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Md. Abdul Latif Sarker

Kyungpook National University <https://orcid.org/0000-0001-7911-3689>

Md. Fazlul Kader

University of Chittagong <https://orcid.org/0000-0002-7139-5965>

Md. Mostafa Kamal Sarker

Robert Gordon University <https://orcid.org/0000-0002-4793-6661>

Moon Ho Lee

Chonbuk National University

Dong Seog Han (✉ dshan@knu.ac.kr)

Kyungpook National University <https://orcid.org/0000-0002-7769-0236>

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A Black Hole-Aided Deep-Helix Channel Model for DNA

Md. Abdul Latif Sarker^{1,5}, Md. Fazlul Kader², Md. Mostafa Kamal Sarker³, Moon Ho Lee⁴, and Dong Seog Han^{5*}

¹Centre for ICT & Automotive Convergence, Kyungpook National University, Daegu 41566, Korea

²Department of Electrical and Electronic Engineering, University of Chittagong, Chittagong, Bangladesh

³ National Subsea Centre, Robert Gordon University, Aberdeen AB10 7GJ, United Kingdom

⁴ Division of Electronics Engineering, Chonbuk National University, Jeonju-si 54896, Korea

⁵School of Electronics Engineering, Kyungpook National University, Daegu 41566, Korea

email: {latifsarker, dshan}@knu.ac.kr, f.kader@cu.ac.bd, m.kamal.sarker@gmail.com, and moonho@jbnu.ac.kr

*corresponding. dshan@knu.ac.kr

In this article, we present a *black-hole-aided deep-helix* (bh-dh) channel model to enhance information bound and mitigate a multiple-helix directional issue in Deoxyribonucleic acid (DNA) communications. The recent observations^{1,2,3,4} of DNA do not match with Shannon^{5,6,7} bound due to their multiple-helix^{8,9} directional issue. Hence, we propose a bh-dh channel model in this paper. The proposed bh-dh channel model follows a similar fashion of DNA and enriches the earlier DNA observations^{1,2,3,4} as well as achieving a composite like information bound. To do successfully the proposed bh-dh channel model, we first define a *black-hole-aided Bernoulli-process* and then consider a symmetric bh-dh channel model. After that, the geometric and graphical insight shows the resemblance of the proposed bh-dh channel model in DNA and Galaxy layout. In our exploration, the proposed bh-dh symmetric channel geometrically sketches a *deep-pair-ellipse* when a *deep-pair* information bit or digit is distributed in the proposed channel. Furthermore, the proposed channel graphically shapes as a *beautiful circulant ring*. The ring contains a *central-hole*, which looks like a *central-black-hole*^{10,11} of a Galaxy. The coordinates of the *inner-ellipses* denote a *deep-double* helix and the coordinates of the *outer-ellipses* sketch a *deep-parallel* strand. Finally, the proposed bh-dh symmetric channel significantly outperforms the traditional binary^{5,6,7} symmetric channel and is verified by computer simulations in terms of Shannon entropy and capacity bound.

Black¹² hole plays an important role in the part of nature, for example, a large galaxy contain a supermassive^{10,11} black-hole at its center. It is not completely black and the area of each individual black hole cannot decrease^{13,14}. A white¹⁵ hole also play a similar manner in the part of nature. There is a connection investigated^{15,16} between holes (black or white) and entropy. *It is to be noted that a black or white hole is a constant blind source dependence natural strategy, which is widely used to expand the space of a region and help execute a multiple-helix directional issue in the part of nature. Since DNA^{1,2,3,4} is a part of nature, it carries out a multiple-helix^{8,9} directional issue. Hence, a black-hole strategy is not only present in the Galaxy, its existence is also noticeable in the DNA or other parts of nature such as a Cyclone.*

However, DNA is a fundamental appearance of life, which assists to trace out the hereditary information within chromosomes. It is mainly an information stream that relies on the nitrogenous bases^{1,2,17,18}: adenine (*A*), cytosine (*C*), guanine (*G*), and thymine (*T*). The quantity of each nucleobase is the key factor to trace out the hereditary information within chromosomes. The main idea of DNA was first introduced in 1869 by Friedrich Miescher. Almost 7 to 8 decades later, many scientists were investigated the physical structure of DNA molecules. For example, in 1950, Erwin Chargaff's team investigated the composition of human DNA and proposed Chargaff's rules of DNA base pairing¹⁷. Zamenhof et al showed that different living organisms contain different quantities of each DNA nucleobase¹⁸. In 1953, Linus Pauling and Robert Corey studied an unsatisfactory structure of nucleic acid in which their model consisted of three intertwined chains with the phosphates near the fiber axis and the bases on the outside^{2,3}. Around the same time, the X-ray fiber diffraction pattern was obtained by Rosalind Franklin¹⁹, and finally, on April 25, 1953, James Watson and Francis Crick suggested a satisfactory right-handed double¹-helix structure of DNA. After Watson and Crick model, several researchers have significantly studied the physical structure of DNA, for example, a twisted⁹-circular structure and a four⁴-stranded structure. It is observed that, most of the relevant above literature's investigated the DNA model utilizing the chemical and biological method. To the best of our knowledge, the concept of the proposed *black-hole-aided deep-helix* (bh-dh) channel model for DNA communications has not yet been studied in the literature.

In this paper, we propose a bh-dh channel model to expand information bound and help conquer a multiple-helix directional issue for DNA communications. The geometrical form of the proposed bh-dh channel is a deep-pair ellipse, which graphically illustrates as a beautiful circulant ring. It is observed that the proposed bh-dh channel model follows a similar fashion of DNA and enriches the Watson-Crick¹ DNA model as well as achieving a composite like information bound. In general, a stochastic process is a Bernoulli process. Hence, we first define a *black hole-aided Bernoulli* (bh-B) process. Based on the bh-B process, we then consider a symmetric bh-dh channel model and measure the entropy and capacity of the channel. Through computer simulations, we finally verify the effectiveness of the proposed bh-dh symmetric channel with the conventional binary^{5,6,7} symmetric channel, in terms of Shannon entropy and capacity bound.

Proposed bh-dh channel model: In this Section, we will design the proposed bh-dh channel considering a black hole-aided technique. To do successfully design the proposed bh-dh channel, we call **Proposition 1** as below:

Proposition 1: A black or white hole is a constant blind source, which produces a central-hole to help expand a circular symmetric channel space.

Proof of Proposition 1: To proof **Proposition 1**, we first define a black-hole-aided Bernoulli (bh-B) process and then consider a bh-dh symmetric channel model to analyze the geometric and graphical illustration as bellow:

Definition of a bh-B process: Consider η be a constant blind or *black-hole* source and $x \in \mathcal{X}$ be a binary or a discrete random source, which construct a composite input source x_d . The composited input source x_d satisfy $\eta \leq x_d \leq 1 + \eta$ with

$0 \leq x \leq 1$. Now, using^{20,21}, we define a bh-B multiplicative form as follows.

$$\Pr(X_d = x_d) = \begin{cases} \frac{(1-p_d)^\eta}{p_d^\eta} p_d^{x_d} (1-p_d)^{1-x_d} & 0 < \eta \leq 1 \text{ for the right-handed bh-B case,} \\ \frac{(1-p_d)^{-\eta}}{p_d^{-\eta}} p_d^{x_d} (1-p_d)^{1-x_d} & -1 \leq \eta < 0 \text{ for the left-handed bh-B case,} \\ p^x (1-p)^{1-x} & \eta = 0 \text{ for otherwise,} \end{cases} \quad (1)$$

where, p_d denotes the composited probability mass function satisfy $\eta \leq p_d \leq 1 + \eta$ with $0 \leq p \leq 1$ and p denotes the probability mass function.

A symmetric bh-dh channel model: If we set $x \in \{0, 1\}$ in (1), then using^{6,7} and (1), we can model a symmetric bh-dh channel with transition matrix $\mathbf{P}_d$, which is given by

$$\mathbf{P}_d = \begin{bmatrix} 1 - p_d & p_d \\ p_d & 1 - p_d \end{bmatrix}. \quad (2)$$

It is noted that the proposed channel $\mathbf{P}_d$ leads two different circulant matrices when two different bits or digits are applied in (2). We also noted that if p_d satisfies $0 \leq p_d \leq 1$ in (2), then the $\mathbf{P}_d$ matrix be a circulant and if p_d satisfies $p_d < 0$ or $p_d > 1$ in (2), then the $\mathbf{P}_d$ matrix become an inverse-circulant. Now, we analyze the geometrical and graphical illustration of the proposed $\mathbf{P}_d$ matrix utilizing the DNA examples as follows.

(a) Geometrical construction of the proposed bh-dh symmetric channel: Let a black-hole constant $\eta = TG - AC$, where η is measured by the DNA nitrogenous bases^{1,12,13} (C, G, A and T) following the DNA physical structure. According to C, G, A and T values, we provide the following examples:

Example 1: For a Human^{18,22} DNA case, if any Human contains the percentage of the DNA nucleobases are $A = 29.3$, $T = 30.0$, $C = 20.0$ and $G = 20.7$, we then obtain a black-hole constant $\eta_h = 0.35$.

Example 2: For an Octopus^{18,22} DNA case, if any Octopus contains the percentage of the DNA nucleobases are $A = 33.2$, $T = 31.6$, $C = 17.6$ and $G = 17.6$, we then get a black-hole constant $\eta_o = -0.28$.

Example 3: For a hole-free case, if any organisms or viruses satisfy the percentage of the DNA nucleobases are $A = T$ and $C = G$, then the black-hole constant η be zero, i.e., $\eta_f = 0$.

Now, we set $p \in \{0, 1\}$, $\eta_h = 0.35$, $\eta_o = -0.28$ and $\eta_f = 0$ in (2) and make TABLE I. From (1), (2) and TABLE I, we can summarize the following points as:

- (i) The proposed $\mathbf{P}_d$ matrix pursues a right-handed DNA channel when η satisfies $0 < \eta \leq 1$ with $TG > AC$.
- (ii) The proposed $\mathbf{P}_d$ matrix becomes a left-handed DNA channel when η satisfies $-1 \leq \eta < 0$ with $TG < AC$ and
- (iii) Otherwise $\mathbf{P}_d$ matrix be a $\mathbf{P}_s$ when $\eta = 0$ with $TG = AC$ and $\mathbf{P}_s$ denotes the central hole-free^{5,6,7} symmetric circulant channel.

(b) Graphical illustration of the proposed bh-dh symmetric channel: According to the geometrical construction of the bh-dh symmetric channel, we analyze the graphical illustration in this subsection. The graphical illustration of the proposed bh-dh symmetric channel model is depicted in Fig. 1. In Fig. 1, we observe that the area of both inner and outer ellipses depend

TABLE I: Geometrical form of the proposed bh-dh symmetric channel matrix

Example 1 at $p \in \{0, 1\}$ and $\eta_h = 0.35$ when $TG > AC$	
$\mathbf{P}_d = \begin{pmatrix} 0.65 & 0.35 \\ 0.35 & 0.65 \end{pmatrix}_{p=0}$	$\mathbf{P}_d = \begin{pmatrix} -0.35 & 1.35 \\ 1.35 & -0.35 \end{pmatrix}_{p=1}$
Example 2 at $p \in \{0, 1\}$ and $\eta_o = -0.28$ when $TG < AC$	
$\mathbf{P}_d = \begin{pmatrix} 1.28 & -0.28 \\ -0.28 & 1.28 \end{pmatrix}_{p=0}$	$\mathbf{P}_d = \begin{pmatrix} 0.28 & 0.72 \\ 0.72 & 0.28 \end{pmatrix}_{p=1}$
Example 3 at $p \in \{0, 1\}$ and $\eta_{wl} = 0$ when $TG = AC$	
$\mathbf{P}_s = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{p=0}$	$\mathbf{P}_s = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{p=1}$

on p_d values, where constant η generates a *center-hole* in Fig. 2. In Fig. 2, we also observe that the proposed bh-dh symmetric channel model conducts a deep-pair ellipse when a deep-pair information bit or digit is applied in (2). For example, when we set $\eta = 0$ and a 1×198 binary-chain in (2), the proposed bh-dh symmetric channel model turns back into a binary symmetric channel and draws a central hole-free circle in Fig. 2a. In contrast, when we set $\eta = 0.35$ and a 1×198 binary-chain in (2), the proposed bh-dh symmetric channel model illustrates a *beautiful-circulant-ring* with central-hole as shown in Fig. 2b.

The most important graphical view describes in Fig. 3 and Fig. 4. Fig. 3 shows the graphical inspection of the proposed bh-dh channel model, which follows the physical structure of DNA. An X-ray pattern of the double¹-helix DNA model is investigated in Fig. 3a. A polar diagram depicts the proposed bh-dh channel model using by a 1×81 binary-chain in Fig. 3b. A 1×81 and 1×91 discrete-chain is applied with the proposed bh-dh channel model in Fig. 3c and Fig. 3d, respectively.

Fig. 4 presents the graphical exposition of the Milky way²³, the galaxy^{10,11} and the proposed bh-dh channel model. A black hole around the Milky way as shown in Fig. 4a. A polar diagram depicts the proposed bh-dh channel model utilizing a 1×198 binary-chain with $\eta = 0.35$ in Fig. 4b. a supermassive^{10,11} black hole residing at the galaxy's core in Fig. 4c. A polar diagram shows the proposed bh-dh channel model considering a 1×270 binary-chain with $\eta = 0.6180$. It is noticed that the area of the *center-hole* relies on the η values as shown in Fig. 4b and Fig. 4b.

According to the definition of a bh-B process, geometrical and graphical illustration, we can see that (2) is a bh-dh symmetric channel, which has expanded a circular symmetric space by producing a central hole. Thus, **Proposition 1** has been proven.

Information entropy and capacity bound of the proposed bh-dh channel model: Particularly, the information capacity is the maximum amount of information that can pass through a channel without error. In contrast, information entropy is a function of a transitional error probability that is usually measured uncertainty in Shannons⁵ (bits) or natural units (nats) or decimal digits (dits). Since the Shannon bound does not match with the DNA information bound, we investigate the entropy and capacity bound of the proposed bh-dh channel model in this Section.

Consider random input and output variables X and Y for a communication channel where $x \in \mathcal{X}$ and $y \in \mathcal{Y}$. Hence, the well-known capacity formula^{5,6,7} is given by

$$C_s = \max_{p(x)} I(X; Y). \quad (3)$$

where $p(x)$ denotes the input distribution function, C_s satisfy $0 \leq C_s \leq 1$ with $0 \leq x \leq 1$ and $0 \leq p \leq 1$, and $I(X; Y)$

Fig. 1: The graphical illustration of the proposed bh-dh channel model

Fig. 2: The polar diagram illustrates a central hole-free circle and a beautiful circulant ring: (a) binary symmetric channel model with 1×198 binary-chain and $\eta = 0$, and (b) proposed bh-dh symmetric channel with 1×198 binary-chain and $\eta = 0.35$.

denotes the mutual information⁶, that is given by

$$\begin{aligned}
 I(X; Y) &= H(X) - H(X|Y) \\
 &= H(Y) - H(Y|X) \\
 &= H(Y) - \sum p(x)H(Y|X = x) \\
 &\leq 1 - H_s(p),
 \end{aligned} \tag{4}$$

where $H(X)$ and $H(Y)$ are the marginal entropies, $H(X|Y)$ and $H(Y|X)$ are the conditional entropies, the p is the error

Fig. 3: The graphical exposition of the DNA and the proposed bh-dh channel (a) X-ray diffraction pattern of the double¹-helix DNA model, (b) the proposed bh-dh channel model with 1×81 binary-chain and $\eta = 0.35$, (c) the proposed bh-dh channel model with 1×81 discrete-chain and $\eta = 0.35$, (d) the proposed bh-dh channel model with 1×91 discrete-chain and $\eta = 0.35$.

probability satisfy $\sum_{x \in \{0,1\}} p(x) = 1$, and the Shannon entropy $H_s(p)$ is given by^{6,7} the following equation

$$H_s(p) = -p \log_2 p - (1-p) \log_2 (1-p) \quad \text{bits}, \quad (5)$$

where $H_s(p)$ satisfy $0 \leq H_s(p) \leq 1$. Similarly, based on (1), (3) and (4), we can write the following capacity formula for the proposed bh-dh channel model as

$$C_d = H_d(Y_d) - H_d(p_d) = \begin{cases} 1 - H_d(p_d, rh) & \text{for right-handed DNA case when } 0 < \eta \leq 1 \text{ with } TG > AC, \\ 1 - H_d(p_d, lh) & \text{for left-handed DNA case when } -1 \leq \eta < 0 \text{ with } TG < AC, \\ 1 - H_s(p) & \text{for Shannon case when } \eta = 0 \text{ with } TG = AC, \end{cases} \quad (6)$$

where $H_d(Y_d) = 1$ when $p_d = 1/2$ is applied in (6), $H_d(p_d)$ denotes the entropy of the proposed bd-dh channel model. Similarly in (5), the $H_d(p_d)$ is given by

$$H_d(p_d) = -p_d \log_2 p_d - (1-p_d) \log_2 (1-p_d) \quad \text{bits}. \quad (7)$$

Fig. 4: The graphical illustration of the Milky way, the galaxy and the proposed bh-dh channel (a) a block hole around the Milky²³ way, (b) the proposed bh-dh channel model with 1×198 binary-chain and $\eta = 0.35$, (c) a supermassive^{10,11} black hole residing at the galaxy's core, and (d) the proposed bh-dh channel model with 1×270 binary-chain and $\eta = 0.6180$.

TABLE II: The maximum expandable entropy and capacity bound of the proposed bh-dhD channel model

p	$H_s(p)$	C_s	η_{rh}	η_{lh}	$p_{d,rh}$	$p_{d,lh}$	$H_d(p_{d,rh})$	$H_d(p_{d,lh})$	$C_{d,rh}$	$C_{d,lh}$
0.00	0.00	1.00	1.00	-1.00	1.00	-1.00	0.00	-2.00	1.00	3.00
0.50	1.00	0.00			1.50	-0.50	-1.37	-1.37	2.37	2.37
1.00	0.00	1.00			2.00	0.00	-2.00	0.00	3.00	1.00

TABLE III: The entropy and capacity of the proposed bh-dh channel based on Human^{18,22} and Octopus^{18,22} DNA case

p	η_h	η_o	$p_{d,h}$	$p_{d,o}$	$H_d(p_{d,h})$	$H_d(p_{d,o})$	$C_{d,h}$	$C_{d,o}$
0.00	0.35	-0.28	0.35	-0.28	0.93	-0.97	0.0659	1.97
0.50			0.85	0.22	0.61	0.76	0.39	0.24
1.00			1.35	0.72	-1.11	0.85	2.11	0.15

Results and Discussion: In this section, we compare the proposed bh-dh symmetric channel model over the binary-symmetric channel via computer simulations. Throughout the simulations, we consider a variable p for both right and left-handed η cases in TABLE II and TABLE III. In TABLE II, when we set $p = 0$ for both $\eta = 1$ and $\eta = -1$ in (6) and (7), the measured entropy $H_d(p_d)$ is given by 0 bit and -2 bit and the measured capacity C_d is given by 1 bit and 3 bits, respectively. In contrast,

Fig. 5: The maximum expandable entropy and capacity bound of the proposed bh-dh channel model: (a) Shannon entropy bound in green color when $\eta = 0$, left-handed bh-dh channel entropy in magenta color when $\eta = -1$, and right-handed bh-dh channel entropy in blue color for when $\eta = 1$ (b) Shannon capacity bound in green color when $\eta = 0$, left-handed bh-dh channel capacity in magenta color when $\eta = -1$, and right-handed bh-dh channel capacity in blue color for when $\eta = 1$.

Fig. 6: The entropy and capacity of the proposed bh-dh channel model (Human and Octopus DNA case) (a) left-handed channel entropy when $\eta = -0.28$ (b) right-handed channel entropy when $\eta = 0.35$ (c) left-handed channel capacity when $\eta = -0.28$ (d) right-handed channel capacity when $\eta = 0.35$.

when we set $p = 1$ for both $\eta = 1$ and $\eta = -1$, the measured entropy $H_d(p_d)$ is given by -2 bit and 0 bit and the measured capacity C_d is 3 bits and 1 bit , respectively.

In TABLE III, when we set $p = 0$ for both $\eta_h = 0.35$ and $\eta_o = -0.28$ in (6) and (7), the measured entropy $H_d(p_{d,h})$ is 0.93 bit and $H_d(p_{d,o})$ is -0.97 bit , and the measured capacity $C_{d,h}$ is 0.0659 bit and $C_{d,o}$ is 1.97 bits , respectively. In contrast, when we set $p = 1$ for both $\eta = 0.35$ and $\eta = -0.28$ in (6) and (7), the measured entropy $H_d(p_{d,h})$ is -1.11 bit and $H_d(p_{d,o})$ is 0.85 bit , and the measured capacity $C_{d,h}$ is 2.11 bits and $C_{d,o}$ is 0.15 bit , respectively.

Fig. 4 shows the maximum expandable entropy and capacity bound of the proposed bh-dh symmetric channel model when we set $p = 0 : 0.0001 : 1$, $\eta = 1$ and $\eta = -1$ in (6) and (7). In Fig. 4 (a) and Fig. 4(b), the composited entropy and capacity bound is given by $-2 \leq H_d(p_d) \leq 1$ and $0 \leq C_d \leq 3$ for the proposed bh-dh symmetric channel, respectively where the maximum expandable entropy and capacity is bounded by $-2 \leq H_d(p_d) \leq 0$ and $1 \leq C_d \leq 3$, respectively.

Fig. 5 illustrates the entropy and capacity of the proposed bh-dh channel model using a Human and an Octopus DNA case. Fig. 5a and Fig. 5c shows the entropy and capacity for Octopus DNA case when $p = 0 : 0.0001 : 1$, $\eta_o = -0.28$ is applied in (6) and (7). Fig. 5b and Fig. 5d presents the entropy and capacity for Human DNA case when $p = 0 : 0.0001 : 1$, $\eta_h = 0.35$ is applied in (6) and (7).

Conclusions: In this paper, we proposed the bh-dh channel model to expand information bound and help mitigate the multiple-helix directional issue of DNA. We observe that the geometrical shape of the proposed bh-dh channel comprised a deep-double helix DNA model, which enriched the Watson-Crick DNA model. The graphical insight of the proposed bh-dh channel is also comprised a beautiful circulant ring, which showed a similar exposition of the galaxy. The composited entropy and capacity bound of the proposed bh-dh channel is $-2 \leq H(p_d) \leq 1$ and $0 \leq C_d \leq 3$, which can outperform the Shannon entropy and capacity bound. The maximum expandable entropy and capacity bound of the proposed bh-dh channel model are $-2 \leq H(p_d) \leq 0$ and $1 \leq C_d \leq 3$, respectively. In future work, the proposed bh-dh channel model can be extended further to the DNA multiple-input and multiple-output communication systems.

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