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Research Article

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Posted Date: November 18th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1054612/v1>

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VERTICAL PLANE BUOYANT JETS AND FOUNTAINS IN A UNIFORM STAGNANT AMBIENT REVISITED

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Abstract

Planar, vertical buoyant jets are of particular interest, both for research and practical purposes for being related to the disposal of the effluent from wastewater treatment plants or saline, a by-product from desalination plants into a body of stagnant fluid. Analytical, closed form solution is derived for plane buoyant jets based on a buoyant jet width parameter proposed by List and Imberger (1973), and compares to earlier laboratory experiments satisfactorily. The derived entrainment coefficient as a function of the local Richardson number of the flow, takes two asymptotic values for jet-like and plume-like flows, while in fountains it takes values lower than that in jets. Laboratory experiments were performed to measure the penetration height of vertical plane fountains with initial Froude number in the range 20 to 130 using shadowgraph and Planar Laser Induced Fluorescence (PLIF) techniques. Interest was focused on the maximum and terminal, steady-state penetration height before the flow direction reversed. The flow was found to be in a state of unstable equilibrium, as it deviated from the vertical axis, swinging on either side. The equations of motion have been solved numerically using the derived entrainment coefficient function, and the results are congruent to earlier and present experiment for vertical fountains.

Keywords

Plane buoyant jet; plane fountain; uniform ambient; Froude number; entrainment coefficient, maximum penetration, terminal height

1. INTRODUCTION

Buoyant jets are discharges through holes or slots into a quiescent fluid of different density and are part of the family of free shear flows. This type of flows occur frequently in nature (volcanos, fires) and it are also used by engineers to dilute fluids efficiently into larger volumes by means of outfalls, in heating or cooling systems of enclosed spaces, etc. Buoyant jets enter the recipient with initial velocity, and density that is usually different from that of the surrounding fluid, in other words with non-zero momentum and buoyancy fluxes at the source. The jet volume and momentum fluxes vary throughout the flow field because of the initial buoyancy flux at the source. The buoyancy forces may have the same or opposite direction with respect to the original momentum, so that the flow can be positively or negatively buoyant. In the case of a positively buoyant flow the flow becomes eventually plume-like, while in a negatively buoyant one (fountain) the buoyancy acts against gravity, the jet vertical momentum flux at some elevation above the source becomes zero and the flow direction is reversed.

A two dimensional (2D) positively buoyant jet that discharges vertically in a uniform calm ambient builds up momentum that is due to its buoyancy flux, and eventually becomes a plume when the buoyancy induced momentum is the driving force, if compared to its initial momentum flux. In a two dimensional negatively buoyant jet, the heavier jet fluid enters vertically upwards the ambient fluid and is driven away above the source by its initial momentum, entraining simultaneously ambient fluid. Its course continues until it reaches a maximum height $Z_{\max}$, whereas the jet momentum vanishes due to mixing with lighter ambient fluid and which is named the maximum penetration height of the jet. Then the flow direction reverses and returns to the original elevation which it was ejected from, while the maximum elevation reached by the flow oscillates around the terminal rise height Z_m that is lower than $Z_{\max}$. The volume of the continuously rising fluid blocks the descending flow, eventually driving it to either side of the vertical jet axis. Thus, the vertical jet of negative

buoyancy does not return symmetrically to the outflow elevation due to instability, but like a fountain, it will sometimes fall on the one side of the rising vein and sometimes on the other side. This instability was recently studied by Hunt et al. (2019) who have shown that part of the reversing flow may fall to one side and simultaneously part the rest of the reversing flow to the other side of ejected vein.

The fundamentals of buoyant jets have been investigated by Rouse, Yih and Humphreys (1952), Morton, Taylor and Turner (1956) and Priestley and Ball (1955). Positively buoyant jets out of a slot have been studied in the past by Rouse et al. (1952), Kotsovinos (1975) and Ramaprian and Chandrasekhara (1985, 1989). Chu and Baines (1989) measured directly the mass flux in quasi two dimensional buoyant jets, while Jones and Baddour (1991) have measured the dilution in heated water buoyant jets. Recently, Paillat and Kaminski (2014 a, b) have performed experiments and studied thoroughly the entrainment in plane jets and plumes. Analyzing earlier momentum jet measurements they have found the entrainment coefficient for Gaussian time-averaged velocity distribution to vary in the range 0.03 to 0.072 while their measurements have shown the coefficient to vary from 0.03 to 0.036. These variations have been attributed to the source conditions and the magnitude of Reynolds number. The proposed entrainment coefficient is 0.030 for momentum jets and 0.072 in plumes, results that are different from 0.055 and 0.11 (Kotsovinos, 1975) plume and 0.045, 0.11 by Ramaprian and Chandrasekhara (1985, 1989).

The dilution obtained by buoyant jets is greater if they are injected at an angle with respect to vertical, because it is an increasing function with trajectory length. Thus, using vertical jets one may obtain the minimum dilution as a result, but despite of the weakness of this mechanism, it is quite often used as a method for the disposal of saline from desalination plants. Apart from this practical application, this flow is also of great research interest due to its peculiarities. What particularly determines the vertical jet of negative buoyancy is the

instability of the flow, due to which it swings around the vertical axis, while the terminal rise height Z_m is also fluctuating.

A limited number of experiments regarding vertical two-dimensional negatively buoyant jets are available in the literature. Following the formulation by Turner (1966), Baines et al. (1990) and Zhang and Baddour (1997) have measured the maximum and terminal rise height of vertical fountains at large Froude numbers in a uniform ambient. Bloomfield and Kerr (1998) investigated fountains in a linear density stratified ambient fluid. Srinarayana et al. (2010) measured the terminal height in fountains with low initial Reynolds number and Froude number in the range 0.4 to 42. The rise height at low Froude numbers did not comply with dimensional arguments. Recently Hunt et al. (2019) and Papakonstantis and Mylonakou (2021) have measured the terminal height of a 2D vertical fountain. The first in an experiment confined by side walls with aspect ratios of the slot width to slot length 300 and 150, while the last did not use side walls and the slot aspect ratio was 100.

Earlier investigations of two-dimensional vertical, negatively buoyant jets, showed significant differences in the results. There is disagreement regarding the structure of the vein and the measurement of the penetration height, that is the inconsistency in the parameterization among the different researchers, making it difficult to compare the results. This inconsistency stems from the fact that Baines, et al. (1990), Zhang and Baddour, (1997), Srinarayana, et al. (2010) and Hunt, et al. (2019) used the half-width of the slot to calculate the dimensionless parameters. This methodology, however, goes against the physical characteristic scales derived by dimensional analysis and used by earlier researchers like Kotsovinos (1975) and Fischer et al. (1979) and the most recent one by Papakonstantis and Mylonakou (2021). Despite this significant difference, their experiments are used to compare the results with appropriate modifications. It should be noted here that the Froude number used in some of these investigations has been limited to around 50.

Scope of this work is to derive closed form solution for vertical, positively buoyant plane jets that apply in the full extend, pure jets, transition and plumes. In fact from the equations of motion we will derive explicit relationships for the volume and momentum flux as a function of the distance from the source using a width parameter proposed by List and Imberger (1973). Then we will derive an equation for the entrainment coefficient that can be used for the computation of such flows. We will present experiments of vertical fountains regarding the penetration elevation for a wide range of initial Froude numbers, and attempt to verify the findings using the entrainment coefficient function derived for positively buoyant jets, via a one-dimensional mathematical model that was developed to determine the terminal penetration height.

2. THEORY

2.1 Analytical solution of vertical 2D buoyant jets

The flow under investigation shown in figure 1 is a vertical plane (two-dimensional, 2D) buoyant jet of density ρ_o , flowing through a slot of width d and infinite length with uniform velocity W into a homogeneous, calm ambient fluid of density ρ_a . The flow is assumed to be turbulent from the source, the density difference between jet and ambient small, and the pressure hydrostatic throughout the flow field.

Figure 1. Schematic of a two-dimensional jet.

The initial kinematic parameters of the flow i.e. the volumetric flow rate q , the specific (per unit mass) momentum flux M and the specific buoyancy flux B expressed per unit length of the slot are

$$q = Wd, \quad M = qW \quad \text{and} \quad B = \frac{\rho_a - \rho_o}{\rho_o} gq = g_o' q \quad (1)$$

with dimensions L^2/T , L^3/T^2 and L^3/T^3 respectively (L and T refer to length and time respectively).

If $\rho_o = \rho_a$ then $B=0$ and the flow is characterized as a pure jet, whilst if $\rho_o < \rho_a$ and $M=0$ the flow is characterized as a pure plume. If $\rho_o \neq \rho_a$ and $M \neq 0$ the flow is characterized as a buoyant jet (forced plume) that is positively buoyant when $\rho_o < \rho_a$ and negatively buoyant when $\rho_o > \rho_a$. If $\rho_o < \rho_a$ and the initial momentum M is dominant, the flow behaves as jet-like, while far from the source where the momentum built from buoyancy is much greater than the initial M the flow is plume-like. The regime of the flow where the initial momentum M is in balance with the momentum produced by the buoyancy is characterized as a buoyant jet (forced plume), and is located in between the two asymptotic jet-like and plume-like regimes. When $\rho_o > \rho_a$ and the initial momentum M is dominant, the flow is characterized as a fountain.

In buoyant 2D jets we make the assumption that the distributions of the time-averaged (mean) velocity w and excess density $\Delta\rho$ have either top-hat or Gaussian distributions. Assuming top-hat distributions with jet half-width b and velocity w , the specific mass $\mu(z)$, momentum $m(z)$ and buoyancy $\beta(z)$ fluxes of the jet at elevation z above the source are written as, $\mu(z) = 2bw$, $m(z) = 2bw^2$ and $\beta(z) = 2bwg_o'$ respectively, where $g_o' = (\Delta\rho/\rho_o)g$ is the mean deficit or excess buoyancy at elevation z . Then the equations of motion are written as (α is the entrainment coefficient)

$$\frac{d\mu}{dz} = 2\alpha \frac{m}{\mu}, \quad \frac{dm}{dz} = \frac{\mu\beta}{m}, \quad \frac{d\beta}{dz} = 0 \quad (2)$$

From the initial flow parameters two characteristic length scales l_q and l_m can be defined (Fischer, et al., 1979) as

$$l_q = \frac{q^2}{M} = d \quad \text{and} \quad l_m = \frac{M}{B^{2/3}}, \quad (3)$$

the latter being a measure for determining the jet-like (momentum driven) or plume-like (buoyancy driven) character of the flow. When l_m is large (z/l_m is small) the flow is driven by the initial jet momentum, while when l_m is small (z/l_m is large) the flow is driven by the momentum produced from buoyancy force.

List and Imberger (1973) have used dimensional arguments and shown that in 2D vertical, positively buoyant jets the dimensionless mononym $C_p = \mu/(zm)^{1/2}$ is invariant. This has been plotted in figure 2 from the measurements by Kotsovinos (1975) and Ramaprian and Chandrasekhara (1985, 1989) hereinafter referred to as RC, and takes an average value around 0.58 throughout the full range of a buoyant jet, i.e. jet-like, plume-like and transition. Making the assumption that the buoyancy flux $\beta(z)$ is conserved throughout the flow field and using the buoyant jet invariant C_p , we can derive an analytical, closed form solution for buoyant jets as it will be shown next.

Figure 2 Plane buoyant jet width parameter C_p plotted against z/l_m from experiments by Kotsovinos (1975) and Ramaprian and Chandrasekhara (1985, 1989).

Hence, substituting $\mu/m^{1/2}=C_p z^{1/2}$ into the momentum equation ($\beta=\text{constant}$), it can be modified as

$$m^{1/2} \frac{dm}{dz} = \frac{\mu\beta}{m^{1/2}} = C_p \sqrt{z}\beta$$

which upon integration gives $m^{3/2} = C_p \beta z^{3/2} + c$, c being the integration constant. For $z=0$ $m=M$ leading to $c=M^{3/2}$. After some manipulation and considering the momentum length scale $l_m = M / B^{2/3}$, the normalized specific momentum flux m/M can be computed from

$$\frac{m}{M} = \left[1 + C_p \left(\frac{z}{l_m} \right)^{3/2} \right]^{2/3} \quad (4)$$

In jets, ($z/l_m \rightarrow 0$) $m/M=1$, while in plumes (z/l_m large) $m/M \approx \left[C_p (z/l_m)^{3/2} \right]^{2/3} = C_p^{2/3} (z/l_m)$, hence after some algebra $m \approx \left[C_p (z/l_m)^{3/2} \right]^{2/3} = C_p^{2/3} B^{2/3} z$. Equation (4) is plotted for comparison along with earlier experimental measurements in figure 3 and it is congruent with the measurements of RC (1989), but deviates from those by Kotsovinos (1975). Substituting m into the momentum equation and solving for $\mu(z)$ one has that

$$\frac{\mu\beta^{1/3}}{M} = C_p \left(\frac{z}{l_m} \right)^{1/2} \left[1 + C_p \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3} \quad (5)$$

In jets, ($z/l_m \rightarrow 0$) $\mu\beta^{1/3}/M \approx C_p (z/l_m)^{1/2} \Rightarrow \mu = C_p (zM)^{1/2}$, while in plumes (z/l_m large), $\mu\beta^{1/3}/M \approx C_p^{4/3} (z/l_m) \Rightarrow \mu = C_p^{4/3} B^{1/3} z$.

Equation (5) is plotted for comparison along with earlier experiments in figure 4 and it is in agreement with measurements of Kotsovinos (1975), RC (1989) and Chu & Baines (1989).

The local Richardson number $Ri(z) = \mu^3 \beta / m^3$ (Kotsovinos, 1975) is computed using equations (4) and (5) as

$$Ri(z) = \frac{\mu^3 \beta}{m^3} = C_p^3 \left(\frac{z}{l_m} \right)^{3/2} \left[1 + C_p \left(\frac{z}{l_m} \right)^{3/2} \right]^{-1}. \quad (6)$$

Figure 3 Normalized plane buoyant jet momentum flux against the dimensionless elevation. Comparison between theory (equation 4) and experiments by Kotsovinos (1975) and Ramaprian and Chandrasekhara (1989).

From equation (6) the asymptotically constant plume Richardson number is $Ri_p = C_p^2$. Solving the volume conservation equation (1) for the entrainment coefficient a we find that

$$\alpha = \frac{1}{4m} \frac{d\mu^2}{dz} = \frac{C_p^2}{4} \left\{ 1 + C_p \left(\frac{z}{l_m} \right)^{3/2} \left[1 + C_p \left(\frac{z}{l_m} \right)^{3/2} \right]^{-1} \right\} = \frac{C_p^2}{4} \left\{ 1 + \frac{Ri(z)}{C_p^2} \right\}. \quad (7)$$

In jets, ($z/l_m \rightarrow 0$) $a_j = C_p^2/4$, while in plumes (z/l_m large) $a_p = C_p^2/2$, hence $a_p = 2a_j$ as shown in Kotsovinos (1975) but higher (0.0595 and 0.119) than 0.055 and 0.11 respectively he has proposed.

Figure 4 Normalized plane buoyant jet volume flux against the dimensionless elevation. Comparison between theory (equation 5) and experiments by Kotsovinos (1975), Ramaprian and Chandrasekhara (1989).

Note that the parameters of the flow $m(z)$, $\mu(z)$, $Ri(z)$ and $a(z)$ can be computed throughout the buoyant jet flow-field, from jets to plumes and transition, and depend only upon the initial kinematic flow parameters M and B and the invariant width parameter C_p . The asymptotic solution derived by List & Imberger (1973) utilizes also the asymptotic plume Richardson number besides C_p for the calculations, while in the present work we have derived Ri_p . Comparison between the findings of the analytical solution presented in the previous paragraphs and earlier investigations is shown in Table 1. The asymptotic constants for jets and plumes are computed for $C_p=0.58$ (from the measurements of Kotsovinos, 1975 and RC 1985 and 1989) and $C_p=0.54$ proposed by Fischer et al (1979), and compare satisfactorily with measurements. The entrainment coefficient and plume Richardson number deviate from the values measured by Paillat and Kaminski (2014 a, b), perhaps because of the initial low Reynolds number of their experiments.

Assuming Gaussian distributions for the time-averaged (mean) velocity and tracer concentration or density deficit, the normalized centerline velocity w_c versus the normalized distance z/l_m from the source can be computed from (see Appendix)

$$\frac{w_c}{B^{1/3}} = \sqrt{2} \frac{m(z)}{B^{1/3} q(z)} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{l_m} \right)^{-1/2} \left[1 + C_p \left(\frac{1 + \lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3} \quad (8)$$

where $\lambda = b_c/b$, the ratio of the 1/e time-averaged concentration to stream-wise velocity distribution widths. The dimensionless minimum dilution $S_c = C/c_c$ along jet axis is computed from (see Appendix)

$$S_c \sqrt{\frac{l_q}{z}} = C_p \frac{\lambda}{\sqrt{1 + \lambda^2}} \left[1 + C_p \left(\frac{1 + \lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3}. \quad (9)$$

Table 1 Comparison between computed asymptotic jet and plume constants and the values proposed by Fisher et al. (1979), and from experiments by Kotsovinos (1975) and Ramaprian and Chandrasekhara (1985, 1989). Entrainment coefficient values are for top-hat velocity and concentration distributions.

	Jet	Plume	Jet	Plume	Jet	Plume	Plume
C_p	$\frac{m}{M}$	$\frac{m}{B^{2/3} z} = C_p^{2/3}$	$\frac{\mu}{(zM)^{1/2}} = C_p$	$\frac{\mu}{B^{1/3} z} = C_p^{4/3}$	$a_j = \frac{C_p^2}{4}$	$a_p = \frac{C_p^2}{2}$	$R_p = C_p^2$
0.58	1	0.70	0.58	0.484	0.084	0.168	0.34
0.54 (Fischer)	1	0.66	0.54	0.440	0.073	0.146	0.29
Fischer (1979)			0.500	0.340			0.40
Kotsovinos (1975)		0.400	0.515	0.365	0.078	0.156	0.38
R&C ¹ (1985, 1989)		0.740	0.410	0.480	0.064	0.159	0.28
Chu & Baines (1989)			0.488	0.437			
P & K ² (2014, a, b)					0.042	0.102	0.140

¹R&C, Ramaprian and Chandrasekhara, ²P&K, Paillat and Kaminski

The analytical solutions from equations (8) and (9) using $C_p=0.58$ from Kotsovinos (1975) and an average value for $\lambda \approx 1.30$ are plotted in figures 5 and 6 respectively for comparison with laboratory measurements. It is evident that the theoretical findings are congruent with experiments.

Figure 5 Normalized plane buoyant jet time- averaged centerline velocity against dimensionless elevation.

Figure 6 Normalized plane buoyant jet time- averaged centerline concentration against dimensionless elevation.

2.2 Plane (2D) vertical fountain

The maximum or the steady state penetration height of a vertical negatively buoyant 2-D jet is a function of the initial flow parameters, that is, $Z=f(q,M,B)$. Neglecting the initial flow rate q far from the slot, dimensional arguments (Turner, 1966) lead to $Z(|B|^{2/3}/M)=\text{constant}$, hence

$$\frac{Z_{\max}}{l_m} = C_1 \text{ and } \frac{Z_m}{l_m} = C_2 \quad (10)$$

C_1 and C_2 being constants to be evaluated. The system of differential equations (2) was solved numerically using a 4th order Runge-Kutta solver. We assumed that the jet virtual origin is

located at distance $z_0=q/C_p M^{1/2}=1.72l_q=1.72d$ above the nozzle (List and Imberger, 1973), and the entrainment coefficient at elevation z is computed from equation (7) as a function of the local Richardson number $Ri(z)=\mu^3\beta/m^3$.

In equation (7) substituting a negative buoyancy in $Ri(z)$, the entrainment coefficient will become lower than α_j for jets as it has been discussed earlier (Kaminski et al, 2005, Papanicolaou et al, 2008) for round fountains. As the flow slows down, the local negative Richardson number increases indefinitely, meaning that for $Ri(z)>C_p^2$ the entrainment coefficient becomes negative, which is against the physics. Hence, for $Ri(z)>C_p^2$ in the computation we restrain the entrainment coefficient to zero, and obtain for large Froude numbers $m(z)=0$ at $z/l_m=1.36$.

At this point we should note that since the momentum computed from equation (4) is the result of integration of the equations of motion, keeping the negative sign in the buoyancy flux one has that $m=0$ when

$$1 + C_p \left(\frac{z}{l_m} \right)^{3/2} = 0 \Leftrightarrow z = \left(-\frac{1}{C_p} \frac{M^{3/2}}{B} \right)^{2/3} = 1.44l_m; \quad l_m = \frac{M}{|B|^{2/3}} \quad (11)$$

3. EXPERIMENTAL

The experiments were carried out at the Applied Hydraulics Laboratory of the National Technical University of Athens. Measurements were made via two flow visualization techniques, the shadowgraph and the Planar Laser-Induced Fluorescence (PLIF) methods, and have been presented in Minos and Papanicolaou (2020). The experimental apparatus is shown schematically in Figure 7.

The jet plenum is mounted at the bottom of a transparent dispersion tank made of tempered glass, with cross-section inside dimensions 1.00 m $\times$ 0.80 m, 0.70 m depth and a peripheral v-notch type overflow (Michas, 2008), which is filled with fresh water. The jet assembly

consists of a Perspex box with horizontal dimensions 220 mm $\times$ 100 mm and 100 mm height. At the top of the box along the middle of the long side a 200 mm rectangular slot with rounded lips and variable width is placed, out of which the jet will be ejected vertically. The slot width varied by displacing the one half of the top of the box which is made to slide horizontally on a silicone based gel, in order to control any jet fluid leaks out of the box. At mid height of the box there is a perforated plate with small holes that kill the turbulence of the jet fluid injected at the lower compartment by four symmetrically placed pipes, two on each long side. The box is positioned between two parallel, vertical, Lucite panels, 700 mm $\times$ 600 mm that are placed 200 mm apart, to confine the 2-D jet inside, thus confirming the two-dimensional character of the flow. A grid was drawn on one of the Lucite panels to be used as a length scale calibration for the shadowgraphs. The jet fluid supply system consists of a solution tank where the saltwater solution is prepared. A submersible pump is placed inside the tank, via which the jet fluid solution is conveyed to a constant head supply tank with an overflow that is placed 3 m above the floor, thus providing energy head adequate to obtain the desired flow rate. Two rotameters were used to measure the saltwater jet flow rate. The pipes out of them were connected to a 1'' collector pipe which was eventually split into four 1/2'' pipes of equal length that supplied symmetrically the lower compartment of the jet Perspex box with jet fluid. A schematic of the experimental apparatus is shown in figure 7, along with the slot jet box device where the four supply pipes and the mid-plane perforated plane can be seen.

The jet is investigated using two visualization techniques, the shadowgraph and planar, laser-induced fluorescence (PLIF). The shadowgraph was used to measure the jet penetration height. Reference grid points were drawn on the two side walls of the jet ejection system that are normal to the projection light beam. The projector is positioned at a distance that was several jet slot lengths away. The reference grid points are projected onto the tracing paper on the side glass panel of the dispersion tank, and are used as a scale to measure the projected

penetration height of the jet, that could be located anywhere along the length of the slot. This uncertainty is quantified as an error which varies with the distance from the projection axis. The magnitude of the error is proportional to the horizontal distance between the flow field and the light source. For this reason, the projector was positioned at a distance that was several jet slot lengths away (approximately 5 m).

Figure 7 Schematic illustration of experimental layout and jet ejection chamber with 2-D slot at top, the four supply pipes and the perforated plate above them to destruct the incoming turbulence.

The PLIF technique was implemented as follows. A Diode Pumped Solid State (DPSS) laser with beam intensity of 1 W was used. The beam was directed to a 24 side polygon mirror rotating at a speed of 20000 rpm, thus converting the beam to a vertical laser light sheet 1 mm thick. A rhodamine 6G fluorescent dye at small concentrations is diluted in the jet, that if stimulated by monochromatic radiation of wavelength $\lambda=532$ nm (green color), emits radiation of wavelength around $\lambda=570$ nm (yellow color). Thus the jet dispersion area can be illuminated if the vertical laser light sheet cuts it through. The laser light sheet was positioned normal to the 2-D jet slot, thus illuminating the mid plane of the jet.

The experimental procedure was as follows. The jet fluid was prepared ahead of time, the jet supply system was filled with saltwater of known density, the rotameters were adjusted at the desired flow rates and the globe valve of the supply pipe was shut. Then the dispersion tank was filled with fresh water and left to rest for two hours. Before each experiment we calculated the salt water density from measured temperature and salinity with a portable instrument (YSI Model 30) and verified the result by a hydrometer, and the dispersion tank temperature to compute the ambient water density. The pump was turned on and the supply system was filled with salt water. The light projector or the PLIF sheet was turned on, the video camera was turned on, and finally the globe valve was turned on and the position of the floats in the calibrated rotameters was taken in order to determine the jet flow rate. For the PLIF experiments a small amount of Rhodamine 6G solution was diluted in jet fluid. The video camera was set at either 50 or 25 fps and the recording time was greater than 1 minute.

4. RESULTS

A total of 37 experiments were conducted for different slot widths so that a wide range of initial Froude numbers $Fr=20$ to 130 was obtained, where $Fr^2 = W^2/(g_0'd) = 1/Ri$. The initial conditions and measured parameters are depicted in Table 1. Flapping due to the instability of the fountain was observed as in Hunt (2019) and the flow would return to the ejection elevation on either side. Two instantaneous photos of the flow, one from shadowgraph and one from PLIF imaging, are depicted in Figure 8. The videos from experiments were digitized frame by frame and a mid-grid was made using the marked points of the side Lucite panels of the jet. Then a known characteristic vertical length of the grid was converted to pixels and using Matlab[®] software built in house to identify the instantaneous maximum height in pixels and record it as a function of time. The recorded height was converted to centimeters and normalized with the characteristic length scale.

Figure 8 Instantaneous photos from the videos recorded using shadowgraph (left) and PLIF (right)

From the normalized height plotted versus the normalized time tB/M (t time from the beginning of jet injection), the maximum and terminal heights were computed as shown in Figure 9. This figure shows that the jet at tB/M around 2 attains the maximum rise height $Z_{\max}$. Then the penetration height z/l_m drops fluctuating around the value of 1.15. From Table 2 it is evident that the normalized time tB/M varied from 1.5 to 2.8 with an average value of 2.2.

Table 2 Initial and measured jet parameters

EXP	d cm	g_0' cm/s ²	W cm/s	l_m cm	$Z_{\max}$ cm	$t (Z_{\max})$ s	Z_m cm	tB/M	Re	Fr
Shadowgraph Experiments										
S1	0.20	4.68	28.19	17.93	34.88	15.28	23.04	2.54	564	29.14
S2	0.20	6.19	26.05	13.40	23.85	9.72	17.14	2.31	521	23.42
S3	0.20	6.30	34.83	19.51	33.12	12.92	25.87	2.34	697	31.04
S4	0.20	6.03	27.83	14.88	24.25	10.12	19.17	2.19	557	25.34
S5	0.05	6.33	35.95	12.78	20.31	11.68	14.48	2.06	180	63.91
S6	0.05	6.16	38.06	14.04	22.8	10.2	15.11	1.65	190	68.61
S7	0.05	5.94	42.29	16.55	23.28	11.76	16.98	1.65	211	77.61
S8	0.05	5.75	45.70	18.75	30.31	13.8	21.36	1.74	229	85.23
S9	0.05	6.01	54.26	22.90	36.31	13.94	22.82	1.54	271	99.00
S10	0.05	5.94	65.68	29.77	37.74	17.12	28.01	1.55	328	120.52
S11	0.05	5.74	71.38	34.05	44.84	22.92	32.09	1.84	357	133.30
S12	0.10	6.36	28.84	11.96	22.06	9.82	15.39	2.17	288	36.17
S13	0.10	6.15	32.84	14.54	23.18	14.12	16.89	2.64	328	41.88
S14	0.10	5.91	37.83	18.04	27.26	12.04	20.23	1.88	378	49.23
S15	0.10	6.55	41.40	18.98	31.67	13.96	21.73	2.21	414	51.14
S16	0.10	6.25	46.39	22.80	32.39	13.08	25.29	1.76	464	58.68
S17	0.10	6.39	50.67	25.28	35.29	17.82	26.97	2.25	507	63.39
S18	0.10	6.08	56.38	30.11	40.81	25.28	32.81	2.73	564	72.29
S19	0.15	6.23	24.75	11.32	20.07	9.94	15.01	2.50	371	25.61

S20	0.15	6.04	27.60	13.36	23.93	10.8	17.73	2.36	414	28.99
S21	0.15	6.08	31.12	15.61	27.95	9.58	20.11	1.87	467	32.58
S22	0.15	5.83	35.78	19.35	32.52	12.22	23.76	1.99	537	38.28
S23	0.15	6.17	41.62	22.79	38.11	14.74	26.00	2.18	624	43.27
S24	0.15	5.84	46.45	27.35	43.46	19.48	31.02	2.45	697	49.62
S25	0.25	6.40	24.05	12.69	19.98	8.22	14.60	2.19	601	19.01
S26	0.25	6.13	27.76	15.80	26.91	7.7	19.07	1.70	694	22.42
S27	0.25	6.42	31.40	18.06	28.42	10.26	21.44	2.10	785	24.77
PLIF Experiments										
P1	0.20	6.66	27.83	13.94	23.51	8.3	16.28	1.99	557	24.12
P2	0.20	6.34	35.39	19.83	30.45	16.4	23.90	2.94	708	31.42
P3	0.15	6.61	32.35	15.56	29	15.76	16.51	3.22	485	32.50
P4	0.15	6.53	43.59	23.33	32.16	10.72	25.11	1.61	654	44.04
P5	0.15	6.53	32.35	15.68	25.34	12.14	16.46	2.45	485	32.69
P6	0.15	6.31	37.23	19.34	28.63	13.22	20.85	2.24	558	38.26
P7	0.25	6.38	26.30	14.33	24.91	11	16.66	2.67	658	20.83
P8	0.25	6.07	23.04	12.41	23.81	10.28	14.02	2.71	576	18.71

The dimensionless penetration heights $Z_{\max}/l_M$ and Z_m/l_M are plotted against the initial Froude number in Figure 10. From this Figure it is evident that the terminal height Z_m is equal to $1.15l_M$ for Froude numbers in the range 50 to 100, while it attains higher values for Froude number in the range $25 < Fr < 50$ and lower values for $Fr > 100$. In the same figure it is shown that the maximum normalized penetration height $Z_{\max}/l_m$ varies in the same manner as the terminal height. An average value of $Z_{\max}/l_m$ over the whole Froude range studied is around 1.60. The experiments recorded with PLIF show lower values of Z_m/l_M for $Fr < 50$ which is reasonable since through shadowgraph we observe the penetration height of the full length of the slot. Finally, an average value of the ratio of terminal to the average penetration height $Z_{\max}/Z_m$ is 1.40.

Our results are compared to earlier experiments in Table 3. To do it we had to reformat the Froude number using the slot width d instead of half the slot width used by the other authors, as we did in the present work. There one can see that our data compare well with those by Papakonstantis and Mylonakou (2021) who have used a different 2-D jet generating arrangement in a much bigger dispersion tank. Our data deviate from those of Baines et al

(1990) that are 15% higher, and are quite different from the measurements by Zhang and Baddour (1997) and Hunt et al (2019).

Figure 9 Normalized maximum penetration height versus normalized time (Exp. 15, $Z_{\max}/l_m=1.67$ at $tB/M=2.25$, $Z/l_m=1.14$, $Z_{\max}$ -average for $tB/M>5$ red line).

The present data are plotted against earlier experiments in figure 11. The experiments of Zhang and Baddour (1997) show an increasing trend with increasing Froude number reaching an asymptotic value of 1.58 when $Fr>20$. The average normalized terminal steady state height measured by Hunt et al (2019) increases up to Froude number around 15, then shows decreasing trend, like the present experiment, as a function of the increasing Froude number. This is more evident in the present data when Froude number is greater than 50. In the same figure the results by Hunt et al (2019) regarding terminal penetration height are bigger when the slot length to width ratio is doubled from $L/d=150$ to 300. In the present experiment the slot length to width ratio was variable because we kept the length $L=20$ cm while the width varied from 0.05 cm to 0.25 cm, thus obtaining $L/d= 80$ to 400. Our steady state height results are neat to those by Papakonstantis and Mylonakou (2021) but the maximum height is somewhat higher.

Figure 10 (a) Normalized penetration heights $Z_{\max}/l_m$ and Z_m/l_m , (b) normalized heights $Z_{\max}/d$ and Z_m/d , against $Fr^{4/3}$. SG stands for data from shadowgraph.

Table 3 Normalized maximum and terminal penetration height of available experiments

Researcher	L/d	Fr	Z_{max}/l_m	Z_m/l_m
Baines et al. (1990)	N/A	350 - 2400	---	1.29
Zhang & Baddour (1997)	3.75-125	6 – 80	---	1.59
P&M ¹ (2021)	100	18 - 47	1.47	1.13
Hunt et al. (2019)	300, 150	2.8 – 44.1	---	1.44/1.30
Present	80-400	19– 133	1.58	1.14

¹P&M, Papakonstantis and Mylonakou

Figure 11 Dimensionless maximum and terminal, steady state penetration heights versus Froude number from different investigations and present experiment. P&M stands for Papakonstantis and Mylonakou (2021).

5. CONCLUDING REMARKS

Planar (2D) vertical turbulent positively buoyant jets and (negatively buoyant) fountains which disperse in a calm, uniform ambient have been investigated. A theoretical model has been developed for buoyant jets based on a buoyant jet width parameter proposed by List and Imberger (1973). Closed form equations have been derived for computing the local

momentum and mass fluxes, the Richardson number and entrainment coefficient in terms of the local Richardson number far from the source, which can be applied in the full range, jets, plumes and buoyant jets. To evaluate these flow parameters one must know only the initial volume, momentum and buoyancy fluxes, as well as the width parameter C_p . The equations regarding the local flow parameters are different from those derived from asymptotic theory proposed by List and Imberger, where besides C_p one has to know the constant plume Richardson number, which in the present work has been derived from the analysis presented. In Table 2 one can find the equations regarding the parameters mentioned above and their asymptotic equations that are valid for jet-like flows and plumes, as they have been derived using top-hat velocity and tracer concentration distributions. The equations derived in table 2 are compatible with earlier experiments as shown in figures 3 and 4.

From the equations of motion we have derived in the appendix (table A1) the flow parameters, when Gaussian distributions have been considered for the time-averaged cross-sectional velocity and tracer concentration. The time-averaged velocity and tracer concentration along the axis are evaluated from the local volume and momentum fluxes in table A3, and the closed form equations are congruent with experiments as shown in figures 5 and 6. Note that the asymptotic values of entrainment coefficient for simple jets and pure plumes 0.0595 and 0.119 derived in table A1 are comparable with the values 0.055 and 0.110 proposed by Kotsovinos (1975), 0.045, 0.11 by RC but quite different than 0.030, 0.072 by Paillat and Kaminski (2014 a, b).

A total of 37 experiments have been implemented in the range of initial Froude numbers 19 to 133, and the flow field was recorded in full high definition video. Video recording has been analyzed using Matlab[®] and the time history of the maximum penetration height was extruded. The flow was found to be quite complex because of the interaction of oppositely moving liquids, and this instability drives the descending fluid to oscillate left and right of the injection slot.

The terminal penetration height varied around a value $1.15 l_m$ that is near the measurements by Papakonstantis and Mylonakou (2021), but deviated from the measurements of other authors for Froude numbers from 40 to 100. Also, the maximum penetration height was found around $1.60 l_m$ and occurred around an average dimensionless time $tB/M=2.2$. The ratio of the maximum to the terminal penetration heights is near to 1.40. The experiments visualized by PLIF showed better accuracy and smaller variation with Fr especially for the terminal penetration height, therefore this is considered to be the most accurate for visualizing the experiments among the two. In general, the penetration height appears to decrease with increasing Froude number, observation which is not consistent with dimensional analysis, but it has been documented by other researchers as well and therefore it needs further consideration.

Finally, analytical and numerical solutions of the equations of motion have predicted higher values of the terminal, steady state penetration height than the measured ones.

APPENDIX

On calculation of time-averaged velocity and tracer concentration along the axis in plane buoyant jets.

Let us consider a plane vertical buoyant jet of density ρ_o out of a slot of width d with uniform exit velocity W , which discharges into quiescent ambient fluid of uniform density $\rho_a > \rho_o$. The initial specific mass, momentum and buoyancy fluxes per unit slot length are

$$q = Wd, \quad M = qW, \quad B = \frac{\rho_a - \rho_o}{\rho_o} gq, \quad (A.1)$$

The entrainment equations considering Gaussian time-averaged distributions of vertical velocity and density difference or concentration are

$$\frac{d\mu}{dz} = 2\sqrt{2}\alpha \frac{m}{\mu}, \quad \frac{dm}{dz} = \left(\frac{1 + \lambda^2}{2} \right)^{1/2} \frac{\mu\beta}{m}, \quad \frac{d\beta}{dz} = 0. \quad (A.2)$$

where $\mu(z)$, $m(z)$ and $\beta(z)=B$ are the local specific mass, momentum and buoyancy fluxes respectively, reading

$$\mu(z) = 2 \int_0^{\infty} w(y, z) dy = \sqrt{\pi} b w_c \quad (\text{A.3})$$

$$m(z) = 2 \int_0^{\infty} w^2(y, z) dy = \sqrt{\pi/2} (b w_c^2) \quad (\text{A.4})$$

$$\beta(z) = 2 \int_0^{\infty} w(y, z) g \frac{\Delta \rho(y, z)}{\rho_o} dy = \sqrt{\pi} (\Delta \rho_c / \rho_o) \left(\frac{\lambda}{\sqrt{1 + \lambda^2}} \right) g w_c b \quad (\text{A.5})$$

In equations above w is the time-averaged stream-wise velocity and $\Delta \rho$ the density difference between jet and ambient fluid. Subscript (c) corresponds to time-averaged values along jet axis, $\lambda = b_c/b$ is the ratio of the 1/e-widths of the time-averaged tracer concentration and velocity distributions respectively, and $C_p = \mu / \sqrt{z m}$ the width parameter introduced by List and Imberger (1973).

Working in the same manner as in chapter 2 we obtain the equations for computing momentum and volume flux, the local Richardson number and entrainment coefficient as functions of C_p , λ and z/l_m (see table A.1). The average values of C_p and λ are evaluated from the data by Kotsovinos (1975) and Ramaprian and Chandrasekhara (1985, 1989) to be 0.58 and 1.3 respectively.

The normalized mean centerline velocity distribution versus the normalized distance z/l_m from the source are computed from

$$\frac{m(z)}{q(z)} = \frac{\sqrt{\frac{\pi}{2}} b w_c^2}{\sqrt{\pi} b w_c} \Rightarrow w_c = \sqrt{2} \frac{m(z)}{q(z)} \Leftrightarrow \frac{w_c}{\beta^{1/3}} = \sqrt{2} \frac{m(z)}{\beta^{1/3} q(z)}$$

Hence from table A.1 by substitution of $m(z)$ and $\mu(z)$

$$\frac{w_c}{B^{1/3}} = \sqrt{2} \frac{m(z)}{B^{1/3} q(z)} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{l_m} \right)^{-1/2} \left[1 + C_p \left(\frac{1 + \lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3} \quad (\text{A.6})$$

TABLE A.1 Parameters of plane buoyant jets calculated for Gaussian time-averaged velocity and concentration distributions.

Equation	Jet asymptote	Plume asymptote
$\frac{m}{M} = \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{2/3}$	$m = M$	$\frac{m}{M} = \left[C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \right]^{2/3} \frac{z}{l_m}$
$\frac{\mu \beta^{1/3}}{M} = C_p \left(\frac{z}{l_m} \right)^{1/2} \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3}$	$\frac{\mu}{Q} = C_p \frac{z}{d}$	$\frac{\mu}{Q} = \frac{C_p^{4/3}}{R_o} \left(\frac{1+\lambda^2}{2} \right)^{1/6} \frac{z}{l_m}$
$Ri(z) = C_p^3 \left(\frac{z}{l_m} \right)^{3/2} \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{-1}$	$Ri(z) = C_p^3 \left(\frac{z}{l_m} \right)^{3/2}$	$Ri(z) = C_p^2 \sqrt{\frac{2}{(1+\lambda^2)}}$
$\alpha = \frac{C_p^2}{4\sqrt{2}} \left\{ 1 + \left(\frac{1+\lambda^2}{2} \right)^{1/2} \frac{Ri(z)}{C_p^2} \right\}$	$\alpha_j = \frac{C_p^2}{4\sqrt{2}}$	$\alpha_p = \frac{C_p^2}{2\sqrt{2}}$
$\frac{w_c}{B^{1/3}} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{l_m} \right)^{-1/2} \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3}$	$\frac{w_c}{W} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{d} \right)^{-1/2}$	$\frac{w_c}{B^{1/3}} = \frac{\sqrt{2}}{C_p^{2/3}} \left(\frac{1+\lambda^2}{2} \right)^{1/6}$
$S_c \sqrt{\frac{l_q}{z}} = C_p \frac{\lambda}{\sqrt{1+\lambda^2}} \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3}$	$\frac{c_c}{C} = \frac{\sqrt{1+\lambda^2}}{\lambda C_p} \sqrt{\frac{d}{z}}$	$\frac{c_c}{C} = \frac{2^{1/6}}{C_p^{4/3}} \frac{(1+\lambda^2)^{1/3}}{\lambda} \frac{q}{B^{1/3}} z^{-1}$

In jets ($z/l_m \rightarrow 0$)

$$\frac{w_c}{B^{1/3}} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{l_m} \right)^{-1/2} \Leftrightarrow \frac{w_c}{W} = \frac{\sqrt{2}}{C_p} \left(\frac{z}{d} \right)^{-1/2}$$

while in plumes ($z/l_m \gg 1$)

$$\frac{w_c}{B^{1/3}} = \frac{\sqrt{2}}{C_p^{2/3}} \left(\frac{1+\lambda^2}{2} \right)^{1/6} = \text{constant}$$

as shown by dimensional arguments in Fischer et al (1979). Using tracer conservation equation with initial concentration C one has

$$qC = \int_A \bar{w} \bar{c} dA = \int_0^\infty w_c c_c \exp\left(-\frac{y^2}{b^2} - \frac{y^2}{b_c^2}\right) dy = \int_0^\infty w_c c_c \exp\left(-\frac{y^2}{b^2} - \frac{y^2}{\lambda^2 b^2}\right) dy$$

In the equation above we have omitted the turbulent transport of the tracer estimated around 10% to 15% of the total in round buoyant jets (Papanicolaou and List, 1988, Wang and Law, 2002). After some manipulation we end up with dimensionless minimum dilution $S_c = C/c_c$ along jet axis

$$S_c \sqrt{\frac{l_q}{z}} = C_p \frac{\lambda}{\sqrt{1+\lambda^2}} \left[1 + C_p \left(\frac{1+\lambda^2}{2} \right)^{1/2} \left(\frac{z}{l_m} \right)^{3/2} \right]^{1/3}. \quad (\text{A.7})$$

In jets ($z/l_m \rightarrow 0$)

$$S_c \sqrt{\frac{l_q}{z}} = C_p \frac{\lambda}{\sqrt{1+\lambda^2}} \Leftrightarrow \frac{C}{c_c} = C_p \frac{\lambda}{\sqrt{1+\lambda^2}} \sqrt{\frac{z}{d}} \quad \text{or} \quad \frac{c_c}{C} = \frac{\sqrt{1+\lambda^2}}{\lambda C_p} \sqrt{\frac{d}{z}}$$

while in plumes ($z/l_m \gg 1$)

$$S_c \sqrt{\frac{l_q}{z}} = \frac{C_p^{4/3}}{2^{1/6}} \frac{\lambda}{(1+\lambda^2)^{1/3}} \left(\frac{z}{l_m} \right)^{1/2} \Leftrightarrow \frac{c_c}{C} = \frac{2^{1/6}}{C_p^{4/3}} \frac{(1+\lambda^2)^{1/3}}{\lambda} \frac{q}{B^{1/3}} z^{-1}$$

as shown by dimensional arguments in Fischer et al (1979).

Acknowledgments: The assistance of Dr. P. Dimitriadis for the PLIF arrangement and Technician Mr. Y. Patselis is gratefully acknowledged. An initial version of the paper has been included in the e-Proceedings of the “15th International Conference on Protection and Restoration of the Environment”, 2021, Patras, Greece, ISBN 978-618-82337-2-0.

Availability of data. All the experimental data of this investigation are included in Table 2.

Code Availability. Matlab codes used to extract the terminal height from video frames and Visual Basic code for computing the terminal rise height are available upon reasonable request.

Author’s Contributions. Panos N. Papanicolaou: Conceptualization, Methodology, Formal analysis and investigation, Validation, Supervision, Writing - original draft preparation;
Panagiotis Minos: Formal analysis and investigation, Hardware design, Implementation of experiments, Software, Validation, Visualization; Final approval.

Funding. There was no funding for this work.

Declarations

Conflict of interest. The authors have no conflicts of interest to declare.

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