Influence of reinforcing ribs on the deformed state of ellipsoidal shells under distributed load

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Influence of reinforcing ribs on the deformed state of ellipsoidal shells under distributed load

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Abstract This paper presents the problem of non-stationary oscillations of reinforced ellipsoidal shells, taking into account the discrete location of the ribs. Problem bases on a geometrically nonlinear variant of the Timoshenko theory for shells and rods. A numerical method for solving problems of this class has been developed and substantiated. This article focuses on the location of the reinforcing ribs. Based on the developed numerical method the deformed state of discretely supported ellipsoidal shells for internal, external, and internal-external placement of ribs is investigated. Boundary conditions for rigidly clamped edges of the shell were studied.

Keywords Oscillations - Shell - Discretely reinforced - Non-stationary load - Numerical algorithm.

1 Introduction

Thin-walled structural elements in the form of shells with various shapes are widely used in mechanical engineering, construction, aerospace, shipbuilding. The shell elements, which consist of the shell itself and reinforcing ribs, have become widespread. The method of fastening the ribs ensures their joint work.

When calculating the strength of reinforced shell structures under dynamic loads, it is necessary to determine the deformed state both in the field of changes in the physical and mechanical parameters for the structure, and at a considerable distance from the inhomogeneities. In this case, one of the features of the studied process is the wave nature and nonlinear distribution of physical and mechanical parameters throughout the system. The complexity of the resulting processes necessitates the use of modern numerical methods for solving dynamic problems of shell elements with reinforcing ribs.

According to reviews and monographs, harmonic oscillations for reinforced shells of simple geometry (cylindrical, conical, and spherical) were mainly considered [1- 3, 11 - 12, 17]. Forced oscillations of reinforced shells under pulse loads were studied in [4 -5, 10, 14, 16]. There is almost no work to study the dynamic behavior of reinforced shells with more complex shapes. In this direction, we can distinguish the following works [6 - 9], which present the results of forced oscillations for the shells of rotation in particular ellipsoidal reinforced shells.

In this paper, the equations for non-axisymmetric oscillations of a discretely reinforced ellipsoidal shell are presented. When considering the shell and reinforcing ribs, a refined model for shells and rods of the Timoshenko type is used [5]. The Hamilton-Ostrogradsky variational principle is used to derive the equations of oscillations. Such problems are quite difficult to set and solve. We use a numerical method based on the application of the integration-interpolation method to construct difference schemes for equations with discontinuous coefficients.

The goal is to study the influence of the location reinforcing ribs on the deformed state of ellipsoidal shells under distributed load.

2 Statement of the problem

We consider an ellipsoidal shell of constant thickness h . The shell is attributed to the Gaussian curvilinear coordinate system α_1, α_2 .

The middle surface of a smooth ellipsoidal shell is given by the relations:

$$\begin{aligned}x &= R \sin \alpha_1 \sin \alpha_2; \\y &= R \sin \alpha_1 \cos \alpha_2; \\z &= kR \cos \alpha_1,\end{aligned}\quad (1)$$

where $k = b/a$ – ellipticity parameter, a, b – the half axis of the ellipse.

To build a mathematical model for the deformation of the reinforced ellipsoidal shell, we will use a geometrically nonlinear version of the Timoshenko-type theory for shells in the quadratic approximation [5].

The change in displacement along the thickness of the shell is given in the form:

$$\begin{aligned}u_1^z(s_1, s_2, z) &= u_1(s_1, s_2) + z\varphi_1(s_1, s_2); \\u_2^z(s_1, s_2, z) &= u_2(s_1, s_2) + z\varphi_2(s_1, s_2); \\u_3^z(s_1, s_2, z) &= u_3(s_1, s_2);\end{aligned}\quad (2)$$

where $z \in \left[-\frac{h}{2}, \frac{h}{2}\right]$, $s_1 = \alpha_1 A_1$, $s_2 = \alpha_2 A_2$, A_1, A_2 – coefficients of the first quadratic form for the middle surface of the given ellipsoidal shell.

Deformation relations for a smooth shell according [13]:

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$$\varepsilon_{11} = \frac{\partial u_1}{\partial s_1} + k_1 u_3 + \frac{1}{2} \theta_1^2, \quad (3)$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial s_2} + \frac{1}{A_2} \frac{\partial A_2}{\partial s_1} u_1 + k_2 u_3 + \frac{1}{2} \theta_2^2,$$

$$\varepsilon_{12} = \omega + \theta_1 \theta_2, \quad \varepsilon_{13} = \varphi_1 + \theta_1,$$

$$\varepsilon_{23} = \varphi_2 + \theta_2, \quad \omega = \omega_1 + \omega_2,$$

$$\omega_1 = \frac{\partial u_2}{\partial s_1}, \quad \omega_2 = \frac{\partial u_1}{\partial s_2} - \frac{1}{A_2} \frac{\partial A_2}{\partial s_1} u_2,$$

$$\theta_1 = \frac{\partial u_3}{\partial s_1} - k_1 u_1, \quad \theta_2 = \frac{\partial u_3}{\partial s_2} - k_2 u_2,$$

$$\chi_{11} = \frac{\partial \varphi_1}{\partial s_1}, \quad \chi_{22} = \frac{\partial \varphi_2}{\partial s_2} + \frac{1}{A_2} \frac{\partial A_2}{\partial s_1} \varphi_1,$$

$$\chi_{12} = \tau_1 + \tau_2 + \kappa_1 \omega_1 + \kappa_2 \omega_2, \quad \tau_1 = \frac{\partial \varphi_2}{\partial s_1},$$

$$\tau_2 = \frac{\partial \varphi_1}{\partial s_2} - \frac{1}{A_2} \frac{\partial A_2}{\partial s_1} \varphi_2.$$

Expressions for the forces and moments of the smooth ellipsoidal shell have the form:

$$T_{11} = B_{11} \varepsilon_{11} + B_{12} \varepsilon_{22}, \quad T_{22} = B_{21} \varepsilon_{11} + B_{22} \varepsilon_{22}, \quad (4)$$

$$T_{12} = S + k_2 H, \quad T_{21} = S + k_1 H$$

$$T_{13} = B_{13} \varepsilon_{13}, \quad T_{23} = B_{23} \varepsilon_{23},$$

$$S = B_s \varepsilon_{12}, \quad M_{11} = D_{11} \chi_{11} + D_{12} \chi_{22},$$

$$M_{22} = D_{21} \chi_{11} + D_{22} \chi_{22}, \quad M_{12} = M_{21} = H,$$

$$H = D_s \chi_{12},$$

where

$$D_s = G_{12} \frac{h^3}{12}, \quad B_s = G_{12} h, \quad (5)$$

$$B_{11} = \frac{E_1 h}{1 - \nu_{12} \nu_{21}}, \quad B_{12} = \frac{E_1 \nu_{21} h}{1 - \nu_{12} \nu_{21}},$$

$$B_{21} = \frac{E_2 \nu_{12} h}{1 - \nu_{12} \nu_{21}}, \quad B_{22} = \frac{E_2 h}{1 - \nu_{12} \nu_{21}},$$

$$B_{13} = G_{13} h k^2, \quad B_{23} = G_{23} h k^2,$$

$$D_{11} = \frac{E_1 h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{12} = \frac{E_1 \nu_{21} h^3}{12(1 - \nu_{12} \nu_{21})},$$

$$D_{21} = \frac{E_2 \nu_{12} h^3}{12(1 - \nu_{12} \nu_{21})}, \quad D_{22} = \frac{E_2 h^3}{12(1 - \nu_{12} \nu_{21})}.$$

In formulas (5) the values of E_1 , E_2 , G_{12} , G_{13} , G_{23} , ν_{12} ,

ν_{21} – physical characteristics of the shell material; k^2 – the shear coefficient for the shell.

We accept the following hypothesis to build a mathematical model of deformation for the j -th reinforcing rib, directed along the axis α_2 . The cross-section of the rib is not deformed. We use the geometrically nonlinear theory of Tymoshenko's rods.

The change in displacements along the cross-section of the j -th reinforcing rib is given by an approximation:

$$u_{1j}^{xz}(x, s_2, z) = u_{1j}(s_2) + z \varphi_{1j}(s_2), \quad (6)$$

$$u_{2j}^{xz}(x, s_2, z) = u_{2j}(s_2) + z \varphi_{2j}(s_2),$$

$$u_{3j}^{xz}(x, s_2, z) = u_{3j}(s_2),$$

where u_{1j} , u_{2j} , u_{3j} , φ_{1j} , φ_{2j} are the components for the generalized vector of displacements for the cross-section gravity center of the j -th rib.

We assume that the shell and discrete reinforcing elements are rigidly connected.

The contact conditions between the components of generalized displacement vector for the cross-sections gravity centers of the j -th rib, which is directed along the axis α_2 , and the components of generalized displacement vector for the original middle surface of the smooth ellipsoidal shell have the form:

$$u_{1j}(s_2) = u_1(s_{1j}, s_2) \pm h_{cj} \varphi_1(s_{1j}, s_2), \quad (7)$$

$$u_{2j}(s_2) = u_2(s_{1j}, s_2) \pm h_{cj} \varphi_2(s_{1j}, s_2),$$

$$u_{3j}(s_2) = u_3(s_{1j}, s_2),$$

$$\varphi_{1j}(s_2) = \varphi_1(s_{1j}, s_2),$$

$$\varphi_{2j}(s_2) = \varphi_2(s_{1j}, s_2),$$

where $h_{cj} = 0,5(h + h_j)$ – distance from the middle surface of the smooth ellipsoidal shell to the line for the centers of gravity for the cross-sections for j -th rib; h_j – the height of the reinforcing j -th rib; s_{1j} – coordinate for the design line of the cross-sections gravity centers of the j -th rib on the coordinate middle surface of the shell.

The sign “+” corresponds to the case of external ribs, and the sign “-” is chosen in the case of internal ribs.

Deformation relations for the j -th rib according to [13]:

$$\varepsilon_{22j} = e_{22j}, \quad e_{22j} = \frac{\partial u_2}{\partial s_2} \pm h_{cj} \frac{\partial \varphi_2}{\partial s_2} + k_2 u_3, \quad (8)$$

$$\varepsilon_{21j} = \theta_{2j}, \quad \varepsilon_{23j} = \varphi_2 + \theta_{1j},$$

$$\theta_{1j} = \frac{\partial u_3}{\partial s_2} - k_{2j}(u_2 \pm h_{cj} \varphi_2), \quad \theta_{2j} = \frac{\partial u_1}{\partial s_2} \pm h_{cj} \frac{\partial \varphi_1}{\partial s_2},$$

$$\chi_{21j} = \frac{\partial \varphi_1}{\partial s_2}, \quad \chi_{22j} = \frac{\partial \varphi_2}{\partial s_2}.$$

Expressions for the forces and moments for the j -th rib have the form:

$$T_{21j} = G_j F_j \varepsilon_{21j}, \quad T_{22j} = E_j F_j \varepsilon_{22j}, \quad (9)$$

$$T_{23j} = G_j F_j k_j^2 \varepsilon_{23j}, \quad M_{21j} = G_j I_{crj} \chi_{21j},$$

$$M_{22j} = E_j I_{2j} \chi_{22j}.$$

In formulas (9) the value of k_j^2 – the shear coefficient for the reinforcing rib; F_j , I_{2j} , I_{crj} – geometric and physical characteristics of transverse elements.

3. Equation of oscillations

The Hamilton-Ostrogradsky variational principle is used to derive oscillation equations for a discretely supported structure. We obtain two groups of equations after performing the operations of variation and integration, taking into account the contact conditions (7).

The equations of the shell in a smooth region:

$$\begin{aligned} \frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 T_{11}) - \frac{\partial A_2}{\partial s_1} T_{22} \right] + k_1 \bar{T}_{13} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 T_{21}) &= \rho h \frac{\partial^2 u_1}{\partial t^2}; \\ \frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 T_{12}) - \frac{\partial A_2}{\partial s_1} T_{21} \right] + k_2 \bar{T}_{23} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 T_{22}) &= \rho h \frac{\partial^2 u_2}{\partial t^2}; \\ \frac{1}{A_2} \frac{\partial}{\partial s_1} (A_2 \bar{T}_{13}) - k_1 T_{11} - k_2 T_{22} + P_3 + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 \bar{T}_{23}) &= \rho h \frac{\partial^2 u_3}{\partial t^2}; \\ \frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 M_{11}) - \frac{\partial A_2}{\partial s_1} M_{22} \right] - T_{13} + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 M_{21}) &= \rho \frac{h^3}{12} \frac{\partial^2 \varphi_1}{\partial t^2}; \\ \frac{1}{A_2} \left[\frac{\partial}{\partial s_1} (A_2 M_{12}) + \frac{\partial A_2}{\partial s_1} M_{21} \right] + \frac{1}{A_1} \frac{\partial}{\partial s_2} (A_1 M_{22}) - T_{23} &= \rho \frac{h^3}{12} \frac{\partial^2 \varphi_2}{\partial t^2}. \end{aligned} \quad (10)$$

The equations for the j -th rib:

$$\begin{aligned} \frac{\partial T_{21j}}{\partial s_2} + [T_{11}]_j &= \rho_j F_j \left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_1}{\partial t^2} \right), \\ \frac{\partial T_{22j}}{\partial s_2} + k_{2j} T_{23j} + [S]_j &= \rho_j F_j \left(\frac{\partial^2 u_2}{\partial t^2} \pm h_{cj} \frac{\partial^2 \varphi_2}{\partial t^2} \right), \\ \frac{\partial T_{23j}}{\partial s_2} - k_{2j} T_{22j} + [T_{13}]_j &= \rho_j F_j \frac{\partial^2 u_3}{\partial t^2}, \\ \frac{\partial M_{21j}}{\partial s_2} \pm h_{cj} \frac{\partial T_{21j}}{\partial s_2} + [M_{11}]_j &= \rho_j F_j \left(\pm h_{cj} \frac{\partial^2 u_1}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j} \right) \frac{\partial^2 \varphi_1}{\partial t^2} \right), \\ \frac{\partial M_{22j}}{\partial s_2} - T_{23j} \pm h_{cj} \left(\frac{\partial T_{22j}}{\partial s_2} + k_{2j} T_{23j} \right) + [H]_j &= \\ = \rho_j F_j \left(\pm h_{cj} \frac{\partial^2 u_2}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j} \right) \frac{\partial^2 \varphi_2}{\partial t^2} \right). \end{aligned} \quad (11)$$

The notation of the type $[S]_j$ corresponds to the total effect of the force and moments for the smooth ellipsoidal shell on the j -th reinforcing rib in the oscillations equations of the discrete reinforcing ribs (11).

The oscillation equations (10) and (11) are complemented by the corresponding natural boundary and initial conditions.

4. Numerical algorithm

The equations (10) and (11) are a system of nonlinear partial differential equations. This system has spatial discontinuities along s_2 . The numerical algorithm is built: a solution is sought in a smooth region of shell and on lines of j -th reinforcing rib. We use an integral-interpolation method for spatial coordinates and an explicit approximation in time [15].

The displacement vector is approximated at integer points of the difference grid. The deformation values are approximated at the half-integer points of the grid.

The transition from a continuous to a finite-difference system is performed in two stages. The first stage consists of the finite-difference approximation of the equations of oscillations in the forces-moments.

Operating of integrating equations (10), using an explicit approximation in the time coordinate, we obtain the following difference equations in a smooth region of an ellipsoidal shell:

$$\begin{aligned} \frac{1}{A_{2l}} \left(\frac{A_{2l+1/2} T_{11l+1/2,m}^n - A_{2l-1/2} T_{11l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2l}} \frac{A_{2l+1/2} - A_{2l-1/2}}{\Delta s_1} T_{22l,m}^n + \\ + \frac{1}{A_{1l}} \frac{A_{1l} T_{21l,m+1/2}^n - A_{1l} T_{21l,m-1/2}^n}{\Delta s_2} + k_{1l} T_{13l,m}^n &= \rho h (u_{1l,m}^n)_{it}; \quad (12) \\ \frac{1}{A_{2l}} \left(\frac{A_{2l+1/2} T_{12l+1/2,m}^n - A_{2l-1/2} T_{12l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2l}} \frac{A_{2l+1/2} - A_{2l-1/2}}{\Delta s_1} T_{21l,m}^n + \\ + \frac{1}{A_{1l}} \left(\frac{A_{1l} T_{22l,m+1/2}^n - A_{1l} T_{22l,m-1/2}^n}{\Delta s_2} \right) + k_{2l} T_{23l,m}^n &= \rho h (u_{2l,m}^n)_{it}; \\ \frac{1}{A_{2l}} \left(\frac{A_{2l+1/2} T_{13l+1/2,m}^n - A_{2l-1/2} T_{13l-1/2,m}^n}{\Delta s_1} \right) - k_{1l} T_{11l,m}^n + \\ + \frac{1}{A_{1l}} \left(\frac{A_{1l} T_{23l,m+1/2}^n - A_{1l} T_{23l,m-1/2}^n}{\Delta s_2} \right) - k_{2l} T_{22l,m}^n + P_{3l,m}^n &= \rho h (u_{3l,m}^n)_{it}; \\ \frac{1}{A_{2l}} \left(\frac{A_{2l+1/2} M_{11l+1/2,m}^n - A_{2l-1/2} M_{11l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2l}} \frac{A_{2l+1/2} - A_{2l-1/2}}{\Delta s_1} M_{22l,m}^n + \\ + \frac{1}{A_{1l}} \left(\frac{A_{1l} M_{21l,m+1/2}^n - A_{1l} M_{21l,m-1/2}^n}{\Delta s_2} \right) - T_{13l,m}^n &= \frac{\rho h^3}{12} (\varphi_{1l,m}^n)_{it}; \\ \frac{1}{A_{2l}} \left(\frac{A_{2l+1/2} M_{12l+1/2,m}^n - A_{2l-1/2} M_{12l-1/2,m}^n}{\Delta s_1} \right) - \frac{1}{A_{2l}} \frac{A_{2l+1/2} - A_{2l-1/2}}{\Delta s_1} M_{21l,m}^n + \\ + \frac{1}{A_{1l}} \left(\frac{A_{1l} M_{22l,m+1/2}^n - A_{1l} M_{22l,m-1/2}^n}{\Delta s_2} \right) - T_{23l,m}^n &= \frac{\rho h^3}{12} (\varphi_{2l,m}^n)_{it}. \end{aligned}$$

In difference equations (12), the components of the generalized displacements vector of the middle surface for the smooth ellipsoidal shell $\bar{U} = (u_1, u_2, u_3, \varphi_1, \varphi_2)^T$ are assigned to whole nodes of the different grid $\bar{U}_{l,m} = (u_{1l,m}, u_{2l,m}, u_{3l,m}, \varphi_{1l,m}, \varphi_{2l,m})^T$ in spatial coordinates.

Operating of integrating equations (11), using an explicit approximation in the time coordinate, we obtain the following difference equations for j -th reinforcing rib:

$$\begin{aligned} \frac{T_{21jm+1/2}^n - T_{21jm-1/2}^n}{\Delta s_2} + [T_{11}]_j^n &= \rho_j F_j \left[(u_{1m}^n)_{it} \pm h_{ci} (\varphi_{1m}^n)_{it} \right], \\ \frac{T_{22jm+1/2}^n - T_{22jm-1/2}^n}{\Delta s_2} + k_{2jm} T_{23jm}^n + [S]_j^n &= \\ &= \rho_j F_j \left[(u_{2m}^n)_{it} \pm h_{ci} (\varphi_{2m}^n)_{it} \right], \\ \frac{T_{23jm+1/2}^n - T_{23jm-1/2}^n}{\Delta s_2} - k_{2jm} T_{22jm}^n + [T_{13}]_j^n &= \rho_j F_j (u_{3m}^n)_{it}, \\ \frac{M_{21jm+1/2}^n - M_{21jm-1/2}^n}{\Delta s_2} \pm h_{cj} \frac{T_{21jm+1/2}^n - T_{21jm-1/2}^n}{\Delta s_2} + [M_{11}]_j^n &= \\ &= \rho_j F_j \left[\pm h_{cj} (u_{1m}^n)_{it} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j} \right) (\varphi_{1m}^n)_{it} \right], \\ \frac{M_{22jm+1/2}^n - M_{22jm-1/2}^n}{\Delta s_2} - T_{23jm}^n \pm h_{cj} \left(\frac{T_{22jm+1/2}^n - T_{22jm-1/2}^n}{\Delta s_2} + k_{2jm} T_{23jm}^n \right) &+ \\ + [H]_j^n &= \rho_j F_j \left[\pm h_{cj} (u_{2m}^n)_{it} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j} \right) (\varphi_{2m}^n)_{it} \right]. \end{aligned} \quad (13)$$

The components of the generalized displacements vector of the mass centers in the cross-sections for the j -th reinforcing rib $\bar{U}_j = (u_{1j}, u_{2j}, u_{3j}, \varphi_{1j}, \varphi_{2j})^T$ are assigned to integer nodes of the difference mesh in spatial coordinates.

In the second stage of approximation, the equations consist of finite-difference approximation of forces-moments and corresponding deformations.

We use the necessary conditions of stability

$$\Delta t \leq 2 / \omega,$$

where $\omega = \max(\omega_0, \omega_j)$, $j = 1, J$; ω_0, ω_j – frequency of natural oscillations the discrete-difference system for the smooth shell and the j -th reinforcing rib.

5. Numerical example

We investigated the change in deformation for discretely reinforced ellipsoidal in the region:

$$D = \{\alpha_{10} \leq \alpha_1 \leq \alpha_{1N}, \alpha_{20} \leq \alpha_2 \leq \alpha_{2N}\}.$$

The load on the shell $P_3(\alpha_1, \alpha_2, t)$ has the form:

$$P_3(\alpha_1, \alpha_2, t) = A \cdot \sin \frac{\pi}{T} [\eta(t) - \eta(t - T)],$$

$$A = 10^6 \Pi a; \quad T = 50 \cdot 10^{-6} c.$$

The boundary conditions are as follows

$$\bar{U}(\alpha_{10}, \alpha_2) = \bar{U}(\alpha_{1N}, \alpha_2) = 0; \quad \bar{U}(\alpha_1, \alpha_{20}) = \bar{U}(\alpha_1, \alpha_{2N}) = 0.$$

The generalized displacement vector has zero components at $t = 0$.

The geometrical parameters of the shell and rib:

$$\begin{aligned} \alpha_{10} &= \frac{\pi}{12}; \quad \alpha_{1N} = \pi - \frac{\pi}{12}; \quad \alpha_{20} = -\frac{\pi}{2}; \quad \alpha_{2N} = \frac{\pi}{2}; \\ \frac{b}{a} &= 1,5; \quad \frac{a}{h} = 30; \\ h_j &= 4 \cdot h; \quad F_j = 4 \cdot h^2. \end{aligned}$$

The physical and mechanical parameters:

$$\begin{aligned} E &= 7 \cdot 10^{10} \Pi a; \quad \nu = 0,33; \quad \rho = 2,7 \cdot 10^3 \kappa z / M^3; \\ E_j &= E_1; \quad \rho_j = \rho. \end{aligned}$$

The location of the reinforcing ribs was investigated for the following 5 cases.

1. External placement of ribs on the shell (curve 1)

$$\alpha_{1j} = \frac{\pi}{4} j, \quad j = \overline{1,3}.$$

2. Internal placement of ribs in the shell (curve 2)

$$\alpha_{1j} = \frac{\pi}{8} + \frac{\pi}{4} j, \quad j = \overline{0,3}.$$

3. External placement of all ribs on the shell (curve 3)

$$\alpha_{1j} = \frac{\pi}{8} + \frac{\pi}{8} j, \quad j = \overline{0,6}.$$

4. Internal placement of all ribs in the shell (curve 4)

$$\alpha_{1j} = \frac{\pi}{8} + \frac{\pi}{8} j, \quad j = \overline{0,6}.$$

5. External placement of ribs $\alpha_{1j} = \frac{\pi}{4} j$ ($j = \overline{1,3}$) on the

shell and internal placement of ribs $\alpha_{1j} = \frac{\pi}{8} + \frac{\pi}{4} j$ ($j = \overline{0,3}$) in the shell. This is a curve 5.

Calculations were performed on the time interval $t_N = 20T$. Fig. 1 – 9 show the deformations ε_{22} , that make it possible to analyze the deformed state of the reinforced ellipsoidal shell.

We investigated the change in the amount of deformation at the most characteristic points of the reinforced shell: at the points of ribs placement and between the ribs.

Fig. 1 shows the changes of deformations ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{8}, \alpha_2 = 0 \right)$, where the rib is placed.

Fig. 1 Deformations $\varepsilon_{22}\left(\frac{\pi}{8};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{\pi}{8};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{\pi}{8};0\right) = 4,17 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{\pi}{8};0\right) = 4,56 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{\pi}{8};0\right) = 4,72 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{\pi}{8};0\right) = 3,86 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{\pi}{8};0\right) = 4,29 \cdot 10^{-4}.$$

The value of deformation $\varepsilon_{22}^4\left(\frac{\pi}{8};0\right)$ is the smallest, and $\varepsilon_{22}^3\left(\frac{\pi}{8};0\right)$ - the largest, which is 1,2 times more:

$$\frac{\varepsilon_{22}^3\left(\frac{\pi}{8};0\right)}{\varepsilon_{22}^4\left(\frac{\pi}{8};0\right)} = \frac{4,72 \cdot 10^{-4}}{3,86 \cdot 10^{-4}} \approx 1,2$$

The average value of the achieved deformations ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{8}, \alpha_2 = 0\right)$ is equal to

$$\bar{\varepsilon}_{22}\left(\frac{\pi}{8};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{\pi}{8};0\right) = 4,32 \cdot 10^{-4}.$$

The changes of deformations ε_{22} between the ribs at the point $\left(\alpha_1 = \frac{3\pi}{16}, \alpha_2 = 0\right)$ are shown in Fig. 2

Fig. 2 Deformations $\varepsilon_{22}\left(\frac{3\pi}{16};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{3\pi}{16};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{3\pi}{16};0\right) = 7,53 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{3\pi}{16};0\right) = 5,70 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right) = 9,50 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{3\pi}{16};0\right) = 5,86 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{3\pi}{16};0\right) = 7,36 \cdot 10^{-4}.$$

The value of deformation $\varepsilon_{22}^2\left(\frac{3\pi}{16};0\right)$ is the smallest, and $\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)$ - the largest, which is 1,7 times more:

$$\frac{\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^2\left(\frac{3\pi}{16};0\right)} = \frac{9,50 \cdot 10^{-4}}{5,70 \cdot 10^{-4}} \approx 1,7$$

The average value of deformations ε_{22} at the point $\left(\alpha_1 = \frac{3\pi}{16}, \alpha_2 = 0\right)$ is equal to

$$\bar{\varepsilon}_{22}\left(\frac{3\pi}{16};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{3\pi}{16};0\right) = 7,19 \cdot 10^{-4}.$$

Fig. 3 represents the changes of deformations ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{4}, \alpha_2 = 0\right)$, where the rib is placed.

Fig. 3 Deformations $\varepsilon_{22}\left(\frac{\pi}{4};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{\pi}{4};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{\pi}{4};0\right) = 2,61 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{\pi}{4};0\right) = 4,63 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{\pi}{4};0\right) = 3,96 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{\pi}{4};0\right) = 3,34 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{\pi}{4};0\right) = 3,35 \cdot 10^{-4}.$$

After analyzing the value of deformation ε_{22} we can conclude that $\varepsilon_{22}^1\left(\frac{\pi}{4};0\right)$ is the smallest, and $\varepsilon_{22}^2\left(\frac{\pi}{4};0\right)$ - the largest, which is 1,8 times more:

$$\frac{\varepsilon_{22}^2\left(\frac{\pi}{4};0\right)}{\varepsilon_{22}^1\left(\frac{\pi}{4};0\right)} = \frac{4,63 \cdot 10^{-4}}{2,61 \cdot 10^{-4}} \approx 1,8$$

After calculating the average value of deformation ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{4}, \alpha_2 = 0\right)$ we receive:

$$\bar{\varepsilon}_{22}\left(\frac{\pi}{4};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{\pi}{4};0\right) = 3,58 \cdot 10^{-4}.$$

Graphically, the results of calculations for deformations ε_{22} between the ribs at the point $\left(\alpha_1 = \frac{5\pi}{16}, \alpha_2 = 0\right)$ are presented in Fig. 4

Fig. 4 Deformations $\varepsilon_{22}\left(\frac{5\pi}{16};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{5\pi}{16};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{5\pi}{16};0\right) = 4,15 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{5\pi}{16};0\right) = 3,30 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{5\pi}{16};0\right) = 3,23 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{5\pi}{16};0\right) = 2,84 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{5\pi}{16};0\right) = 2,84 \cdot 10^{-4}.$$

After analyzing the results, we can conclude that the value of deformation $\varepsilon_{22}^4\left(\frac{5\pi}{16};0\right)$ or $\varepsilon_{22}^5\left(\frac{5\pi}{16};0\right)$ is the smallest, and $\varepsilon_{22}^1\left(\frac{5\pi}{16};0\right)$ - the largest, which is 1,5 times more:

$$\frac{\varepsilon_{22}^1\left(\frac{5\pi}{16};0\right)}{\varepsilon_{22}^4\left(\frac{5\pi}{16};0\right)} = \frac{4,15 \cdot 10^{-4}}{2,84 \cdot 10^{-4}} \approx 1,5$$

Using the found values of deformation ε_{22} at the point $\left(\alpha_1 = \frac{5\pi}{16}, \alpha_2 = 0\right)$ we calculate the average value of deformation at this point:

$$\bar{\varepsilon}_{22}\left(\frac{5\pi}{16};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{5\pi}{16};0\right) = 3,27 \cdot 10^{-4}.$$

The results of deformation calculations ε_{22} at the point $\left(\alpha_1 = \frac{3\pi}{8}, \alpha_2 = 0\right)$ in the rib are shown in Figure 5.

Fig. 5 Deformations $\varepsilon_{22}\left(\frac{3\pi}{8};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{3\pi}{8};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{3\pi}{8};0\right) = 3,23 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{3\pi}{8};0\right) = 3,03 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{3\pi}{8};0\right) = 2,87 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{3\pi}{8};0\right) = 3,64 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{3\pi}{8};0\right) = 3,22 \cdot 10^{-4}.$$

Analyzing the value of deformations at the point $\left(\alpha_1 = \frac{3\pi}{8}, \alpha_2 = 0\right)$, we can say that the value of deformation $\varepsilon_{22}^3\left(\frac{3\pi}{8};0\right)$ is the smallest, and $\varepsilon_{22}^4\left(\frac{3\pi}{8};0\right)$ - the largest, which is 1,3 times more:

$$\frac{\varepsilon_{22}^4\left(\frac{3\pi}{8};0\right)}{\varepsilon_{22}^3\left(\frac{3\pi}{8};0\right)} = \frac{3,64 \cdot 10^{-4}}{2,87 \cdot 10^{-4}} \approx 1,3$$

At the point $\left(\alpha_1 = \frac{3\pi}{8}, \alpha_2 = 0\right)$ the average value of deformations ε_{22} is equal to

$$\bar{\varepsilon}_{22}\left(\frac{3\pi}{8};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{3\pi}{8};0\right) = 3,2 \cdot 10^{-4}.$$

The changes of deformations ε_{22} between the ribs at the point $\left(\alpha_1 = \frac{7\pi}{16}, \alpha_2 = 0\right)$ are shown in Fig. 6.

Fig. 6 Deformations $\varepsilon_{22}\left(\frac{7\pi}{16};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{7\pi}{16};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{7\pi}{16};0\right) = 3,43 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{7\pi}{16};0\right) = 3,84 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{7\pi}{16};0\right) = 3,51 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{7\pi}{16};0\right) = 4,07 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{7\pi}{16};0\right) = 3,49 \cdot 10^{-4}.$$

We can conclude that the value of deformation $\varepsilon_{22}^1\left(\frac{7\pi}{16};0\right)$ is the smallest, and $\varepsilon_{22}^4\left(\frac{7\pi}{16};0\right)$ - the largest, which is 1,2 times more:

$$\frac{\varepsilon_{22}^4\left(\frac{7\pi}{16};0\right)}{\varepsilon_{22}^1\left(\frac{7\pi}{16};0\right)} = \frac{4,07 \cdot 10^{-4}}{3,43 \cdot 10^{-4}} \approx 1,2$$

The average value of deformations ε_{22} at the point $\left(\alpha_1 = \frac{7\pi}{16}, \alpha_2 = 0\right)$ is equal to

$$\bar{\varepsilon}_{22}\left(\frac{7\pi}{16};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{7\pi}{16};0\right) = 3,67 \cdot 10^{-4}.$$

Fig. 7 shows the changes of deformation ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{2}, \alpha_2 = 0\right)$ in the rib.

Fig. 7 Deformations $\varepsilon_{22}\left(\frac{\pi}{2};0\right)$ on time t

The largest absolute values of deformations $\varepsilon_{22}\left(\frac{\pi}{2};0\right)$ for given cases:

$$\varepsilon_{22}^1\left(\frac{\pi}{2};0\right) = 4,18 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{\pi}{2};0\right) = 3,47 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{\pi}{2};0\right) = 5,71 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{\pi}{2};0\right) = 3,75 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{\pi}{2};0\right) = 5,76 \cdot 10^{-4}.$$

After analyzing the results, we can say that the value of deformation $\varepsilon_{22}^2\left(\frac{\pi}{2};0\right)$ is the smallest, and $\varepsilon_{22}^5\left(\frac{\pi}{2};0\right)$ - the largest, which is 1,7 times more:

$$\frac{\varepsilon_{22}^5\left(\frac{\pi}{2};0\right)}{\varepsilon_{22}^2\left(\frac{\pi}{2};0\right)} = \frac{5,76 \cdot 10^{-4}}{3,47 \cdot 10^{-4}} \approx 1,7$$

We calculate the average value of deformations ε_{22} at the point $\left(\alpha_1 = \frac{\pi}{2}, \alpha_2 = 0\right)$:

$$\bar{\varepsilon}_{22}\left(\frac{\pi}{2};0\right) = \frac{1}{5} \sum_{n=1}^5 \varepsilon_{22}^n\left(\frac{\pi}{2};0\right) = 4,57 \cdot 10^{-4}.$$

Therefore, the largest difference for deformations ε_{22} is manifested for cases of external and internal placement of ribs (cases 1 and 2) at the point $\left(\alpha_1 = \frac{\pi}{4}, \alpha_2 = 0\right)$:

$$\frac{\varepsilon_{22}^2\left(\frac{\pi}{4};0\right)}{\varepsilon_{22}^1\left(\frac{\pi}{4};0\right)} \approx 1,8$$

After analyzing the results for case 1, we can conclude that the largest value of deformation between the ribs

$\varepsilon_{22}^1\left(\frac{3\pi}{16};0\right)$ is 1,8 times more than the largest value of deformation in the rib $\varepsilon_{22}^1\left(\frac{\pi}{2};0\right)$:

$$\frac{\varepsilon_{22}^1\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^1\left(\frac{\pi}{2};0\right)} = \frac{7,53 \cdot 10^{-4}}{4,18 \cdot 10^{-4}} \approx 1,8$$

Considering the results for case 2, we can say that the largest value of deformation between the ribs $\varepsilon_{22}^2\left(\frac{3\pi}{16};0\right)$ is 1,2 times more than the largest value of deformation in the rib $\varepsilon_{22}^2\left(\frac{\pi}{4};0\right)$:

$$\frac{\varepsilon_{22}^2\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^2\left(\frac{\pi}{4};0\right)} = \frac{5,70 \cdot 10^{-4}}{4,63 \cdot 10^{-4}} \approx 1,2$$

Discussing the results for case 3, we can conclude that the largest value of deformation between the ribs $\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)$ is 1,7 times more than the largest value of deformation in the rib $\varepsilon_{22}^3\left(\frac{\pi}{2};0\right)$:

$$\frac{\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^3\left(\frac{\pi}{2};0\right)} = \frac{9,50 \cdot 10^{-4}}{5,71 \cdot 10^{-4}} \approx 1,7$$

After analyzing the results for case 4, we can assume that the largest value of deformation between the ribs $\varepsilon_{22}^4\left(\frac{3\pi}{16};0\right)$ is 1,5 times more than the largest value of deformation in the rib $\varepsilon_{22}^4\left(\frac{\pi}{8};0\right)$:

$$\frac{\varepsilon_{22}^4\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^4\left(\frac{\pi}{8};0\right)} = \frac{5,86 \cdot 10^{-4}}{3,86 \cdot 10^{-4}} \approx 1,5$$

According to results for case 5, we can conclude that the largest value of deformation between the ribs $\varepsilon_{22}^5\left(\frac{3\pi}{16};0\right)$ is 1,3 times more than the largest value of deformation in the rib $\varepsilon_{22}^5\left(\frac{\pi}{2};0\right)$:

$$\frac{\varepsilon_{22}^5\left(\frac{3\pi}{16};0\right)}{\varepsilon_{22}^5\left(\frac{\pi}{2};0\right)} = \frac{7,36 \cdot 10^{-4}}{5,76 \cdot 10^{-4}} \approx 1,3$$

Based on the above, we conclude:

1) the largest values of deformations between the rib are

$$\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right) = 9,5 \cdot 10^{-4};$$

2) the largest value of deformation in the rib is

$$\varepsilon_{22}^5\left(\frac{\pi}{2};0\right)=5,76 \cdot 10^{-4};$$

3) the smallest value of deformation between the rib is

$$\varepsilon_{22}^5\left(\frac{5\pi}{16};0\right)=2,84 \cdot 10^{-4};$$

4) the smallest value of deformation in the rib is

$$\varepsilon_{22}^1\left(\frac{\pi}{4};0\right)=2,61 \cdot 10^{-4}.$$

Let's carry out the comparative analysis of deformations ε_{22} for the considered cases of the ribs locations.

The average values of deformations ε_{22} for case 1 are equal to

$$\bar{\varepsilon}_{22}^1\left(\frac{\pi}{8};\frac{\pi}{4};\frac{3\pi}{8};\frac{\pi}{2}\right)=3,75 \cdot 10^{-4} \text{ (in the ribs),}$$

$$\bar{\varepsilon}_{22}^1\left(\frac{3\pi}{16};\frac{5\pi}{16};\frac{7\pi}{16}\right)=5,04 \cdot 10^{-4} \text{ (between the ribs).}$$

After calculating the average values of deformations ε_{22} for case 2 we receive

$$\bar{\varepsilon}_{22}^2\left(\frac{\pi}{8};\frac{\pi}{4};\frac{3\pi}{8};\frac{\pi}{2}\right)=3,92 \cdot 10^{-4} \text{ (in the ribs),}$$

$$\bar{\varepsilon}_{22}^2\left(\frac{3\pi}{16};\frac{5\pi}{16};\frac{7\pi}{16}\right)=4,28 \cdot 10^{-4} \text{ (between the ribs).}$$

Using the found values of deformation ε_{22} for case 3 we calculate the average values of deformations

$$\bar{\varepsilon}_{22}^3\left(\frac{\pi}{8};\frac{\pi}{4};\frac{3\pi}{8};\frac{\pi}{2}\right)=4,32 \cdot 10^{-4} \text{ (in the ribs),}$$

$$\bar{\varepsilon}_{22}^3\left(\frac{3\pi}{16};\frac{5\pi}{16};\frac{7\pi}{16}\right)=5,41 \cdot 10^{-4} \text{ (between the ribs).}$$

For case 4 the average values of deformations ε_{22} are equal to

$$\bar{\varepsilon}_{22}^4\left(\frac{\pi}{8};\frac{\pi}{4};\frac{3\pi}{8};\frac{\pi}{2}\right)=3,65 \cdot 10^{-4} \text{ (in the ribs),}$$

$$\bar{\varepsilon}_{22}^4\left(\frac{3\pi}{16};\frac{5\pi}{16};\frac{7\pi}{16}\right)=4,26 \cdot 10^{-4} \text{ (between the ribs).}$$

We can conclude that the average values of deformations ε_{22} for case 5 are equal to

$$\bar{\varepsilon}_{22}^5\left(\frac{\pi}{8};\frac{\pi}{4};\frac{3\pi}{8};\frac{\pi}{2}\right)=4,16 \cdot 10^{-4} \text{ (in the ribs),}$$

$$\bar{\varepsilon}_{22}^5\left(\frac{3\pi}{16};\frac{5\pi}{16};\frac{7\pi}{16}\right)=4,56 \cdot 10^{-4} \text{ (between the ribs).}$$

Therefore, analyzing the average values found, we can say the following. In both cases: at the location of the rib and between the ribs, the largest values of deformations for case 3 are when all the ribs are placed external the ellipsoidal shell. The smallest deformations for case 4, when all the ribs are placed internally in the shell.

Due to the symmetry along the coordinate α_1 , the dependencies are given in the range $\alpha_{10} \leq \alpha_1 \leq \frac{\pi}{2}$ in the section $\alpha_2 = 0$.

Fig. 8 shows the deformations ε_{22} at the largest time $t = 20T$.

Fig. 8 Deformations ε_{22} on spatial coordinate α_1 in cross-section $\alpha_2 = 0$ the time $t = 20T$

The largest absolute values of the deformations ε_{22} for given cases the time $t = 20T$:

$$\varepsilon_{22}^1\left(\frac{5\pi}{16};0\right)\Bigg|_{t=20T} = 4,37 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{5\pi}{16};0\right)\Bigg|_{t=20T} = 2,54 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)\Bigg|_{t=20T} = 5,53 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{3\pi}{8};0\right)\Bigg|_{t=20T} = 4,3 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{5\pi}{16};0\right)\Bigg|_{t=20T} = 4,33 \cdot 10^{-4}.$$

The largest value of deformation at time $t = 20T$ is obtained between the rib at the point $\left(\alpha_1 = \frac{3\pi}{16}, \alpha_2 = 0\right)$:

$$\varepsilon_{22}^3\left(\frac{3\pi}{16};0\right)\Bigg|_{t=20T} = 5,53 \cdot 10^{-4}.$$

The deformations ε_{22} the time $t = 13T$ are shown in Fig. 9.

Fig. 9 Deformations ε_{22} on spatial coordinate α_1 in cross-section $\alpha_2 = 0$ the time $t = 13T$

The largest absolute values of the deformations ε_{22} for

given cases the time $t = 13T$:

$$\varepsilon_{22}^1\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 7,53 \cdot 10^{-4}; \quad \varepsilon_{22}^2\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 5,7 \cdot 10^{-4};$$

$$\varepsilon_{22}^3\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 9,5 \cdot 10^{-4};$$

$$\varepsilon_{22}^4\left(\frac{3\pi}{8}; 0\right)\bigg|_{t=13T} = 5,86 \cdot 10^{-4}; \quad \varepsilon_{22}^5\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 7,36 \cdot 10^{-4}.$$

The largest value of deformation the time $t = 13T$ is obtained between the rib at the point $\left(\alpha_1 = \frac{3\pi}{16}, \alpha_2 = 0\right)$:

$$\varepsilon_{22}^3\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 9,5 \cdot 10^{-4}.$$

In Fig. 8-9 we can visually determine the location of the reinforcing ribs and their impact on the deformation of discretely reinforced ellipsoidal shells.

6. Conclusions

The problems for the deformed state of discretely reinforced ellipsoidal shells under the action of non-stationary loads are considered. The equations of oscillations for shells with corresponding boundary and initial conditions are given. The numerical algorithm based on a finite-difference approximation of the original equations in spatial and temporal coordinates is constructed to solve the problems. Numerical results for five different cases of the ribs placement on the discretely reinforced ellipsoidal shells and their comparative analysis are given.

After analyzing the calculated values for deformation ε_{22} , we can conclude that the largest deformation for the reinforced ellipsoidal shell is achieved at the point between the ribs $\left(\alpha_1 = \frac{3\pi}{16}, \alpha_2 = 0\right)$ the time $t = 13T$ for case 3 is

$$\text{equal to } \varepsilon_{22}^3\left(\frac{3\pi}{16}; 0\right)\bigg|_{t=13T} = 9,5 \cdot 10^{-4}.$$

Based on the obtained results, it can be concluded that of the five cases under consideration, the largest values of deformations ε_{22} are achieved for case 3. In case 3 all ribs are external placement on the ellipsoidal shell. The smallest values of deformations are observed for case 4. In case 4 all ribs are placed internally in the shell. The largest deformation is achieved at the time $t = 13T$.

Conflict of Interest

The authors declare that they have no conflict of interest.

Data Availability Statements

The datasets generated and analyzed during the current study are available in the Science Data Bank repository, [http://www.doi.org/10.11922/sciencedb.01334.\(2021-11-15\)](http://www.doi.org/10.11922/sciencedb.01334.(2021-11-15)).

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