Sliding Mode Control of Interval Type-2 T-S Fuzzy Systems with Redundant Channels

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Abstract This paper investigates the sliding mode control (SMC) of interval type-2 (IT2) T-S fuzzy systems. The measurement outputs are propagated via redundant channels for reducing the probability of packet loss and improving the reliability of data transmission. A key feature for the above problem is that the premise variables and the measurement signals may not be available by the controller, which brings difficulty to stabilize the nonlinear systems. Accordingly, a crucial issue is how to synthesize an implementable SMC law under the redundant channels. To this end, the characteristic of the redundant channels is firstly analyzed and the model of available measurement output signals is established. By employing these available measurements as the premise variables and utilizing the upper and lower bounds of the system membership functions (MFs), new MFs are constructed and the sliding mode controller is synthesized. By introducing some null terms carrying the information of MFs, sufficient conditions are derived in terms of nonlinear matrix inequalities to ensure the stochastically ultimate boundedness of the closed-loop system and the reachability of the specified sliding surface. Besides, a binary genetic algorithm (GA) is introduced to solve the nonlinear criteria via the objective function reflecting the control performance. Finally, a numerical example illustrates the effectiveness of the proposed methods.

Keywords: Interval type-2 T-S fuzzy system, redundant channel transmission, sliding mode control, optimization algorithm.

1 Introduction

As one of the frequently confronted network-induced phenomena, packet loss has gained considerable attention in the past decades. Lots of efforts have been devoted to compensating its negative influences such as system performance degradation or instability [1, 2]. For example, a redundant channel transmission (RCT) strategy was proposed in [3] to bring down the probability of packet dropout and enhance the system performance. Compared with the data transmitted via a single channel, the RCT allows two or more accesses available for data propagation. If one channel undergoes the failure data exchange, the redundant channels may function to deliver the information, thereby improving the reliability of the signal communication. Based on the RCT, some initial works have been published on the estimate/filtering [4–6] and control [7] problems under packet loss. In [5], the authors investigated the distributed filtering of the Markovian jump systems. Two independent paths were considered in the communication network towards performance benefit. Taking the advantage of the sliding mode control (SMC) approach (e.g., fast response, insensitivity to matched disturbances and uncertainties as discussed in [8–11]), Song, et al. [7] investigated the control design of the fast sampling singularity perturbed systems. A sliding function was constructed by taking the special structure of RCT into account and the SMC law was designed relying on the measurement outputs of N channels.

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Takagi-Sugeno (T-S) fuzzy system is powerful in representing nonlinear complex systems [12–14]. Its main idea is to express nonlinear plants by “blending” several local linear subsystems via membership functions, which measure the contributions of each subsystem. Based on this fuzzy modeling approach, some preliminary efforts have been devoted to nonlinear systems with redundant channels [15, 16]. For example, the authors in [15] focused on the control design of fuzzy systems subject to packet loss, in which the controller was designed via the parallel distributed compensation (PDC) strategy, endowing the controller with the same premise variables as the ones in the plant. However, it is worth noting that the premise variables of the fuzzy system under consideration may not be available for this controller due to the packet dropout. This means that the controller based on the frequently utilized PDC strategy may be physically infeasible.

On the other hand, one should note that the above T-S fuzzy systems, also termed as type-1 fuzzy systems, do well in handling nonlinearities but are not competent for directly dealing with uncertainties. This urges the study of interval type-2 (IT2) T-S fuzzy systems [17–20]. In the IT2 T-S fuzzy model, uncertain parameters are absorbed via the upper and lower membership functions (MFs), thus the MFs for the system model become uncertain. This special structure brings challenges for control design of IT2 T-S fuzzy systems, since the uncertain MFs for the fuzzy system cannot be directly applied. In view of this, MFs distinct from the ones of the fuzzy system are usually required for an implementable controller. In [17], for the first time, the IT2 T-S fuzzy model under the model-based framework was proposed and the MFs of the control law were designed through the average normalized membership of upper and lower MFs. By introducing predefined nonlinear coefficients, the authors in [21] established new MFs to develop fuzzy controllers. In general, the imperfect premise concept [22] and membership-function-dependent analysis [23] play an important role to investigate the stability analysis and control synthesis problems in IT2 fuzzy-model-based control systems. When signals are transmitted via an unreliable network, the control problem for IT2 T-S fuzzy systems under packet loss has also attracted some research interest [24, 25]. However, the predictive controller in [25] ignored the effects of packet dropout on premise variables. Besides, the above results on IT2 T-S fuzzy systems dealt with packet dropout in a passive way, that is, only when packet dropout happened, some compensating measures would be taken. Apparently, a more positive alternative is using the RCT to reduce the probability of data loss as in [7] on singularly perturbed

systems and [15, 16] on T-S fuzzy systems. Thus, a potential topic is to design a reliable controller for IT2 T-S fuzzy systems via redundant communication channels.

Based on the above discussions, this paper aims to study the SMC problem of IT2 T-S fuzzy systems via redundant channels. Due to the characteristic of IT2 T-S fuzzy systems as pointed out in the above analysis, the existing results as in [7, 15, 16] cannot be directly applied. To this end, the model of available measurement output signals is established for the redundant channels. By introducing two predefined nonlinear functions, an implementable SMC law is synthesized under the constraint that only the bounds of system MFs are known. With the aid of some null terms, sufficient criteria are obtained for the stochastically ultimate boundedness of the resultant closed-loop IT2 T-S fuzzy system and the reachability of the prescribed sliding surface. By constructing a minimization problem reflecting the SMC performance, a binary genetic algorithm (GA) is utilized to handle the control design under nonlinear matrix inequalities. Finally, an example verifies the proposed SMC design method.

Notations: In this paper, $\lambda_{\min}(\Omega)$ ($\lambda_{\max}(\Omega)$) denotes the smallest (largest) eigenvalue of Ω ; $\mathbb{R}$ refers to the real number set. $(*)$ means that the matrix can be deduced via symmetry for symmetric matrices. $\|\cdot\|$ is the Euclidean norm. $E\{a \mid b\}$ stands for the conditional expectation of stochastic variable a on stochastic variable b and $E\{a\}$ represents the expectation of a . The convex hull of points $x, y \in \mathbb{R}^n$ is $\text{co}\{x, y\} = \{\vartheta_1 x + \vartheta_2 y \mid \vartheta_1 + \vartheta_2 = 1\}$. In addition, if not specifically defined a matrix, it is supposed to have compatible dimension.

2 System Description and Problem Formulation

2.1 IT2 T-S fuzzy systems

Consider an s -rule IT2 T-S fuzzy system as follows:

$$x(k) = \sum_{i=1}^s w_i(\xi(k)) [A_i x(k) + D_i f(x(k)) + B_i(u(k)) + d(k, x(k))], \quad (1)$$

$$y_1(k) = C_1 x(k), \quad (2)$$

$$\vdots$$

$$y_N(k) = C_N x(k), \quad (3)$$

where $x(k) \in \mathbb{R}^n$ denotes the state, $u(k) \in \mathbb{R}^m$ refers to the control input, $y_i(k) \in \mathbb{R}^p$ ($i = 1, 2, \dots, N$) are the measurement outputs from different sensors, A_i , D_i , B_i and $C_1, \dots, C_N$ are known constant matrices, the

nonlinear function $f(x(k))$ in the local model satisfies $\|f(x(k))\|^2 \leq \epsilon \|Ex(k)\|^2$ with ϵ being a known scalar and E being a known matrix, $d(k, x(k))$ is the disturbance satisfying $\|d(k, x(k))\| \leq \bar{d}$, where $\bar{d}$ is a known positive constant. For the fuzzy model (1), the MFs $w_i(\xi(k))$ are formulated as

$$w_i(\xi(k)) = \underline{w}_i(\xi(k))\underline{\beta}_i(\xi(k)) + \bar{w}_i(\xi(k))\bar{\beta}_i(\xi(k)), \quad (4)$$

where $\underline{w}_i(\xi(k))$ and $\bar{w}_i(\xi(k))$ are the known lower and upper MFs computed by

$$\begin{aligned} \underline{w}_i(\xi(k)) &= \prod_{\ell=1}^r \underline{\mu}_{M_\ell^i}(\chi_\ell(\xi(k))) \geq 0, \\ \bar{w}_i(\xi(k)) &= \prod_{\ell=1}^r \bar{\mu}_{M_\ell^i}(\chi_\ell(\xi(k))) \geq 0, \\ \sum_{i=1}^s w_i(\xi(k)) &= 1 \end{aligned}$$

with $\bar{\mu}_{M_\ell^i}(\chi_\ell(\xi(k)))$ and $\underline{\mu}_{M_\ell^i}(\chi_\ell(\xi(k)))$, respectively, denoting the upper and lower grade of membership satisfying $0 \leq \underline{\mu}_{M_\ell^i}(\chi_\ell(\xi(k))) \leq \bar{\mu}_{M_\ell^i}(\chi_\ell(\xi(k))) \leq 1$ and M_ℓ^i being the fuzzy set of the i th fuzzy rule corresponding to the ℓ th premise. Moreover, the uncertain coefficients $\underline{\beta}_i(\xi(k))$ and $\bar{\beta}_i(\xi(k))$ in (4) satisfy $\underline{\beta}_i(\xi(k)) \in [0, 1]$, $\bar{\beta}_i(\xi(k)) \in [0, 1]$ and $\underline{\beta}_i(\xi(k)) + \bar{\beta}_i(\xi(k)) = 1$. Apparently, due to the uncertain coefficients, the MFs $w_i(\xi(k))$ in (4) are unknown and cannot be used by the controller. This is just the feature and difficulty in controlling IT2 T-S fuzzy systems.

2.2 Redundant channels

As shown in Fig. 1, the communication paths are composed of N independent links, where the first one is the primary channel and the others are redundant channels. The RCT structure is effective for overcoming the effect of the packet dropout and improving the transmission reliability of the measurement signals (this will be further discussed in Remark 1 and illustrated by the simulation later). In this case, the actual measurement output $z(k)$ received at the controller side is modeled as:

$$\begin{aligned} z(k) &= \theta_1(k)y_1(k) + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \theta_q(k)) \right] \theta_p(k)y_p(k) \right\} \\ &\quad + \left[\prod_{q=1}^N (1 - \theta_q(k)) \right] z(k-1), \end{aligned} \quad (5)$$

where $\theta_i(k)$ ($i = 1, \dots, N$) are mutually independent Bernoulli processes. For $\theta_i(k) = 0$, it means that the

packet is loss, while $\theta_i(k) = 1$ implies a successful data transmission. The probability distribution of $\theta_i(k)$ is expressed as:

$$\text{Prob}\{\theta_i(k) = 1\} = \bar{\theta}_i, \quad \text{Prob}\{\theta_i(k) = 0\} = 1 - \bar{\theta}_i, \quad (6)$$

where $\bar{\theta}_i$ is a known constant taking value in $[0, 1]$.

Fig. 1: The control systems with redundant channels

Denoting

$$\begin{aligned} \Omega_1(k) &= \theta_1(k)C_1 + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \theta_q(k)) \right] \theta_p(k)C_p \right\}, \\ \Omega_2(k) &= \prod_{q=1}^N (1 - \theta_q(k)), \end{aligned} \quad (7)$$

the variable $z(k)$ is further expressed as:

$$z(k) = \Omega_1(k)x(k) + \Omega_2(k)z(k-1). \quad (8)$$

Remark 1 It can be seen from (5) that if a packet is successfully transmitted via the primary channel, i.e., $\theta_1(k) = 1$, we have $z(k) = y_1(k)$. If $\theta_1(k) = 0$, i.e., the packet loss happens in the first channel, the measurement will be obtained from the other channel j satisfying $\theta_i(k) = 0$ ($i = 1, 2, \dots, j-1$) and $\theta_j(k) = 1$, that is, $z(k) = y_j(k)$. Therefore, the RCT can effectively reduce the probability of packet loss from $1 - \bar{\theta}_1$ to $\prod_{q=1}^N (1 - \bar{\theta}_q)$, and improve the network reliability, which will be further illustrated in the simulation. Moreover, according to the model (5) or (8), if the transmissions via all channels fail, i.e., $\theta_i(k) = 0$ for $i = 1, 2, \dots, N$, the measurement at instant $k-1$ will be exploited to compensate for the effect of packet dropout.

Definition 1 [26] The systems (1)-(3) with $u(t) = 0$ are stochastically ultimate bounded (SUB) if there exists a positive scalar $\bar{h}$ such that the condition

$$\lim_{k \rightarrow \infty} E\|x(k)\|^2 \leq \bar{h} \quad (9)$$

holds for any initial condition $x(0) \neq 0$.

This work aims to synthesize a feasible SMC law for achieving the SUB of the resultant closed-loop IT2 T-S fuzzy systems under redundant channels.

3 SMC Law Design

Now, considering the structure of redundant channels (5) or (8), we design the following sliding function

$$s(k) = G\bar{\theta}_1 y_1(k) + G \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p y_p(k) \right\} + G \left[\prod_{q=1}^N (1 - \bar{\theta}_q) \right] z(k-1), \quad (10)$$

where G is the sliding gain to be designed later.

Denoting

$$\bar{\Omega}_1 = \bar{\theta}_1 C_1 + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p \right\}, \quad \bar{\Omega}_2 = \prod_{q=1}^N (1 - \bar{\theta}_q), \quad (11)$$

and using (2)-(3), the sliding function $s(k)$ is further expressed as:

$$s(k) = G\bar{\Omega}_1 x(k) + G\bar{\Omega}_2 z(k-1). \quad (12)$$

In what follows, a desired SMC law will be synthesized. As discussed in the Introduction section, new MFs are necessary for the controller design due to the following facts: i) the MFs for the fuzzy model $w_i(\xi(k))$ ($i = 1, 2, \dots, s$) are uncertain; ii) the premise variable $\xi(k)$ may be unavailable at the controller side when it is transmitted via unreliable links from the sensor to the controller. For the former, a feasible way is to use the bounds of the system MFs $w_i(\xi(k))$ ($i = 1, 2, \dots, s$). For the latter, which is usually neglected in some existing works, one solution is to replace the premise variable $\xi(k)$ by available information. Since only $z(k)$, instead of $y_i(k)$, $i = 1, \dots, N$, is available at the controller side under the redundant channels, we construct the following MFs with $z(k)$ as the premise variable for the controller:

$$h_j(z(k)) = \frac{\underline{\nu}_j(z(k))\underline{w}_j(z(k)) + \bar{\nu}_j(z(k))\bar{w}_j(z(k))}{\sum_{j=1}^s [\underline{\nu}_j(z(k))\underline{w}_j(z(k)) + \bar{\nu}_j(z(k))\bar{w}_j(z(k))]}, \quad (13)$$

where $\underline{\nu}_j(z(k))$ and $\bar{\nu}_j(z(k))$ are the predefined nonlinear functions satisfying $\underline{\nu}_j(z(k)) + \bar{\nu}_j(z(k)) = 1$ and $\underline{\nu}_j(z(k)), \bar{\nu}_j(z(k)) \in [0, 1]$.

Based on the above MFs $h_j(z(k))$ in (13), we synthesize the SMC law as follows:

$$u(k) = \sum_{j=1}^s h_j(z(k)) F_j z(k) - \bar{d} \text{sgn}(\hat{s}(k)), \quad (14)$$

where $\hat{s}(k)$ is the sliding variable $s(k)$ in (12) with $\Omega_1(k)$ and $\Omega_2(k)$ replaced $\bar{\Omega}_1$ and $\bar{\Omega}_2$, respectively.

Substituting (8) and (14) into (1) yields the following closed-loop IT2 T-S fuzzy systems:

$$x(k+1) = \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \left\{ (A_i + B_i F_j \bar{\Omega}_1) x(k) + B_i F_j \bar{\Omega}_2 z(k-1) + B_i \hat{d}(k) + D_i f(x(k)) - B_i F_j \tilde{\Omega}_1(k) x(k) - B_i F_j \tilde{\Omega}_2(k) z(k-1) \right\} \quad (15)$$

with $\tilde{\Omega}_l(k) \triangleq \bar{\Omega}_l - \Omega_l(k)$ ($l = 1, 2$) and $\hat{d}(k) = d(k) - \bar{d} \text{sgn}(\hat{s}(k))$.

4 Stability and Reachability Analysis

In this part, a lemma is firstly provided to facilitate the analysis on the stability and reachability. By introducing some null terms, sufficient criteria are deduced subsequently for ensuring the SUB of the resultant closed-loop IT2 T-S fuzzy system (15) and the reachability of the specified sliding surface $s(k) = 0$.

Lemma 1 For any matrix $W \geq 0$, the following inequalities holds:

$$E\{\Omega_1^T(k) W \Omega_1(k)\} = \bar{\theta}_1 C_1^T W C_1 + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T W C_p \right\}, \quad (16)$$

$$E\{\Omega_2^2(k) W\} = \bar{\Omega}_2 W, \quad E\{\Omega_2(k) \Omega_1^T(k) W\} = 0, \quad (17)$$

$$E\{\tilde{\Omega}_1^T(k) W \tilde{\Omega}_1(k)\} = -\bar{\Omega}_1^T W \bar{\Omega}_1 + \bar{\theta}_1 C_1^T W C_1 + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T W C_p \right\}, \quad (18)$$

$$E\{\tilde{\Omega}_2^2(k) W\} = (\bar{\Omega}_2 - \bar{\Omega}_2^2) W, \quad (19)$$

$$E\{\tilde{\Omega}_2(k) \tilde{\Omega}_1^T(k) W\} = -\bar{\Omega}_2 \bar{\Omega}_1^T W. \quad (20)$$

Proof: For all $i = 1, 2, \dots, N$, one has

$$E\{\theta_i^2(k)\} = \bar{\theta}_i, \quad E\{\theta_i(k)(1 - \theta_i(k))\} = 0, \quad (21)$$

$$E\{(1 - \theta_1(k))^2 \cdots (1 - \theta_{i-1}(k))^2 \theta_i^2(k)\} = (1 - \bar{\theta}_1) \cdots (1 - \bar{\theta}_{i-1}) \bar{\theta}_i. \quad (22)$$

Then, Eqs. (16)-(17) hold according to the conclusions (21)-(22). Furthermore, recalling $\bar{\Omega}_l(k) \triangleq \bar{\Omega}_l - \Omega_l(k)$ and using (16)-(17), one has

$$\begin{aligned}
& E\{\tilde{\Omega}_1^T(k)W\tilde{\Omega}_1(k)\} \\
&= E\{(\bar{\Omega}_1 - \Omega_1(k))^T W(\bar{\Omega}_1 - \Omega_1(k))\} \\
&= -\bar{\Omega}_1^T W \bar{\Omega}_1 + E\{\Omega_1^T(k)W\Omega_1(k)\} \\
&= -\bar{\Omega}_1^T W \bar{\Omega}_1 + \bar{\theta}_1 C_1^T W C_1 \\
&\quad + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T W C_p \right\}, \\
& E\{\tilde{\Omega}_2^2(k)W\} = E\{(\bar{\Omega}_2 - \Omega_2(k))^2\}W \\
&= [-\bar{\Omega}_2^2 + E\{\Omega_2^2(k)\}]W = (\bar{\Omega}_2 - \bar{\Omega}_2^2)W, \\
& E\{\tilde{\Omega}_2(k)\tilde{\Omega}_1^T(k)W\} \\
&= E\{(\bar{\Omega}_2 - \Omega_2(k))(\bar{\Omega}_1 - \Omega_1(k))^T\}W \\
&= -\bar{\Omega}_2 \bar{\Omega}_1^T W,
\end{aligned}$$

which implies that Eqs. (18)-(20) hold. This completes the proof. $\square$

Now, the following null terms equipped with any symmetric matrices Γ_i will serve the analysis later [22]:

$$\begin{aligned}
& \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) (w_j(\xi(k)) - h_j(z(k))) \Gamma_i, \\
&= \sum_{i=1}^s w_i(\xi(k)) \left(\sum_{j=1}^s w_j(\xi(k)) - \sum_{j=1}^s h_j(z(k)) \right) \Gamma_i \\
&= 0,
\end{aligned} \tag{23}$$

which is ensured by the fact that $\sum_{j=1}^s w_j(\xi(k)) = \sum_{j=1}^s h_j(z(k)) = 1$.

Theorem 1 (Stability) *Given scalars τ_i satisfying*

$$h_i(z(k)) - \tau_i w_i(\xi(k)) > 0,$$

with $0 < \tau_i \leq 1$, $i = 1, 2, \dots, s$, if there are matrices $P > 0$, $Q > 0$, F_j , $\Gamma_i = \Gamma_i^T$ and scalars $\mu_i > 0$, $\lambda_i > 0$ satisfying

$$\begin{bmatrix} -\mu_i I & * \\ D_i & -P^{-1} \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -\lambda_i I & * \\ B_i & -P^{-1} \end{bmatrix} < 0, \tag{25}$$

$$\Phi_{ij} - \Gamma_i < 0, \tag{26}$$

$$\tau_i \Phi_{ii} + \Gamma_i - \tau_i \Gamma_i < 0, \tag{27}$$

$$\tau_j \Phi_{ij} + \Gamma_i - \tau_j \Gamma_i + \tau_i \Phi_{ji} + \Gamma_j - \tau_i \Gamma_j < 0 \tag{28}$$

with

$$\Phi_{ij} \triangleq \begin{bmatrix} \Phi_{ij}^{11} & * \\ \Phi_{ij}^{21} & \Phi_{ij}^{22} \end{bmatrix},$$

$$\begin{aligned}
\Phi_{ij}^{11} &\triangleq \begin{bmatrix} -P + 4\epsilon\mu_i E^T E + \bar{\theta}_1 C_1^T Q C_1 \\ + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T Q C_p \right\} \\ 0 \\ 2(A_i + B_i F_j \bar{\Omega}_1) \end{bmatrix}, \\
\Phi_{ij}^{21} &\triangleq \begin{bmatrix} \sqrt{2\bar{\theta}_1} B_i F_j C_1 & 0 & 0 \\ \sqrt{2(1 - \bar{\theta}_1)} \bar{\theta}_2 B_i F_j C_2 & 0 & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{2 \prod_{q=1}^{N-1} (1 - \bar{\theta}_q)} \bar{\theta}_N B_i F_j C_N & 0 & 0 \\ 0 & \sqrt{2\bar{\Omega}_2} B_i F_j & 0 \end{bmatrix}, \\
\Phi_{ij}^{22} &\triangleq \text{diag}\{-P^{-1}, \dots, -P^{-1}\}_{N+1},
\end{aligned}$$

the resultant closed-loop IT2 T-S fuzzy system (15) is SUB with the bound $\mathbb{D}$ as:

$$\mathbb{D} \triangleq \left\{ \|\eta(k)\| \leq 2\bar{d} \sqrt{\frac{(1+m) \max_i \{\lambda_i\}}{\delta}} \right\}, \tag{29}$$

where δ is a positive scalar.

Proof. Applying Schur complement to the expressions (24)-(25) yields

$$D_i^T P D_i \leq \mu_i I, \tag{30}$$

$$B_i^T P B_i \leq \lambda_i I. \tag{31}$$

By means of the equality (23), one can obtain

$$\begin{aligned}
& \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \Phi_{ij} \\
&= \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) (h_j(z(k)) - \tau_j w_j(\xi(k))) \\
&\quad + \tau_j w_j(\xi(k)) \Phi_{ij} + \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) (w_j(\xi(k)) \\
&\quad - h_j(z(k)) + \tau_j w_j(\xi(k)) - \tau_j w_j(\xi(k))) \Gamma_i.
\end{aligned} \tag{32}$$

Reshuffling the above terms yields

$$\begin{aligned}
& \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \Phi_{ij} \\
&= \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) (h_j(z(k)) - \tau_j w_j(\xi(k))) (\Phi_{ij} - \Gamma_i) \\
&\quad + \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) w_j(\xi(k)) (\tau_j \Phi_{ij} + \Gamma_i - \tau_j \Gamma_i) \\
&= \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) (h_j(z(k)) - \tau_j w_j(\xi(k))) (\Phi_{ij} - \Gamma_i)
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^s w_i^2(\xi(k))(\tau_i \Phi_{ii} + \Gamma_i - \tau_i \Gamma_i) \\
& + \sum_{i=1}^s \sum_{j>i} w_i(\xi(k))w_j(\xi(k))(\tau_j \Phi_{ij} + \Gamma_i - \tau_j \Gamma_i \\
& + \tau_i \Phi_{ji} + \Gamma_j - \tau_i \Gamma_j). \tag{33}
\end{aligned}$$

Note that $h_i(z(k)) - \tau_i w_i(\xi(k)) > 0$ for given τ_i ($i = 1, 2, \dots, s$) [22]. If the inequalities (26)-(28) hold, it follows from (33) that

$$\sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\Phi_{ij} < 0. \tag{34}$$

Furthermore, applying Schur complement to the expression (34) leads to

$$\sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\hat{\Phi}_{ij} < 0. \tag{35}$$

with

$$\begin{aligned}
\hat{\Phi}_{ij} & \triangleq \begin{bmatrix} \hat{\Phi}_{ij}^{11} & * \\ \hat{\Phi}_{ij}^{21} & \hat{\Phi}_{ij}^{22} \end{bmatrix}, \\
\hat{\Phi}_{ij}^{11} & \triangleq 4(A_i + B_i F_j \bar{\Omega}_1)^T P (A_i + B_i F_j \bar{\Omega}_1) + \bar{\theta}_1 C_1^T Q C_1 \\
& + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T Q C_p \right\} \\
& + 2\bar{\theta}_1 C_1^T F_j^T B_i^T P B_i F_j C_1 \\
& + 2 \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T F_j^T B_i^T P B_i F_j C_p \right\} \\
& - P + 4\epsilon \mu_i E^T E, \\
\hat{\Phi}_{ij}^{21} & \triangleq 4\bar{\Omega}_2 F_j^T B_i^T P (A_i + B_i F_j \bar{\Omega}_1), \\
\hat{\Phi}_{ij}^{22} & \triangleq 4\bar{\Omega}_2^2 F_j^T B_i^T P B_i F_j + 2\bar{\Omega}_2 F_j^T B_i^T P B_i F_j + \bar{\Omega}_2 Q - Q.
\end{aligned}$$

Define $V_1(k) = x^T(k)Px(k) + z^T(k-1)Qz(k-1)$ as the Lyapunov candidate function. For the first term of function $V_1(k)$, it yields from Eq. (15) that

$$\begin{aligned}
& E\{x^T(k+1)Px(k+1)|x(k), z(k-1)\} \\
\leq & E\left\{ \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \left\{ (A_i + B_i F_j \bar{\Omega}_1)x(k) \right. \right. \\
& + B_i F_j \bar{\Omega}_2 z(k-1) + B_i \hat{d}(k) + D_i f(x(k)) \\
& - B_i F_j \bar{\Omega}_1(k)x(k) - B_i F_j \bar{\Omega}_2(k)z(k-1) \left. \right\}^T P \\
& \times \left\{ (A_i + B_i F_j \bar{\Omega}_1)x(k) + B_i F_j \bar{\Omega}_2 z(k-1) + B_i \hat{d}(k) \right. \\
& + D_i f(x(k)) - B_i F_j \bar{\Omega}_1(k)x(k) - B_i F_j \bar{\Omega}_2(k)z(k-1) \left. \right\} \Big\}.
\end{aligned}$$

Noting that $(a+b)^2 \leq 2a^2 + 2b^2$ and $E\{\tilde{\Omega}_1(k)\} = E\{\tilde{\Omega}_2(k)\} = 0$, the previous expression can be further developed as

$$\begin{aligned}
& E\{x^T(k+1)Px(k+1)|x(k), z(k-1)\} \\
\leq & E\left\{ \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \left\{ 2[(A_i + B_i F_j \bar{\Omega}_1)x(k) \right. \right. \\
& + B_i F_j \bar{\Omega}_2 z(k-1) + D_i f(x(k))]^T P [(A_i + B_i F_j \bar{\Omega}_1)x(k) \\
& + B_i F_j \bar{\Omega}_2 z(k-1) + D_i f(x(k))] + 2[B_i F_j \bar{\Omega}_1(k)x(k) \\
& + B_i F_j \bar{\Omega}_2(k)z(k-1)]^T P [B_i F_j \bar{\Omega}_1(k)x(k) \\
& + B_i F_j \bar{\Omega}_2(k)z(k-1)] + 2\hat{d}^T(k)B_i P B_i \hat{d}(k) \left. \right\} \Big\}. \tag{36}
\end{aligned}$$

By employing expressions (18)-(20) in Lemma 1 and inequalities (30)-(31), it follows from (36) that

$$\begin{aligned}
& E\{x^T(k+1)Px(k+1)|x(k), z(k-1)\} \\
\leq & \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \left\{ 4[(A_i + B_i F_j \bar{\Omega}_1)x(k) \right. \\
& + B_i F_j \bar{\Omega}_2 z(k-1)]^T P [(A_i + B_i F_j \bar{\Omega}_1)x(k) \\
& + B_i F_j \bar{\Omega}_2 z(k-1)] + 4\epsilon \mu_i x^T(k)E^T E x(k) \\
& + 2x^T(k) \left\{ -\bar{\Omega}_1^T F_j^T B_i^T P B_i F_j \bar{\Omega}_1 \right. \\
& + \bar{\theta}_1 C_1^T F_j^T B_i^T P B_i F_j C_1 \\
& + 2 \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T F_j^T B_i^T P B_i F_j C_p \right\} \left. \right\} x(k) \\
& + 2z^T(k-1)(\bar{\Omega}_2 - \bar{\Omega}_2^2)F_j^T B_i^T P B_i F_j z(k-1) \\
& + 4(1+m)\lambda_i \bar{d}^2 \\
& - 4x^T(k)\bar{\Omega}_2 \bar{\Omega}_1^T F_j^T B_i^T P B_i F_j z(k-1) \left. \right\} \\
= & \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \left\{ 4[(A_i + B_i F_j \bar{\Omega}_1)x(k) \right. \\
& + B_i F_j \bar{\Omega}_2 z(k-1)]^T P \\
& \times [(A_i + B_i F_j \bar{\Omega}_1)x(k) + B_i F_j \bar{\Omega}_2 z(k-1)] \\
& + 4\epsilon \mu_i x^T(k)E^T E x(k) + 2x^T(k) \\
& \times \left\{ \bar{\theta}_1 C_1^T F_j^T B_i^T P B_i F_j C_1 + 2 \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \right. \right. \\
& \times \bar{\theta}_p C_p^T F_j^T B_i^T P B_i F_j C_p \left. \right\} \left. \right\} x(k) \\
& + 2z^T(k-1)\bar{\Omega}_2 F_j^T B_i^T P B_i F_j z(k-1) \\
& + 4(1+m)\lambda_i \bar{d}^2 - [x^T(k) \ z^T(k-1)] \\
& \times \begin{bmatrix} \sqrt{2}\bar{\Omega}_1^T F_j^T B_i^T \\ \sqrt{2}\bar{\Omega}_2 F_j^T B_i^T \end{bmatrix} P \begin{bmatrix} \sqrt{2}B_i F_j \bar{\Omega}_1 & \sqrt{2}B_i F_j \bar{\Omega}_2 \end{bmatrix}
\end{aligned}$$

$$\times \begin{bmatrix} x(k) \\ z(k-1) \end{bmatrix} \Bigg\}. \quad (37)$$

Thus, for $P > 0$, we have

$$\begin{aligned} & E\{x^T(k+1)Px(k+1)|x(k), z(k-1)\} \\ & \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \Bigg\{ 4[(A_i + B_i F_j \bar{\Omega}_1)x(k) \\ & + B_i F_j \bar{\Omega}_2 z(k-1)]^T P[(A_i + B_i F_j \bar{\Omega}_1)x(k) \\ & + B_i F_j \bar{\Omega}_2 z(k-1)] + 4\epsilon \mu_i x^T(k) E^T E x(k) \\ & + 2x^T(k) \Bigg\{ \bar{\theta}_1 C_1^T F_j^T B_i^T P B_i F_j C_1 \\ & + 2 \sum_{p=2}^N \Bigg\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T F_j^T B_i^T P B_i F_j C_p \Bigg\} \Bigg\} \\ & \times x(k) + 2z^T(k-1) \bar{\Omega}_2 F_j^T B_i^T P B_i F_j z(k-1) \\ & + 4(1+m)\lambda_i \bar{d}^2 \Bigg\}. \quad (38) \end{aligned}$$

Similarly, for the second term of Lyapunov function $V_1(k)$, it follows from expressions (8) and (16)-(17) in Lemma 1 that

$$\begin{aligned} & E\{z^T(k)Qz(k)|x(k), z(k-1)\} \\ & = E\{[\Omega_1(k)x(k) + \Omega_2(k)z(k-1)]^T Q[\Omega_1(k)x(k) \\ & + \Omega_2(k)z(k-1)]\} \\ & = x^T(k) \Bigg\{ \sum_{p=2}^N \Bigg\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T Q C_p \\ & + \bar{\theta}_1 C_1^T Q C_1 \Bigg\} \Bigg\} x(k) + z(k-1)^T \bar{\Omega}_2 Q z(k-1). \quad (39) \end{aligned}$$

By utilizing (38)-(39), for Lyapunov function $V_1(k)$, one has

$$\begin{aligned} & E\{\Delta V_1(k)|x(k), z(k-1)\} \\ & = E\{x^T(k+1)Px(k+1)|x(k), z(k-1)\} \\ & + E\{z^T(k)Qz(k)|x(k), z(k-1)\} \\ & - x^T(k)Px(k) - z^T(k-1)Qz(k-1) \\ & \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\eta^T(k)\hat{\Phi}_{ij}\eta(k) \\ & + 4(1+m)\lambda_i \bar{d}^2. \quad (40) \end{aligned}$$

Owing to the expression (35), there exists a positive scalar δ such that

$$\sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\eta^T(k)\hat{\Phi}_{ij}\eta(k) < -\delta\|\eta(k)\|^2. \quad (41)$$

Therefore, it follows from inequalities (40)-(41) that

$$\begin{aligned} & E\{\Delta V_1(k)|x(k), z(k-1)\} \\ & \leq -\delta\|\eta(k)\|^2 + 4\max\{\lambda_i\}(1+m)\bar{d}^2. \quad (42) \end{aligned}$$

Taking expectations on both sides of (42) yields

$$\begin{aligned} & E\{\Delta V_1(k)\} \\ & \leq -\delta E\{\|\eta(k)\|^2\} + 4\max\{\lambda_i\}(1+m)\bar{d}^2, \quad (43) \end{aligned}$$

which implies $E\{\Delta V_1(k)\} < 0$, once the system states escape from the region $\mathbb{D}$ defined in (29). In other words, the function $V_1(k)$ monotonously declines (in the mean square) outside the region $\mathbb{D}$. So, the state trajectories will enter into the region $\mathbb{D}$, and then, the SUB of the resultant closed-loop IT2 T-S fuzzy system (15) is ensured. $\square$

Remark 2 As discussed in the Introduction, the MFs of the system (1) and the ones for the controller (14) are unmatched due to the special structure of IT2 T-S fuzzy systems and the unreliable channels. These different MFs simultaneously exists in the closed-loop systems (15). By using the null terms in (23) and the inequalities $h_i(z(k)) - \tau_i w_i(\xi(k)) > 0$ involving in these MFs, the favorable symmetry property is satisfied as shown in Eq. (33) and the relaxed stability conditions can be derived.

In what follows, the reachability of the specified sliding surface $s(k) = 0$ will be further analyzed.

By means of (1), (8), (12) and (14), one can obtain

$$\begin{aligned} s(k+1) & = \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \Bigg\{ [G\bar{\Omega}_1 A_i \\ & + \bar{G}_{ij}\bar{\Omega}_1]x(k) + \bar{G}_{ij}\bar{\Omega}_2 z(k-1) \\ & + G\bar{\Omega}_1 D_i f(x(k)) + G\bar{\Omega}_1 B_i \hat{d}(k) \\ & - \bar{G}_{ij}\bar{\Omega}_1(k)x(k) - \bar{G}_{ij}\bar{\Omega}_2(k)z(k-1) \Bigg\} \quad (44) \end{aligned}$$

with $\bar{G}_{ij} \triangleq (G\bar{\Omega}_1 B_i F_j + G\bar{\Omega}_2)$.

Theorem 2 (Reachability) *Given scalars τ_i satisfying $h_i(z(k)) - \tau_i w_i(\xi(k)) > 0$ ($0 < \tau_i \leq 1$, $i = 1, 2, \dots, s$), if there exist scalars $\mu_i > 0$, $\lambda_i > 0$, $\zeta_i > 0$, $\gamma_i > 0$ and matrices $P > 0$, $Q > 0$, $\bar{Z} > 0$, G , F_j , $\Xi_i = \Xi_i^T$ satisfying (24)-(25) and the following inequalities*

$$\begin{bmatrix} -\zeta_i I & * \\ G\bar{\Omega}_1 B_i & -\bar{Z} \end{bmatrix} < 0, \quad (45)$$

$$\begin{bmatrix} -\gamma_i I & * \\ G\bar{\Omega}_1 D_i & -\bar{Z} \end{bmatrix} < 0, \quad (46)$$

$$\Pi_{ij} - \Xi_i < 0, \quad (47)$$

$$\tau_i \Pi_{ii} + \Xi_i - \tau_i \Xi_i < 0, \quad (48)$$

$$\tau_j \Pi_{ij} + \Xi_i - \tau_j \Xi_i + \tau_i \Pi_{ji} + \Xi_j - \tau_i \Xi_j < 0, \quad (49)$$

with

$$\Pi_{ij} \triangleq \begin{bmatrix} \Pi_{ij}^{11} & * & * & * \\ \Phi_{ij}^{21} & \Phi_{ij}^{22} & * & * \\ \Pi_{ij}^{31} & 0 & -\bar{Z} & * \\ \Pi_{ij}^{41} & 0 & 0 & \Pi_{ij}^{44} \end{bmatrix},$$

$$\Pi_{ij}^{11} \triangleq \Phi_{ij}^{11} + 4\epsilon\gamma_i E^T E,$$

$$\Pi_{ij}^{31} \triangleq [2(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) \ 2\bar{G}_{ij} \ 0],$$

$$\Pi_{ij}^{32} \triangleq [2\bar{G}_{ij} \ 0 \ 0 \ 0 \ \cdots \ 0 \ 0],$$

$$\Pi_{ij}^{41} \triangleq \begin{bmatrix} \sqrt{2\bar{\theta}_1}\bar{G}_{ij}C_1 & 0 & 0 \\ \sqrt{2(1-\bar{\theta}_1)\bar{\theta}_2}\bar{G}_{ij}C_2 & 0 & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{2\prod_{q=1}^{N-1}(1-\bar{\theta}_q)\bar{\theta}_N}\bar{G}_{ij}C_N & 0 & 0 \\ 0 & \sqrt{2\bar{\Omega}_2}\bar{G}_{ij} & 0 \end{bmatrix},$$

$$\Pi_{ij}^{44} \triangleq \text{diag}\{-\bar{Z}, \dots, -\bar{Z}\}_{N+1},$$

the states of the closed-loop IT2 T-S fuzzy system (15) will be forced into the following domain around the specified sliding surface $s(k) = 0$ in the mean square sense:

$$\mathbb{O} \triangleq \{s(k) \mid \|s(k)\| \leq \sigma\} \quad (50)$$

$$\text{with } \sigma \triangleq \frac{\sqrt{8(1+m)\max_i\{\lambda_i + \gamma_i\}}}{\lambda_{\min}\{Z\}}\bar{d} \text{ and } Z = \bar{Z}^{-1}.$$

Proof. Applying Schur complement to the inequalities (45)-(46) yields

$$B_i^T \bar{\Omega}_1^T G^T Z G \bar{\Omega}_1 B_i \leq \zeta_i I, \quad (51)$$

$$D_i^T \bar{\Omega}_1^T \bar{G}^T Z G \bar{\Omega}_1 \bar{D}_i \leq \gamma_i I. \quad (52)$$

Replacing Γ_i by Ξ_i in (23) and following the similar lines from (32) to (35), we have by (47)-(49)

$$\sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \hat{\Pi}_{ij} < 0, \quad (53)$$

where

$$\hat{\Pi}_{ij} \triangleq \begin{bmatrix} \hat{\Pi}_{ij}^{11} & * \\ \hat{\Pi}_{ij}^{21} & \hat{\Pi}_{ij}^{22} \end{bmatrix},$$

$$\hat{\Pi}_{ij}^{11} \triangleq \hat{\Phi}_{ij}^{11} + 4(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1)^T Z (G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) \\ + 2\bar{\theta}_1 C_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_1 + 4\epsilon\gamma_i E^T E \\ + 2 \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_p \right\},$$

$$\hat{\Pi}_{ij}^{21} \triangleq \hat{\Phi}_{ij}^{21} + 4\bar{\Omega}_2 \bar{G}_{ij}^T Z (G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1),$$

$$\hat{\Pi}_{ij}^{22} \triangleq \hat{\Phi}_{ij}^{22} + (4\bar{\Omega}_2^2 + 2\bar{\Omega}_2) \bar{G}_{ij}^T Z \bar{G}_{ij}.$$

Set $V_2(k) = V_1(k) + s^T(k) Z s(k)$ as the Lyapunov function. By means of (44), one has for the second term of $V_2(k)$

$$\begin{aligned} & E\{s^T(k+1) Z s(k+1) | x(k), z(k-1), s(k)\} \\ & \leq E \left\{ \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \left\{ [G\bar{\Omega}_1 A_i \right. \right. \\ & \quad + \bar{G}_{ij}\bar{\Omega}_1] x(k) + \bar{G}_{ij}\bar{\Omega}_2 z(k-1) + G\bar{\Omega}_1 D_i f(x(k)) \\ & \quad + G\bar{\Omega}_1 B_i \hat{d}(k) - \bar{G}_{ij}\bar{\Omega}_1(k) x(k) - \bar{G}_{ij}\bar{\Omega}_2(k) z(k-1) \Big\} \\ & \quad \times Z \left\{ [G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1] x(k) + \bar{G}_{ij}\bar{\Omega}_2 z(k-1) \right. \\ & \quad + G\bar{\Omega}_1 D_i f(x(k)) + G\bar{\Omega}_1 B_i \hat{d}(k) - \bar{G}_{ij}\bar{\Omega}_1(k) x(k) \\ & \quad \left. \left. - \bar{G}_{ij}\bar{\Omega}_2(k) z(k-1) \right\} \right\}. \end{aligned} \quad (54)$$

By means of expressions (18)-(20) in Lemma 1 and inequalities (51)-(52), we can derive from (54) that

$$\begin{aligned} & E\{s^T(k+1) Z s(k+1) | x(k), z(k-1), s(k)\} \\ & \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \left\{ 4[(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) x(k) \right. \\ & \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)]^T Z [(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) x(k) \\ & \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)] + 4\epsilon\gamma_i x^T(k) E^T E x(k) \\ & \quad + 2x^T(k) \left\{ -\bar{\Omega}_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} \bar{\Omega}_1 + \bar{\theta}_1 C_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_1 \right. \\ & \quad + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_p \right\} \Big\} \\ & \quad + 2z^T(k-1) (\bar{\Omega}_2 - \bar{\Omega}_2^2) \bar{G}_{ij}^T Z \bar{G}_{ij} \bar{z}(k-1) \\ & \quad \left. - 4x^T(k) \bar{\Omega}_2 \bar{\Omega}_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} z(k-1) + 4\zeta_i \bar{d}^2 (1+m) \right\}. \\ & = \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k)) h_j(z(k)) \left\{ 4[(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) x(k) \right. \\ & \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)]^T Z [(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1) x(k) \\ & \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)] + 4\epsilon\gamma_i x^T(k) E^T E x(k) \\ & \quad + 2x^T(k) \left\{ \bar{\theta}_1 C_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_1 \right. \\ & \quad + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_p \right\} \Big\} \\ & \quad + 2z^T(k-1) \bar{\Omega}_2 \bar{G}_{ij}^T Z \bar{G}_{ij} \bar{z}(k-1) + 4\zeta_i \bar{d}^2 (1+m) \\ & \quad - [x^T(k) \ z^T(k-1)] \begin{bmatrix} \sqrt{2}\bar{\Omega}_1^T \bar{G}_{ij}^T \\ \sqrt{2}\bar{\Omega}_2 \bar{G}_{ij}^T \end{bmatrix} \\ & \quad \times Z \begin{bmatrix} \sqrt{2}\bar{G}_{ij} \bar{\Omega}_1 & \sqrt{2}\bar{\Omega}_2 \bar{G}_{ij} \end{bmatrix} \begin{bmatrix} x(k) \\ z(k-1) \end{bmatrix} \Big\}. \end{aligned} \quad (55)$$

Owing to $Z > 0$, the expression (55) can be further developed as

$$\begin{aligned}
& E\{s^T(k+1)Zs(k+1)|x(k), z(k-1), s(k)\} \\
& \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k)) \left\{ 4[(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1)x(k) \right. \\
& \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)]^T Z[(G\bar{\Omega}_1 A_i + \bar{G}_{ij}\bar{\Omega}_1)x(k) \\
& \quad + \bar{G}_{ij}\bar{\Omega}_2 z(k-1)] + 4\epsilon\gamma_i x^T(k)E^T E x(k) \\
& \quad + 2x^T(k) \left\{ \bar{\theta}_1 C_1^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_1 \right. \\
& \quad \left. + \sum_{p=2}^N \left\{ \left[\prod_{q=1}^{p-1} (1 - \bar{\theta}_q) \right] \bar{\theta}_p C_p^T \bar{G}_{ij}^T Z \bar{G}_{ij} C_p \right\} \right\} \\
& \quad \left. + 2z^T(k-1)\bar{\Omega}_2 \bar{G}_{ij}^T Z \bar{G}_{ij} \bar{z}(k-1) + 4\zeta_i \bar{d}^2 (1+m) \right\}. \tag{56}
\end{aligned}$$

Furthermore, one can obtain from (38)-(40) and (56) that

$$\begin{aligned}
& E\{\Delta V_2(k)|x(k), z(k-1), s(k)\} \\
& = E\{\Delta V_1(k)|x(k), z(k-1)\} \\
& \quad + E\{s^T(k+1)Zs(k+1)|x(k), z(k-1), s(k)\} \\
& \quad - s^T(k)Zs(k) \\
& \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\eta^T(k)\hat{\Pi}_{ij}\eta(k) \\
& \quad + 4(1+m)(\lambda_i + \gamma_i)\bar{d}^2 - s^T(k)Zs(k) \\
& \leq \sum_{i=1}^s \sum_{j=1}^s w_i(\xi(k))h_j(z(k))\eta^T(k)\hat{\Pi}_{ij}\eta(k) \\
& \quad - \frac{1}{2}s^T(k)Zs(k) - \left[\frac{1}{2}\lambda_{\min}\{Z\}\|s(k)\|^2 \right. \\
& \quad \left. - 4(1+m)\max_i\{\lambda_i + \gamma_i\}\bar{d}^2\right]. \tag{57}
\end{aligned}$$

When $\|s(k)\| > \sigma$ with σ defined below (50), we have $E\{\Delta V_2(k)|x(k), z(k-1), s(k)\} < 0$ by (53). Taking expectations on both sides of $E\{\Delta V_2(k)|x(k), z(k-1), s(k)\} < 0$ leads to $E\{\Delta V_2(k)\} < 0$. This implies that outside the domain $\mathbb{O}$ defined in (50), the Lyapunov function $V_2(k)$ is monotonically decreasing in the mean-square sense. Thus, it can be concluded that the states can be driven into the sliding domain $\mathbb{O}$ in the mean square. $\square$

5 Solving Algorithm

In order to guarantee the SUB of the resultant closed-loop IT2 T-S fuzzy system (15) and the reachability of the prescribed sliding surface $s(k) = 0$, the sufficient criteria (24)-(28) and (45)-(49) in both Theorems 1 and

2 should be ensured. However, it should be pointed out that these conditions are difficultly solved due to the coupling terms P , P^{-1} and $\bar{G}_{ij} \triangleq (G\bar{\Omega}_1 B_i F_j + G\bar{\Omega}_2)$. Aiming at decoupling these terms, this work proposes a minimization problem:

$$\min_{P, G} \sigma = \frac{\sqrt{8(1+m)\max_i\{\lambda_i + \gamma_i\}}}{\lambda_{\min}\{Z\}} \bar{d} \tag{58}$$

subject to (24)-(28), (45)-(49),

which can be solved by using the notable binary-coded genetic algorithm (GA) as follows.

GA-based solving algorithm:

- **Step 1. Parameter encoding:** Denote the Lyapunov variable $P = [\rho_{u_2 v_2}]_{n \times n}$ and the sliding gain matrix $G = [g_{u_1 v_1}]_{m \times n}$ with $\frac{2mn+n^2+n}{2}$ independent variables. These variables are further written into the row vector $\omega \in \mathbb{R}^{1 \times \frac{2mn+n^2+n}{2}}$ as follows:

$$\begin{aligned}
[G, P^{-1}] & \mapsto \omega \\
& \triangleq [g_{11} \ \cdots \ g_{1n} \ g_{21} \ \cdots \ g_{2n} \ g_{31} \ \cdots \ g_{mn} \\
& \quad \wp_{11} \ \cdots \ \wp_{1n} \ \wp_{22} \ \cdots \ \wp_{2n} \ \wp_{33} \ \cdots \ \wp_{nn}].
\end{aligned}$$

The elements $g_{u_1 v_1}$ and $\wp_{u_2 v_2}$ in ω are encoded as binary strings with lengths $l_{g_{u_1 v_1}}$ and $l_{\wp_{u_2 v_2}}$, respectively, over the ranges $g_{u_1 v_1} \in [\underline{g}_{u_1 v_1}, \bar{g}_{u_1 v_1}]$ and $\wp_{u_2 v_2} \in [\underline{\wp}_{u_2 v_2}, \bar{\wp}_{u_2 v_2}]$. Note that $\wp_{u_2 v_2} > 0$ for $u_2 = v_2$ due to $P^{-1} > 0$. Then, the precisions $q_{g_{u_1 v_1}}$ and $q_{\wp_{u_2 v_2}}$ can be yielded via the linearly mapped coding method as

$$q_{g_{u_1 v_1}} = \frac{\bar{g}_{u_1 v_1} - \underline{g}_{u_1 v_1}}{2^{l_{g_{u_1 v_1}}} - 1}, \quad q_{\wp_{u_2 v_2}} = \frac{\bar{\wp}_{u_2 v_2} - \underline{\wp}_{u_2 v_2}}{2^{l_{\wp_{u_2 v_2}}} - 1}. \tag{59}$$

- **Step 2. Population initialization:** Produce an initial population with $\bar{N}$ chromosomes ω_ι ($\iota = 1, 2, \dots, \bar{N}$) stochastically.
- **Step 3. Fitness function and assignment:** For every sliding gain $G = [g_{u_1 v_1}]$ and the Lyapunov variable $P = [\wp_{u_2 v_2}]$, decode the initial population generated in Step 2 into their real values. Check whether the conditions (24)-(28), (45)-(49) are feasible. If these criteria are infeasible, assign a sufficiently small scalar to the fitness function. Otherwise, the fitness function is defined as $\mathcal{F}(P^{-1}, G) = 1/\sigma$.
- **Step 4. Performing genetic operations:** The next generation can be obtained through genetic operations involving Selection, Crossover, and Mutation. Denote p_m and p_c as the single bit mutation probability and the single-point crossover probability, respectively.

- **Step 5. Stop criterion:** Repeat Steps 3 and 4 in each generation until reaching the maximum generation $\mathcal{G}_{max}$. The real values of the sliding gain G and the Lyapunov variable P are decoded via the best chromosome ω_b .
- **Step 6. SMC law design:** The SMC law (14) can be constructed via the sliding gain G and the control gain F_i obtained in Step 5.

Remark 3 It can be seen from (24)-(28), (45)-(49) and (50) that the value of σ may be indirectly affected by the choices of P and G via the decision variables Z , λ_i and γ_i . By solving the above optimization problem (58), a smaller sliding domain $\mathbb{O}$ defined in (50) may be obtained and a better SMC performance can be expected.

6 Simulation

Consider the following nonlinear system:

$$\begin{cases} x_1(k+1) = -0.5x_1(k) + \alpha \sin^2(x_1(k))x_1(k) \\ \quad - 0.3x_2(k) - u(k) - d(k, x(k)), \\ x_2(k+1) = 0.1x_1(k) + 0.5x_2(k) + u(k) + d(k, x(k)) \\ \quad + \sin(x_1(k)) - 2/\pi x_1(k), \end{cases} \quad (60)$$

where $d(k, x(k)) \triangleq \bar{d} \sin(k)$ with $\bar{d} = 0.001$, $x_1(k) \in [-\pi/2, \pi/2]$, $x(k) \triangleq [x_1(k), x_2(k)]^T$, and the uncertain parameter α satisfies $\alpha \in [1, 2]$. Denote

$$f(x(k)) \triangleq \sin(x_1(k)) - 2/\pi x_1(k).$$

According to Ref. [27], one has

$$\sin(x_1(k)) \in \text{co}(2/\pi x_1(k), x_1(k))$$

for $x_1(k) \in [-\pi/2, \pi/2]$, which implies

$$f(x(k)) \in \text{co}(0, (1 - 2/\pi)x_1(k)).$$

Thus, we have $\|f(x(k))\|^2 \leq (1 - 2/\pi)^2 \|x_1(k)\|^2$ and the parameters ϵ , E can be selected as $\epsilon = (1 - 2/\pi)^2$, $E = [1 \ 0]$.

Denote $\phi(x_1(k)) = \alpha \sin^2(x_1(k))$, whose maximum value ϕ_{max} and the minimum value ϕ_{min} are, respectively, calculated as $\phi_{max} = 2$ and $\phi_{min} = 0$. The upper and lower MFs are selected as

$$\bar{w}_1(x_1(k)) = \frac{2\sin^2(x_1(k)) - \hat{\phi}_{min}}{\hat{\phi}_{max} - \hat{\phi}_{min}},$$

$$\underline{w}_1(x_1(k)) = \frac{\sin^2(x_1(k)) - \hat{\phi}_{min}}{\hat{\phi}_{max} - \hat{\phi}_{min}},$$

$$\bar{w}_2(x_1(k)) = 1 - \underline{w}_1(x_1(k)), \quad \hat{\phi}_{max} = \phi_{max} + 0.02,$$

$$\underline{w}_2(x_1(k)) = 1 - \bar{w}_1(x_1(k)), \quad \hat{\phi}_{min} = \phi_{min} - 0.02.$$

Thus, the nonlinear plant (60) is represented as a 2-rule T-S fuzzy system as follows:

$$x(k) = \sum_{i=1}^2 w_i(x_1(k)) [A_i x(k) + D_i f(x(k)) + B_i(u(t) + d(k, x(k)))], \quad (61)$$

where

$$A_1 = \begin{bmatrix} -0.5 + \hat{\phi}_{max} & -0.3 \\ 0.1 & 0.5 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.5 + \hat{\phi}_{min} & -0.3 \\ 0.1 & 0.5 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

and the MFs are given as $w_1(x_1(k))$ and $w_2(x_1(k)) = 1 - w_1(x_1(k))$.

It is assumed that two communication channels are employed to transmit the measurement output with the output matrices $C_1 = [1 \ 0]$ and $C_2 = [0.8 \ 0]$. As discussed in the Introduction, the MFs $w_i(x_1(k))$ ($i = 1, 2$) of the system (61) are uncertain, so we design the MFs of the SMC law as

$$h_1(z(k)) = \frac{0.7\underline{w}_1(z(k)) + 0.3\bar{w}_1(z(k))}{\varpi(k)}, \quad (62)$$

$$h_2(z(k)) = \frac{0.3\underline{w}_2(z(k)) + 0.7\bar{w}_2(z(k))}{\varpi(k)} \quad (63)$$

with $\varpi(k) = 0.7\underline{w}_1(z(k)) + 0.3\bar{w}_1(z(k)) + 0.3\underline{w}_2(z(k)) + 0.7\bar{w}_2(z(k))$. According to the inequalities introduced in Theorems 1 and 2, the parameters τ_1 and τ_2 are set as $\tau_1 = 0.019$ and $\tau_2 = 0.108$ satisfying $h_i(z(k)) - \tau_i w_i(\xi(k)) > 0$, $i = 1, 2$.

Next, we utilize the proposed GA-based algorithm to solve the optimization problem (58) with the corresponding parameters given as: the population size $\bar{N} = 80$, the maximum generation $\mathcal{G}_{max} = 100$, the mutation and crossover probabilities $p_m = 0.05$ and $p_c = 0.8$, respectively, the parameter ranges $g_{11} \in [-1, 1]$, $\wp_{11}, \wp_{22} \in [0.001, 1]$ and $\wp_{12} \in [-1, 1]$, the lengths of binary strings $l_{g_{11}} = 11$, $l_{\wp_{11}} = l_{\wp_{22}} = 10$ and $l_{\wp_{12}} = 10$, respectively, the probability $\bar{\theta}_1 = 0.8$ and $\bar{\theta}_2 = 0.7$. And then, the following optimal solutions are obtained:

$$P = \begin{bmatrix} 6.7503 & 2.6431 \\ 2.6431 & 2.1591 \end{bmatrix}, \quad G = -0.6541,$$

$$F_1 = 0.1818, \quad F_2 = 0.1812.$$

For initial states $x(0) = [1 \ -1]^T$, we conduct 20 individual experiments for showing the stochastic characteristic of the packet dropout obeying Bernoulli processes. The validity of the design strategy in this work is illustrated by Figs. 2-4, where the SUB (Fig. 2) of

Fig. 2: The evolution of $x(k)$ Fig. 5: The evolution of $x(k)$ with only one channel Fig. 3: The sliding variable $s(k)$ Fig. 6: The control $u(k)$ with only one channel Fig. 4: The control $u(k)$

the resultant closed-loop IT2 T-S fuzzy system and the reachability (Fig. 3) of a sliding domain around the sliding surface $s(k) = 0$ can be achieved via the SMC law (Fig. 4).

As a comparison, we further consider the case that only one communication channel is utilized. In this case, the actual measurement will be $z(k) = \theta_1(k)y_1(k) + (1 - \theta_1(k))z(k-1)$. The corresponding results are shown in Figs. 5-6, from which it is seen that the control performances may become worse using only one channel.

7 Conclusion

This work has addressed the SMC of IT2 T-S fuzzy systems under redundant channels. The main challenge has been the premise variables and the measurement outputs unavailable for the controller. An implementable SMC scheme has been designed by constructing the premise variables and the MFs of the controller. The

corresponding conditions on the stability and reachability analysis have been derived. Moreover, an optimizing algorithm via GA has been provided for solving the proposed control design problem subject to nonconvex constraints.

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Data availability statements All data generated or analyzed during this study are included within the article.

Compliance with ethical standards

Conflict of interest All the authors in this manuscript declare that they have no conflict of interest.

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