

Tax Competition between Symmetric Countries for Large Asymmetric Investors

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Tax Competition between Symmetric Countries for Large Asymmetric Investors

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Abstract

In a dynamic two-period game between two symmetric countries, we show that a unique subgame-perfect equilibrium arises during the initial stage of the game. A mixed taxation regime arises in the equilibrium where one country adopts a non-preferential taxation regime while its competitor adopts a preferential taxation regime. The country with a non-preferential taxation regime earns a higher tax revenue compared to the country with a preferential taxation regime. A tax holiday does not arise during the initial stage of the game when the size of the mobile capital base that enters during the later stage is considerably larger than the size of the mobile capital base that enters the economy during the initial stage. We provide the complete characterization and proof of the uniqueness of the mixed strategy Nash equilibrium.

JEL classification: F21, H21, H25, H87

Keywords: Preferential Taxation, Non-preferential Taxation, Dynamic Tax Competition, Tax Holidays.

1 Introduction

It is well documented that the corporate tax rates in most countries are decreasing over time. Policymakers and governments across the world are concerned

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with the falling tax revenues from corporate taxation. Authors argue that the reason for low tax rates on most corporate incomes is the falling cost of capital relocation¹. Corporations shift their earnings from the host countries to tax havens to avoid paying taxes. When such tax avoidance is restricted, corporations move their capital to another country that offers lower tax rates. This results in competition among countries for attracting mobile foreign capital². This competition for mobile capital bases leads to a "race to bottom" in corporate taxation where in many countries the tax rate on certain capital bases is close to zero.

Alarmed by falling tax rates and tax revenues various measures have been taken by individual countries through supranational agencies such as Organization for Economic Co-operation and Development (OECD) to reduce competition for mobile capital bases that relocates merely to gain from lower tax rates³. Various measures have been taken by OECD to reduce "tax competition" among independent jurisdictions. One particular measure that has been discussed widely among academicians and policymakers is the "non-preferential" taxation regime. It is argued that countries are increasingly adopting "preferential" taxation regimes where the tax rate on a less mobile domestic capital base is relatively large, but the tax rate on a more mobile capital base is small. OECD encourages countries to adopt non-preferential taxation regimes which restrict countries from setting different tax rates depending on the origin, vintage, or mobility of the capital.

There is a large theoretical as well as empirical literature on tax competition⁴. The debate is not over whether competing countries earn higher tax revenues under a non-preferential taxation regime or a preferential taxation regime⁵. Authors compare tax revenues under two taxation regimes: one where competing countries jointly adopt non-preferential taxation regimes with the alternatives that they continue to exercise discriminating preferential taxation. Depending on the characteristics of tax bases authors have found rationals for jointly adopting non-preferential taxation⁶.

Most papers on tax competition are static. The literature on dynamic tax competition is scarce. Notable papers are Konrad and Kovenock (2009), Kishore (2018, 2021, 2021), Arcalean (2018). Konrad and Kovenock (2009) look at tax competition between two countries under a non-preferential taxation regime when the country that attracts the investor during the initial period has agglomeration advantages during the later period⁷. But Konrad and Kovenock

¹For a survey on capital mobility and tax competition see Fuest, Huber, and Mintz (2003); Zodrow (2010)

²For empirical evidence see for example; Philipp (2021) and Devereux (2008)

³See for example OECD. 1997, Model Tax Convention on Income and on Capital (Paris:OECD Committee on Fiscal Affairs)

⁴For a survey on international tax competition see Keen and Konrad (2013)

⁵See for example; Janeba and Smart (2003), Wilson (2005), Kishore (2019), Wang (2004), Wilson(1999, 2005), Janeba and Smart (2003), Mongrain and Wilson (2018)

⁶To see whether a country has incentives to commit to a non-preferential taxation regime in the absence of any competition see Kishore and Roy(2014), and Kishore(2017, 2021)

⁷See Baldwin and Krugman (2004) for a static model of tax competition when one country

(2009) did not analyze whether countries will commit to non-preferential taxation regimes when they are allowed to choose taxation regimes unilaterally. Kishore (2018) looks at a dynamic two periods model where competing countries compete over one investor in each period with one unit of capital. At the start of the game, competing countries choose whether to commit to a non-preferential or a preferential taxation regime. Kishore (2018) finds that a mixed taxation regime where one country adopts a preferential and the other adopts a non-preferential regime, or a non-preferential taxation regime where both countries adopt non-preferential taxation regimes can arise in equilibrium. Kishore (2021) looks at a case similar to Konrad and Kovenock (2009) where the country that attracts the investor during the initial period has an agglomeration advantage during the later period. Kishore (2021) also looks at the case when competing countries choose taxation regimes sequentially. Kishore (2021) finds that when agglomeration effects are not very large, the first move chooses a non-preferential taxation regime. On the other hand, when agglomeration effects are large, the first mover chooses a preferential taxation regime. Kishore (2021) looks at the case when two investors enter during the initial period. Kishore (2021) finds that the competition over multiple strategic investors during the initial period reduces competition and increases tax revenues.

In the papers discussed above, the investors that enter the economy during the initial period and during the later period own identical units of capital. In this paper, the investor that enters during the later period can be larger or smaller compared to the investor that enters during the initial period. When the investor that enters during the later period owns more capital, the competition to attract new investments reduces tax rates in the later period. This makes a country that adopts a non-preferential regime a more attractive destination for investments during the initial period. On the other hand, the return on investment in a country with a preferential taxation regime is fully expropriated during the later period. Therefore, when the size of the capital base that enters the economy during the later period is considerably large, a country with a non-preferential taxation regime has to offer relatively small tax rebates during the initial period to attract the investor. In particular, when the size of the capital base that enters during the later period is large enough, the country with a non-preferential taxation regime sets a strictly positive tax rate during the initial period and attracts the investor.

Another implication of this result is that, when at least one country adopts a non-preferential taxation regime, competition over a much larger mobile capital base during the later period mitigates the hold-up problem. The result is similar to Kehoe (1989), who finds that competition between governments can solve the hold-up problem.

has an agglomeration advantage

2 Model

We consider two identical countries/jurisdictions indexed by $i \in (A, B)$. Economy lasts for two periods, 1 and 2. Countries (A, B) compete to attract investments from outside their jurisdictions. Competing countries have no domestic capital at the beginning of period 1. At the beginning of period 1, a single investor (who owns a unit of capital) enter the market outside of the jurisdictions of competing countries. At the beginning of period 2, an investor with β units of capital enters the market outside of the jurisdictions of country A and B . For simplicity, we assume that outside the two competing countries, the return on invested capital is equal to 0. The return on investments in country A and B is equal to r in each period. We assume that investments in country A (country B) are fully sunk. We also assume that there is no cost of relocating to country A or country B from outside of two countries. We assume that competing countries cannot commit to future tax rates. Therefore, at the beginning of each period, competing countries announce tax rates applicable for that period.

We analyze this dynamic tax competition between two symmetric countries when at the beginning of the game countries simultaneously choose whether to commit to a non-preferential or a preferential regime. If a country commits to a non-preferential taxation regime, it cannot set discriminating tax rates depending on the origin of the capital (domestic and foreign) or capital bases of different vintages (old investments and potential new investments). Under a preferential taxation regime a country is free to set different tax rates for different capital bases. We assume that governments maximize tax revenues, and investors maximize net returns on investments after tax payments. We further assume neither governments not investors discount future income. The stages of the game can be described as below:

Stage 1: Competing countries simultaneously decide whether to adopt a non-preferential or a preferential taxation regime for the entire duration of the game. The same is observed by governments and investors. We consider equilibria in pure strategies at this stage of the game.

Stage 2: At the beginning of period 1, competing countries simultaneously announce tax rate applicable in period 1. Even if a country adopts a preferential taxation regime at the initial stage it announces a single tax rate because there is no domestic capital base. The investor observes the prevailing tax rates and taxation regimes, and make an investment in country A or country B .

Stage 3: At the beginning of period 2, competing countries announce tax rates applicable for period 2. If a country has domestic capital (investments from period 1) and has a preferential taxation regime, then it announces tax rates applicable for the domestic and foreign capital bases. A country announces a single tax rate when it has no domestic capital (no investments during the initial period), or it adopts a non-preferential taxation regime at the initial stage. The new investor observe prevailing tax rates and make an investment either in country A or B . Governments receive taxes at the end of period 2.

We look at the subgame-perfect Nash equilibrium of this three-stage dynamic game.

3 Non-preferential Regime

In this section we consider the case when competing countries jointly adopt non-preferential taxation regime at the initial stage of the game. First, we look at the outcome of period 2.

3.1 Non-preferential Regime: Period Two

Without a loss of generality, suppose country A attracts the investor in period 1. Let t_a and t_b be the prevailing tax rates in period 2 in country A and country B , respectively. The tax revenue of country A in period 2, TNP_a^2 , is

$$TNP_a^2 = \begin{cases} (1 + \beta)t_a, & \text{if } t_a \leq t_b \\ t_a, & \text{if } t_a > t_b \end{cases} \quad (1)$$

Here we assume that when the investor is indifferent between investing in country A and country B , it invests in country A . When country A sets a tax rate t_a such that $t_a \leq t_b$, it receives taxes from the domestic capital base and also attracts the new investor. When country A sets a tax rate larger than the tax rate of country B , it receives taxes only from the domestic capital base. The tax revenue of country B in period 2, TNP_b^2 , is

$$TNP_b^2 = \begin{cases} \beta t_b, & \text{if } t_b < t_a \\ 0, & \text{if } t_b \geq t_a \end{cases} \quad (2)$$

Because country B has no domestic capital base, it only receives taxes when it attracts the new investor, i.e., when $t_a < t_b$. Lemma 1 describes the equilibrium outcome in period 2.

Lemma 1 *A pure strategy Nash equilibrium does not exist in period 2 when both countries adopt non-preferential taxation regimes. However, a unique mixed strategy Nash equilibrium exists. The equilibrium tax revenues of country A and B are 1 and $\frac{\beta}{1+\beta}$, respectively. The competing countries randomize over the common support $[\frac{1}{1+\beta}, 1]$. The distributions of taxes over the supports of country A and country B are $F_a \equiv 1 - \frac{1}{t_b(1+\beta)}$ and $F_b \equiv 1 + \frac{1}{\beta}(1 - \frac{1}{t_a})$, respectively. There is a probability mass, m , at the supremum of the support of country A , where $m \equiv \frac{1}{1+\beta}$. The expected tax rate in country A is, $E(t_a) \equiv \frac{1}{1+\beta} \log(1 + \beta) + \frac{1}{1+\beta}$. The expected tax rate in country B is, $E(t_b) \equiv \frac{1}{\beta} \log(1 + \beta)$.*

Proof. See Appendix.

Note that country A can obtain a tax revenue of 1 by setting the maximum tax rate on the domestic capital base and forgo foreign investments. If country sets a tax rate t_a and attracts the new investor, the total tax revenue is $t_a(1 + \beta)$. Therefore, the minimum tax rate country A sets in period 2 is $\frac{1}{1+\beta}$. Country B has no domestic capital in period 2. Therefore, country B is willing to undercut

the tax rate of country A when the tax rate is strictly positive. If country B sets the tax rate $\frac{1}{1+\beta}$, it attracts the new investor with probability 1 and receive the amount $\frac{\beta}{1+\beta}$ in tax revenues. Next, we look at the outcome in period 1.

3.2 Non-preferential Taxation: Period One

The tax revenue of a country in period 2 is 1 when it attracts the investor in period 1. On the other hand, the tax revenue is equal to $\frac{\beta}{1+\beta}$ when it does not attract the investor. Therefore, the revenue gain in period 2 from attracting the investor in period 1 is $\frac{1}{1+\beta}$. Therefore, the maximum tax rebate a country offers in period 1 to attract the investor is equal to $(\frac{1}{1+\beta})$. The investor invests in one of the two countries when both countries set the tax rate equal to $-(\frac{1}{1+\beta})$. If the investor invests in country i then the sum of the tax revenues from two period, TR_i , is $\frac{\beta}{1+\beta}$. If a country does not attract the investor in period 1 then it receives taxes only in period 2, and that is also equal to $\frac{\beta}{1+\beta}$. Lemma 2 describes the outcome of tax competition in period 2.

Lemma 2 *When both countries jointly adopt non-preferential taxation regime, the tax revenue of the competing countries is equal to $\frac{\beta}{1+\beta}$. In a pure strategy Nash equilibrium, the unique tax rate competing countries set in period 1 is equal to $-(\frac{1}{1+\beta})$.*

Proof. Proof is obvious.

In the next section we analyze the case when one country adopts a non-preferential regime, and the other country adopts a preferential taxation regime.

4 Mixed Taxation

Without loss of generality suppose country A adopts a non-preferential taxation regime, and country B adopts a preferential taxation regime. First, we look at the outcomes in period 2.

4.1 Mixed Taxation: Period Two

When country A adopts a non-preferential taxation regime then the outcome in period 2 is described by Lemma 1. Now we consider the case when country B attracts the investor in period 1. Country B sets different tax rates on the domestic capital base (investments from period 1) and the foreign capital base (new investments in period 2). The domestic capital base is immobile, therefore, country B sets the tax rate of 1. Because the new investor is perfectly mobile, a Bertrand type competition drives down the tax rate on new investments equal to 0. Therefore, country B receives 1 and 0 as tax revenues from the domestic and foreign capital bases, respectively. Lemma 3 describes the outcome formally.

Lemma 3 *Under a mixed taxation regime, the tax revenue of the country with a preferential taxation regime is equal to 1 when it attracts the investor in period 1. In a unique pure strategy Nash equilibrium it sets tax rates 1 and 0 on the domestic and the foreign capital base, respectively. The tax revenue of the other country is equal to 0. It sets the tax rate of 0 on the foreign capital base.*

Next we look at the outcome in period 1.

4.2 Mixed Taxation: Period One

When country A (the country with a non-preferential taxation regime) attracts the investor then its tax revenues in period 2 is 1, and the tax revenue of country B (the country with a preferential taxation regime) is equal to $\frac{\beta}{1+\beta}$. When country B attracts the investor in period 1 then its tax revenues in period 2 is 1, and the tax revenue of country A is equal to 0. The revenue gain to country A in period 2 when it attracts the investor in period 1 is 1. The revenue gain to country B in period 2 when it attracts the investor in period 1 is $1 - \frac{\beta}{1+\beta} \equiv \frac{1}{1+\beta}$. Therefore, country A is willing to offer a larger tax rebate in period 1 to attract the investor.

If the investor invests in country A then the expected amount it pays in taxes in period 2, $E_2(t)$, is $\frac{1}{1+\beta}(1 + \log(1 + \beta))$. $E_2(t)$ is strictly less than 1 when $\beta > 0$, and it monotonically decreases when β increases. On the other hand, if the investor invests in country B then it pays 1 in taxes in period 2. Therefore, country A can offer a smaller tax rebate of $(1 - \frac{1}{1+\beta}(1 + \log(1 + \beta)))$, in period 1 compared to country B and attract the investor.

The minimum tax rate country B sets in period 1 is equal to the revenue gain from attracting the investor in period 2, that is, $-(\frac{1}{1+\beta})$. Therefore, the minimum tax rate country A sets in period 1, t_{min}^a , is $-(\frac{1}{1+\beta}) + (1 - \frac{1}{1+\beta}(1 + \log(1 + \beta)))$. After rearranging we obtain:

$$t_{min}^a = -(\frac{1}{1+\beta}(1 + \log(1 + \beta) - \beta)). \quad (3)$$

The minimum tax rate, t_{min}^a , is depicted below in Figure 1.

From figure (1), we observe that country A sets the tax rate, t_{min}^a , in period 1 and attracts the investor. Therefore, the outcome in period 2 is described by lemma 1. The tax revenue of country A in period 2 is 1. Therefore, the total tax revenue of country A over two periods, TRM_a , is equal to:

$$TRM_a = 1 - (\frac{1}{1+\beta}(1 + \log(1 + \beta) - \beta)). \quad (4)$$

Country B does not attract the investor with a positive probability in period 1 and receives taxes only in period 2. As described in lemma 1, the tax revenue of country B is $\frac{\beta}{1+\beta}$.

Figure (2) depicts the total tax revenues of country A and country B . When the size of the mobile capital base in period 2, β , is strictly positive, the tax

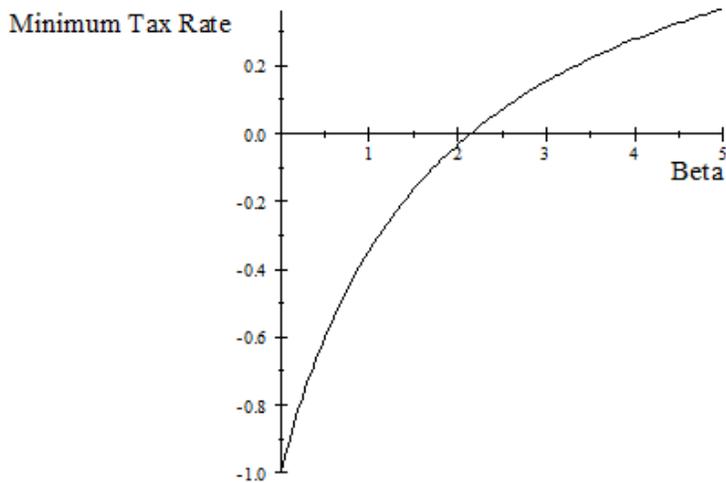


Figure 1: The tax rate in period 1 under a mixed taxation regime

revenues of two countries monotonically increases when β increases. The tax revenue of country A (the country with a non-preferential taxation) is strictly higher than the tax revenue of country B . Moreover, the difference between tax revenues of country A and country B also increases when β increases. The competition for the foreign mobile capital base is high when the β is large. Due to increase in competition, the tax rates in period 2 falls. When the expected tax rates in period 2 is smaller, country A offers comparatively smaller tax rebate in period 1 and attracts the investor. This increases the tax revenue of country A . Lemma 3 describes the equilibrium formally.

Lemma 3 *Under a mixed taxation regime the equilibrium tax revenues of country A and country B are TRM_a and $\frac{\beta}{1+\beta}$, respectively. Country A offers a tax rebate of t_{min}^a in period 1 and attracts the investor. The total tax revenues of both countries are increasing functions of the size of the mobile capital base in period 2.*

5 Preferential Taxation

In this section, we consider the case when both countries adopt preferential taxation regimes. The outcomes are relatively simple, therefore, we do not discuss this scenario in detail. It is instructive to discuss the outcome under a mixed taxation regime discussed in section (4). The outcome in period 2 is similar to the case under a mixed taxation regime when the country with a preferential taxation regime (country B) attracts the investor in period 1. Country B re-

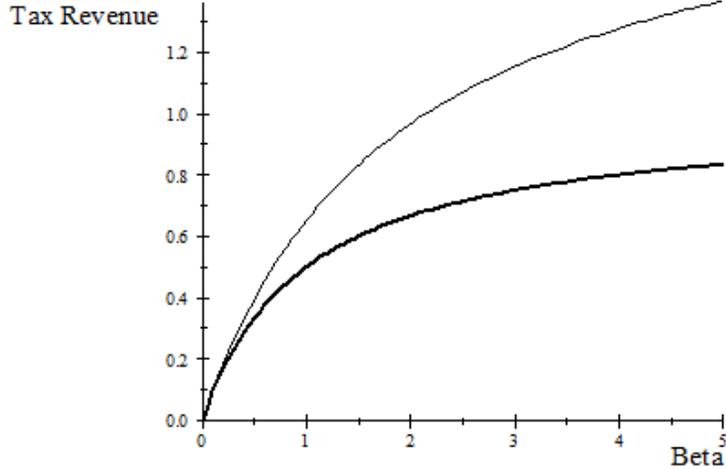


Figure 2: The tax revenues of country A (thin line) and country B under a mixed taxation regime.

ceives the tax revenue of 1 from the domestic capital base, and the tax revenue from the foreign capital is 0. Therefore, the minimum tax rebate competing countries offers in period 1 is equal to 1. Therefore, the gain in tax revenues in period 2 are completely offset by the tax rebates they offer in period 1. Lemma 4 describes the equilibrium tax revenues of competing countries when both countries adopt preferential taxation regimes.

Lemma 4 *The tax revenue of competing countries is equal to zero when both countries adopt preferential taxation regimes during the initial stage of the game.*

In the next section, we compare the outcomes under three taxation regimes.

6 Comparison

From lemma 1 we observe that when both countries adopt non-preferential taxation regime, the equilibrium tax revenue of competing countries is equal to $\frac{\beta}{1+\beta}$. From lemma 3 we observe that under a mixed taxation regime, the equilibrium tax revenue of the country with a preferential taxation regime is equal to $\frac{\beta}{1+\beta}$. Therefore, the tax revenue of a country with a preferential taxation regime under a mixed taxation regime is equal to that a country earns when both countries jointly adopt non-preferential taxation regimes. It is also clear from lemma 3 that the equilibrium tax revenue of the country with a non-preferential taxation regime under a mixed taxation regime is larger than the country with a preferential taxation regime. Figure (2) also depicts the equilibrium tax revenues of the

country with a non-preferential taxation regime under a mixed taxation regime, and the tax revenue of a country when both countries adopt non-preferential taxation regime.

The equilibrium tax rate in period 1 when both countries jointly adopt non-preferential regimes is equal to $-\frac{1}{1+\beta}$. The equilibrium tax rate in period 1 under a mixed taxation regime, t_{min}^a , is given by (3). Under both taxation regimes, the equilibrium tax rates in period 1 increase with β . Figure (3) below depicts the equilibrium tax rates in period 1 under a mixed taxation regime and a non-preferential taxation regime. We observe that the equilibrium tax rate in period 1 is strictly lower under a mixed taxation regime compared to a non-preferential taxation regime when the size of the mobile capital base in period 2 (β) is strictly positive. Proposition 1 describes the subgame-perfect outcome of the game formally. The proof is obvious from the above discussion.

Proposition 1 *In a unique subgame-perfect outcome of the game in pure strategies at the initial stage, one of the competing countries adopts a non-preferential taxation regime, and the other adopts a preferential taxation regime.*

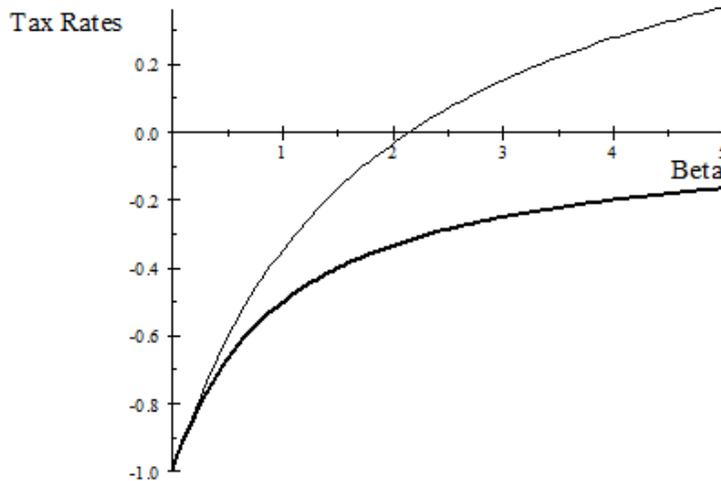


Figure 3: The equilibrium tax rates in period 1 under a mixed taxation regime (thin line), and under a non-preferential taxation regime.

7 Conclusion

In a dynamic game where competing countries simultaneously adopt a preferential or a non-preferential taxation regime, we show that in a unique subgame-perfect equilibrium a mixed taxation regime arises where one of the competing countries adopts a non-preferential taxation regime, and the other adopts a

preferential taxation regime. The tax revenue of a country with a preferential regime under a mixed taxation regime is equal to that a country earns when both countries jointly adopt a non-preferential taxation regime. The equilibrium tax revenues of competing countries increase monotonically with the increase in the size of the mobile capital base that enters the market during the later period. The equilibrium tax rate during the initial period is larger under a mixed taxation regime compared to a scenario when both countries jointly adopt non-preferential regimes. Moreover, the equilibrium tax rate in period 1 is strictly positive when the size of the mobile capital base that enters the market during the later period is considerably large. This result is novel as far as we know. A future study should look at the case of multiple large investors during the initial as well as later stages of the game. Future works should also try to answer the question not addressed in this paper; under which taxation strategies a country attracts more foreign investments.

”Data sharing is not applicable to this article as no new data were created or analyzed in this study.”

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8 Appendix

Proof of Lemma 1. It is straight forward to verify that for the given strategy pair in lemma 1, competing countries earn equal tax revenues everywhere on their support. Moreover, no country can set a tax rate lower than $\frac{1}{1+\beta}$ and do better. We follow Narasimhan (1988) to prove the uniqueness of the mixed strategy Nash equilibrium. We prove lemma 1 in *Steps 1 - 5*.

Step 1. There is no Nash equilibrium in pure strategies.

Proof. Suppose (t_a^*, t_b^*) is an equilibrium pair of Nash strategies. Then there is no t_b such that $TNP_b^2(t_a^*, t_b) > TNP_i^2(t_a^*, t_b^*)$.

Suppose there is a symmetric pure strategy Nash equilibrium such that $t_a^* = t_b^* > 0$. From (1) we have

$$TNB_b^2(t_a^*, t_b^*) = 0 \quad (5)$$

Let $t_b = t_b^* - \epsilon, \epsilon > 0$. Then

$$TNB_b^2(t_a^*, t_b) = \beta t_b > 0. \quad (6)$$

Since $t_b^* > 0$, such an ϵ always exists. Therefore, there is no pure strategy Nash equilibrium such that $t_a^* = t_b^*$. Second, suppose (t_a^*, t_b^*) is an equilibrium pair of Nash strategies such that $t_a^* > t_b^* \geq 0$. From (1) we have

$$TNB_b^2(t_a^*, t_b^*) = \beta t_b^*. \quad (7)$$

take $t_b = t_b^* + \frac{t_a^* - t_b^*}{2}$. Note that $t_b < t_a^*$. From (1) we have

$$TNB_b^2(t_a^*, t_b) = \beta t_b. \quad (8)$$

As $TNB_b^2(t_a^*, t_b) > TNB_b^2(t_a^*, t_b^*)$, the strategy pair (t_a^*, t_b^*) such that $t_a^* > t_b^* \geq 0$ is not a Nash equilibrium. Similarly, we can show that a strategy pair (t_a^*, t_b^*) such that $t_b^* > t_a^*$ is not a Nash equilibrium. Therefore, there is no Nash equilibrium in pure strategies. In *step 2* we show that the support of the mixed strategy Nash equilibrium is convex. Let strategy sets S_a^* and S_b^* constitute the supports of the mixed strategy Nash equilibrium.

Step 2. The strategy sets S_a^ and S_b^* are convex.*

Proof. First, we show that $T = S_a^* \cap S_b^*$ is convex.

T is convex. Let $\hat{T} = \inf(T)$ and $\hat{T} = \sup(T)$. To show that T is convex, we show that there is no "holes" in T . That is, there is no interval $I = (T^k, T^h)$ such that, for $\hat{T} < T^k < T^h < \hat{T}$ and for $T \in I, T \notin T$. This can happen one of the competing countries has the support over T and the other does not or neither countries has support over the interval I .

First, we show that if country A sets $t_a \in I$ with probability zero, then so does country B . Let t^1 and t^2 be defined as

$$t^1 \in S_a^* \text{ and } t^1 = \sup(T|T < T^k)$$

$$t^2 \in S_a^* \text{ and } t^2 = \inf(T|T > T^h)$$

The tax revenue of country B in period 2 when it sets $t_b \in I$ is $TNB_b^2 \equiv \beta t_b(1 - F_a(t_b))$ is increasing in t_b for $t_b \in I$ because country A sets a tax rate in I with zero probability. Therefore, country B is better off setting t^2 with probability $[F_b(t^2) - F_b(t^1)]$ and has no mass over the set I . Similarly, we can also show that when country B does not set $t_b \in I$ with a positive probability then so does country A .

Next, we consider the case that neither country is randomizing over the set I . The tax revenue of country B when it sets t^1 is

$$TNB_b^2 = t^1 \beta (1 - F_a(t^1)).$$

The tax revenue of country B when it sets t^2 is

$$TNB_b^2 = t^2 \beta (1 - F_a(t^2)).$$

Because country A does not set $t_a \in I$ with a positive probability, we have $F_a(t^1) = F_a(t^2)$. Country B is better off setting t^2 compared to t^1 because $t^2 > t^1$. In a mixed strategy Nash equilibrium, a country should obtain an equal tax revenue everywhere on the support. Therefore, we have a contradiction.

Similarly, we can show that there are no holes in $T' = S_i^* - S_i^* \cap S_j^*$.

Step 3. Neither country can have a mass point in the interior or at the lower boundary of the other's support, nor can either country have a mass point at the upper boundary of other's support if that boundary is a mass point for the other firm.

Proof. Let $\hat{T}_i = \inf(S_i^*)$ and $\hat{T}_i = \sup(S_i^*)$. Note that country A can obtain a tax revenue of 1 with certainty when it sets $t_a = 1$. Therefore, the lowest tax rate country A sets in any mixed strategy Nash equilibrium is strictly positive. Therefore, the lowest tax rate country B sets in any equilibrium is also strictly positive. Therefore, we have $\hat{T}_i > 0$. Assume to the contrary that country B sets t_b^* , $\hat{T}_a < t_b^* < \hat{T}_a$, with probability m_b . We can show that country A can increase its tax revenue by changing its strategy.

From *step 2* we know that there are no "holes" in the strategy set of country $i \in (A, B)$. Consider the tax revenues of country A when it sets $(t_b^* - \epsilon)$ and $(t_b^* + \epsilon)$, $\epsilon > 0$. These are respectively given by

$$(t_b^* - \epsilon) + \beta(t_b^* - \epsilon)[1 - F_b((t_b^* - \epsilon))] \quad (9)$$

and

$$(t_b^* + \epsilon) + \beta(t_b^* + \epsilon)[1 - F_b((t_b^* + \epsilon))] \quad (10)$$

Subtracting (10) from (9) we obtain

$$-2\epsilon - 2\beta\epsilon + \beta t_b^* m + \beta\epsilon(F_b(t_b^* - \epsilon) + F_b(t_b^* + \epsilon)). \quad (11)$$

For a small enough $\epsilon > 0$, this is strictly positive, suggesting that country A can increase its revenue by shifting some mass to the lower of t_b^* from the above of t_b^* . Contradicting that we have a mixed strategy Nash equilibrium. Now suppose $t_b^* = \hat{T}_a$ and country A has a mass point at \hat{T}_a . Again country A can do better by shifting the mass below t_b^* , and setting t_b^* with zero probability. A similar argument can also be given when $t_b^* = \hat{T}_a$. The only case left is when only one country has a mass point at the *supremum* of the support.

Step 4. The strategy sets S_i^ and S_j^* are identical when neither country has a mass point. If country i has a mass point at \hat{T}_i , then country j set \hat{T}_i with zero density.*

Proof. Consider the case of no mass points. Assume to the contrary that $S_a^* \subset S_b^*$. From the earlier discussion we know that the interval where country A has no support will either be on the upper or the lower end. If it to the lower end the tax revenue of country B then country A move probability mass up and set $\inf(T_b^*)$ and increase its tax revenue. Suppose the interval of no probability mass to the upper end of the support of country B . In this case country B can move probability mass distributed over the set $(\sup(T_a), \sup(T_b))$ to $\sup(T_b)$ and do better. But in this case the support of country B is not convex.

Now consider the case when one of the countries has a mass point at the *supremum* of its support. It is easy to argue that the other country does better by setting a tax rate arbitrarily close but lower than the *supremum*. We also know that the strategy sets are convex. Therefore, we argue that the strategy sets are identical. A country can have a mass point only at the *supremum* of its support, and that is possible when the other country does not have a probability mass at the *supremum* of the support. In the next step we argue that the *supremum* of the support is equal to 1, that is the maximum permissible tax rate.

Step 5. $\sup(S_a^) = \sup(S_b^*) = 1$.*

Proof. In our proposed strategies for a Nash equilibrium, country A has a mass point at the *supremum* of its support. Assume to the contrary that $\sup(S_a^*) = r < 1$. When country A sets $t_a = r$, country B undercuts the tax rate of country A with probability one. From (1) we have, The tax revenue of country A in this case is r . A country earns an equal tax revenue everywhere on its support. Therefore, the equilibrium tax revenue of country A is equal to r . Consider a strategy such that country A sets $t_a = 1$ with probability one. Irrespective of the strategies of country B , the tax revenue of country A is 1, and that is strictly greater than r . Contradicting that we have a mixed strategy Nash equilibrium. Therefore we argue that $\sup(S_a^*) = \sup(S_b^*) = 1$.

From *steps (1 - 5)* it is clear that the *supremum* of equilibrium strategies is equal to 1. It is also easy to argue that the minimum tax rate country A sets in

any mixed strategy Nash equilibrium is $\frac{1}{1+\beta}$. Moreover, it is easy to argue that country A can have a mass point at 1, but country B cannot have a mass point at 1. If country B has a mass point at 1 then country A cannot have a mass point at 1. In this case country A undercuts the tax rate of country B with probability one. The tax revenue of country B is equal to zero. Contradicting that we have an equilibrium. Therefore, we conclude that the strategies described in lemma 1 constitutes a unique mixed strategy Nash equilibrium of the game.