Synchronization Mode Transitions Induced by Chaos in Modified Morris-Lecar Neural Systems with Weak Coupling

Tianyu Li

Central China Normal University

Guowei Wang

Central China Normal University

Dong Yu

Central China Normal University

Qianming Ding

Central China Normal University

Ya Jia (✉ jjay@mail.ccnu.edu.cn)

Central China Normal University School of Physics <https://orcid.org/0000-0002-2818-9074>

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Synchronization mode transitions induced by chaos in modified Morris-Lecar neural systems with weak coupling

Tianyu Li, Guowei Wang, Dong Yu, Qianming Ding, Ya Jia*

Department of Physics, Central China Normal University, Wuhan 430079, China

Abstract: Based on a modified Morris–Lecar neural model, the synchronization modes transitions between two coupled neurons or star–coupled neural network connected by weak electrical and chemical coupling are respectively investigated. For the two coupled neurons, by increasing the calcium conductivity, it is found that the period–2 synchronization of action potential of neurons is transformed to desynchronization first, and then to period–3 synchronization. By increasing the potassium conductivity, however, the synchronization mode transition is a reversal direction process as mentioned above. The bifurcation analysis of inter–spike interval shows that the synchronization modes transition is induced by the chaos. The stronger the coupling strength is, the smaller the period–2 synchronization region in the parameters plane is, while the larger the period–3 synchronization region will be. For the star–coupled neural network, in the presence of weak electrical coupling, it can exhibit the complete synchronization, desynchronization, and drum head mode states under different parameter values, respectively. In the presence of chemical synapse, however, the completely synchronized state can not be observed in the star–coupled neural network. Our results might provide novel insights into synchronization modes transition and related biological experiments.

Keywords: synchronization, chaos, Morris–Lecar neuron, weak coupling

* Corresponding author: jiay@mail.ccnu.edu.cn

1. Introduction

In recent decades, studying the nervous system through neuronal mathematical models has become a very common and effective method. Morris–Lecar (M–L) neuron model [1] is one of the simplified versions of the classic Hodgkin–Huxley (H–H) model [2–3]. The M–L model can behave like a real neuron by adjusting different system parameters. However, due to the lack of a slow variable, the original two–dimensional M–L model cannot generate bursting behavior. In order to investigate several kinds of bursting in the M–L model, a feedback current which can be treated as a slow variable is introduced by many researchers [4–8]. Also, there are other reasonable ways to introduce slow variables. For example, Hu *et al.* proposed a modified M–L model in which the magnetic flux is introduced to be a slow variable [9]. In addition, a complete electronic circuit of the 2D M–L neuron model has been established and discussed [10], which provides a novel way to study M–L neuron dynamics.

Neuronal activity is related to the transport of sodium and potassium ions across the membrane. Factors such as temperature changes and ion channel blocks have essential influences on neuronal activity. As for a real neuron, with the temperature increasing, it is found that the firing frequency is higher, while the amplitude becomes lower [11], which is consistent with the conclusions drawn in numerical simulation of the H–H model [12]. It is also reported that a higher temperature makes it difficult for a neuron to transmit information along the entire axon [13]. The function of potassium and sodium ion channel can be affected by some toxins. Experimental investigations have shown that the number of operating potassium and sodium ion channel is reduced by tetraethylammonium (TEA) and tetrodotoxin (TTX), respectively [14, 15]. The effect of ion channel blocks is that the number of opened channels or maximum ion conductivity is reduced [16–20]. In terms of neural discharging, the firing activity is enhanced under the potassium ion channel block, while the firing activity is suppressed when the sodium ion channel is blocked [21].

In the real nervous system, there are two types of couplings between two neurons, the electrical synapse [22–24] and the chemical synapse coupling [25, 26]. For coupling topologies of nonlinear systems, such as nearest coupling, global coupling, non–local coupling, were widely investigated [27–31]. In addition, biological experiments and numerical simulations have shown that synchronization is the most common behavior in the nervous system [32–35]. It is found that chaos break can be regarded as a precursor of neural synchronization in Hindmarsh–Rose neuron under electromagnetic induction [36]. Pal *et al.* investigated energy consumption of two

synchronous coupling H–H neuron, and it reported that blockage of ion channels also affects neuronal synchronization [37].

Noise is a manifestation of randomness and fluctuations for a nonlinear system. It is interesting to notice that synchronization can be triggered by different kinds of noise. Wang *et al.* found that a relatively stronger Gaussian white noise can enhance complete synchronization between two M–L neurons [38]. Similarly, an appropriate intensity of bounded noise can induce complete synchronization between two weak coupled FitzHugh–Nagumo (FHN) neurons [39]. It also reported that multiplicative noise enhances synchronization between two thermosensitive FHN neurons, but coupling can damage it [40]. The synchronization is widely studied not only in two neuron coupling systems but also in some complex neural networks. For example, Budzinski *et al.* studied the synchronization of identical thermally sensitive Hodgkin–Huxley neurons in a small–world network, and it found that continue to increase temperature, the synchronization degree becomes worse at first, then it becomes better [41]. In addition, an interesting topology named star–coupled system was proposed by Pecora and Carroll [42–44]. Pecora reported an interesting desynchronization state named drum head mode (DHM), which indicates all spokes are synchronized, but the dynamics of the hub are different with spokes. Usha *et al.* established star–coupled system with Hindmarsh–Rose (HR) neuron model, and it reported that the DHM state could be found in the unidirectional and bidirectional coupling [45, 46].

Although there are some works on the dynamic changes caused by adjusting different ion conductivity, while the effect of adjusting ion conductivity on synchronization modes transition has seldom been studied up to now. As for the star–coupled system, the DHM state has never been observed in weak electrical coupling neurons up to now. Also, investigating the influence of the total number of star–coupled units is still meaningful.

In this paper, a modified M–L neuron model is introduced to study the synchronization modes transition. Section 2 gives a brief introduction of the improved M–L neuron model and electrical and chemical coupling. Next, we discuss synchronization modes transition in two coupled neurons and star–coupled M–L neural network with weak electric or chemical coupling in section 3. Section 4 concludes this study.

2. Model

In order to make the traditional M–L neuron model exhibit bursting behavior, Hu *et al.* proposed a modified M–L model with a slow variable [9]. This modified M–L model is described as follows:

$$\begin{cases}
\frac{dV}{dt} = -g_{Ca}M_{\infty}(V)(V - V_{Ca}) - g_K W(V - V_K) - g_l(V - V_l) + k_1(\alpha + \beta\phi^2)V, \\
\frac{dW}{dt} = \lambda(V)(W_{\infty}(V) - W), \\
\frac{d\phi}{dt} = k_2V + k_3\phi_{\text{ext}},
\end{cases} \quad (1)$$

with

$$\begin{aligned}
M_{\infty}(V) &= \frac{1}{2} \left(1 + \tanh \frac{V - V_a}{V_b} \right), \\
W_{\infty}(V) &= \frac{1}{2} \left(1 + \tanh \frac{V - V_c}{V_d} \right), \\
\lambda(V) &= \frac{1}{3} \left(1 + \cosh \frac{V - V_c}{2V_d} \right),
\end{aligned}$$

where V and W represent membrane potential and recovery variable. ϕ represents magnetic flux which is regarded as a slow variable. The parameters V_{Ca} , V_K , and V_l are reversal potential of calcium, potassium, and leak ion. g_{Ca} , g_K , and g_l are maximal conductivity of calcium, potassium, and leak ion, respectively. The term $k_1(\alpha + \beta\phi^2)V$ presents the feedback current caused by magnetic flux changes, and k_1 represents the feedback gain. k_2 defines the interplay between magnetic flux and membrane potential. The term $k_3\phi_{\text{ext}}$ describes a constant leakage magnetic flux. M_{∞} and W_{∞} describe the steady values of gating variables of calcium and potassium ion, λ_{∞} is the relaxation time. V_a , V_b , V_c and V_d are system parameters.

In order to investigate the synchronization modes transition, two coupling M-L neuron is described as follows:

$$\begin{cases}
\frac{dV_1}{dt} = -g_{Ca}M_{\infty}(V_1)(V_1 - V_{Ca}) - g_K W_1(V_1 - V_K) - g_l(V_1 - V_l) + k_1(\alpha + \beta\phi_1^2)V_1 - g_{(e/c)}\delta_{(e/c)}(V_1 - V_2), \\
\frac{dW_1}{dt} = \lambda(V_1)(W_{\infty}(V_1) - W_1), \\
\frac{d\phi_1}{dt} = k_2V_1 + k_3\phi_{\text{ext}}, \\
\frac{dV_2}{dt} = -g_{Ca}M_{\infty}(V_2)(V_2 - V_{Ca}) - g_K W_2(V_2 - V_K) - g_l(V_2 - V_l) + k_1(\alpha + \beta\phi_2^2)V_2 - g_{(e/c)}\delta_{(e/c)}(V_2 - V_1), \\
\frac{dW_2}{dt} = \lambda(V_2)(W_{\infty}(V_2) - W_2), \\
\frac{d\phi_2}{dt} = k_2V_2 + k_3\phi_{\text{ext}},
\end{cases} \quad (2)$$

where $g_{(e/c)}$ is the coupling intensity of electrical or chemical coupling. $\delta(V_1, V_2)$ defines different types of coupling terms. In this work, The electrical coupling is described as:

$$\delta_e(V_1, V_2) = V_1 - V_2. \quad (3)$$

The chemical coupling is described as follows:

$$\delta_c(V_1, V_2) = \frac{V_1 - V_s}{1 + \exp[-\lambda(V_2 - \theta)]}, \quad (4)$$

where V_s is reversal potential. In this paper, only excitatory synapse is adopted, which requires the value of V_s should be larger than the maximal value of the membrane potential. λ and θ represent the ratio parameter and threshold of chemical coupling, respectively.

To quantitatively determine the degree of synchronization between neurons, a statistic index named similar function is introduced. It is given by

$$S = \sqrt{\frac{\langle (V_1(t) - V_2(t))^2 \rangle}{\sqrt{\langle V_1^2(t) \rangle \cdot \langle V_2^2(t) \rangle}}}, \quad (5)$$

where $\langle \cdot \rangle$ represents average value over time. Based on previous work [38], a relatively smaller S means better synchronization between two neurons. The value $S = 0$ means that the coupled system is completely synchronized.

3. Results and discussions

We have used the Euler algorithm to analyze the dynamics of coupled M–L neurons. The time step h is set as 0.001. Here, g_{Ca} , g_k , and $g_{(e/c)}$ are regarded as control parameters. The remaining system parameter are selected as: $V_{Ca} = 1$, $V_l = -0.5$, $V_K = -0.7$, $V_a = -0.01$, $V_b = 0.15$, $V_c = 0.1$, $V_d = 0.05$, $\alpha = 0.1$, $\beta = -0.09$, $\varphi_{ext} = 0.2$, $k_1 = 1.0$, $k_2 = 0.15$, $k_3 = 0.15$, $V_s = 0.4$, $\lambda = 30$, and $\theta = 0.1$.

3.1 The effect of changing ion conductivity on dynamics of a single M–L neuron.

Before studying synchronization modes transition between neurons, we first discuss the effect of changing different ion conductivity on a single neuron. In the absence of coupling, the firing activity of a single M–L neuron exhibits variable dynamical behaviors by adjusting g_{Ca} and g_k . The bifurcation diagram of inter-spike interval (*ISI*) and maximum / minimum of membrane potential (V_{max} / V_{min}) is shown in Fig. 1. With the g_{Ca} increasing, the period-adding phenomenon is observed, and the firing state is transformed from spiking state to bursting state. As g_{Ca} increases, V_{max} is monotonously enhanced, and V_{min} is reduced. Conversely, the phenomenon of

period-reducing is obtained as g_k increases. It shows that the firing state is transformed from bursting to spiking state, and $V_{\max}$ is decreased with g_k decreasing. One can easily find that increasing g_{Ca} has opposite effects on neural dynamic to increasing g_k .

Fig. 1 The bifurcation diagram of inter-spike interval (ISI) and maximum and minimum value of membrane potential ($V_{\max} / V_{\min}$) versus maximal conductivity of calcium g_{Ca} and potassium g_k . The fixed parameter: **a1 a2** $g_k = 2.0$. **b1 b2** $g_{Ca} = 1.2$. The initial values of (V, W, φ) are (0.1, 0, -0.1).

3.2 Weak electrical coupling for two M-L neurons.

In order to investigate the synchronization modes transition caused by adjusting g_{Ca} and g_k , the coupling intensity g_e is fixed at $g_e = 0.005$. The time series of membrane potential is shown in Fig. 2. Even if the coupling is very weak, the complete synchronization of two M-L neurons can be observed clearly. In Figs. 2a1–a4, with calcium conductivity g_{Ca} increasing, the successive synchronization modes transition is realized. it is found that synchronization mode is transformed from period-2 synchronization ($g_{Ca} = 1.25$) to period-3 desynchronization ($g_{Ca} = 1.305$), then to phase lock desynchronization ($g_{Ca} = 1.33$), and finally to period-3 synchronization ($g_{Ca} = 1.4$). On the contrary, with increases in g_k , synchronization mode is changed from period-3 synchronization ($g_k = 1.33$) to phase lock desynchronization ($g_k = 1.83$), then to period-3

desynchronization ($g_k = 1.85$), finally to period-2 synchronization ($g_k = 2.0$). In terms of synchronization modes transition, it is interesting to notice that increasing g_{Ca} also has opposite effects of increasing g_k .

Fig. 2 Different synchronization modes transition of two M-L neurons with different value of g_{Ca} (left column) and g_k (right column). The electrical coupling intensity $g_e = 0.005$. **a1–a4** $g_{Ca} = 1.25, 1.305, 1.33, 1.40$, $g_k = 2.0$. **b1–b4** $g_k = 1.75, 1.83, 1.85, 2.0$, $g_{Ca} = 1.2$.

Fig. 3 Similar function S (red) and bifurcation diagram (black) of the first neuron associate with g_{Ca} and g_k respectively. **b** and **d** are enlargements of **a** and **c**. The electrical coupling intensity $g_e = 0.005$. **a b** $g_k = 2.0$; **c d** $g_{Ca} = 1.2$.

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Next, we'll show chaos caused by adjusting g_{Ca} or g_k can greatly influence the synchronization of two M-L neurons. Fig. 3 provides similar function S of two neurons and the inter-spike interval (ISI) of V_1 in a picture. It should be mentioned that the ISI of the two neurons are very similar. In Figs. 3a and 3b, one can clearly see that synchronization disappeared at about $g_{Ca} = 1.29$. In addition, from the view of the ISI , the firing state of the first neuron is also transformed from period-2 bursting to chaotic state at $g_{Ca} = 1.29$. At $g_{Ca} = 1.29$, the transition of bursting to chaotic state is the reason for the disappearance of synchronization.

When g_{Ca} is greater than 1.31, the chaotic is transformed to period-2 bursting state. When $g_{Ca} = 1.32$, even the chaotic state disappears, synchronization does not occur. Further increasing g_{Ca} to approximately 1.344, a very narrow chaotic region emerges, and the synchronization of the two neurons is suddenly getting better (Fig. 3b). If g_{Ca} keeps increasing, the value of S slowly decreases to 0. From Figs. 3b and d, the effect of increasing g_k on firing states and synchronization modes transition is investigated. When g_k is increased to 1.797 (before chaotic state), the synchronization has been destroyed at first. After the chaotic region, period-3 bursting can be observed, but synchronization does not show up. As g_k further increases, a relatively wider chaotic region appeared at $g_k = 1.84$, and the value of S began to reduce. When g_k is over 1.86, the chaotic is changed to period-2 bursting state, and S is dropping rapidly to 0. In short, we conclude that chaos is the reason for the establishment and breaking of the synchronization.

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Fig. 4 Similar function S in parameter plane of the electrical coupling intensity g_e and g_{Ca} (or g_k). The fixed parameter: **a** $g_k = 2.0$; **b** $g_{Ca} = 1.2$.

Similar function S associates with electrical coupling strength g_e and two conductivity (g_{Ca} or g_k) is depicted in Fig. 4. From Fig. 4a, as g_e increases, the period-2 synchronization region

becomes smaller, and the critical value of g_{Ca} at synchronization modes transition decreases, which indicates the period-2 synchronization region is shrunk in the parameter plane. A similar conclusion can be drawn in Fig. 4b.

3.3 Weak excitatory chemical coupling for two M-L neurons.

In this part, we will prove that a conclusion similar to weak electrical coupling can be found in two identical M-L neurons connected by excitatory chemical coupling. Here, the initial values of $(V_1(0), W_1(0), \varphi_1(0), V_2(0), W_2(0), \varphi_2(0))$ are $(0.1, 0, -0.1, 0, 0, -0.1)$.

Fig. 5 Different synchronization modes transition of two M-L neurons with different value of g_{Ca} (left column) and g_k (right column).. **a1–a4** $g_{Ca} = 1.25, 1.309, 1.35, 1.40$, $g_k = 2.0$. **b1–b4** $g_k = 1.75, 1.80, 1.843, 2.0$, $g_{Ca} = 1.2$.

The time series of membrane potential is drawn in Fig. 5. We choose the coupling intensity $g_c = 0.005$. The initial transients completely vanished after $t = 1000$, and the system behavior seems to be stable. As shown in Figs. 5a1–a4, with increase in calcium conductivity g_{Ca} , the synchronization modes undergo a successive transitions, i.e., period-2 synchronization ($g_{Ca} = 1.25$) $\rightarrow$ desynchronization ($g_{Ca} = 1.309$) $\rightarrow$ period-3 phase lock ($g_{Ca} = 1.35$) $\rightarrow$ period-3 synchronization. The process of synchronization modes transition by increasing g_k is completely opposite to increasing g_{Ca} .

Fig. 6 Similar function S (red) and bifurcation diagram (black) of the first neuron versus g_{Ca} and g_K , respectively. **b** and **d** are enlargements of **a** and **c**. The chemical coupling intensity $g_c = 0.005$. **a b** $g_K = 2.0$; **c d** $g_{Ca} = 1.2$.

The Similar function S and inter-spike interval (ISI) of the first neuron is provided in Fig. 6. If g_{Ca} is less than 1.31, the two neurons are completely synchronized. However, when $g_{Ca} = 1.31$, the firing state of the first neuron is transformed to chaotic state, and the synchronization disappears. Although the chaos region vanishes at $g_{Ca} = 1.312$, there is no complete synchronization before the next chaotic region arrives. From Fig. 6b, When g_{Ca} is greater than 1.355, S starts to decrease gradually. At $g_{Ca} = 1.37$, S decreases to 0, and perfect synchronization is formed. We also studied the influence of adjusting g_K on synchronization modes transition (Figs. 6c and d). It found that increasing g_K almost has the opposite effect as increasing g_{Ca} .

Fig. 7 Similar function S in parameter plane of the chemical coupling intensity g_c and g_{Ca} (or g_k). The fixed parameter: **a** $g_k = 2.0$; **b** $g_{Ca} = 1.2$.

Similar function S versus parameter plane in (g_c, g_{Ca}) and (g_c, g_k) is depicted in Fig. 7. In Figs. 7a and b, it is found that as g_c increases, the period-2 synchronization region shrinks, but the period-3 synchronization region expands in the parameter plane. In short, whether it is weak electrical or chemical coupling, adjusting g_{Ca} (or g_k) exerts similar effect on synchronization modes transformation.

3.4 Weak electrical coupling for star-coupled neural network.

In this section, synchronization modes transition is discussed in the star-coupled system. The star-coupled M-L neural network can be described as follows:

$$\begin{cases} \frac{dV_i}{dt} = -g_{Ca}M_{\infty}(V_i)(V_i - V_{Ca}) - g_KW_i(V_i - V_K) - g_l(V_i - V_l) + k_1(a + \beta\phi_i^2)V_i - g_{(e/c)}\sum_{\substack{j=1 \\ i \neq j}}^N C(i,j)\delta_{(e/c)}(V_i, V_j), \\ \frac{dW_i}{dt} = \lambda(V_i)(W_{\infty}(V_i) - W_i), \\ \frac{d\phi_i}{dt} = k_2V_i + k_3\phi_{ext}, \end{cases} \quad (6)$$

where N is the total number of neurons, $i, j = 1, 2, \dots, N$. $C(i, j)$ represents the $N \times N$ connectivity matrix. If the i th neuron receives synaptic effect from j th neuron, $C(i, j) = 1$; otherwise $C(i, j) = 0$.

Fig. 8 Topological structure of star-coupled neural network. The 1st to $N-1$ th units named nodes are arranged on the spokes of star, and the N th unit is regarded as the hub.

First, it is necessary to introduce the structure and characteristics of the star-coupled system. The topological structure is illustrated in Fig. 8. The 1st to N-1th and Nth units are named nodes and hub, respectively. All the nodes are connected with the hub bidirectionally, and there is no interaction between any two nodes. Next, system parameters for all the units are complete the same except the initial values. In the star-coupled neural network, there are three different modes. The first one is the complete synchronization, which means both nodes and hub are synchronized. The second one is a special desynchronized mode named drum head mode (DHM) [42–44], i.e., all the nodes are synchronized, but the hub exhibits different behavior. The third is the desynchronized mode: not all nodes are synchronized.

In order to distinguish the different modes of this system, we introduce two kinds of similar function. The first one is the average similar function of all star-coupled neurons S_{all} , which is represented by:

$$S_{\text{all}} = \frac{\sum_{i=1}^N \sum_{\substack{j=1 \\ i \neq j}}^N \sqrt{\frac{\langle (V_i(t) - V_j(t))^2 \rangle}{\langle V_i^2(t) \rangle \cdot \langle V_j^2(t) \rangle}}}{(N-1)^2}. \quad (7)$$

The second is the average similar function of all the nodes, which is represented by:

$$S_{\text{nodes}} = \frac{\sum_{i=1}^{N-1} \sum_{\substack{j=1 \\ i \neq j}}^{N-1} \sqrt{\frac{\langle (V_i(t) - V_j(t))^2 \rangle}{\langle V_i^2(t) \rangle \cdot \langle V_j^2(t) \rangle}}}{(N-2)^2}. \quad (8)$$

If S_{all} is very close to 0, the neurons in network are in the complete synchronized mode; If S_{nodes} is very close to 0 while S_{all} is significantly greater than 0, it exhibits the DHM. Otherwise, the neurons in the network are in the desynchronized mode.

Fig. 9 The collective dynamics of star-coupled neural network with weak electrical coupling at $t = 2000$. The unit number is plotted along the X-axis, and the corresponding time series of membrane potentials are plotted along the Y-axis. The red point represents the hub. Electrical coupling intensity $g_e = 0.001$ **a1–a3**, $g_k = 2.0$, $g_{Ca} = 1.3425, 1.343, 1.346$; **b1–b3**, $g_{Ca} = 2.0$, $g_k = 1.796, 1.797, 1.799$.

In this section, the weak electrical coupling is used to connect star-coupled M-L neurons. All system parameters are the same as above. We still regard g_{Ca} , g_k , and g_e as control parameters. The initial values of $(V_i(0), W_i(0), \phi_i(0))$ are $(0.01 \times i, 0, 0)$.

The value of membrane potential of all units at $t = 2000$ is provided in Fig. 9. Coupling intensity and the number of neurons in star-coupled neurons are fixed: $g_e = 0.001$ and $N = 20$, respectively. The 20th neuron is regarded as the hub. The initial transients completely died out at this moment, and star-coupled system appears to be stable. From left column, it is found that with g_{Ca} increasing, the star-coupled system shows complete synchronized mode ($g_{Ca} = 1.3425$), then desynchronization ($g_{Ca} = 1.343$), finally DHM ($g_{Ca} = 1.346$). As for increasing g_k , the system goes through the opposite process. The system exhibits DHM at $g_k = 1.796$, desynchronized mode at $g_k = 1.797$, complete synchronization at $g_k = 1.799$.

Fig. 10 a b Bifurcation diagram (ISI) of the 1st neuron in electrical star-coupled network versus g_{Ca} and g_k , respectively. **c d** Similar function of all neurons S_{all} and nodes S_{nodes} versus g_{Ca} and g_k , respectively. Fixed parameter: $g_e = 0.001$ **a c** $g_k = 2.0$; **b d** $g_{Ca} = 1.2$.

In this section, we'll show synchronization modes transition of star-coupled neurons is caused by chaos (or bifurcation). It should be mentioned that bifurcation diagrams of all nodes are very similar. We provide bifurcation diagram (ISI) of the 1st neuron and two similar functions (S_{all} and S_{nodes}) in Fig. 10. From Figs. 10a and c, when g_{Ca} is less than 1.3426, both S_{all} and S_{nodes} are very close to 0. When g_{Ca} is slightly greater than 1.3427, the firing state is transformed to chaotic state, and complete synchronized mode disappears (Figs. 11a and c). Continue to increase g_{Ca} , the complete synchronization or DHM does not appear until $g_{Ca} = 1.3436$. If g_{Ca} is slightly greater than 1.3437, the firing state is transformed to period-3 bursting, and DHM is observed.

For the case of increasing g_k (Figs. 10b and d), the star-coupled system exhibits the DHM when g_k is less than 1.7966. When g_k is slightly greater than 1.7967, period-3 is turned to the period-4 bursting, and the DHM is transformed to desynchronized state. As g_k increases, there exists multiple DHM regions, which indicates that synchronization modes transition of the star-coupled system is more sensitive to changes in g_k than g_{Ca} . If g_k is slightly greater than 1.7982, period-3 is changed to period-2 bursting, and the desynchronized mode is transformed to complete synchronization (Fig. 11).

Fig. 11 Enlargement of Fig. 10. Here, we only show a branch of the bifurcation diagram (ISI).

We have drawn the parameter plane of (g_e, g_{Ca}) and (g_e, g_k) in Fig. 12. Here, complete synchronized, DHM, and desynchronized modes are represented by red, white and black, respectively. As electrical coupling intensity g_e is increased, regions of the complete synchronized and DHM regions are expanded. Influence by adjusting the number of neurons N on this system is

also investigated. From the top column of Fig. 12, it is found that a star-coupled system with more neurons has a relatively smaller region of complete synchronization. Additionally, increasing N exerts little effect on the DHM region. A similar conclusion can be drawn from the bottom column of Fig. 12.

Fig. 12 Parameter space. The black, red and white regions indicate desynchronized, completely synchronized and DHM states, respectively. The number of star-coupled neurons from left to right to is $N = 20, 25, 30$. **a1–a3** $g_k = 2.0$; **b1–b3** $g_{Ca} = 1.2$.

Fig. 13 The collective dynamics of star-coupled M-L neurons with weak excitatory chemical coupling at $t = 2000$. The unit number is plotted along the X-axis, and the corresponding time series of membrane potentials are plotted along the Y-axis. The red point represents the hub. Electrical coupling intensity $g_c = 0.0005$. **a1–a3**, $g_k = 2.0$, $g_{Ca} = 1.342, 1.343, 1.344$; **b1–b3**, $g_{Ca} = 2.0$, $g_k = 1.796, 1.798, 1.80$.

3.5 Weak excitable chemical coupling for star-coupled neural network.

In this section, the star-coupled M-L system is built by weak excitable chemical coupling. The initial values of $(V_i(0), W_i(0), \phi_i(0))$ are still set as $(0.01 \times i, 0, 0)$. The collective dynamics of all units at $t=2000$ is plotted in Fig. 13. We fixed coupling intensity $g_c = 0.0005$ and total number $N = 20$. From the left column of Fig. 13, the star-coupled system undergoes transition between with g_{Ca} increasing, i.e., the DHM ($g_{Ca} = 1.342$) is changed to desynchronized mode ($g_{Ca} = 1.343$) then turn back to the DHM ($g_{Ca} = 1.344$). The right column shows the same process (Figs. 13b1, b2 and b3).

Fig. 14 a b Bifurcation diagram (ISI) of the 1st neuron in weak excitatory chemical star-coupled network versus g_{Ca} and g_k , respectively. **c d** Similar function of all neurons S_{all} and nodes S_{nodes} versus g_{Ca} and g_k respectively. Fixed parameter: $g_c = 0.0005$, **a c** $g_k = 2.0$; **b d** $g_{Ca} = 1.2$.

The bifurcation diagram (ISI) of the 1st neuron and two similar functions (S_{all} and S_{nodes}) are drawn in Fig. 14. When g_{Ca} is less than 1.3428, the system exhibits the DHM. As g_{Ca} increases, the DHM disappears, and the firing state of 1st neuron is transformed to chaotic state. The DHM reappears at $g_{Ca} = 1.3434$, where chaotic state is changed to period-3 bursting. The analogous phenomenon can be found in Figs. 14b and d.

Fig. 15 Parameter space. The black, white regions indicate desynchronized and DHM states, respectively. The number of star-coupled neurons from left to right column is $N = 20, 22, 25$. **a1–a3** $g_k = 2.0$; **b1–b3** $g_{Ca} = 1.2$.

The parameter plane of (g_c, g_{Ca}) and (g_c, g_k) is plotted in Fig. 15. One can clearly observe that if g_{Ca} (or g_k) is fixed, a higher value of g_c makes the DHM region larger. However, with the number of nodes increasing, the DHM region is reduced significantly. The absence of the complete synchronization state in a weak chemical coupled system could be a consequence of the nonlinear feedback form of the coupling.

4. Conclusion

In this work, the relationship between chaos and synchronization modes transition with weak electrical and excitatory chemical coupling is investigated. The main conclusion of the two coupled neurons is as follows: If the electrical coupling intensity g_c is fixed, increasing g_{Ca} makes the system undergo a successive transition: from period-2 synchronization to desynchronization, and finally to period-3 synchronization. The phenomenon of synchronization mode transition is caused by chaos. The desynchronized region is mainly sandwiched between two chaotic regions. An opposite phenomenon of increasing g_{Ca} can be observed by increasing g_k . In addition, a relatively stronger electrical coupling makes the period-2 synchronization region smaller but the period-3 synchronization region larger in the parameter plane. For weak excitatory chemical coupling, the phenomenon and conclusions are very similar to those of electrical coupling.

For a star-coupled neural network with electrical synapse, it was found that: Continue to increase g_{Ca} , complete synchronization is changed to desynchronization, finally to the DHM. It is the first time that the DHM has been found in an star-coupled neural network with electrical synapse. These phenomena of synchronization modes transition are created by chaos (or bifurcation). An opposite modes transition of increasing g_{Ca} can be observed by increasing g_k . A relatively stronger electrical coupling expands the region of complete synchronization and DHM, while the completely synchronized region shrinks as the number of nodes increases.

For a star-coupled neural network with weak excitatory chemical synapse, it was found that: by increasing g_{Ca} , the DHM is changed to desynchronization, and finally to DHM. In this case, we failed to observe the complete synchronization. In addition, a relatively greater value of g_C expands the parameter region of the DHM, but the DHM region shrinks as the number of nodes increases.

We have investigated the synchronization mode transition with electrical and excitatory chemical coupling. To study the effects of inhibitory synapses on the above-mentioned topologies is still meaningful. Also, both various noise and time delay of information transmission [47, 48] should also be introduced to further investigation.

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Data Availability Statement

All data generated or analysed during this study are included in this published article.

Conflict of interest

The authors declare that they have no potential conflict of interest.

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