i=m+1}^n \overline{\overline{A_i}} \right) d\nu \right|
\end{aligned}$$

From [10], we know

$$\sum_{i=m}^{n-1} \overline{\overline{A_i}} = G(\underline{\varphi}_{n-1}, \overline{\varphi}_{n-1}) - G(\underline{\varphi}_{m-1}, \overline{\varphi}_{m-1}).$$

So,

$$\begin{aligned}
\|\overline{\varphi_n} - \overline{\varphi_m}\| &= \max_{\forall x \in [a, b]} \left| \int_a^r k(w, \nu) (G(\underline{\varphi}_{n-1}, \overline{\varphi}_{n-1}) - G(\underline{\varphi}_{m-1}, \overline{\varphi}_{m-1})) d\nu \right| \\
&\leq \int_a^r |k(w, \nu)| |G(\underline{\varphi}_{n-1}, \overline{\varphi}_{n-1}) - G(\underline{\varphi}_{m-1}, \overline{\varphi}_{m-1})| d\nu \\
&\leq j \|\overline{\varphi_n} - \overline{\varphi_m}\|
\end{aligned}$$

Take $n = m + 1$, we have

$$\begin{aligned}
\|\overline{\varphi_n} - \overline{\varphi_m}\| &\leq j \|\overline{\varphi_m} - \overline{\varphi_{m-1}}\| \\
&\leq j \|\overline{\varphi_{m-1}} - \overline{\varphi_{m-2}}\| \\
&\quad \vdots \\
&\leq j \|\overline{\varphi_1} - \overline{\varphi_0}\|
\end{aligned}$$

and

$$\begin{aligned}
\|\overline{\varphi}_n - \overline{\varphi}_m\| &\leq \|\overline{\varphi}_{m+1} - \overline{\varphi}_m\| + \|\overline{\varphi}_{m+2} - \overline{\varphi}_{m+1}\| + \cdots + \|\overline{\varphi}_n - \overline{\varphi}_{n-1}\| \\
&\leq |j^m + j^{m+1} + \cdots + j^{n-1}| \|\overline{\varphi}_1 - \overline{\varphi}_0\| \\
&\leq |j^1 + j^2 + \cdots + j^{n-m-1}| \|\overline{\varphi}_1 - \overline{\varphi}_0\| \\
&\leq j^m \left[\frac{1-j^{n-m}}{1-j} \right] \|\overline{\psi}_1(w, \alpha)\|
\end{aligned}$$

Since $0 < j < 1$, we have $1 - j^{n-m} < 1$ then $\|\overline{\varphi}_n - \overline{\varphi}_m\| \leq \max_{\forall x \in [a, b]} |\overline{\psi}_1(w, \alpha)|$. But $|\overline{\psi}_1(w, \alpha)| < \infty$. So as

$m \rightarrow \infty, \|\overline{\varphi}_n - \overline{\varphi}_m\| \rightarrow 0$. We finalised that $\overline{\varphi}_n$ is Cauchy alignment, so $\overline{\psi}(w, \alpha) = \lim_{n \rightarrow \infty} \overline{\psi}(w, \alpha)$.

Likewise, we admit $\underline{\varphi}_n$ is a Cauchy alignment. Then, we address by $\underline{\psi}(w, \alpha) = \lim_{n \rightarrow \infty} \underline{\psi}(w, \alpha)$. Therefore,

$\underline{\psi}(w, \alpha) = \lim_{n \rightarrow \infty} \underline{\psi}(w, \alpha)$. So the series is converges.

VI. NUMERICAL EXAMPLE

Here, we show the iterative effectiveness of ADM nonlinear Volterra fuzzy delay integro-differential equations with a analytical precedent is outlined. Give us a chance to study the nonlinear Volterra fuzzy delay integro-differential equation has the accompanying structure

$$\begin{aligned}
\underline{\psi}'(r, \alpha) &= \frac{\alpha^2}{2} + \alpha e^r - \frac{\alpha^2}{2} e^{2r} + \int_0^r \underline{\psi}^2\left(\frac{t}{2}, \alpha\right) dt \\
\overline{\psi}'(r, \alpha) &= 2e^r - \alpha e^r + 2 - 2\alpha + \frac{\alpha^2}{2} + 2\alpha e^{2r} + \int_0^r \overline{\psi}^2\left(\frac{t}{2}, \alpha\right) dt
\end{aligned} \tag{6.1}$$

with initial conditions $\underline{\psi}(0; \alpha) = \alpha$ and $\overline{\psi}(0; \alpha) = 2 - \alpha$. Has the exact solution,

$\underline{\zeta}(r; \alpha) = \alpha e^r, \overline{\zeta}(r; \alpha) = (2 - \alpha)e^r$. On using the series (4.4) and (4.5) into the right and left span of Eq.(6.1) respectively gives,

$$\begin{aligned}
\underline{\psi}'(r, \alpha) &= \alpha + \frac{\alpha^2}{2} r + \alpha e^r - \frac{\alpha^2}{4} e^{2r} + \int_0^r \underline{A}_n dt \\
\overline{\psi}'(r, \alpha) &= 2 - \alpha + 2e^r - \alpha e^r + 2r - 2\alpha r \\
&\quad + \frac{\alpha^2}{2} r - e^{2r} + \alpha e^{2r} - \frac{\alpha^2}{4} e^{2r} + \int_0^r \overline{A}_n dt
\end{aligned} \tag{6.2}$$

On solving the Eq. (6.2) with the considered iterative method, will give the inexact solutions and the outcomes are organized in Table 1. It shows the numerical results at $r = 0.5$ which is very close to the exact solution.

Table 1. Observation between Exact and Approximate values

α	Exact Solution		Approximate Solution		Error Estimation	
	$\underline{\zeta}(r)$	$\bar{\zeta}(r)$	$\underline{\psi}(r)$	$\bar{\psi}(r)$	$\underline{E}(r)$	$\bar{E}(r)$
0	0	3.29744	0	3.27581	0	0.02158
0.2	0.32974	2.96770	0.32851	2.92819	0.00123	0.03946
0.4	0.65949	2.63795	0.65455	2.59895	0.00493	0.03896
0.6	0.98923	2.30821	0.97811	2.28761	0.01111	0.02057
0.8	1.31898	1.97847	1.29919	1.94392	0.01976	0.03452
1.0	1.64872	1.64872	1.61779	1.61779	0.03090	0.03090

Fig.2. Graphical representation of solutions at $x=0.5$

Estimated arrangements of nonlinear Volterra Fuzzy delay integro-differential conditions are discovered utilizing ADM from the investigation. Fig 2, demonstrates the graphical portrayal of similarity among estimated and precise arrangements. It built up the union quality of the proposed technique.

VII. CONCLUSION

In this paper, a surmised arrangement of nonlinear Volterra Fuzzy postponement integro-differential conditions is discovered utilizing ADM is indented from the examination. For delineation, a precedent is chosen to demonstrate the computational exactness. It might be inferred that the Adomian disintegration strategy is exceptionally incredible and proficient in finding surmised answers for wide classes of issues. The arrangements acquired by the present strategy are consistently focalized. As appeared in Tables 1. The present investigation has affirmed that the Adomian disintegration strategy offers huge points of interest as far as its direct pertinence, its computational viability, and its exactness to explain the unequivocally straight conditions.

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