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DEVELOPMENT OF A SEMI-ANALYTICAL DYNAMIC THRUST FORCE MODEL

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ABSTRACT

A moving water mass generates force which is exerted on its moving path. Cyclone generated storm surge or earthquake generated tsunami are specific examples of moving water mass the generates force along the coasts. In addition to human lives, these moving water masses cause severe damages to the coastal infrastructure due to tremendous force exerted on these structures. To assess the damage on these infrastructures, an essential parameter is the resultant force exerted on these structures. To evaluate the damages, there is hardly any quantitative method available to compute this force. In this paper we have developed a semi-analytical model, named as Dynamic Force Model (DFM), by using Variational Iteration Method to compute this force. As governing equations, we have used the Saint Venant equations which are basically 1D shallow water equations derived from the Navier-Stokes equations. The verified, calibrated and validated DFM is applied in Bangladesh coastal zone to compute dynamic thrust force due to tropical cyclone SIDR.

Keywords: Semi-analytical method, Variational Iteration Method, Moving water mass, Thrust force, Navier-Stokes equations, Saint-Venant equations.

INTRODUCTION

A moving water mass generates force due to momentum created by the water mass itself and the acceleration due its movement. Specific examples of moving water mass are surge waves generated by cyclones and tsunami waves generated by earthquakes that strike the coast with considerable magnitude of force. Though the sources of these moving water masses (for example storm surge and tsunami) are different, the physical characteristics of their wave propagation (in deep water), nonlinear transformation (in shallow water) and runup (land inundation) are identical (Yim, 2005). The rapid changes in hydrodynamics in near shore substantially damage infrastructures of coastal regions. So, from the major concepts of coastal engineering, storm loading (Clark, 1982) (wind load, waves, hydrostatic and hydrodynamic loads) and tsunami induced forces are necessary in a coastal building code. Water related loads accompanying a coastal storm are often not adequately addressed in coastal building code, with wind generated/ induced waves producing the most critical and complex forces to which the coast and its structures are subjected (Clark, 1982). Few

current building codes of coastal structures such as the city and county of Honolulu Building Code (CCH); the 1997 Uniform Building Code (UBC 97); the 2000 International Building Code (IBC 2000), the SEI/ASCE 7-02 (ASCE 7), and the Federal Emergency Management Agency Coastal Construction Manual (FEMA CCM) incorporates the effects of hydrodynamic loads (surge/tsunami induced)(Pacheco and Robertson, 2005). The generalized equations of contributing force components of these codes are discussed in Yeh and Robertson, 2005; Palermo, 2009; Yim, 2005. In these codes, computation of hydrostatic force is the multiplicative combination of gravitational acceleration and square of specific energy. Hydrodynamic force is calculated as the drag force acting on the direction of the uniform flow. But these codes have over-simplified the process of estimating the hydrodynamic loads (Yim, 2005). The forces are calculated for an instantaneous velocity of moving water mass for design purposes of coastal infrastructures. But these codes neglect the non-linearity of the variation of the total force generated during a cyclone or Tsunami.

Apart from structural measures, numerous earthen embankment structures like levees, dams, dikes etc. have been built by river and coastal line around the world. Because these structures can sustain only limited safety levels and are subject to decay, they may fail owing to various triggering mechanisms (CA 1985; Foster et al. 2000; Allsop et al. 2007), especially with a high probability of failure under extreme conditions such as storm surge and tsunami. Embankment failures are heavily influenced by the structures' materials and configurations along with all other impacting forces and other environmental factors. The major causes of earthen embankment failure during storm surge are overtopping, foundation and structural defects, and piping, although additional failure causes may exist for individual structures (ASCE/EWRI Task Committee on Dam/Levee Breaching, 2011). There are many studies focusing on failure due to overtopping of flow (Pugh 1985; Ralston 1987; Powledge et al. 1989; Singh 1996; Visser 1998; Hanson et al. 2005). The investigation report of the performance of the New Orleans flood protection systems during Hurricane Katrina made reference to a number of cases where signs of under-seepage piping erosion is thought to be responsible for the failure of the levees (Seed et al. 2006). Wolff (1997) analyzed that the probability of levee failure due to under-seepage is among the main causes of levee failures and increases as the floodwater elevation increases. Piping may occur because of the presence of seepage flow through weak layers, cracks, structural joints, tree roots, and animal burrows in the embankment, without the necessity for the water level to have reached the full height of the embankment (Foster et al. 2000; Fell et al. 2003; Richards and Reddy 2007). Rapid rise in water level and the prolonged immersion in high water also facilitate embankment failure during storm surge (Luo et al, 2016). The former induces a tremendous impact effect, which significantly decreases the resisting forces against seepage failure by increasing erosion power of seepage flow (Luo et al., 2013, 2016). Extensive research has been carried out focusing on seepage failure mainly focuses on steady flow condition (Sellmeijer, 1988, Sellmeijer and Koenders , 1991, Asaoka and Kodaka, 1992,Fontana, 2008, Maknoon and Mahdi, 2010) and transient flow condition (Ozkan, 2003, Ozkan et al., 2008, El Shamy and Aydin , 2008, Awal et al., 2011, Luo et al., 2013 and 2016). While these studies focus mainly on geotechnical failure of an earthen embankment, there is lack of study computing initial thrust force exerted on the embankment in presence of which, later geotechnical failures are initiated.

In this study the momentum equation popularly known as the Saint-Venant equation which is derived from the Navier-Stokes equation is semi-analytically solved to compute hydrodynamic force. This model simulates the hydrodynamic force generated by moving

water mass. In this study, the resultant force generated in the system is termed as dynamic thrust force and the model developed to compute this thrust force is termed as Dynamic thrust Force Model (DFM).

ANALYTICAL SOLUTION METHODS OF PARTIAL DIFFERENTIAL EQUATIONS

A wide varieties of problems in fluid mechanics are expressed by partial differential equations and they are nonlinear. Nonlinearity exists everywhere, and nature is nonlinear in general. There are many methods, such as perturbation methods, to solve nonlinear partial differential equations (PDEs). A method known as the method of separation of variables is perhaps one of the oldest systematic methods for solving partial differential equations including the wave equation. The wave equation and its methods of solution attracted the attention of many famous mathematicians including Leonhard Euler (1707–1783), James Bernoulli (1667–1748), Daniel Bernoulli (1700–1782), J.L. Lagrange (1736–1813), and Jacques Hadamard (1865–1963). They discovered solutions in several different forms of partial differential equations. A well-known analytical method is Decomposition Method which is established by Adomian. Special attention should be paid to Adomian's decomposition method (Adomian, 1983, 1988) and Liao's homotopy analysis method (Liao, 1997). With these methods, most PDEs can be approximately solved without linerization or weak linerization or small perturbations. However, the approximation obtained by Adomian's method could not always satisfy all its boundary conditions, leading to error near boundaries. A successful approximation of solution for partial differential equations is established with no boundary problems by Ji-Huan He (He, 1997, 1998, 1999a, 1999b) which is known as Variational Iteration Method (VIM). This method is based on analytically solving the equations that result from discretizing the spatial coordinates of a partial differential equation (Vidts and White, 1992).

Governing equations used in this study are the Saint-Venant equations derived from Navier-Stokes equations with one dimensional shallow water approximation. The equations are characteristically non-linear partial differential equations. Functional relations for some of the variables in the governing equations are not available. So, the equations are solved semi-analytically by applying coupled VIM and discretization method. In this study, this coupled solution method is termed as 'semi-analytical'.

MODEL DEVELOPMENT

Assumptions to the development of Saint-Venant Equations

In Fluid Dynamics, the Saint-Venant equations (Raisinghania, 2003) were formulated in the 19th century by two mathematicians, Adhémar Jean Claude Barréde Saint-Venant and Bousinnesque (Raisinghania, 2003). Saint-Venant equations are derived from Navier-Stokes equations (Vreukdenhil, 1994) for shallow water conditions (Kubatko, 2005). The Navier-Stokes equations represent a general model used to model fluid flows (Bessona et al., 2007; Bulatov, 2013). On the other hand, a general flood wave for 1-D situation (Bessona et al., 2007; Bulatov, 2013) can be described by the Saint-Venant equations.

During development of the Saint Venant equations, following assumptions (Vreukdenhil, 1994) were made:

- Flow is one-dimensional.

- 133 • Flow is unsteady and non-uniform.
- 134 • Hydrostatic pressure prevails and vertical accelerations are negligible.
- 135 • Streamline curvature is small.
- 136 • Bottom slope of the channel is small.
- 137 • The fluid is incompressible.
- 138 • The gravity force is the only force which is taken into account. So, the influence
- 139 of the Coriolis force is neglected.
- 140

141 In addition to the above assumption, following assumptions are made for this study:

- 142 • The steady uniform state of flow is expressed by Manning's equation which is used
- 143 as the initial condition for solution.
- 144 • Manning's coefficient n is used to describe the resistance effects.

145 These two additional assumptions do not affect the basic governing equations.

146 Conservation of mass

147 Law of mass conservation says that mass can neither be created nor destroyed. When this
 148 law is applied in any control volume of fluid (water), it means net rate of change of mass is
 149 equal to the net change of mass due to inflow and outflow.

150 For fluid dynamics, the continuity equation (Vreukdenhil, 1994) becomes

$$151 \quad \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) = 0 \quad (1)$$

152 Where h is the water depth and u is the flow velocity.

153 Conservation of momentum

154 This law states that the rate of change of momentum (mass times acceleration) in the control
 155 volume is equal to the net forces acting on the control volume.

156 Inside a control volume, conservation of momentum can be expressed (Vreukdenhil, 1994)
 157 as:

$$\begin{aligned}
 158 \quad & \frac{d}{dt} [\text{Momentum}] \\
 159 \quad & = \text{Momentum flux entering at } x - \text{Momentum flux exiting at } (x + dx) \\
 160 \quad & + \text{Pressure force in the rear} - \text{Pressure force ahead} \\
 161 \quad & + \text{Downslope gravitational force} - \text{Frictional force along the bottom} \\
 162 \quad & \quad \quad \quad (2)
 \end{aligned}$$

163 The momentum is the mass times the velocity, i.e. $(\rho dv)u = \rho A u dx$, whereas the
 164 momentum flux is the mass flux times the velocity, i.e. $(\rho A u)u = \rho A u^2$. The pressure force

165 F_p acting at each end of the control volume is obtained from the integration of the depth-
 166 dependent pressure over the cross section.

$$167 \quad \text{Pressure force} = F_p = \iint p dA = \int_0^h p(z)w(z)dz \quad (3)$$

168 in which $p(z)$ and $w(z)$ are the pressure and channel width at level z , with z varying from
 169 zero at the bottom-most point to h at the surface. Under the assumption of a hydrostatic
 170 balance, the pressure increases linearly with depth according to

$$171 \quad p(z) = \rho g(h - z) \quad (4)$$

172 discounting the atmospheric pressure which acts all around and has no net effect on the flow.
 173 The pressure force is thus equal to:

$$174 \quad F_p = \int_0^h \rho g(h - z)w(z)dz \quad (5)$$

175 and is a function of how filled the channel is. In other words, it is a function of depth h .
 176 Taking the h derivative (which will be needed later), we have:

$$\begin{aligned} 177 \quad \frac{dF_p}{dh} &= [\rho g(z - h)w(z)]_{z=h} + \int_0^h \rho g w(z)dz \\ 178 \quad &= \rho g \int_0^h w(z)dz = \rho g A \end{aligned} \quad (6)$$

179 The gravitational force is the weight of the water slice projected along the x -direction, which
 180 is mg ($= \rho v$) times the sine of the slope angle θ ,

$$181 \quad \text{Gravitational Force} = [(\rho dv)g] \sin \theta = \rho Ag S_0 dx \quad (7)$$

182 The frictional force is the bottom stress τ_b multiplied by the wetted surface area

$$183 \quad \text{Frictional Force} = \tau_b p dx \quad (8)$$

184 The bottom stress is proportional to the square of the velocity. Invoking a drag coefficient
 185 C_d , we write:

$$186 \quad \text{Bottom stress} = \tau_b = C_d \rho u^2 \quad (9)$$

187 which resembles a Reynolds stress ($\tau = -\rho u'w'$), with the turbulent fluctuations u' and
 188 w' each proportional to the average velocity u . The frictional force exerted on the slice of
 189 water is then

$$190 \quad \text{Frictional Force} = \tau_b p dx = C_d \rho u^2 p dx \quad (10)$$

191 We now gather the momentum budget as

$$\begin{aligned} 192 \quad & \frac{[\rho A u dx]_{at \ (t+dt)} - [\rho A u dx]_{at \ t}}{dt} = \rho A u^2 \Big|_{at \ x} - \rho A u^2 \Big|_{at \ (x+dx)} + F_p \Big|_{at \ x} - F_p \Big|_{at \ (x+dx)} \\ & + \rho g A S_0 dx - C_d \rho u^2 p dx \end{aligned}$$

$$193 \quad (11)$$

194 In differential form,

$$195 \quad \frac{\partial}{\partial t}(\rho A u) + \frac{\partial}{\partial x}(\rho A u^2) = -\frac{\partial F_p}{\partial x} + \rho g A S_0 - C_d \rho p u^2 \quad (12)$$

196 The gradient of the pressure force becomes

$$197 \quad \frac{\partial F_p}{\partial x} = \frac{dF_p}{dh} \frac{\partial h}{\partial x} = \rho g A \frac{\partial h}{\partial x} \quad (13)$$

198 Which simplifies into

$$199 \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} + g S_0 - C_d \frac{u^2}{R_h} \quad (14)$$

200 In this equation the ratio of the cross-sectional area A over the wetted perimeter p , which
201 has the dimension of a length, is defined as

$$202 \quad R_h = \frac{A}{p} \quad (15)$$

203 This is called hydraulic radius. Because we consider wide water body, which is much wider
204 than the depth, the wetted perimeter is generally not much more than the width ($p \approx W$),
205 and so the hydraulic radius is approximately

$$206 \quad R_h = \frac{A}{W} \quad (16)$$

207 Proposed Governing Equations

208 If we include the cyclonic wind term in the momentum equation and consider $\rho = \rho_w =$
209 *the density of water* and $\rho_a =$ *air density*, Equation (14) becomes

$$210 \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -g \frac{\partial h}{\partial x} + g S_0 - C_d \frac{u^2}{R_h} + \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \quad (17)$$

211 Where the last term is derived from wind shear stress, $\tau_w = \rho_a C_w u_w^2$ i.e. wind force $F_w =$
212 $\tau_w B dx$, thus $F_w = \rho_a C_w u_w^2 B dx$; where u_w is the wind velocity, C_w is wind drag
213 coefficient, B is width of channel and ρ_a is the density of air.

Equations (14) and (17) are called the Saint-Venant equation i.e. 1D shallow water equation for open channel flow.

In present study, all these equations are solved semi-analytically to compute spatial variation of forces exerted by the wind and water mass.

Variational Iteration Method (VIM)

In 1978, Inokuti (Inokuti, 1978) proposed a general Lagrange multiplier method to solve non-linear problems, which was first proposed to solve problems in quantum mechanics (Inokuti, 1978). The main feature of the method is as follows: the solution of a mathematical problem with linearization assumption is used as initial approximation or trial-function, then a precise approximation at some special point can be obtained. Considering the following general non-linear system,

$$Lu(t) + Nu(t) = g(t) \quad (18)$$

where L is a linear operator, N is a non-linear operator and $g(t)$ is a known analytical function.

Above method was modified into an iteration method (He, 1997; Finlayson, 1972; He, 1998; He, 1999a; Mungkasi and Wiryanto, 2015) in the following way:

$$u_{n+1} = u_n + \int_0^t \lambda \left[Lu_n(\xi) + Nu_n(\xi) - g(\xi) \right] d\xi \quad (19)$$

where λ is a general Lagrange multiplier (Inokuti, 1978), which can be identified optimally via the Variational theory (Inokuti, 1978; He, 1999b; Nayfeh, 1985; Finlayson, 1972), the subscript n denotes the n^{th} approximation, and u_n is considered as a restricted variation (Finlayson, 1972), i.e. $\delta \overline{u_n} = 0$

The method is shown to solve effectively, easily, and accurately a large class of non-linear problems with approximations converging rapidly to accurate solutions.

VIM is now widely used by many researchers to solve linear and nonlinear partial differential equations. The method introduces a reliable and efficient process for a wide variety of scientific and engineering applications, linear or nonlinear, homogeneous or non-homogeneous, equations and systems of equations as well. It was shown by many authors (He, 1997, 1998, 1999a, 1999b; Abdou and Soliman, 2005) that this method is more powerful than existing techniques such as the Adomian method (Adomian, 1988; Hagedorn, 1981) or perturbation method (Holmes, 2009). The method gives rapidly convergent successive approximations of the exact solution if such a solution exists; otherwise a few approximations can be used for numerical purposes. The existing numerical techniques suffer from restrictive assumptions that are used to handle nonlinear terms. The VIM has no specific requirements, such as linearization, small parameters, Adomian polynomials, etc. for nonlinear operators. Another important advantage is that the VIM method is capable of greatly reducing the size of calculation while still maintaining high accuracy of the solution. Moreover, the power of the method gives it a wider applicability in handling large number of analytical and numerical applications.

252 Solution of the governing equation

253 Equations (14) and (17) which are known as Saint Venant equations can be re-written as

254
$$\frac{\partial u_n}{\partial t} + u \frac{\partial u_n}{\partial x} + g \frac{\partial h}{\partial x} - g s_0 + C_d \frac{u_n^2}{R_h} = 0 \quad (20)$$

255 And

256
$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} - g s_0 + C_d \frac{u^2}{R_h} - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 = 0 \quad (21)$$

257 Where ρ_a is air density, ρ_w is water density, u_w is cyclonic wind speed, u is water velocity,
258 C_d is water drag coefficient and C_w is wind drag coefficient.

259 We express the steady uniform state of flow by Manning's flow equation (Chow, 1959) as
260 an initial condition for the Equations (20) and (21), which means

261
$$u_0(x, 0) = \frac{1}{n} R_h^{2/3} S_0^{1/2} \quad (22)$$

262 where n is Manning's roughness coefficient and R_h is hydraulic radius of the channel.

263 The correction functional for Equation (21) is

264
$$u_{n+1} = u_n + \int_0^t \lambda(\xi) \left[\frac{\partial u_n}{\partial \xi} + u \frac{\partial u_n}{\partial x} + g \frac{\partial h}{\partial x} - g s_0 + C_d \frac{u_n^2}{R_h} - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (23)$$

265 The stationary condition is given by

266
$$\lambda'(\xi) = 0$$

267
$$1 + \lambda'(\xi) \Big|_{\xi=t} = 0 \quad (24)$$

268 Equation (24) gives $\lambda = -1$ and substituting the value of the Lagrange multiplier into the
269 correction functional, Equation (23) gives the iteration formula.

270
$$u_{n+1} = u_n - \int_0^t \left[\frac{\partial u_n}{\partial \xi} + u \frac{\partial u_n}{\partial x} + g \frac{\partial h}{\partial x} - g s_0 + C_d \frac{u_n^2}{R_h} - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (25)$$

271 The zeroth approximations $u_0(x, 0) = \frac{1}{n} R_h^{2/3} S_0^{1/2}$ are selected by using the given initial
272 conditions. Following are the successive approximations,

273
$$u_0(x, 0) = \frac{1}{n} R_h^{2/3} S_0^{1/2} \quad (26)$$

$$u_1 = u_0 - \int_0^t \left[\frac{\partial u_0}{\partial \xi} + u_0 \frac{\partial u_0}{\partial x} + g \frac{\partial h}{\partial x} - g s_0 + C_d \frac{u_0^2}{R_h} - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (27)$$

$$u_1 = \frac{1}{n} R_h^{2/3} S_0^{1/2} - \int_0^t \left[\frac{\partial}{\partial \xi} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right) + \frac{1}{n} R_h^{2/3} S_0^{1/2} * \frac{\partial}{\partial x} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right) + g \frac{\partial h}{\partial x} - g S_0 + C_d \frac{1}{R_h} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right)^2 - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (28)$$

$$u_1 = \frac{1}{n} R_h^{2/3} S_0^{1/2} - \int \left[g \frac{\partial h}{\partial x} - g S_0 + C_d \frac{1}{R_h} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right)^2 - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (29)$$

$$\text{as } \frac{\partial}{\partial x} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right) = \frac{\partial}{\partial \xi} \left(\frac{1}{n} R_h^{2/3} S_0^{1/2} \right) = 0$$

is the steady and uniform condition.

$$u_2 = u_1 - \int_0^t \left[\frac{\partial u_1}{\partial \xi} + u_1 \frac{\partial u_1}{\partial x} + g \frac{\partial h}{\partial x} - g s_1 + C_d \frac{u_1^2}{R_h} - \frac{\rho_a}{\rho_w R_h} C_w u_w^2 \right] d\xi \quad (30)$$

Using Equation (29), we get the 2nd iterated formula as

$$\begin{aligned} u_2 \approx & \frac{1}{n} R_h^{2/3} S_0^{1/2} - \frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) + \\ & \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \\ & \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \\ & - \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\ & + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\ & \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right)^2 \Delta \xi + \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \end{aligned} \quad (31)$$

286 Definite integral

287 Given a function $f(x)$ that is continuous on the interval $[a,b]$ we divide the interval into n
 288 subintervals of equal width Δx , and from each interval select a point, x_i . Then the definite
 289 integral of $f(x)$ from a to b is

$$290 \quad \int_a^b f(x)dx \approx \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x \quad (32)$$

291 Using definite integral, we have

$$292 \quad \int_0^t g \frac{\partial h}{\partial x} d\xi \approx \sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi \text{ and } \int_0^t u_w d\xi \approx \sum_{\xi=0}^t u_w \Delta \xi \quad (33)$$

293 Equation (31) is simplified into

$$\begin{aligned} 294 \quad u_2 \approx & \frac{1}{n} R_h^{2/3} S_0^{1/2} - \frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) + \\ & \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \\ & \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \\ & - \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\ & + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\ & \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right)^2 \Delta \xi + \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \end{aligned} \quad (34)$$

296 Equation (34) is the approximate solution of Saint-Venant equation (Equation (21) to
 297 compute velocity u (including water depth and wind speed), i.e. $u \approx u_2$

298 Therefore,

$$\begin{aligned}
u \approx & \frac{1}{n} R_h^{2/3} S_0^{1/2} - \frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) + \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \\
& \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \\
& - \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\
& + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\
& \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right)^2 \Delta \xi + \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi
\end{aligned}$$

299

300

(35)

301 If we do not consider any wind term in the momentum equation, the solution becomes

$$\begin{aligned}
u \approx & \frac{1}{n} R_h^{2/3} S_0^{1/2} - \frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi \right) + \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 \right) \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi \right) - \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t \right) + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 \right) \\
& - \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t \right)^2 \Delta \xi
\end{aligned}$$

302

303

(36)

304 Equation (36) is the approximate solution of Saint-Venant equation (when wind speed is
305 absent) given by Equation (20).

306 Computation of Dynamic Thrust Force

307 Let $u(x, t)$ denotes the water velocity (computed by using water depth and wind speed as
308 input) at spatial location x and temporal state t .

309 If a water particle travels along the curve $x(t)$ through the water body, the rate of change of
310 velocity creates two kinds of accelerations: (a) Local acceleration and (b) Convective
311 acceleration

312 The acceleration for water particle can be calculated by using

$$a = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \quad (37)$$

where, $\frac{\partial u}{\partial t}$ is called local acceleration i.e temporal variation and $u \frac{\partial u}{\partial x}$ is called convective acceleration when the particles move through regions with spatially varying velocity.

Since the water under consideration is moving, it is acted upon by external forces which will follow Newton's second law. i.e. $F = ma$

Therefore, the force for moving fluid can be calculated as

$$F = \rho A \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) \quad (38)$$

which can also be considered as the dynamic thrust force due to cyclone generated surge wave.

Using Equation (35), we can compute the local acceleration as

$$\begin{aligned} \frac{\partial u}{\partial t} \approx & -\frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) + \\ & \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \\ 323 \quad & \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) - \left(g \frac{\partial h}{\partial x} - g S_0 + \frac{C_d}{n^2} R_h^{1/3} S_0 - \frac{C_w \rho_a}{\rho_w R_h} u_w^2 \right) + \\ & \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\ & \frac{C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right)^2 \end{aligned} \quad (39)$$

Again, using Equation (35), we can compute the convective acceleration as

$$\begin{aligned}
\frac{\partial u}{\partial x} \approx & -\frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^3 h}{\partial x^3} \Delta \xi^3 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial^2 u_w}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \left(\frac{\partial u_w}{\partial x} \right)^2 \Delta \xi^2 \right) + \\
& \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) \\
& \left(\sum_{\xi=0}^t g \frac{\partial^3 h}{\partial x^3} \Delta \xi^3 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial^2 u_w}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \left(\frac{\partial u_w}{\partial x} \right)^2 \Delta \xi^2 \right) \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - \sum_{\xi=0}^t g \frac{\partial z}{\partial x} \Delta \xi + \frac{C_d}{n^2} R_h^{1/3} S_0 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\
& - \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \\
& \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) - \\
& \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - \sum_{\xi=0}^t g \frac{\partial z}{\partial x} \Delta \xi + \frac{C_d}{n^2} R_h^{1/3} S_0 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\
& \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right)
\end{aligned}$$

326

327

(40)

328 Now we can write Equation (38) as

$$\begin{aligned}
F = \rho A \Bigg[& \left\{ -\frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) + \right. \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \right. \\
& \left. \left. \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) - \right. \\
& \left(g \frac{\partial h}{\partial x} - g S_0 + \frac{C_d}{n^2} R_h^{1/3} S_0 - \frac{C_w \rho_a}{\rho_w R_h} u_w^2 \right) + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\
& \left. \frac{C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right)^2 \right\} + u \left\{ -\frac{1}{n} R_h^{2/3} S_0^{1/2} \right. \\
& \left(\sum_{\xi=0}^t g \frac{\partial^3 h}{\partial x^3} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial^2 u_w}{\partial x^2} \Delta \xi^2 - \frac{2C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w \left(\frac{\partial u_w}{\partial x} \right)^2 \Delta \xi^2 \right) + \\
& \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) + \\
& \left(\sum_{\xi=0}^t g \frac{\partial^3 h}{\partial x^3} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial^2 u_w}{\partial x^2} \Delta \xi^2 - \frac{2C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w \left(\frac{\partial u_w}{\partial x} \right)^2 \Delta \xi^2 \right) \\
& \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi + \frac{C_d}{n^2} R_h^{1/3} S_0 - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) - \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) + \\
& \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) - \frac{2C_d}{R_h} \\
& \left. \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{C_w \rho_a}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) \right\} \Bigg]
\end{aligned}$$

329

330

(41)

331 Where,

$$\begin{aligned}
 u \approx & \frac{1}{n} R_h^{2/3} S_0^{1/2} - \frac{1}{n} R_h^{2/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi^2 - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi^2 \right) + \\
 & \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) \\
 & \left(\sum_{\xi=0}^t g \frac{\partial^2 h}{\partial x^2} \Delta \xi - \frac{2\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w \frac{\partial u_w}{\partial x} \Delta \xi \right) \\
 & - \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right) \\
 & + \frac{2C_d}{n} R_h^{-1/3} S_0^{1/2} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi^2 - g S_0 t^2 + \frac{C_d}{n^2} R_h^{1/3} S_0 t^2 - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi^2 \right) - \\
 & \frac{2C_d}{R_h} \left(\sum_{\xi=0}^t g \frac{\partial h}{\partial x} \Delta \xi - g S_0 t + \frac{C_d}{n^2} R_h^{1/3} S_0 t - \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi \right)^2 \Delta \xi + \frac{\rho_a C_w}{\rho_w R_h} \sum_{\xi=0}^t u_w^2 \Delta \xi
 \end{aligned}
 \tag{42}$$

334 Equations (41) and (42) represent the dynamic thrust force due to momentum created by
 335 the wind and water mass. In present study, these are used to compute the dynamic thrust
 336 force exerted by the cyclone generated storm surge. The model represented by Equations
 337 (41) and (42) is named as Dynamic Force Model (DFM).

338 Equations (41) and (42) contain the derivatives $\frac{\partial h}{\partial x}$, $\frac{\partial u_w}{\partial x}$, $\frac{\partial^2 h}{\partial x^2}$ and $\frac{\partial^2 u_w}{\partial x^2}$ which can not be
 339 directly solved analytically. In this study, these derivatives are solved in their discretized
 340 form as:

$$341 \quad \frac{\partial h}{\partial x} = \frac{h_i - h_{i-1}}{\Delta x} \tag{43}$$

$$342 \quad \frac{\partial u_w}{\partial x} = \frac{u_{w_i} - u_{w_{i-1}}}{\Delta x} \tag{44}$$

$$343 \quad \frac{\partial^2 h}{\partial x^2} = \frac{h_i - 2h_{i-1} + h_{i+1}}{2\Delta x} \tag{45}$$

$$344 \quad \frac{\partial^2 u_w}{\partial x^2} = \frac{u_{w_i} - 2u_{w_{i-1}} + u_{w_{i+1}}}{2\Delta x} \tag{46}$$

345 where $i = 1, 2, 3, \dots$

346 Discretized solutions represented by Equations (43) to (46) are substituted into the
 347 analytical solutions represented by Equations (41) and (42). This approach is termed as
 348 semi-analytical. Due to this reason, DFM is a semi-analytical model.

In this study, the discretized equations (43)-(46) are solved by applying the numerical model Delft 3D (Deltares, 2014). To make the DFM directly coupled with Delft 3D, water depth h , wind velocity u_w and discretization interval Δx is directly inserted in Equations (41) and (42) from the computational domain and simulation results of Delft 3D. It is to be noted here that this kind of direct coupling with a numerical model (the way it is done in this study) is not mandatory for DFM. The discretized equations (43)-(46) can be solved by any method with a reasonable computation of h , u_w and Δx .

MODEL VERIFICATION

In this study, a Dynamic Force Model (DFM) is developed by semi-analytically solving the Saint-Venant equations (1D shallow water equations) which are derived from Navier-Stokes equations. Model verification essentially means model algorithm and code are correctly implemented (Trucano et al., 2006). In this study, the solution algorithm and model code are verified by comparing the semi-analytical model results with the numerical solution. We use Variational Iteration Method as the analytical method. To verify this semi-analytical solution, the numerical method selected is the Finite Difference Method. The reflexive condition is used as boundary conditions during the numerical solution. Using MATLAB, we have prepared two scripts for the semi-analytical solution and the numerical solution separately. During comparison, only the velocities computed from the DFM is compared with the numerical solution. This essentially serves the purpose of model verification.

Numerical solution using finite difference method

Finite difference method is a numerical method for the solutions of shallow water equation. The standard definition of finite difference method in elementary calculus is the following

$$\frac{\partial u}{\partial x} = \frac{u(x + \Delta x) - u(x)}{\Delta x} \text{ and } \frac{\partial u}{\partial t} = \frac{u(t + \Delta t) - u(t)}{\Delta t} \quad (47)$$

In this study, the Saint-Venant equation is solved by applying the finite difference method. The solution is given as:

$$u_i = \frac{u_{i-1} + u_{i+1}}{2} - \frac{dt}{2dx} (F_{i+1} - F_{i-1}) + \frac{S_{i+1} + S_{i-1}}{2} dt; \quad i = 1, 2, 3, 4, 5, \dots \quad (48)$$

To verify the DFM algorithm and code, the velocity field computed by Equation (35) is compared with the velocity field computed from numerical solution (Equation (48)) in a hypothetical channel where water is flowing from upstream to downstream.

Hypothetical channel description

A hypothetical channel of 5.1km long, 100m width and 5m deep is considered for verification of the model (Figure-1). Winds with variable speeds are assumed to blow over the channel. Steady and uniform state of flow represented by Manning's equation i.e. $u_0 = \frac{1}{n} R_h^{2/3} S_0^{1/2}$ is used as initial condition, where R_h is the hydraulic radius and S_0 is the bed slope of the channel.

Figure-1: Hypothetical channel

3 different cases (Case-1, Case-2 and Case-3) are constructed to compare the velocities computed by DFM with the numerical solution. In Case-1, water depth and water surface slope are kept constant (Figure-2) with no wind is blowing over the channel. This results almost constant water velocity along the channel. Computed water velocities from the DFM and the numerical solution are shown in Figure-3. The result shows a near perfect agreement between the DFM and the numerical solution with $R^2=0.96$.

Figure-2: Water depths $h(m)$ in different positions $x(km)$ along the horizontal for Case-1.

Figure-3: Comparison of longitudinal velocity profile $u(\text{m/s})$ between the DFM and the numerical solution for Case-1.

In Case-2, water depth is varying along the channel, but no wind is blowing (Figure-4). This results variable water surface gradient along the channel. Comparison of computed water velocity between the DFM and the numerical solution is shown in Figure-5. The result shows a slight decrease of agreement between the DFM and the numerical solution compared to Case-1 (R^2 decreases from 0.96 to 0.91).

Figure-4: Water depths $h(\text{m})$ in different positions $x(\text{km})$ along the horizontal for Case-2.

Figure-5: Comparison of longitudinal velocity profile $u(\text{m/s})$ between the DFM and the numerical solution for Case-2.

In Case-3, wind with variable speed is assumed to blow over the channel where water depth was initially constant (Figure-6). Variable wind speed results change in water surface gradient along the channel. Comparison of water velocity between the DFM and the numerical solution for Case-3 is shown in Figure-7. The result shows further decrease of agreement between the DFM and the numerical solution compared to Case-1 and Case-2 ($R^2=0.82$ in Case-3 compared to 0.96 in Case-1 and 0.91 in Case-2).

The verification results show systematic deviation of numerical solution from the analytical solution with the increase of non-linearity in the system, which is an inherent drawback of numerical solutions. This shows that both the solution algorithm and code are appropriately implemented in DFM.

Figure-6: Variable wind Speed u_w (m/s) in different positions x (m) along the channel for Case-3.

Figure-7: Comparison of longitudinal velocity profile u (m/s) between the DFM and the numerical solution for Case-3.

MODEL CALIBRATION

By model calibration, we generally mean selecting the appropriate model parameters that represents experimental or field conditions (Trucano et al., 2006). DFM has two parameters that needs to be calibrated. These are: wind drag coefficient C_w (see Equation 21) and Manning's n (see Equation 22). As mentioned before, the discretized Equations (43)-(46) are solved by coupling the DFM with Delft 3D. The field application of DFM is made by coupling the field simulation result of Delft 3D. To make DFM compatible with Delft 3D,

same Manning's n is used both for DFM and Delft 3D. As solution algorithm between DFM and Delft 3D is different, for wind drag coefficient C_w , an empirical equation is derived between C_w and wind velocity u_w which is used for different field application of DFM. To derive this empirical equation, values of the wind drag coefficient used in a particular application of Delft 3D (which is 0.002) and simulated water velocity from the same application of Delft 3D is used. To make values of C_w used in Delft 3D compatible with the DFM, following empirical equation is used:

$$C_w = 0.002 \times \left(\frac{u}{u_d} \right) \quad (49)$$

Where, C_w is a wind drag coefficient used as the calibration parameter of DFM, u is the simulated water velocity for a particular application of Delft 3D and u_d is the computed velocity for the same application of DFM. After series of model runs with DFM, relation between wind drag coefficient C_w and wind velocity u_w is established and is shown in Figure-8.

Figure-8: Relation between wind drag coefficient C_w and wind velocity u_w computed by applying DFM

Using the relation in Figure-8, following empirical equation between C_w and u_w is developed:

$$C_w = 0.000004u_w^2 + 0.002 \quad (50)$$

During subsequent model application by DFM, Equation (50) is used to compute the wind drag coefficient for specific cyclone wind speed represented by u_w .

The transfer of momentum from air to the ocean surface produces a wave field that imposes shear stress which ultimately causes the rise of water level (Powell et al, 2003). The drag coefficient C_w controls the transfer of momentum from air to water surface in the governing momentum equation (Donelan, et al., 1996). It mainly depends on wind speed. In modeling storm surge, this coefficient is a crucial parameter (Drews, 2013). Past studies show that

this co-efficient either remains constant or increases with the increase of wind speed (Charnock, 1955; Large and Pond, 1981; Smith and Banke, 1975; Bender et al., 2007; Kim et al., 2008; Kohno and Higaki, 2006; Zhizhua et al., 2010). In the present study, it is found that wind drag co-efficient has a non-linear relation with the wind speed (see Figure-8 and Equation (50)).

MODEL VALIDATION

Model validation means ability of the model to represent the real world for which the model is developed (Trucano et al., 2006). As real-world data of thrust force for any moving water mass is not available, validation of DFM is performed from two different perspectives. These are:

1. Validation of thrust force of DFM by comparing velocity-force curve generated for tsunami and cyclonic storm surge.
2. Validation of flow field of DFM with flow field simulation from calibrated and validated numerical model Delft 3D.

Validation with the tsunami force

Tsunami is a small amplitude long wave created by lifting of water mass above the sea bottom due to rapture of the sea floor created mainly by earthquake (Dutykh and Dias, 2008). The potential energy of this displaced water mass converts into kinetic energy (Dutykh and Dias, 2008) and propagates with tremendous velocity (as high as 800 km/hr) in deep sea. This wave when reaches to the coast, is slowed down but amplified due to shallow water effect and strikes the coastal structures with the same energy it has when it was created in the deep sea. On the other hand, cyclone generated storm surge is an elevated sea surface above the tide level due to lower atmospheric pressure and cyclonic wind. The storm surge acts simultaneously with tide and wind waves. The energy created due to storm surge is proportional to the tidal range of storm tide (Lewis et al., 2017). Similar to tsunami, the storm surge is amplified near the coast due to shallow water effect. The tsunami wave contains energy due to displacement of sea water mass above the sea-bed, whereas the storm surge contains energy created due to difference of sea surface elevation measured by tidal range of storm tide (Dutykh and Dias, 2008; Lewis et al., 2017). Energy of tsunami wave is several times higher than storm surge (water depth above sea-bed can be as high as 4000m, whereas, tidal range of a storm tide can vary between 6m to 7m depending on the tidal amplitude and strength of cyclone). After reaching the coast, both the tsunami wave and storm surge strikes the coastal structure with a force which is directly proportional to the energy (Venkoba, 2017). This means that after reaching the coast, the water mass created due to tsunami and moving with a certain velocity will exert more force on a coastal structure compared to a moving water mass moving with the same velocity but created due to cyclonic storm surge. The reason is – the moving water mass created by tsunami contains more energy compared to the moving water mass created by storm surge (Dutykh and Dias, 2008; Lewis et al., 2017) and force is directly proportional to energy (Venkoba, 2017).

In the validation exercise of DFM force, we generate velocity-force curve for both tsunami and storm surge. For tsunami, the velocity-force curve is generated by using the relation developed by Murata, et al. (2011) based on Japan tsunami study. They expressed the water

510 velocity during tsunami wave propagation as u and the drag force of tsunami as F_d acting
511 on coastal structures as:

$$512 \quad u = 1.1\sqrt{gh} \quad (51)$$

$$513 \quad F_d \cong 0.61\gamma_w C_d h^2 B \quad (52)$$

514 Where, γ_w is the specific weight of seawater, C_d is a drag coefficient, h is water depth and
515 B is the breadth of a building in the flow direction. Velocity-force curve for tsunami wave
516 is generated by calculating force with the following parameter values suggested by Murata,
517 et al. (2011):

$$518 \quad \gamma_w = 10.03 \text{ kN/m}^3$$

$$519 \quad C_d = 2.2$$

$$520 \quad B = 1.0\text{m to compute the force per unit length of the structure.}$$

521 Values of h in Equation (52) is calculated from Equation (51) by assuming different ranges
522 of velocity u .

523 For storm surge, the velocity-force curve is generated by applying DFM in the hypothetical
524 channel which was used for model verification (Figure-1). In this case, we used case-3
525 model setup (Figure-6), but to get a reasonable range of force to generate velocity-force
526 curve, we have used a higher range of wind speed compared to the wind speed range shown
527 in Figure-6. To generate velocity-force curve, different ranges of velocity are computed in
528 different sections of the hypothetical channel by applying DFM. These velocities are used
529 in Equation (38) to compute the force numerically. The numerical solution of Equation (38)
530 is equivalent to the semi-analytical solution of Equation (41).

531 Comparison of velocity-force curve between tsunami and storm surge is shown in Figure-
532 9. The results show that for the same velocity, tsunami force is several times higher than
533 storm surge force. With increasing velocity, this difference also increase. We have already
534 discussed why tsunami force should be higher than storm surge force. The velocity-force
535 curve in Figure-9 shows that DFM realistically computes force due to cyclonic storm surge.

536

537

538

539 Figure-9: Comparison of velocity-force curve between tsunami and storm surge.

540

541 Validation with Delft 3D simulation

542 In this validation approach of DFM, velocity field simulated by DFM is compared with the
 543 velocity field simulated by Delft 3D. The study area for this comparison is the coastal zone
 544 of Bangladesh (Figure-10) for the tropical cyclone event SIDR that made landfall in
 545 Bangladesh coast on November 15, 2007. Comparison of surge velocities between Delft 3D
 546 and DFM at the time of landfall is shown in Figure-11.

Figure-10: Coastal zone of Bangladesh.

Figure-11: Comparison of surge velocities between DFM and Delft 3D.

To quantify the visual qualitative color comparison, a one-to-one function is defined among the colors between the two maps. In this way, it is possible to quantify the overlapped color and deviated color between the two maps. The overlapped color shows the similarity, and the deviated color shows the dissimilarity between the maps. The comparison is shown in Table-1. The comparison shows that surge velocities computed by DFM is 97% similar to that computed by Delft 3D. It is noted here that Delft 3D applied in this study is a validated storm surge model in the coastal zone of Bangladesh (Haque et al., 2018). From this perspective, this result shows the field validation of one important parameter of DFM.

Table-1: Map Comparison

Models	In Percentage (%)	
	Overlapped Colors	Deviated Colors
Delft3D – DFM	97%	3%

562 The DFM model has numerous prospects in improving coastal resilience in particular and
 563 structural resilience in general. The model is not only able to compute dynamic thrust force
 564 due to cyclone generated storm surge, but also can compute the dynamic thrust force exerted
 565 by any moving water mass on a structure in its flow path (for example due to tidal wave,
 566 wind wave, flows in ocean, estuaries and rivers, tsunami etc.). The output from the model
 567 can be used as one of the design criteria of infrastructures on which force is exerted by the
 568 moving water mass. As the DFM is capable of calculating spatial and temporal changes of
 569 thrust force, it can also be considered as one of the monitoring parameters of extreme
 570 climatic conditions related to moving water. As an illustration, the model is applied to
 571 compute dynamic thrust force due to tropical cyclone SIDR.

572
 573 Tropical cyclone SIDR is considered as one of the devastating cyclones that made landfall
 574 on November 15, 2007 at the east of Sundarban in Bangladesh coast. During the time of
 575 landfall, the maximum wind speed of this cyclone was 215 km/hr (Islam, et.al. 2011). This
 576 cyclone is believed to generate a maximum surge depth of 6m (GoB, 2008; Sakib, et al.
 577 2016). Simulated thrust force due to cyclone SIDR at the time of landfall is shown in Figure-
 578 12. Table-2 shows the maximum thrust force at different coastal districts in Bangladesh (the
 579 district names are shown in Figure-12) due to cyclone SIDR.

580 Table-2: Maximum thrust force at different coastal districts of Bangladesh due to cyclone
 581 SIDR.

District	Maximum thrust force (F) in kN/m
Patuakhali	110.5
Barguna	86.6
Bhola	54.1
Pirojpur	49.6
Jhalokati	44.5
Barisal	40.2
Bagerhat	33.2
Noakhali	17.9
Khulna	8.4
Chittagong	6.9
Lakshmipur	6.3
Cox's Bazar	4.8
Satkhira	4.3
Shariatpur	3.1
Narail	2.4
Gopalganj	2.2
Chandpur	2.1
Feni	1.9
Jessore	0.3

583 According to simulation results of DFM, the coastal districts of Patuakhali, Barguna, and
 584 Bhola (see Figure-12) are impacted with very high magnitude of thrust force (magnitude
 585 greater than 50 kN/m, see Table-2). In the post-disaster damage and loss assessment report,
 586 these districts are also identified as the worst affected coastal districts in terms of economic
 587 and infrastructure loss (GoB, 2008). Due to anti-clockwise rotational impacts of cyclones in
 588 northern hemisphere, locations on the right side of cyclone tracks are the most affected.
 589 According to DFM simulation, coastal districts of Patuakhali, Barguna and Bhola are the
 590 worst affected regions which are situated at right side of the cyclone track and at the same
 591 time close to the path of cyclone track. On the other hand, although Pirojpur, Jhalokati and
 592 Bagerhat are situated close to the cyclone track (see Figure-12), magnitude of thrust forces
 593 are relatively less in these districts (less than 50 kN/m) because these districts are located at
 594 the left side of the cyclone track. The red lines in Figure-12 shows the coastal embankments
 595 (locally known as polders) to protect the lands from surge wave. DFM simulated thrust force
 596 inside the protected land is only due to cyclone wind, not due to combined impact of cyclone
 597 wind and moving surge. But the thrust force in the unprotected land is due to combined
 598 impacts of moving surge and cyclone wind. Thrust force simulated by DFM can be applied
 599 to assess probability of failure of embankments during propagation of surge wave and
 600 possible damage of households both inside and outside the protected land. These data can
 601 also be used to further strengthen these structures or during construction of new structures
 602 in the coastal region.

Figure-12. Computed dynamic thrust force due to cyclone SIDR at the time of landfall in the coastal zone of Bangladesh.

607 CONCLUSION

608 A Dynamic thrust Force Model (DFM) is developed by semi-analytically solving the Saint-
609 Venant equations. The model can dynamically compute the thrust force due to any moving
610 water mass including cyclone generated storm surge. As an analytical method, Variational
611 Iteration Method (VIM) is used to solve the equations. During solution, steady and uniform
612 state of flow computed by Manning's equation is considered as an initial condition. Thrust
613 force is computed from local and convective accelerations of moving water.

614 The model is verified (verification of solution algorithm and model code) by applying the
615 model in a hypothetical channel on which wind is blowing. The computed water velocity by
616 semi-analytical solution is compared with a finite difference solution. The verification
617 results show systematic deviation of numerical solution from the analytical solution with
618 increasing non-linearity in the system, which is an inherent drawback of numerical
619 solutions. This shows model algorithm and code are appropriately implemented. To apply
620 the model in simulating dynamic thrust force due to cyclone generated storm surge,
621 Manning's n and wind drag coefficient are used as the calibration parameters. For the DFM,
622 an empirical equation is developed by relating wind drag co-efficient with wind speed which
623 can be used for any specific application (related to wind driven water flow) of the model. It
624 is found that wind drag coefficient increases non-linearly with the increase of wind speed.
625 Thrust force computed by DFM is validated by comparing the model result with the tsunami
626 data studied in Japan. On the other hand, surge velocity computed by DFM is validated by
627 comparing the model result with the surge velocity computed by field validated numerical
628 model Delft 3D. The verified, calibrated and validated DFM is applied to simulate dynamic
629 thrust force in Bangladesh zone due to tropical cyclone SIDR. This specific application
630 shows importance of thrust force as a parameter to assess performance of coastal
631 infrastructure.

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639 ETHICAL STATEMENT

640 The manuscript submitted in the Journal of Natural Hazard, Springer is entitled
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