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K-clique percolation in free association networks. The mechanism behind the 7 ± 2 law?

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ABSTRACT

It is important to reveal the mechanisms of propagation in different cognitive networks. In this study we discuss the k -clique percolation phenomenon on the free association networks including "English Small World of Words project" (SWOW-EN). We compare different semantic networks and networks of free associations for different languages. Surprisingly it turned out that k -clique percolation for all $k < k_c = (6 - 7)$ is possible on free association networks of different languages. Our analysis suggests the new universality patterns for a community organization of free association networks. We conjecture that our result can provide the qualitative explanation of the Miller's 7 ± 2 rule for the capacity limit of working memory. The new model of network evolution extending the preferential attachment is suggested which provides the observed value of k_c .

Introduction

Networks represent powerful models for exploring different cognitive systems and processes^{1,2}. For example, in^{3,4} the authors propose multiplex network model of the formation of mental lexicon and early word acquisition. In⁵ the structural properties of semantic networks for low and high creativity people are discussed. In⁶⁻¹⁰ network-based methods are used for simulation the mechanisms of solving Remote Associates Tests, allowing to estimate a human's creative potential¹¹.

Complex networks often exhibit meso-scale or global characteristics of structural order. Certain networks exhibit community structure, in which densely connected communities of nodes exhibit sparse or weak inter-community connections. In semantic networks, one word can belong to several communities, so standard community detection methods are not applicable. We investigate the community organization of the free association network focusing at one described in¹², known as "English Small World of Words project" (SWOW-EN). This network differs from other datasets in a higher density, which is achieved by the presence of links of weak association strength. The dense network structure allows us to study k -clique community organization of larger k . We compare its properties with different semantic networks of the English and Dutch languages and networks of free associations.

This study is mainly focused at the percolation analysis of the free association networks. The percolation approach was used to quantify the flexibility of one or another network characteristics of semantic network¹³⁻¹⁵. In¹³ flexibility of thought is investigated by percolation analysis and the cognitive declines due to aging have been discussed. In context of the creativity theory the percolation analysis has been discussed in^{5,13,14} and it was demonstrated that the semantic network of the high-creative group broke apart slower than that of less-creative group. It was also shown via percolation approach in¹⁴ that across the lifespan the mental lexicon is fragile against the combined semantic and phonological attacks.

More general phenomenon involves the percolation of k -clique introduced in^{16,17}. For the random Erdos-Renyi ensemble the critical link probability for any k can be found analytically however for real networks an estimation of the critical threshold for k -clique percolation is a nontrivial problem. In the cognitive networks the k -clique percolation has been recently discussed in¹⁸ for the problem of aging in the semantic memory.

In this study we investigate a k -clique percolation in the free association networks and question if there is some upper bound k_c when no k -clique percolation exists for $k > k_c$. A bit surprisingly it turned out that there is the sharp bound at $k_c = 6 - 7$ both for English and Dutch languages.

The sharp bound for clique percolation certainly provides the information concerning the structural organization of free association networks. However it certainly also influences the effectiveness of the processes on the network since k -clique percolation is a particular dynamical process. Only the k -cliques with $k < k_c$ can propagate effectively through the free association network. The discussion concerning the distinction between a structure and a process in semantic networks can be

found in [19,20](#).

The test protocols for free associations allow very short time intervals for answering hence we can consider them as a kind of probe of working memory. On the other hand the limitation of the working memory capacity is well-known phenomenon^{21,22} and the person can remember simultaneously only a finite number of items of different nature although there is some mild dependence on a nature of item. This phenomenon is known as Miller's 7 ± 2 law. We conjecture that our finding could serve as potential explanation of the mechanism behind the Miller's law. Indeed we have to remember the k -linked items for some short period of time. This can be considered as the k -clique percolation process in some effective "working memory network".

Looking at the mechanism responsible for the limit of working memory capacity the natural question concerns the evolutionary origin of the particular value of k_c and the rules of evolution which bring the network to this particular value of k_c . We suggest new rule of network evolution which can be considered as the modification of preferential attachment when a new node it attached to both nodes connected by link. It turns out that this new mechanism provides the desired value $k_c = (5 - 6)$ for different sizes of the network.

Results

K-clique community organization of semantic networks

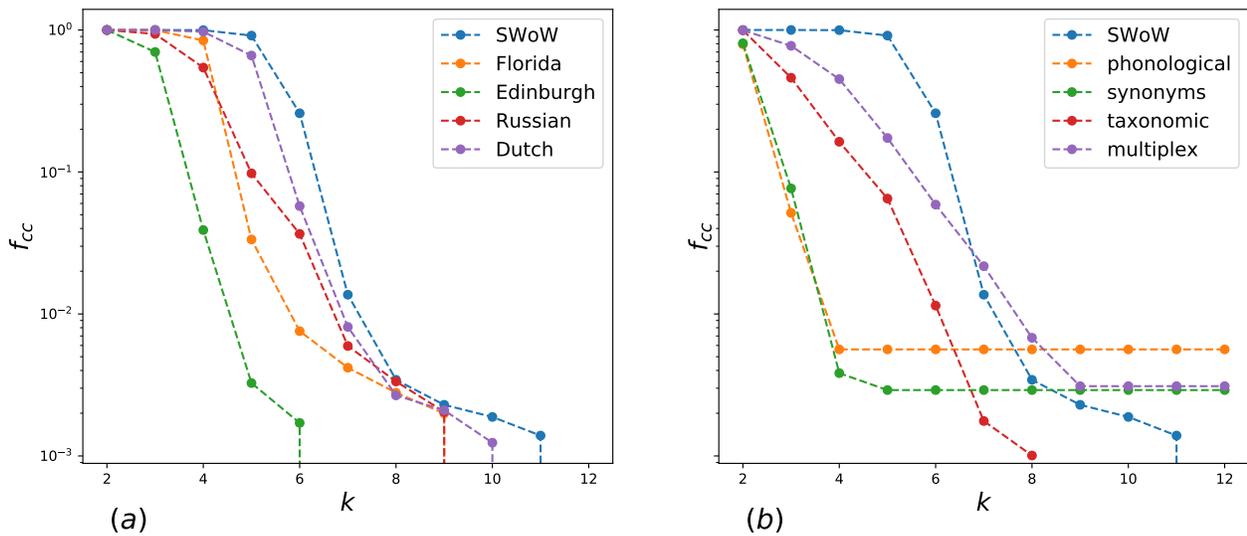


Figure 1. (a) The size of k -clique percolation cluster in dependence on the value k for different free association datasets. (b) The size of k -clique percolation cluster in dependence on the value k for different English semantic networks.

We calculated the fraction of nodes f_{cc} , included in k -clique percolation cluster for different values k . The dependencies for different free association datasets are presented in Fig.1(a). Firstly, note that almost all words are included in 3-clique percolation cluster, the existence of this cluster explains the high transitivity and average clustering coefficient, see Table 2. Secondly, all free association networks, except Edinburgh dataset demonstrates k -clique percolation for large k , i.e., the clusters of $k = 5$ and $k = 6$ contain essential fraction of words and for SWoW-EN dataset - almost all words. We suggest, that this feature is result of higher density of SWoW-EN dataset, which is achieved by including weak associations. A more detailed analysis of percolation depending on the association strength is presented in the Section 5.

In Fig.1(b) the dependencies for semantic networks of different nature are presented. In contrast to the networks of free associations, phonological and synonymous networks form a 3-clique percolation cluster only partially, and clusters of higher orders are completely absent, despite the fact that these networks are characterized by higher values of transitivity and clustering. We also calculated the respective dependence for the so-called multiplex network, in which we considered three layers: phonological, taxonomic and synonyms. For such a network, we observe clusters of the order of 4 and 5. Thus, we can assume that the variety of links, weak association in SWoW-EN, or different types of links in the multiplex network ensure the existence of high-order clique clusters.

Structural features and clustering in SWOW-EN network

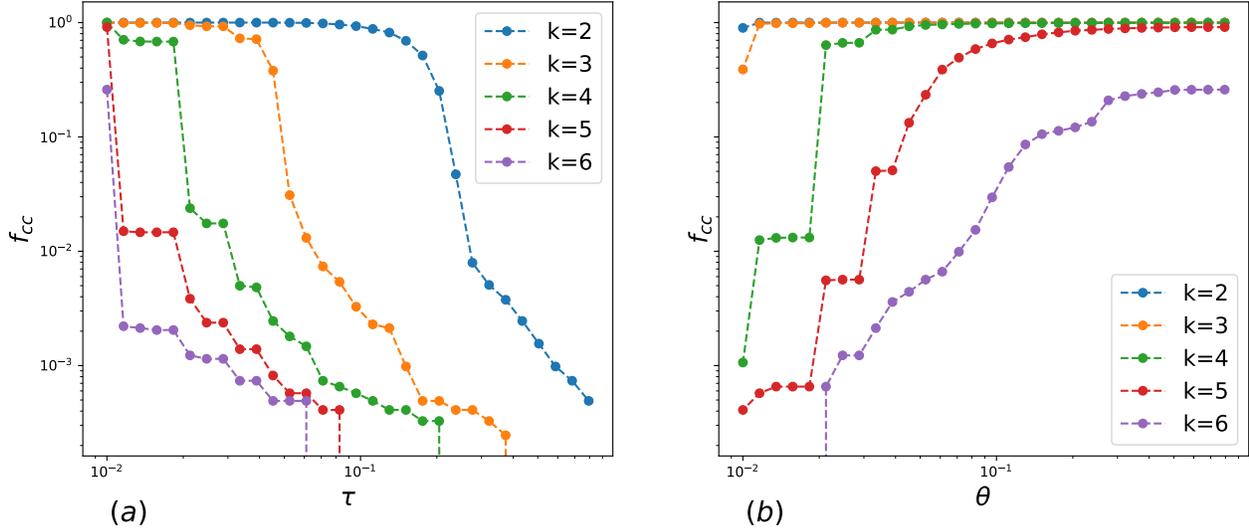


Figure 2. (a) The size of k -clique percolation cluster in dependence on the threshold τ for different values k in SWOW-EN. (b) The size of k -clique percolation cluster in dependence on the threshold θ for different values k in SWOW-EN.

We analyze k -clique community clusters in dependence on the association strength. For this aim we perform following numerical experiments. In first simulation we take a threshold τ for association strength and delete all links of weights *less* than the threshold. Fig.2(a) presents the fraction of nodes including in K -clique community cluster of $k = 2, 3..6$ in dependence on the threshold τ . We observe, that k -clique community clusters of higher order ($k = 5$ and $k = 6$) exist only for initial network state and almost disappear at a small threshold. Percolation clusters for $k = 3$ and $k = 4$ include all vertices up to sufficiently high threshold τ , indicating the stability of network community organisation. The second simulation is following. We establish a threshold θ for association strength and delete all links of weights *more or equal* than the value θ , i.e. we analyze a subgraph of weak associations. The respective dependencies for different k are depicted in Fig.2(b). 3-clique percolation cluster is not sensitive to the threshold θ and exists for all weak subgraphs. The percolation clusters for $k = 4$ and $k = 5$ include all words for high threshold and abruptly decrease at small values θ . Interestingly, that for $k = 6$ even for very high θ the percolation cluster contains only some part of nodes. This result shows that strong free associations can be considered as "core" links, which are involved in few cliques, providing intersection of cliques in percolation cluster. While weak associations form rather a "shell" of clique community, see Fig.3.

This assumption is confirmed by the study of the distribution of triangles belonging to the links depending on association strength. We introduce an *edge clustering coefficient* for as follows:

$$C_{ij} = \frac{N_T(ij)}{\min(k_i, k_j) - 1},$$

where $N_T(ij)$ is the number of triangles, containing the edge (i, j) , k_i, k_j are the degrees of i and j nodes respectively. Like a clustering coefficient of a node, the value C_{ij} shows the fraction of triangles and lies in the range $[0, 1]$. Note that the introduced clustering coefficient C_{ij} correlates with a topological overlap for nodes i and j in case of their adjacency²⁹. We found the clustering coefficient for each edge in the free association network SWOW-EN, sorted them and splitted into $n = 100$ intervals of equal size. For each intervals $l, l = 1, 2, \dots, n$ we calculated the average values for the clustering coefficient $\langle c_{ij}^l \rangle$ and for the association strengths $\langle w_{ij}^l \rangle$. Fig.4(a) presents the dependence of the average association strength on the respective clustering coefficient in given interval. One can see that the dependence is fitted by the curve $\log \langle w_{ij} \rangle = 5.1 \cdot \langle c_{ij} \rangle - 4.4$. Thus, we observe a positive correlation between the number of triangles, belonging to a link and its association strength. Note that this correlation was not discussed before and it is interesting by itself and can be used in modeling the human lexicon. Besides, we introduce *k-clique number* of an edge as the maximal clique size, containing the edge. In Fig.4(b) the distributions of k -clique numbers are presented for the weakest association links, i.e. $w_{ij} = 0.01$ and for the strongest association links, $w_{ij} > 0.1$.

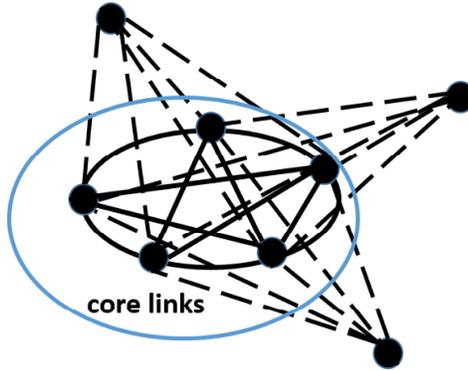


Figure 3. (a) Three k -cliques are adjacent ($k = 6$) through the central $(k-1)$ -clique, which could be considered as a "core".

Simulation of clique organization in free association networks

Network models of language structure are discussed in^{30,31}. Particularly, Dorogovtsev and Mendes³¹ proposed a stochastic theory of the evolution of human language, which treats language as a self-organizing network of interacting words. It is well known that language evolves, then the question is what kind of growth (in the sense of increase of lexical repertoire) leads to a self-organized structure with characteristic scale-free degree distribution. Dorogovtsev and Mendes' scheme of the language network growth is following. A new word is connected to some old one i with the probability proportional to its degree k_i (Barabasi and Albert's preferential attachment); additionally, at each time step, c new edges randomly emerge between old words, where c is a constant coefficient that characterizes a particular network. This model explained very well power law degree distribution and small world properties of semantic networks.

To describe clique organization in semantic networks we propose a new model based on Dorogovtsev and Mendes' mechanism, presented in Fig.5(a). In our model a new word is connected to $2m$ existing *linked* words i and j with the probability proportional to the sum degree $k_i + k_j$, forming a triangular; in addition, at each time step, we add c new edges randomly between old words. The network evolution begins with an initial small Erdos-Renyi random graph $G(l, p_0)$.

Table 1. Structural properties of simulated networks.

| Nodes | Edges | Density | Transitivity | Clustering | $p_c(2)$ | $p_c(3)$ |
|-------|--------|---------|--------------|------------|----------------------|----------|
| 2 000 | 23 213 | 0.0116 | 0.048 | 0.175 | $5 \cdot 10^{-4}$ | 0.0158 |
| 4 000 | 46 783 | 0.0058 | 0.028 | 0.158 | $2.5 \cdot 10^{-4}$ | 0.0111 |
| 6 000 | 69 307 | 0.0039 | 0.016 | 0.172 | $1.66 \cdot 10^{-4}$ | 0.0091 |
| 8 000 | 91 275 | 0.0028 | 0.010 | 0.187 | $1.25 \cdot 10^{-4}$ | 0.0079 |

We simulated the networks of different sizes with the model parameters $m = 4$, $c = 4$, $l = 20$, $p_0 = 0.1$. Structural properties of the networks are summarized in Table 1. All networks are sparse with the density, $p_c(2) < \rho < p_c(3)$, and high clustered. Degree distributions of the networks are fitted by power law $p = Cd^{-\gamma}$ with $\gamma = 2.6$, see Fig.6(a). The size of k -clique percolation cluster in dependence on the value k demonstrates the same behaviour as observed for free association networks (Fig.6(b)) and does not depend on the network size. Thus, the assumption of preferential attachment to an edge rather than a single word may explain clique organization in free association networks.

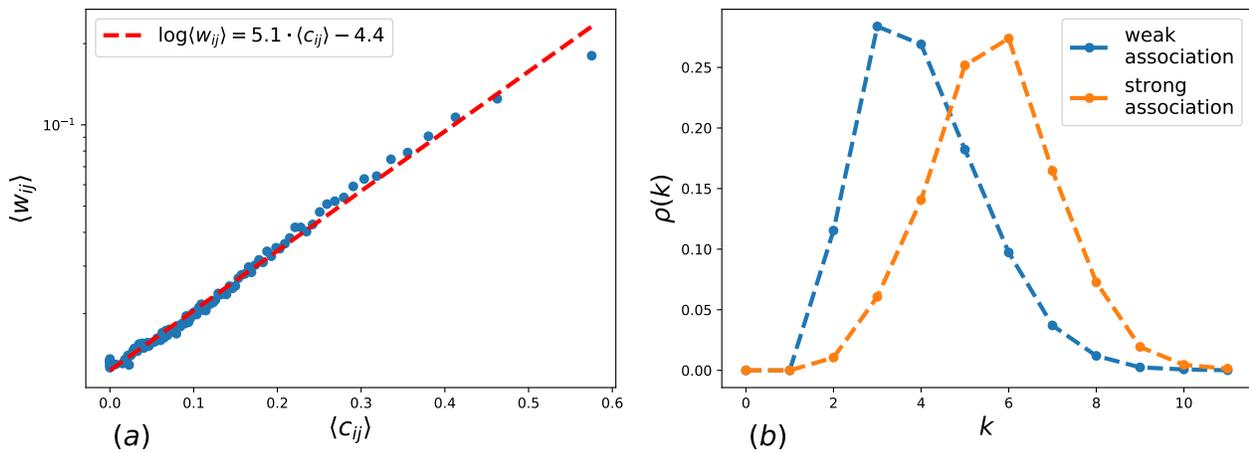


Figure 4. (a) The dependence of average association strengths on the edge clustering. (b) The distribution of maximal clique sizes for weak and strong associations.

Discussion: towards the explanation of Miller's 7 ± 2 rule?

In this Section we shall make the conjecture concerning more general meaning of our findings. It was remarked long time ago²¹ that many phenomena concerning the consuming information by the human brain for a short period of time have a natural restriction of the number of controlled items. This number is estimated by the Miller's 7 ± 2 rule which implies the restricted ability of brain to handle with the information processing. There were a few attempts to apply the underlying network structure to explain the Miller rule for the limit of capacity of working memory²².

From the psychological viewpoint three groups of mechanisms behind the limit capacity have been suggested (see,³² for the review). First approach implies the short time decay of the groups of items because of some reason. Second mechanism of limited resource claims that there are not enough resources for higher capacity. A resource is considered as some limited quantity that enables a cognitive function or process. According to the third mechanism our ability to hold several representations available at the same time is limited by mutual destructive interference between these representations. As an example of this mechanism one could have in mind the interference of frequency bands in brain activity. Indeed it is known that a few bands are simultaneously involved in the processing of working memory. None of these mechanisms can be considered as fully satisfactory. Another network motivated approach³³ utilizes the mathematical result concerning the plane colouring by four colors. This idea was conjectured to be relevant for the smaller critical number of items discussed in²².

Can we gain some new insight concerning the mechanism behind the Miller's rule from our study? Let us assume that the free association tests are the specific probes of the working memory. This assumption has been discussed in the literature before (see, for instance³⁴) and it is natural since the time allowed for the performance of tests is quite restricted. Hence let us assume that networks of free associations carry the information about the working memory and their structure code the information concerning the group of related stimuli during the test. We conjecture that keeping the information about the linked group of k stimuli is encoded in the k -clique percolation. Hence our finding that the k -clique percolation for the SWOW-EN and Dutch networks is possible only for $k \leq 6$ can be interpreted as the example of the Miller's rule.

One may concern that in the working memory setup we have "percolation in time" to keep the group of stimuli as a whole for some period of time. On the other hand in the clique percolation we have a "percolation on the network" keeping the clique intact when moving along the graph. However to some extent the formation of the association network can be considered as the growing network model. This viewpoint if true suggests the new perspective of explanation of the Miller's rule for all behavioural situations when the network description is available. One has to estimate the maximum size of percolating clique in the particular network architecture. It provides the answer for the capacity of the working memory which ensure the propagation of the linked group of items in time.

One could question why only the low size clique percolation in the human brain is possible although naively we could expect that the brain would prefer the higher working memory capacity. In particular a limit of the working memory capacity for human is higher than of other animals³⁵ and it is assumed that higher working memory corresponds to higher intellect. The answer certainly should involve some evolutionary arguments and at least two alternative scenarios are possible. First, presumably the network architecture admitting higher clique percolation is in contradiction with some other vital properties of

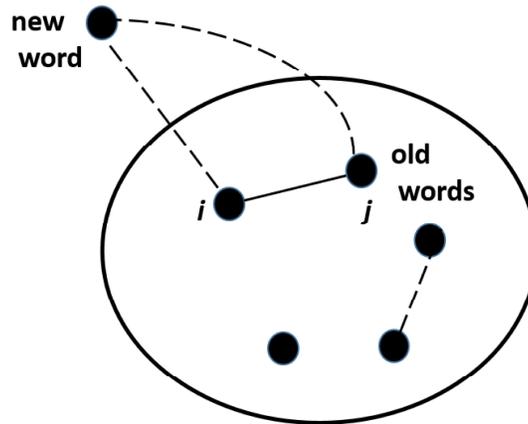


Figure 5. Network model description: a new word is connected to a link (i, j) by preferential attachment; in addition, random links between old words emerge. Existing links are depicted by solid line, new links are dashed.

a brain encoded in connectome architecture.

Secondly, we could assume that the particular evolutionary rules (see,³⁶ for the review) for the corresponding network dynamically bring it to the particular capacity limit somewhat in the spirit of self-organized criticality. In the previous Section we have supported this possibility suggesting the non-conventional version of the preferential attachment procedure which indeed yields the reasonable value of $k_c = (5 - 6)$.

Methods

Data description

The free association network SWOW-EN is a weighted directed network with $N = 12\,217$ stimuli words. Stimulus materials (cue words) were constructed using a snowball sampling method, allowing authors¹² to include both frequent and less frequent cues at the same time. The final set consists of 12 292 cues (stimuli), the weight of the link indicates fraction of the experiment participants which gave this particular response to a cue (i.e. the conditional probability of a response given a cue). Therefore, the total weight of links going out of each node is less or equal to 1. For our analysis we consider the network as undirected, attributing the greatest weight to an edge in the case of a bidirectional association.

Also we analyze the free association network, based on the South Florida free association data base²³ and the free association network, known as the Edinburgh Associative Thesaurus²⁴.

Besides, we use networks, containing taxonomic relations (e.g. “A is a type of B”), synonymous relation (e.g. “A also means B”) and phonological similarities. All data were retrieved from Wolfram Research²⁵, which mostly coincides with WordNet 3.0²⁶. Finally, we study free association networks for Russian and Dutch languages. We used Russian thesaurus²⁷ and Dutch association data²⁸, removing words, that have no associations. Table 2 summarize the basic structural properties of used networks.

K-clique percolation

We begin with a few definitions laying down the fundamentals of k -clique percolation^{16,17}. K -clique is a complete (fully connected) subgraph of k vertices. We say, that two k -cliques are *adjacent* if they share $k - 1$ vertices, i.e., if they differ only in a single vertex. A subgraph, which is the union of a sequence of adjacent k -cliques is called *k -clique chain* and two k -cliques are *k -clique-connected*, if there exists at least one k -clique chain containing the two k -cliques. Finally, *k -clique percolation cluster* is defined as a maximal k -clique-connected subgraph, i.e., it is the union of all k -cliques that are k -clique-connected to a particular k -clique.

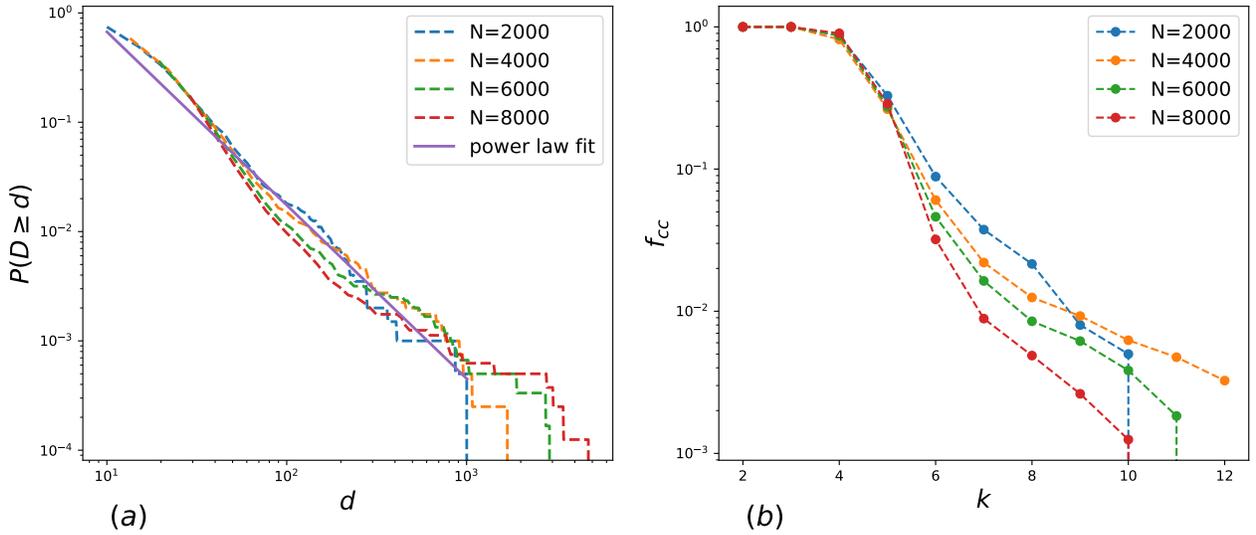


Figure 6. (a) Complementary cumulative degree distribution function for simulated networks of different sizes; (b) The size of k -clique percolation cluster in dependence on the value k for networks of different sizes.

Table 2. Structural properties of semantic networks.

| Network | Nodes | Edges | Density | Transitivity | Clustering | $p_c(2)$ | $p_c(3)$ |
|----------------------------|--------|---------|---------|--------------|------------|---------------------|----------|
| SWOW-EN free association | 12 217 | 352 403 | 0.0047 | 0.052 | 0.113 | $8.2 \cdot 10^{-5}$ | 0.0064 |
| Florida free association | 5 019 | 55 246 | 0.0044 | 0.083 | 0.186 | $2.0 \cdot 10^{-4}$ | 0.0100 |
| Edinburgh free association | 6 437 | 36 921 | 0.0017 | 0.059 | 0.124 | $1.6 \cdot 10^{-4}$ | 0.0088 |
| Taxonomic | 7 943 | 42 042 | 0.0013 | 0.048 | 0.093 | $1.3 \cdot 10^{-4}$ | 0.0079 |
| Synonyms | 6 526 | 13 134 | 0.0006 | 0.284 | 0.344 | $1.5 \cdot 10^{-4}$ | 0.0088 |
| Phonological | 4 618 | 15 447 | 0.0014 | 0.345 | 0.246 | $2.2 \cdot 10^{-4}$ | 0.0104 |
| Multiplex | 8 383 | 68 505 | 0.0019 | 0.112 | 0.283 | $1.2 \cdot 10^{-4}$ | 0.0078 |
| RUS thesaurus | 5 377 | 51 191 | 0.002 | 0.067 | 0.163 | $1.9 \cdot 10^{-4}$ | 0.0096 |
| Dutch Data | 10 486 | 207 810 | 0.0038 | 0.067 | 0.163 | $9.5 \cdot 10^{-5}$ | 0.0069 |

The Erdosh–Renyi random graphs show a series of interesting transitions when the probability p of two nodes being connected is increased. For $k = 2$ the transition is well known and manifested by the appearance of a giant component in a network at critical probability $p_c(k = 2) = \frac{1}{N}$, where N is the number of nodes. For each k one can find a certain threshold probability $p_c(k)$ above which the k -cliques organize into a giant community¹⁷:

$$p_c(k) = \frac{1}{[N(k-1)]^{\frac{1}{k-1}}}.$$

Table 2 contains the values $p_c(k)$ with $k = 2, 3$ for random networks of the same size as semantic networks. We found, that network density, i.e. the observed link probability, for all datasets satisfy the inequality $p_c(2) < \rho < p_c(3)$. That is, if links in a semantic network were formed randomly, then all the vertices are included in the percolation cluster of $k = 2$, that is, one connected component, but do not form a cluster of $k = 3$.

Conclusion

In this paper we have analysed the k -clique percolation in the free association networks and semantic networks for few languages. There are few findings of our study which seem to be important. First, using the traditional approach we have

investigated the structural network properties via the percolation theory and made a few new observations

- There is the critical value k_c for the maximal size of a percolating k -clique for the SWOW-EN network and semantic networks. The higher clique with $k > k_c$ can not percolate through these networks
- The density of the analyzed network does not allow the percolation of $k > 2$ cliques if the network is considered as random. This means that our study confirms the non-randomness of the free association networks
- Imposing the thresholds on the link weights we investigated the role of weak and strong associations on the k -clique size and percolation. It is found that the strong associations play the key role in the k -clique percolating cluster while the weak associations provide a kind of shadow which however is necessary ingredient supporting the observations made in¹⁰. The clear-cut dependence between the averaged local weights and the local connectivity has been found.

Secondly we assume that our findings provide the additional information not only on the structure but also for the processing on the network. Namely the free association networks can be considered as the peculiar probe of the working memory. Therefore the critical k_c for the k -clique percolation we have found presumably can be interpreted as the limitation of capacity of the working memory and therefore can be new nontrivial qualitative mechanism behind the Miller's law. This can be further checked by investigating a threshold in k -clique percolation for other cognitive networks involving short-time performances probing working memory.

It would be interesting to elaborate further the possible origin of k_c actual value. Presumably it can be established evolutionary³⁶ as a optimal result of competition between the clique percolation related to the working memory and another properties of the connectome responsible for important cognitive properties. Another possibility demonstrated in our study is that the specific version of the preferential attachment evolution mechanism yields the critical value $k_c = (5 - 6)$ via a kind of self-organized critically. It would be interesting to test our new evolutionary rule in the other cognitive processes.

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Author contributions statement

O.V. conceived and conducted the numerical experiments, A.G. suggested an explanation for Miller's law. All authors reviewed the manuscript.