New Optical Soliton Solutions For The Variable Coefficients Nonlinear Schrodinger Equation

Yongyi Gu

Nanjing University

Najva Aminakbari (✉ najvaaminakbari@yahoo.com)

Guangdong University of Finance and Economics <https://orcid.org/0000-0002-4242-0762>

Research Article

Keywords: Optical Solitons, Bernoulli (G'/G)-expansion method, Nonlinear Schrödinger equation, Variable coefficients, Dynamic structures

Posted Date: November 16th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1039799/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Version of Record: A version of this preprint was published at Optical and Quantum Electronics on March 24th, 2022. See the published version at <https://doi.org/10.1007/s11082-022-03645-4>.

New optical soliton solutions for the variable coefficients nonlinear Schrödinger equation

Yongyi Gu^{1,2} · Najva Aminakbari³

Received: date / Accepted: date

Abstract This paper is appropriated to seek new optical soliton solutions of nonlinear Schrödinger equation (NLSE) with time-dependent coefficients which describes the dispersion decreasing fiber. By proposing Bernoulli (G'/G) -expansion method, where $G = G(\zeta)$ satisfies Bernoulli equation, some periodic wave, bright and dark soliton solutions are successfully achieved. In addition, 3D, line and contour maps graphs of the obtained results under effect of different values of coefficients are illustrated to have acceptable conception of dynamic structures.

Keywords Optical Solitons · Bernoulli (G'/G) -expansion method · Nonlinear Schrödinger equation · Variable coefficients · Dynamic structures

1 Introduction

The variable coefficients nonlinear Schrödinger equation (VcNLSE) is one of the main topic in modern physics to pragmatically describe many nonlinear behaviors in nonlinear optical fibers, Bose-Einstein condensation, plasma and water waves (Ponomarenko and Agrawal, , 2007; He et al., , 2009; Yu and Yan, , 2014; Yan et al., , 2015; Yao et al., , 2016; Kaur and Wazwaz, , 2019). Several studies have been done to peruse on solutions of VcNLSE; for example, Pérez-García et al. (Pérez-García et al., , 2006) used similarity transformations connecting NLSE involved time-varying coefficients with the autonomous cubic NLSE. He et al. (He et al., , 2014) studied rogue wave solutions in NLSE with variable coefficients. Kedziora et al. (Kedziora et al., , 2015) by using Lax pair and Darboux transformation formalisms could find the solutions of VcNLSE. Liu et al. (Liu et al., , 2019) achieved the distinguished types of nonautonomous complex wave solutions, which include bright

N. Aminakbari
E-mail: najvaaminakbari@yahoo.com

¹ Department of Mathematics, Nanjing University, Nanjing 210093, China

² School of Statistics and Mathematics, Guangdong University of Finance and Economics, Guangzhou 510320, China

³ School of Mathematics and Information Science, Guangzhou University, Guangzhou 510006, China

and dark soliton solutions. Guo and Liu (Guo and Liu, , 2020) by analyzing the dynamical properties of the related Hamiltonian obtained some new dark solitons, bubble solutions and periodic solutions.

In the past few years studying on different types of VcNLSE with coefficients which depend on the evolution variable has been of great interest to scientists. For instance, stabilized solitons, which appear in multidimensional VcNLSE equations when the coefficient is controlled appropriately, that is exist in optical applications (Bergé et al., , 2000), according to these projects, it was realized that there is a similar phenomenon arises in the context of mean field models of Bose-Einstein condensation (Abdullaev et al., , 2003; Saito and Ueda, , 2003; Montesinos, , 2004). The studies of the nonlinear waveforms for the nonlinear Schrödinger equations with external potentials has important role in the rapid development of the Bose-Einstein condensates (Bao and Cai, , 2012; Feng, , 2016; Feng and Zhang, , 2018; Wazwaz, , 2009). For the nonlinear Schrödinger equations with external potential, Zhang (Zhang, , 2000) studied the existence of the condensations in a critical value. Bao and Cai (Bao and Cai, , 2012) accomplished numerical studies on the nonlinear Schrödinger equations with singular potential modelling the dipolar Bose-Einstein condensation for ground states.

In this research, we explain the optical soliton solutions of the following form VcNLSE with time-dependent coefficients (Serkin and Hasegawa, , 2000; Kruglov et al., , 2005; Zeng et al., , 2016) by proposing Bernoulli (G'/G)-expansion method:

$$i\psi_t = -\frac{1}{2}\beta(t)\psi_{xx} + i\gamma(t)\psi - g(t)|\psi|^2\psi, \quad (1)$$

here $\psi(x, t)$ is a complex wave function of x and t , the dispersion parameter $\beta(t)$, the non-linearity parameter $g(t)$ and gain/loss parameter $\gamma(t)$ ($> 0, < 0$) are real functions of time. This proposed method is inspired by the (G'/G)-expansion and the generalized (G'/G)-expansion methods (Wang et al., , 2008; Zhang et al., , 2008), which is based on the ideas that traveling wave solutions of a nonlinear evolution equation (NLEE) can be explained by a polynomial in (G'/G), and $G = G(\zeta)$ satisfies a second order linear ordinary differential equation with constant coefficients, while in Bernoulli (G'/G)-expansion method, we express $G = G(\zeta)$ as a Bernoulli equation with variable coefficients. Although many useful methods are developed to investigate NLEEs (Zhao, , 2021; Hosseini et al., , 2020; Bulut et al., , 2018; Khan and Akbar, , 2016; Roshid and Rahman, , 2014), our method enrich the studies of this area and the results show that the proposed method is effectual to search exact solutions of many other nonlinear differential equations with variable coefficients.

This paper is structured into: In section 2, the Bernoulli (G'/G)-expansion method is described. In section 3, we give the application of this method by applying to Eq. (1) in two different conditions. In section 3 dynamic behaviors of the begotten results are illustrated. Some conclusion and explanation are expressed at the end.

2 Description of the Bernoulli (G'/G)-expansion method

A NLEE with dependent and independent variables, respectively u and $X = (x, t)$, is given by

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0. \quad (2)$$

By using transformation, Eq. (2) can be reduced to an ordinary differential equation (ODE), which the solutions can be expressed by

$$u = \sum_{i=1}^n a_i(X) \left(\frac{G'}{G} \right)^i + a_0(X), \quad a_n(X) \neq 0, \quad (3)$$

where $a_0(X)$, $a_i(X)$ ($i = 1, 2, \dots, n$) should be determined, and $\zeta = \zeta(X)$ which in this paper we consider it as $\zeta = b(x - ct)$, where b and c are constants. Function $G = G(\zeta)$ satisfies the following Bernoulli equation:

$$G' + p(\zeta)G = q(\zeta)G^2, \quad (4)$$

where $p(\zeta)$ and $q(\zeta)$ should be specified.

By balancing the highest order nonlinear term and the highest order derivative of u in ODE, integer n will be achieved. Substituting (3) along with Eq. (4) into ODE and collecting the same order of G together will get a set of algebra equations of variable coefficients that can be determined by use of Maple, while general solutions of (G'/G) are given as

$$\left(\frac{G'}{G} \right) = - \frac{\left(\cosh \left(\int p(\zeta) d\zeta \right) - \sinh \left(\int p(\zeta) d\zeta \right) \right) q(\zeta)}{\int \left(\cosh \left(\int p(\zeta) d\zeta \right) - \sinh \left(\int p(\zeta) d\zeta \right) \right) q(\zeta) d\zeta - C_1} - p(\zeta), \quad (5)$$

where C_1 is an arbitrary constant. Substituting these results into (3), we have exact solutions of Eq. (1).

3 Applications

The first step to find exact solutions of Eq. (1) is done by applying the following transformation

$$\psi(x, t) = \phi(\zeta) e^{ikx - i\mu t}, \quad (6)$$

where ϕ is amplitude function, k and μ are phase constants. By substituting (6) into (1), for real and imaginary parts respectively, we have

$$\mu \phi - \frac{1}{2} k^2 \beta(t) \phi + \frac{1}{2} \beta(t) \phi_{xx} + g(t) \phi^3 = 0, \quad (7)$$

$$\phi_t + k \beta(t) \phi_x - \gamma(t) \phi = 0. \quad (8)$$

Balancing between ϕ_{xx} and ϕ^3 in (7), can get $n = 1$ for (3), which leads to ϕ as follows:

$$\phi = a_0(t) + a_1(t) \left(\frac{G'}{G} \right), \quad a_1(t) \neq 0, \quad (9)$$

where $G = G(\zeta)$ satisfies Eq. (4). In this research, we have studied on two different conditions to derive exact solutions of Eq. (1):

First condition: Substituting (9) into (7) and (8), collecting coefficients with the same order of G^j , ($j = 0, 1, 2, 3$) to zero, yields a system of differential equations as follows:

$$\begin{aligned}
& 3g(t)a_0(t)a_1^2(t)p^2(\zeta) - a_1^3(t)p^3(\zeta)g(t) - 3g(t)a_0^2(t)a_1(t)p(\zeta) + 0.5a_1(t)p(\zeta)\beta(t)k^2 \\
& - 0.5\beta(t)a_1(t)p''(\zeta)b^2 + a_0^3(t)g(t) - 0.5a_0(t)\beta(t)k^2 - a_1(t)p(\zeta)\mu + a_0(t)\mu = 0, \\
& a_1(t)q(\zeta)\beta(t)p^2(\zeta)b^2 - a_1(t)q(\zeta)\beta(t)k^2 - a_1(t)q(\zeta)\beta(t)p'(\zeta)b^2 - 2a_1(t)\beta(t)p(\zeta)q'(\zeta)b^2 \\
& + 6a_1^3(t)p^2(\zeta)q(\zeta)g(t) - 12a_1^2(t)p(\zeta)q(\zeta)a_0(t)g(t) + 6a_1(t)q(\zeta)a_0^2(t)g(t) \\
& + 2a_1(t)q(\zeta)\mu + a_1(t)\beta(t)q(\zeta)b^2 = 0, \\
& 3q'(\zeta)a_1(t)q(\zeta)\beta(t)b^2 - 3p(\zeta)a_1(t)q^2(\zeta)\beta(t)b^2 - 6p(\zeta)g(t)a_1^3(t)q^2(\zeta) \\
& + 6a_0(t)g(t)a_1^2(t)q^2(\zeta) = 0, \\
& g(t)a_1^3(t)q^3(\zeta) + \beta(t)a_1(t)q^3(\zeta)b^2 = 0, \\
& a_1(t)p'(\zeta)bc - \beta(t)a_1(t)p'(\zeta)bk + a_1(t)p(\zeta)\gamma(t) - a_1'(t)p(\zeta) - a_0(t)\gamma(t) + a_0'(t) = 0, \\
& q(\zeta)a_1(t)p(\zeta)bc - q(\zeta)a_1(t)p(\zeta)\beta(t)bk + a_1(t)q'(\zeta)\beta(t)bk - a_1(t)q'(\zeta)bc - \gamma(t)q(\zeta)a_1(t) \\
& + a_1'(t)q(\zeta) = 0, \\
& \beta(t)a_1(t)q^2(\zeta)bk - a_1(t)q^2(\zeta)bc = 0,
\end{aligned}$$

which have solutions

$$a_0(t) = \pm \frac{1}{2} C_2 \frac{\left(\pm 2C_4 cb + \sqrt{-2c^2k^2 + 4ck\mu} \right)}{cb} e^{\int \gamma(t) dt}, \quad a_1(t) = C_2 e^{\int \gamma(t) dt}, \quad \beta(t) = \frac{c}{k},$$

$$g(t) = -\frac{cb^2}{kC_2^2 e^{2\int \gamma(t) dt}}, \quad p(\zeta) = \pm \frac{\pm C_4 cb + \sqrt{-2c^2k^2 + 4ck\mu}}{cb}, \quad q(\zeta) = C_3 e^{C_4 \zeta},$$

where C_2, C_3 and C_4 are arbitrary constants. $\gamma(t)$ is an arbitrary function. In this study, that is considered as follows

$$\gamma(t) = \gamma_0 e^{\rho t},$$

with γ_0 and ρ as constants.

Substituting the above results into (9) and by use of (6) and (5), we have:

$$\begin{aligned}
\psi_1(x, t) = \pm & \frac{\left(C_3 \left(\cosh \left(\frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} \zeta \right) \pm \sinh \left(\frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} \zeta \right) \right) \pm \frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} C_1 \right)}{\left(C_3 \left(\cosh \left(\frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} \zeta \right) \pm \sinh \left(\frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} \zeta \right) \right) \mp \frac{\sqrt{-2c^2k^2 + 4ck\mu}}{cb} C_1 \right)} \\
& \cdot \frac{\sqrt{-2c^2k^2 + 4ck\mu} C_2}{2cb} e^{\int \gamma_0 e^{\rho t} dt} e^{ikx - i\mu t}, \quad (10)
\end{aligned}$$

where ζ is defined by $b(x - ct)$.

Second condition: Considering $b = 1$, $c = 0$ and $k = 0$, and substituting (9) into (7) and (8), collecting coefficients with the same order of G^j , ($j = 0, 1, 2, 3$) to zero, yields a system of

differential equations as follows:

$$\begin{aligned}
& a_0(t) \mu - a_1(t) p(x) \mu - 0.5 \beta(t) a_1(t) p''(x) - a_1^3(t) p^3(x) g(t) + 3 a_1^2(t) p^2(x) a_0(t) g(t) \\
& - 3 a_1(t) p(x) a_0^2(t) g(t) + a_0^3(t) g(t) = 0, \\
& 2 a_1(t) q(x) \mu + p^2(x) a_1(t) q(x) \beta(t) - 2 p(x) q'(x) a_1(t) \beta(t) + 6 a_0^2(t) g(t) a_1(t) q(x) \\
& + q''(x) a_1(t) \beta(t) + 6 p^2(x) g(t) a_1^3(t) q(x) - 12 p(x) a_0(t) g(t) a_1^2(t) q(x) - p'(x) a_1(t) q(x) \beta(t) = 0, \\
& 3 q'(x) a_1(t) q(x) \beta(t) - 3 p(x) a_1(t) q^2(x) \beta(t) - 6 p(x) g(t) a_1^3(t) q(x) + 6 a_0(t) g(t) a_1^2(t) q^2(x) = 0, \\
& g(t) a_1^3(t) q^3(x) + \beta(t) a_1(t) q^3(x) = 0, \\
& \gamma(t) a_1(t) p(x) - a_1'(t) p(x) - \gamma(t) a_0(t) + a_0'(t) = 0, \\
& a_1'(t) q(x) - \gamma(t) a_1(t) q(x) = 0.
\end{aligned}$$

Solving this system of differential equations, gives us two different cases:

Case 1.

$$p(x) = p, \quad q(x) = q, \quad a_1(t) = C_2 e^{\int \gamma(t) dt}, \quad a_0(t) = \frac{1}{2} p a_1(t),$$

$$\beta(t) = \frac{4\mu}{p^2}, \quad g(t) = -\frac{4\mu}{p^2 a_1^2(t)},$$

where p, q, μ and C_2 are arbitrary constants.

Substituting the above results into (9) and by use of (6) and (5), for wave function, we have:

$$\psi_2(x, t) = -\frac{1}{2} \frac{C_2 p (p C_1 - (\cosh(px) - \sinh(px)) q)}{p C_1 + (\cosh(px) - \sinh(px)) q} e^{\int \gamma_0 e^{p t} dt} e^{-i \mu t}. \quad (11)$$

Case 2.

$$p(x) = \frac{1}{2} \frac{\sinh\left(\frac{1}{2} \frac{C_4}{C_3} + \frac{1}{2} \frac{x}{C_3}\right)}{C_3 \cosh\left(\frac{1}{2} \frac{C_4}{C_3} + \frac{1}{2} \frac{x}{C_3}\right)} + C_5, \quad q(x) = \frac{C_6 e^{C_5 C_4} e^{C_5 x}}{\cosh\left(\frac{1}{2} \frac{C_4 + x}{C_3}\right)},$$

$$a_1(t) = C_2 e^{\int \gamma(t) dt}, \quad a_0(t) = C_5 a_1(t), \quad \beta(t) = 4\mu C_3^2, \quad g(t) = -\frac{4\mu C_3^2}{a_1^2(t)},$$

where C_2, C_3, C_4, C_5, C_6 and μ are arbitrary constants. By setting these results into (9) and by use of (6) and (5), wave function can be obtained as:

$$\psi_3(x, t) = -\frac{1}{2} \frac{C_2 \left(-2 C_3 C_6 e^{C_5 C_4} \cosh\left(\frac{1}{2} \frac{C_4 + x}{C_3}\right) + C_1 \sinh\left(\frac{1}{2} \frac{C_4 + x}{C_3}\right) \right)}{C_3 \left(-2 C_3 C_6 e^{C_5 C_4} \sinh\left(\frac{1}{2} \frac{C_4 + x}{C_3}\right) + C_1 \cosh\left(\frac{1}{2} \frac{C_4 + x}{C_3}\right) \right)} e^{\int \gamma_0 e^{p t} dt} e^{-i \mu t}. \quad (12)$$

Fig. 1 $|\psi_1|$ is solution of Eq. (1) for $c = -2.5$ when (a1-a3) $\gamma_0 = -0.2, \rho = 0.11$, (b1-b3) $\gamma_0 = 0, \rho = 0$, (c1-c3) $\gamma_0 = 0.2, \rho = 0.11$

Fig. 2 $|\psi_1|$ is solution of Eq. (1) for $c = 0.1$ when (a1-a3) $\gamma_0 = -0.2, \rho = 0.11$, (b1-b3) $\gamma_0 = 0, \rho = 0$, (c1-c3) $\gamma_0 = 0.2, \rho = 0.11$

4 Dynamic structures

In this section we will demonstrate the dynamic structures of our results to get better understanding of wave solutions. Figure 1 illustrates the wave propagation of wave function $\psi_1(x, t)$, by using $\mu = 1.2, k = 1.1, p = 2, C_1 = -1.1, C_2 = 0.2, C_3 = 1.1, b = 1.5$ and $c = -2.5$ with three different values of γ_0 and ρ as follows: Fig. 1 (a1- a3) describe 3D, line and contour maps graphs for $\gamma_0 = -0.2, \rho = 0.11$. Fig. 1 (b1- b3) describe 3D, line and

Fig. 3 $|\psi_1|$ is solution of Eq. (1) for $c = 2.5$ when (a1-a3) $\gamma_0 = -0.2, \rho = 0.11$, (b1-b3) $\gamma_0 = 0, \rho = 0$, (c1-c3) $\gamma_0 = 0.2, \rho = 0.11$

Fig. 4 $|\psi_2|$ is solution of Eq. (1) when (a1-a3) $\gamma_0 = -0.2, \rho = 0.11$, (b1-b3) $\gamma_0 = 0, \rho = 0$, (c1-c3) $\gamma_0 = 0.2, \rho = 0.11$

contour maps graphs for $\gamma_0 = 0, \rho = 0$, which show us the results when $\gamma(t) = 0$. We also represent in Fig. 1 (c1- c3), 3D, line and contour maps when $\gamma_0 = 0.2, \rho = 0.11$.

Figure 2 and Figure 3 are displayed to show the important role of constant c in dynamic behaviors of wave function $\psi_1(x, t)$, respectively by setting $c = 0.1$ and $c = 2.5$ while other parameters are same as Figure 1. As shown in Figure 1 by choosing $c = -2.5$ the wave function demonstrates the periodic behaviors, in Figure 2 for $c = 0.1$ we have solitary waves and by setting $c = 2.5$ we have a different kind of periodic wave plots.

Fig. 5 $|\psi_3|$ is solution of Eq. (1) when (a1-a3) $\gamma_0 = -0.2, \rho = 0.11$, (b1-b3) $\gamma_0 = 0, \rho = 0$, (c1-c3) $\gamma_0 = 0.2, \rho = 0.11$

Figure 4 exposes the solution of $\psi_2(x, t)$ in Eq. (1) by using $\mu = 1.2, p = 2, q = 1.1, C_1 = 0.01$ and $C_2 = 1.41$ with three different values of γ_0 and ρ as follows: Fig. 4 (a1- a3) describe 3D, line and contour maps graphs for $\gamma_0 = -0.2, \rho = 0.11$. Fig. 4 (b1- b3) describe 3D, line and contour maps graphs for $\gamma_0 = 0, \rho = 0$, which show us the results when $\gamma(t) = 0$. We also show in Fig. 4 (c1- c3), 3D, line and contour maps graphs by setting $\gamma_0 = 0.2, \rho = 0.11$.

In Figure 5, wave function $\psi_3(x, t)$ is displayed, by setting $\mu = 1.2, C_1 = 0, C_2 = 1.1, C_3 = 1.1, C_4 = 0.01, C_5 = 0.01$ and $C_6 = 1.3$ with three different values of γ_0 and ρ as follows: Fig.5 (a1-a3) describe 3D, line and contour graphs for $\gamma_0 = -0.2, \rho = 0.11$. Fig. 5 (b1- b3) describe $\gamma_0 = 0, \rho = 0$, which give us the results when $\gamma(t) = 0$. Also, Fig. 5 (c1- c3), illustrate by considering $\gamma_0 = 0.2, \rho = 0.11$.

For these figures to have good point of view, $+/-$ standard deviation are represented in line plots.

5 Conclusion

In this article, we have presented Bernoulli (G'/G) -expansion method to find optical soliton solutions of VcNLSE, based on expressing the exact solutions of NLEE by a polynomial, which is defined as a fractional function of the Bernoulli equation with p and q as variable coefficients. To our knowledge, the begotten results have not been reported in another literatures. This method could easily explian exact solutions of VcNLSE in two conditions, and also it can be applied to the other NLEEs with space-dependent or time- and space-dependent coefficients.

In addition, to complete our research, we have illustrated the dynamic structures of the obtained wave functions by providing an arbitrary function with different values of the free parameters and we could show that by considering negative values of γ_0 , the wave functions are decreasing by the time and for positive values of γ_0 , the wave functions are increasing as time go by.

Acknowledgements This work is supported by the National Natural Science Foundation of China (11901111).

References

- Bergé, L., Mezentsev, V. K., Rasmussen, J. J., Christiansen, P. L., and Gaididei, Y. B. Self-guiding light in layered nonlinear media. *Optics Letters*, 25(14):1037, (2000). doi: 10.1364/ol.25.001037.
- Serkin, V. N. and Hasegawa, A. Novel soliton solutions of the nonlinear Schrödinger equation model. *Physical Review Letters*, 85(21):4502–4505, (2000). doi: 10.1103/physrevlett.85.4502.
- Zhang, J. Stability of attractive Bose–Einstein condensates. *Journal of Statistical Physics*, 101(3/4):731–746, (2000). doi:10.1023/a:1026437923987.
- Abdullaev, F. K., Caputo, J. G., Kraenkel, R. A., and Malomed, B. A. Controlling collapse in Bose-Einstein condensates by temporal modulation of the scattering length. *Physical Review A*, 67(1):013605, (2003). doi:10.1103/physreva.67.013605.
- Saito, H. and Ueda, M. Dynamically stabilized bright solitons in a two-dimensional Bose-Einstein condensate. *Physical Review Letters*, 90(4):040403, (2003). doi: 10.1103/physrevlett.90.040403.
- Montesinos, G. Stabilization of solitons of the multidimensional nonlinear Schrödinger equation: matter-wave breathers. *Physica D: Nonlinear Phenomena*, 191(3-4):193–210, (2004). doi:10.1016/j.physd.2003.12.001.
- Kruglov, V. I., Peacock, A. C., and Harvey, J. D. Exact solutions of the generalized nonlinear Schrödinger equation with distributed coefficients. *Physical Review E*, 71(5):056619, (2005). doi:10.1103/physreve.71.056619.
- Pérez-García, V. M., Torres, P. J., and Konotop, V. V. Similarity transformations for nonlinear Schrödinger equations with time-dependent coefficients. *Physica D: Nonlinear Phenomena*, 221(1):31–36, (2006). doi:10.1016/j.physd.2006.07.002.
- Ponomarenko, S. A. and Agrawal, G. P. Optical similaritons in nonlinear waveguides. *Optics Letters*, 32(12):1659, (2007). doi:10.1364/ol.32.001659.
- Wang, M., Li, X., and Zhang, J. The (G'/G) -expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. *Physics Letters A*, 372(4):417–423, (2008). doi:10.1016/j.physleta.2007.07.051.
- Zhang, S., Tong, J.-L., and Wang, W. A generalized (G'/G) -expansion method for the mKdV equation with variable coefficients. *Physics Letters A*, 372(13):2254–2257, (2008). doi: 10.1016/j.physleta.2007.11.026.
- He, X.-G., Zhao, D., Li, L., and Luo, H.-G. Engineering integrable nonautonomous nonlinear Schrödinger equations. *Physical Review E*, 79(5):056610, (2009). doi: 10.1103/physreve.79.056610.
- Wazwaz, A.-M. *Partial Differential Equations and Solitary Waves Theory*. Springer Berlin Heidelberg, (2009).
- Bao, W. and Cai, Y. Mathematical theory and numerical methods for Bose-Einstein condensation. *Kinetic and Related Models*, 6:1–135, (2012). doi:10.3934/krm.2013.6.1.
- He, J., Charalampidis, E., Kevrekidis, P., and Frantzeskakis, D. Rogue waves in nonlinear Schrödinger models with variable coefficients: Application to bose–einstein condensates. *Physics Letters A*, 378(5-6):577–583, (2014). doi:10.1016/j.physleta.2013.12.002.
- Roshid, H. and Rahman, M. A. The $\exp(-\phi(\xi))$ -expansion method with application in the (1+1)-dimensional classical Boussinesq equations. *Results in Physics*, 4:150–155, (2014). doi:10.1016/j.rinp.2014.07.006.

- Yu, F. and Yan, Z. New rogue waves and dark-bright soliton solutions for a coupled nonlinear Schrödinger equation with variable coefficients. *Applied Mathematics and Computation*, 233:351–358, (2014). doi:10.1016/j.amc.2014.02.023.
- Kedziora, D. J., Ankiewicz, A., Chowdury, A., and Akhmediev, N. Integrable equations of the infinite nonlinear Schrödinger equation hierarchy with time variable coefficients. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 25(10):103114, (2015). doi:10.1063/1.4931710.
- Yan, Z., Wen, Z., and Konotop, V. V. Solitons in a nonlinear Schrödinger equation with PT-symmetric potentials and inhomogeneous nonlinearity: Stability and excitation of nonlinear modes. *Physical Review A*, 92(2):023821, (2015). doi:10.1103/physreva.92.023821.
- Feng, B. Sharp threshold of global existence and instability of standing wave for the Schrödinger–Hartree equation with a harmonic potential. *Nonlinear Analysis: Real World Applications*, 31:132–145, (2016). doi:10.1016/j.nonrwa.2016.01.012.
- Khan, K. and Akbar, M. A. Solving unsteady Korteweg–de Vries equation and its two alternatives. *Mathematical Methods in the Applied Sciences*, 39(10):2752–2760, (2016). doi:10.1002/mma.3727.
- Yao, Y.-Q., Li, J., Han, W., Wang, D.-S., and Liu, W.-M. Localized spatially nonlinear matter waves in atomic-molecular Bose-Einstein condensates with space-modulated nonlinearity. *Scientific Reports*, 6(1):29566, (2016). doi:10.1038/srep29566.
- Zeng, Z.-F., Liu, J.-G., Jiang, Y., and Nie, B. Transformations and soliton solutions for a variable-coefficient nonlinear Schrödinger equation in the dispersion decreasing fiber with symbolic computation. *Fundamenta Informaticae*, 145(2):207–219, (2016). ISSN 0169-2968. doi:10.3233/FI-2016-1355.
- Bulut, H., Sulaiman, T. A., Baskonus, H. M., and Aktürk, T. On the bright and singular optical solitons to the (2+1)-dimensional NLS and the Hirota equations. *Optical and Quantum Electronics*, 50(3):134, (2018). doi:10.1007/s11082-018-1411-6.
- Feng, B. and Zhang, H. Stability of standing waves for the fractional Schrödinger–Choquard equation. *Computers & Mathematics with Applications*, 75(7):2499–2507, (2018). doi:10.1016/j.camwa.2017.12.025.
- Kaur, L. and Wazwaz, A.-M. Bright – dark optical solitons for Schrödinger–Hirota equation with variable coefficients. *Optik*, 179:479–484, (2019). doi:10.1016/j.ijleo.2018.09.035.
- Liu, J.-G., Osman, M., and Wazwaz, A.-M. A variety of nonautonomous complex wave solutions for the (2+1)-dimensional nonlinear Schrödinger equation with variable coefficients in nonlinear optical fibers. *Optik*, 180:917–923, (2019). doi:10.1016/j.ijleo.2018.12.002.
- Guo, Q. and Liu, J. New exact solutions to the nonlinear Schrödinger equation with variable coefficients. *Results in Physics*, 16:102857, (2020). doi:10.1016/j.rinp.2019.102857.
- Hosseini, K., Mirzazadeh, M., Rabiei, F., Baskonus, H. M., and Yel, G. Dark optical solitons to the Biswas–Arshed equation with high order dispersions and absence of the self-phase modulation. *Optik*, 209:164576, (2020). doi:10.1016/j.ijleo.2020.164576.
- Zhao, X.-H. Dark soliton solutions for a coupled nonlinear Schrödinger system. *Applied Mathematics Letters*, 121:107383, (2021). doi:10.1016/j.aml.2021.107383.