An Investigation Into The Mathematical and Physical Origins of The Fine-Structure Constant

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Abstract

The fine-structure constant, α , unites fundamental aspects of electromagnetism, quantum physics, and relativity. As such, it is one of the most important constants in nature. However, why it has the value of approximately $1/137$ has been a mystery since it was first identified more than 100 years ago. To date, it is an *ad hoc* feature of the Standard Model, as it does not appear to be derivable within that body of work — being determined solely by experimentation. This report presents a mathematical formula for α that results in an exact match with the currently accepted value of the constant. The formula requires that a simple corrective term be applied to the value of one of the factors in the suggested equation. Notably, this corrective term, at approximately 0.023, is similar in value to the electron anomalous magnetic moment value, at approximately 0.0023, which is the corrective term that needs to be applied to the g-factor in the equation for the electron spin magnetic moment. In addition, it is shown that the corrective term for the proposed equation for α can be derived from the anomalous magnetic moment values of the electron, muon, and tau particle — values that have been well established through theory and/or experimentation. This supports the notion that the corrective term for the α formula is also a real and natural quantity. The quantum mechanical origins of the lepton anomalous magnetic moment values suggest that there might be a quantum mechanical origin to the corrective term for α as well. This possibility, as well as a broader physical interpretation of the value of α , is explored.

Keywords: Fine-structure constant; electromagnetic coupling constant; universal physical constants; dimensionless physical constants; mathematical constants; electron spin g-factor; electron, muon, tau anomalous magnetic moment

1. Introduction

The fine-structure constant, α , also called Sommerfeld's constant, and the electromagnetic coupling constant, represents the strength of the interaction between electrically charged elementary particles. At times referred to as a “magic number” and “the most important number in physics,” it unites fundamental aspects of electromagnetism (elementary electric charge, e), quantum physics (reduced Planck’s constant, $\hbar$), and relativity (speed of light, c), as well as the electric permittivity of free space (ϵ_0):

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} . \quad (1)$$

Unfortunately, why α has the value it has, at approximately 1/137, has been a mystery, since it was identified by Arnold Sommerfeld in 1916. A fundamental mathematical formula for the constant leading to a match with experimental results has been elusive, particularly as α has appeared to have no connection to mathematical constants. In the Standard Model, the number currently stands as an *ad hoc* value that must be determined experimentally.

This study presents a mathematical formula for α that does involve mathematical constants, and that leads to an exact match with the current recommended value of α ,

as determined by the Committee on Data for Science and Technology (CODATA, 2018) [1]. It is also shown how the formula is related to the well-established formula of the electron spin magnetic moment, particularly the electron spin g-factor (g_e) component of the equation. It is further shown how the formula is mathematically linked to the anomalous magnetic moment values of the charged leptons — values that have been established through theory and/or experimentation. Additionally, a physical interpretation of the suggested formula for α is presented, including the possibility of there being a quantum mechanical contribution to the constant's value.

2. Analysis and Discussion

The equation for the electron's spin magnetic moment provides important insight into the possible mathematical underpinnings of α . The z-component of the magnetic moment can be calculated as follows:

$$\mu_z = \pm \frac{1}{2} |g_e| \mu_B. \quad (2)$$

where g_e is the electron spin g-factor, a dimensionless value, and μ_B is the Bohr magneton, a unit of magnetic moment.

Paul Dirac predicted g_e to have a value of -2 [2]. However, experimentation has shown the value to actually be closer to $-2.00231930436256(35)$. The correction needed on the value of 2 can be symbolized as follows:

$$-g_e = -(2 + 2\alpha_e), \quad (3)$$

101 where α_e is 0.00115965218128(18) and is referred to as the electron's anomalous
102 magnetic moment. The anomaly arises from quantum effects at the particle level that
103 cause the value of g_e to slightly exceed 2. The full value of g_e can be formulated well
104 through perturbative quantum field theory techniques, thus far matching up to 10
105 significant digits of the experimentally determined value [3, 4, 5, 6].

106 The principal idea here is that one of the factors in the equation for μ_z (specifically g_e)
107 requires the addition of a small corrective, or anomalous, term — 2 times
108 0.00115965218128(18), or 0.00231930436256(35) — to obtain the true value of g_e and
109 thereby μ_z .

110 A similar situation appears to arise in the setting of α . That is, as there is an anomalous
111 value associated with the electron's *magnetic field* that must be accounted for to
112 calculate the accurate value of the spin, there also appears to be an anomalous value
113 associated with the electron's *electric field* that must be accounted for in the calculation
114 of α . The concept of electric field lines can help in initial steps to identify the anomalous
115 electric field value.

116 The electric force between two electrons — or, for ease, an electron and positron,
117 before annihilation — can be depicted via classical field lines, Fig. 1.

Fig. 1 | The electric interaction between an electron and positron (before annihilation), as depicted using classical field lines. On the far side of the particles, there are field lines that do not take part in the interaction, with the field lines simply extending into space, representing a blind spot in the field.

The field lines represent the interaction, and thus the coupling of the particles — i.e., how strong that force is. An increase in the density of the field lines would represent an increase in the coupling or strength of the force. However, as shown in Fig. 1, on the far side of the particles, there are field lines that do not take part in the interaction, with the field lines simply extending into space. This can be viewed on a macroscopic level using the magnetic field analogue of the electric field (Fig. 2).

129

130 **Fig. 2 Shown is a depiction of the magnetic field analogue of the electric field. The**
 131 **drawing shows how iron filings arrange themselves around a magnetic due to the**
 132 **influence of the magnetic field lines. The iron filings effectively map the field lines,**
 133 **showing the blind spot in the field at a macroscopic level, where the field lines in the**
 134 **middle of the far sides of the magnet extend into space with no involvement in the**
 135 **interaction between the magnetic poles.**

136 These non-participating field lines exist in a sort of blind spot in the field. This physical
 137 condition, by its very nature, affects the coupling of the electrons, as not all field lines
 138 are participating in their interaction. It was hypothesized that the blind spot in the
 139 electric field affects the numerical value of α .

140 If the field surrounding each particle were divided into an odd number of sectors, the
 141 non-participating field lines in the blind spot could be relegated to a single region within

the larger field, with an equal portion of the electric field on either side of it. Dividing the field into 3 sectors would be the minimum needed for this purpose, but with this, the blind spot would have to take up nearly a third of the electric field, likely overcompensating for the area. Whatever the best value happens to be, there is also the question of whether it should be considered an *ad hoc* construct or whether the division is something *fundamental* to the electric field. Only the latter would be of benefit in understanding the nature of α .

To help identify an appropriate value to segregate the field lines in the blind spot from the rest of the field, the degrees around the circular field of an unperturbed electron were used. Starting on the far side at the 180-degree mark, in the center of the blind spot, and moving incrementally in 1-degree steps on each side of that mark, fractions of 360 degrees were identified that resulted in a whole *odd* number in the denominator (Fig. 3). As shown in Fig. 3, five numbers (3, 5, 9, 15, 45) were identified up to a span of 120 degrees for the whole blind spot (60 degrees on either side of the center).

$\frac{2}{360}$	$\frac{4}{360}$	$\frac{6}{360}$	$\frac{8}{360}$	...	$\frac{24}{360}$	...	$\frac{40}{360}$	...	$\frac{72}{360}$	...	$\frac{120}{360}$
↓	↓	↓	↓		↓		↓		↓		↓
$\frac{1}{180}$	$\frac{1}{90}$	$\frac{1}{60}$	<div>$\frac{1}{45}$$\frac{1}{15}$$\frac{1}{9}$$\frac{1}{5}$$\frac{1}{3}$</div>								

Fig. 3 | Division of the electric field into an odd number of sectors, to relegate the blind spot in the field to one sector. Starting on the far side at the 180-degree mark of the field, in the center of the blind spot, and moving incrementally in 1-degree steps on each side of that mark, fractions of 360 degrees were identified that resulted in a whole odd number in the denominator. Five numbers (3, 5, 9, 15, 45) were identified up to a span of 120 degrees for the blind spot (60 degrees on either side of the center).

A choice of 9 sectors appears, at least superficially, to be appropriate for several reasons: 1) As noted above, a choice of 3 sectors would be the minimum needed but would overcompensate for the blind spot—which would have to take up a third of the field. Nine is the first multiple of 3 within the identified set and leads to a reasonable size for the blind spot, at a little more than a tenth of the field (1/9).

With 5 sectors, the blind spot would have to take up nearly a quarter of the electric field. Thus, similar to 3, it would likely overcompensate for the area. The values of 15 and 45 would relegate the blind spot to only about 7% and 2% of the field, respectively, likely falling short of the area. Indeed, within the identified set, 9 falls exactly in the middle of the two numbers that would likely overcompensate for the area, and the two that would likely fall short of the area.

With the choice of 9 sectors, and one of them representing the blind spot, 8 sectors would be involved in the coupling per particle, Fig. 4.

Fig. 4 | Division of the electron and positron's electric fields into 9 sectors each. Only about 8 sectors per particle would be involved in the coupling. The sector on the far side of each particle represents a blind spot in the field, where the field lines in that sector largely extend into space with no involvement in the interaction.

Dividing the ensuing value of 16 from the inverse of the numerical value of α revealed a factor that was approximately the product of Euler's number times pi, leading to the following equation:

$$\alpha^{-1} \approx (2)(8)e\pi = 136.635747562777. \quad (4a)$$

189 The fact that two mathematical constants (to an approximation) naturally arose from
190 the choice of 9 (and ultimately 8) sectors per particle also supported the use of 9 for the
191 number of electric field sectors surrounding the particles.

192 However, although the equation produces a result close to the value of α , it does not
193 lead to a match with the 2018 CODATA value of the constant ($\alpha^{-1} = 137.035999084[21]$).
194 It is unlikely, however, that the division of the field would be so precise that exactly 8
195 sectors — no more, no less — would be involved in the coupling per particle upon
196 elimination of the blind spot in the field. Rather, it is more likely the case that the sector
197 value would be 8 plus or minus some small amount, likely due, at least in part, to the
198 quantum fluctuations in the overall electromagnetic field, which would alter any
199 geometric precision.

200 Working backward from the currently accepted value of α , the electric field sector value
201 for each particle would be approximately 8.02343465913, such that equation (4a) can
202 be recast as follows:

$$\alpha^{-1} = (2)(8.02343465913)e\pi. \quad (4b)$$

203
204
205
206 The amount in excess of 8 — at 0.0234346591350 — is here referred to as the
207 anomalous electric field sector value, a correction needed on the value of 8, just as the
208 anomalous magnetic moment value of 0.00231930436182 is needed on the value of 2
209 for g_e . Indeed, the value of the corrective term needed for the sector value is nearly the
210 same as that needed for g_e , mostly differing by a simple factor of 10. Here, the electric

211 field sector value will be referred to as the “s-factor,” S_e . The proposed general equation
212 for α is thus:

$$213 \quad \alpha^{-1} = 2 \left| S_e \right| e\pi. \quad (5)$$

214 The absolute value is used because in electric repulsion the sectors are, in a sense,
215 “missing” (leading to their having a negative value). In electric repulsion, two electrons
216 (starting off close together) would move apart under the force, filling in the *missing*
217 sectors of field lines as they separate. In electric attraction, an electron and positron
218 (starting off some distance apart) would move together under the force, eliminating the
219 *existing* sectors of field lines between them as they unite. – S_e can be thought of as an
220 approximately 8-point “hole” to be filled in repulsion, and S_e as an approximately 8-
221 point “hill” to be leveled in attraction, to allow full coupling to occur.

222 Note that as the anomalous value for g_e is regarded as $2\alpha_e$, where α_e is
223 0.00115965218128(18), the anomalous value for S_e can be regarded as $2(S_e)_\alpha$, where
224 $(S_e)_\alpha$ is 0.0117173295675. Thus, similar to the equation for g_e , where $g_e = 2 + 2\alpha_e$, S_e can
225 be written as follows:

$$226 \quad S_e = 8 + 2(S_e)_\alpha. \quad (6)$$

227

228 The question remains, however, as to whether S_e , and thereby equation (5), is actually
229 fundamental in nature, given the apparent *ad hoc* decision to divide the electric field
230 into 9 sectors to account for the blind spot in the field. It appears that this question can
231 be answered by way of the anomalous value, $(S_e)_\alpha$.

232 As the value of S_e is regarded above as a completely arbitrary choice into which to divide
 233 the electric field, $(S_e)_\alpha$ would be an equally arbitrary value, as it directly stems from that
 234 choice. As such, it would be highly improbable for $(S_e)_\alpha$ to have any connection to
 235 fundamental constants in nature, being much more likely to have nothing to do with
 236 them. However, $(S_e)_\alpha$ can be derived, to several significant digits, by using the values of
 237 the anomalous magnetic moments of the electron, muon, and tau particle (α_e , α_μ , and
 238 α_τ , respectively). The result is achieved through the following power series:

$$239 \quad \frac{(S_e)_\alpha}{10} \approx \frac{C_0(\alpha_e + \alpha_\mu + \alpha_\tau)^0 + C_1(\alpha_e + \alpha_\mu + \alpha_\tau) + C_2(\alpha_e + \alpha_\mu + \alpha_\tau)^2 + \dots}{3} \quad (7)$$

240 where

$$241 \quad \alpha_e = 0.00115965218128(18)$$

$$242 \quad \alpha_\mu = 0.00116592061(41) \text{ (From reference 7)}$$

$$243 \quad \alpha_\tau = 0.001177171(39) \text{ (From reference 8)}$$

$$244 \quad C_0 = 0$$

$$245 \quad C_1 = 1$$

$$246 \quad C_2 = 1 + 10(\alpha_e) = 1.011\underline{5}965218128$$

$$247 \quad C_3 = 1 + 10(\alpha_\mu) = 1.011\underline{6}592061$$

$$248 \quad C_4 = 1 + 10(\alpha_\tau) = 1.011\underline{7}7171$$

$$249 \quad C_5 = 1 + ??? \approx 1.011\underline{8}...$$

$$250 \quad C_6 = 1 + ??? \approx 1.011\underline{9}...$$

$$251 \quad \vdots$$

If pattern holds

252

Using equation (7) and the values above for α_e , α_μ and α_τ leads to an $(S_e)_\alpha$ value of about 0.01171733, compared with 0.0117173295675 from the known value of α , a 99.99999% (7-significant-digit) match. Inputting this result into equations (5) and (6) leads to a calculated α^{-1} value of 137.0359, truncated at the seventh digit due to the low precision level of the α_τ value. Interestingly, an increase in the precision of the value of α_τ through Standard Model calculations is correlated with an increasingly closer match between the value of α as calculated through equations (5), (6), and (7) and the accepted value of the constant (Table 1). It will be interesting to see if increased refinement of the value of α_τ through the Standard Model, or even experimentation, leads to even closer results.

Table 1. The increase in the precision of the value of α_τ through Standard Model calculations is correlated with an increasingly closer match between the calculated value of α using the study equations and the accepted value of the constant

Reference	Value of α_τ Through Standard Model Calculations	Calculated Value of α^{-1} by Inputting the α_τ Value into Equations (5), (6), (7) in This Study*
Samuel & Li, 1991, ref. 9	0.0011773(3)	<u>137.03</u> 6013882 (5 significant digits)
Eidelman & Passera, 2007, ref. 10	0.00117721(5)	<u>137.036</u> 003561 (6 significant digits)
Keshavarzi, Nomura & Teubner, 2020, ref. 8	0.001177171(39)	<u>137.0359</u> 99089 (7 significant digits)

* Significant digits in bold/underline, corresponding to precision of α_τ value.

Note: 2018 CODATA value of $\alpha^{-1} = 137.035999084(21)$.

Equation (7) might not be the only or best equation for showing the relationship between α_e , α_μ , α_τ and $(S_e)_\alpha$. However, it does so to a notable degree, with the result of

270 the expression ultimately leading to a good approximation of the value of α that has
 271 only increased in precision as the precision of the value of α_τ has increased through
 272 Standard Model calculations — with 7 digits of the value of α matched currently.

273 Of course, if future evaluations of α_e , α_μ , and α_τ lead to an exact match with the value of
 274 α through equation (7), this would greatly support the idea of a relationship among the
 275 magnetic moment anomalies and $(S_e)_\alpha$. However, failure to lead to a result beyond a 7-,
 276 8-, or 9-digit match with the value of α could also mean the above equation represents
 277 only a limiting case of such a relationship or that an adjustment to the formula is
 278 needed. Additional support would come from how well the above information
 279 intersects with other areas of physics. For example, as shown in section 2.2 below, the
 280 value of S_e , including its $(S_e)_\alpha$ component, can also be formulated in a way that is
 281 reflective of the quantum electrodynamics (QED) formula for g_e , including its α_e
 282 component. This is also the case for g_μ and g_τ and their anomalous components. This
 283 moderate intersection with QED further suggests there being some degree of a true
 284 mathematical relationship between the magnetic moment anomalies and $(S_e)_\alpha$.

285 As a value stemming from the completely arbitrary decision to divide the electric field
 286 into 9 sectors, having nothing to do with lepton magnetic moment anomalies, there is
 287 no *a priori* reason for $(S_e)_\alpha$ to have *any* connection with α_e , α_μ , or α_τ , unless there is
 288 indeed a preexisting natural relationship among them. Such a link would be highly
 289 improbable as a random occurrence, as completely fabricated numbers cannot typically
 290 be applied in mathematical analyses in any useful way alongside true physical constants
 291 with which they had no original connection.

292 This suggests that like the values of α_e , α_μ and α_τ , the value of $(S_e)_\alpha$ is indeed a real and
293 natural quantity — an anomalous value associated with the electric field of the electron,
294 muon, and tau particle that is mathematically linked to the anomalous values associated
295 with their magnetic fields, as perhaps another example of how electric and magnetic
296 phenomena are linked. This, in turn, would make the value of S_e , at approximately 8,
297 also a natural feature of the electric field, and thereby equation (5) a fundamental
298 equation for α .

299 S_e being a natural value would mean that, similar to the way that the valence shell of
300 atomic nuclei of the main group elements has 8 zones by which the nuclei interact with
301 electrons, there would be 8 zones or sectors in the space surrounding a lepton by which
302 the particle would engage in coupling with another lepton. The main difference is that
303 the lepton, as a single elementary particle, would be interacting with just one other
304 particle, whereas the sectors surrounding an atomic nucleus, which is composed of
305 multiple protons (themselves composed of multiple elementary particles), would be
306 engaged in interactions with multiple (up to 8) electrons. The atom would be in the
307 most energetically stable state when all 8 of the sectors in its valence shell are engaged
308 in interactions with electrons. Similarly, a lepton would be in the most energetically
309 stable state from the standpoint of electromagnetic coupling when all 8 of its field
310 sectors are engaged in the interaction with the other lepton. Thus, there appears to be a
311 certain symmetry between leptons and atoms, with each being surrounded by 8 distinct
312 sectors of space through which interactions occur.

313 Such spatial organization around an electron is not incompatible with the virtual photon
314 cloud concept. Compagno and colleagues noted that it is possible "...to speak of a cloud
315 of virtual photons [surrounding an electron] having a well defined spatial structure" and
316 that generally it is also possible "...to connect some properties of the spatial structure of
317 clouds to the internal dynamics of the source" [11]. In their study of the cloud of virtual
318 photons associated with the hydrogen atom, Passante and colleagues noted that they
319 were "... led to the conclusion that the structure of the virtual cloud of photons in the
320 ground state is in fact an 'inside-out' mapping of the electronic structure of the
321 hydrogen atom" [12].

322 In contrast to the hydrogen atom, elementary particles such as the electron are
323 considered to be point particles, with no internal structure. However, this is a
324 mathematical abstraction, used in large part because of current technological
325 limitations in probing to sizes that would reveal any internal structure. As such, just as it
326 is suggested that the virtual photon cloud associated with the ground state of the
327 hydrogen atom is an "inside-out" mapping of the electronic structure of the atom, so
328 too might the 8-sector electric field, or equivalent virtual photon cloud, of the electron
329 be an "inside-out" mapping of some finer, as yet identified, electronic structure of the
330 elementary particle. Indeed, the identification of the 8 sectors of the electric field might
331 be a first glimpse into that structure.

332 The question of "Why does nature chose 8 specifically?" cannot be immediately
333 answered for either the lepton or the atom. However, this concept would represent yet
334 another "octet rule" in particle physics: In the model above, a lepton couples with

another lepton through 8 electric field sectors (plus the field's anomalous portion). The "Eightfold Way" concerns the organization of hadrons, and the currently established "Octet Rule" concerns the 8-zoned valence shell of at least the main group elements, even when there are many more electrons between the nucleus and the valence shell. Thus, while there are always exceptions, a "theme of 8" appears to be carried through from leptons, to hadrons, to some lepton-hadron interactions.

Although a quantum mechanical analysis would be needed for further evaluation, there are, thus, several clues that the space surrounding an electron does indeed have a discrete, 8-zoned structure, analogous to the valence shell of many atomic nuclei. The most notable support for this concept comes from the fact that the 8-sector model gives rise to the value of $(S_e)_\alpha$, which in turn can be derived to several significant digits from the values of the lepton magnetic moment anomalies, which have been established through theory and/or experimentation. Thus, as the magnetic moment anomalies are true physical quantities, so too are likely the values of $(S_e)_\alpha$ and S_e , which directly stem from them.

Therefore, $(S_e)_\alpha$ and S_e appear to be real, nontrivial components in the mathematics of α . In the following sections, this is explored further, and it is shown how the concepts of $(S_e)_\alpha$ and S_e help to explain additional electromagnetic phenomenon and provide even greater clarity regarding the mathematical and physical origins of α .

2.1 Types of α Values

If all 18 sectors were involved between the two interacting particles, α^{-1} would equal about 153 (from $18e\pi$) instead of about 137 (from $16e\pi$)—indeed, at a superficial level, 153 seems as “unusual” a number as 137. The different scenarios lead to three types of α values, a basic value (α_B) associated with the value 18, encompassing each interacting particle’s full electric field; the true, corrected value of α ; and a reduced value (α_R) that takes the blind spot per field into account but at a gross level (Table 2).

Table 2. Suggested types of the fine-structure constant, α , presented as inverse values

Type	Symbol	Expression	Value
Basic value	α_B^{-1}	$18e\pi$	153.715216008
True (corrected) value	α^{-1}	$16.046869325e\pi$	137.035999084*
Reduced (uncorrected) value	α_R^{-1}	$16e\pi$	136.635747562

*2018 CODATA value.

The basic value could not really exist, as it would be physically impossible for all 18 sectors to be involved in the coupling in any meaningful way. For this to happen, the field would have to be highly contorted to allow the sectors of field lines on the far side of each particle to play a part in the interaction.

As the particles move closer to one another under the electric force of attraction, the field lines involved in the interaction would extend over fewer and fewer sectors of space. Ultimately, there would be zero sectors of space between the particles, and the field line density (number of field lines per sector) would become infinite. As a consequence, the coupling strength would become infinite. This is in accordance with

the Landau pole — the state of infinite coupling strength in quantum theory, and in this work is represented as follows:

$$\alpha^{-1} = 2 \left| 0 \right| e\pi = 0; \alpha = \infty. \quad (8)$$

Thus, the sector value serves as a proxy for field line density, but in the reverse — meaning a *decrease* in the sector value (as an absolute value) corresponds to an *increase* in field line density:

$$\text{Field line density} = \frac{\text{Number of field lines}}{\left| \text{Sector value} \right|}. \quad (9)$$

Thus, as noted above, the sectors represent a threshold, or barrier, that must be surmounted for full, or infinite, coupling to occur. The greater the sector value, the more substantial the barrier will be and the lower the field line density, and consequently, the lower the coupling strength.

The sector value concept can also help explain the change in the value of α at rising energy levels when particles are driven together during collider experiments. That is, the value 1/137 is the approximate value of α at low energy. The constant is said to “run,” or change, as the particles’ energy level changes. When the particles are given higher energy in the collider, the value of α has been measured to rise. At 90 GeV, α has been measured to have a value of approximately 1/128.5 (or 0.007812) [13]. This is the identical value attained when one electric field sector (or its equivalent) is lost in

equation (5): $\alpha^{-1} = (16.0468693182699 - 1)e\pi$. Upon collision, each particle's field would "infiltrate" the other's, with the effective loss of one sector between them.

In electric attraction, the sector value starts off as positive, and an increase in coupling can be viewed as a *decrease* in this value as the particles move closer together. In electric repulsion, the sector value starts off as negative, and an increase in coupling can be viewed as an *increase* in this value as the particles move further apart.

$$\alpha^{-1} = 2 \left| 8.0234346591350 - x \right| e\pi, \quad (10)$$

(attraction)

$$\alpha^{-1} = 2 \left| -8.0234346591350 + x \right| e\pi. \quad (11)$$

(repulsion)

When enough distance is lost in the setting of electric attraction, or gained in the setting of electric repulsion, to fully mitigate the effects of the sector value, the value would go to zero and the strength of the force would reach infinity. Addition or subtraction beyond zero would have no meaning, as you could not remove more sectors than there are to be removed (in electric attraction) or add more sectors around the particles than can be added (in electric repulsion).

2.2 QED-Like Formula for the Anomalous Electric Field Sector Value

From a geometric perspective, the anomalous electric field sector value can be divided into 2 subsections: a small amount at the upper end of the blind spot in the electric field

416 and an equally small amount at the lower end, due to a natural symmetry around the
 417 circular field (Fig. 5). As the full correction in excess of 8 is 0.0234346591350, each
 418 subsection, $(S_e)_\alpha$, would be half of this, or 0.0117173295675.

419

420 **Fig. 5 | Shown are the posited 9 sectors of the electric field surrounding an electron.**
 421 **Eight sectors would be involved in an interaction plus a portion of the ninth — a small**
 422 **amount above the blind spot and a small amount below it, corresponding to 2**
 423 **subsections, each here referred to as $(S_e)_\alpha$, for the anomalous electric field sector**
 424 **value involved in the interaction. Each subsection would be 0.0117173295675.**
 425 **Together, they constitute the full correction of 0.0234346591350 needed on the value**
 426 **8 in the calculation of α .**

427 Thus, as noted above, S_e can be written as $S_e = 8 + 2(S_e)_\alpha$, similar to the equation for g_e ,
 428 where $g_e = 2 + 2\alpha_e$.

429 As with the full value of g_e , the full value of S_e can also be formulated perturbatively. In
 430 the case of g_e , the perturbative method is applied to quantum field theory, specifically
 431 QED. The perturbative formula for the QED contribution to g_e is as follows:

$$\frac{g_e}{2} = 1 + C_1 \left(\frac{\alpha}{\pi} \right) + C_2 \left(\frac{\alpha}{\pi} \right)^2 + C_3 \left(\frac{\alpha}{\pi} \right)^3 + C_4 \left(\frac{\alpha}{\pi} \right)^4 + \dots + non - QED \quad (12)$$

The formal power series of α/π corresponds to quantum corrections as determined through Feynman diagrams, which in turn correspond to real quantum activity at the particle level. The coefficients of the formula (C_i) have been calculated to $(\alpha/\pi)^5$ and have been confirmed experimentally [6]:

$$C_1 = 0.5 \quad C_4 = -1.912...$$

$$C_2 = -0.328 \dots \quad C_5 = 6.737....$$

$$C_3 = 1.181 \dots$$

The class of quantum activities that would yield a correction on S_e is currently not clear. However, S_e can be mathematically formulated in a similar way to g_e by using α_R (again, $1/16\pi$, see Table 2) in the formal power series in place of α :

$$\frac{S_e}{2} = 4 + C_1 \left(\frac{\alpha_R}{\pi} \right) + C_2 \left(\frac{\alpha_R}{\pi} \right)^2 + C_3 \left(\frac{\alpha_R}{\pi} \right)^3 + C_4 \left(\frac{\alpha_R}{\pi} \right)^4 + C_5 \left(\frac{\alpha_R}{\pi} \right)^5 \quad (13)$$

Using mathematical deduction, several sets of coefficients for equation (13) were identified, each leading to approximately the same value for α when the corresponding S_e value was inserted into equation (5). Each solution is also consistent with the 2018 CODATA value of the constant (Table 3).

448 In each set, the base values of C_1 through C_5 are the same, at 0.5 , $4/\pi$, $3/\pi$, $2/\pi$, and $1/\pi$,
 449 respectively, largely following a pattern as multiples of $1/\pi$. C_1 and C_2 in each set have
 450 exponents of 1. Exponents for C_3 through C_5 begin with a value of 3 in each set. Starting
 451 at C_3 , the exponents follow a simple numerical sequence in set 1, the prime number
 452 sequence in set 2 (which actually can be regarded as beginning with C_2), multiples of 3 in
 453 set 3, and every fourth number in set 4. Note that each coefficient must be multiplied by
 454 a factor of 10 before applying it to equation (13).

455 **Table 3. Coefficients for Equation (13), Four Possible Sets***

Coefficient	Set 1	Set 2	Set 3	Set 4
C_1	0.5	0.5	0.5	0.5
C_2	$4/\pi$	$4/\pi$	$4/\pi$	$4/\pi$
C_3	$(3/\pi)^3$	$(3/\pi)^3$	$(3/\pi)^3$	$(3/\pi)^3$
C_4	$(2/\pi)^4$	$(2/\pi)^5$	$(2/\pi)^6$	$(2/\pi)^7$
C_5	$(1/\pi)^5$	$(1/\pi)^7$	$(1/\pi)^9$	$(1/\pi)^{11}$
Calculated α^{-1} value	137.035999085306...	137.035999084705...	137.035999084323...	137.035999084080...

456 CODATA α^{-1} value: 137.035999084(21).

457 *Each coefficient in the table must be multiplied by a factor of 10 before applying to
 458 equation (13).

459
 460 In the above case, the factor of 10 is considered to be part of the coefficient. Setting 10
 461 apart from the coefficients, the equation can be written as follows:

$$\frac{S_e}{2} = 4 + (10) \left[C_1 \left(\frac{\alpha_R}{\pi} \right) + C_2 \left(\frac{\alpha_R}{\pi} \right)^2 + C_3 \left(\frac{\alpha_R}{\pi} \right)^3 + C_4 \left(\frac{\alpha_R}{\pi} \right)^4 + C_5 \left(\frac{\alpha_R}{\pi} \right)^5 \right] \quad (14)$$

Sets 3 and 4 each appear to lead to the closest match with the current CODATA value, excluding its margins of error.

A recent experiment, producing the most precise measurement of α as of 2020, suggests that α^{-1} might have a value closer to 137.035999206(11) [14]. In this setting, the coefficients could be similar to those listed below, still with a pattern among the values:

$$C_1 = 0.5$$

$$C_2 = \frac{4}{\pi}$$

$$C_3 = \frac{\sqrt{8}}{\pi}$$

$$C_4 = \frac{\sqrt[4]{12}}{\pi}$$

$$C_5 = \frac{\sqrt[6]{16}}{\pi}.$$

These coefficients would lead to an α^{-1} value of 137.035999217, at the upper end of the experimental value range.

479 The perturbative treatments of S_e above might serve as purely mathematical formulas.
 480 However, given the similarities of the perturbative formulations for g_e and S_e and the
 481 similarity of the anomalous values — at 0.00231930436 for g_e and 0.023434659 for S_e —
 482 the perturbative formulations for S_e might be associated with actual physical
 483 phenomena at the quantum level, just as the perturbative formulation for g_e is. Indeed,
 484 as shown above in equation (7), the value of $(S_e)_\alpha$ appears derivable by using the values
 485 of α_e , α_{μ} , and α_{τ} , which are themselves derivable through QED. This too supports the
 486 idea of $(S_e)_\alpha$, and thereby of S_e , being associated with actual quantum activity.

487 Note that the ability to identify coefficients by mathematical deduction does not
 488 preclude the role of quantum mechanics in nature. For example, the following
 489 coefficients, determined by mathematical deduction, could be used in equation (12) for
 490 calculating g_e :

491
$$C_1 = 0.5$$

492
$$C_2 = -\frac{1}{3}$$

493
$$C_3 = \frac{\pi^2}{3}$$

494
$$C_4 = -\pi^2$$

495
$$C_5 = 2\pi^2.$$

496

497 They lead to a g_e value of 2.00231930436249, a 99.99999999999% (13-significant-digit)
 498 match with the 2018 CODATA value of 2.00231930436256(35). QED, however, is one of
 499 the most verified physical theories, indicating the existence of actual quantum activity.

500 A better understanding of the quantum activity leading to the anomalous component of
 501 S_e is needed, perhaps leading to one of the above perturbative formulas or a different
 502 one. However, the presented mathematical work does demonstrate that such a
 503 formula, linked to actual quantum mechanical phenomena, might be possible.

504 Whether or not a quantum mechanical basis to α is found, it is clear that the constant
 505 can be well formulated by using mathematical constants in contrast to what is generally
 506 believed today. One of the notable aspects of the formula is its self-consistent nature —
 507 that is, α_R is the number to be corrected and is, at the same time, the tool by which the
 508 correction is attained, as the numerator of the parameter in the power series.

509 Caution should be exercised, however, in attempting to mathematically model whatever
 510 the latest CODATA value happens to be. The CODATA value is simply determined
 511 through the best experimental studies available at the time it is established, and
 512 experimental values have a tendency to shift slightly. Whereas the 2018 CODATA value
 513 for α^{-1} is 137.035999084(21), the 2014 CODATA value was 137.035999139(31), which
 514 would require a different set of coefficients than those above.

515 Also, while the above perturbative formulas are characterized by particular patterns
 516 among the coefficients, a valid formula for a different experimental value for α might

517 not have a pattern (or at least not an obvious one). The principal idea here is that
518 flexibility in any mathematical model is important.

519 If an obvious pattern is detected among the coefficients, and experimental results
520 consistently agree with such a pattern, there is also the possibility that the formula
521 could be used to calculate α to an indefinite number of decimal places, if the expansion
522 diverges. The convergence of such a series would, of course, be invaluable information,
523 as well.

524 In all, the perturbative treatment for S_e shown above in conjunction with equation (5)
525 represents, for the first time in history, a full mathematical expression for α that
526 involves mathematical constants, leads to an exact match with the established value of
527 the constant, has the potential to calculate α to an indefinite number of decimal places,
528 and is linked to a physical aspect of the electric field. This physical aspect concerns field
529 geometry at a gross level (accounting for a blind spot in the field), but likely also
530 quantum activity within the greater electromagnetic field.

531 *2.3 Alternate Formulas for the Anomalous Electric Field Sector Value*

532 As alternate mathematical expressions for calculating a quantity can often be
533 informative, the perturbative solution above was converted into other forms: 1) an
534 alternate perturbative series, with a leading term slightly higher than the value of 4 used
535 above, 2) a related expansion series with non-integer exponents, and 3) a generalized
536 continued fraction. Each offers additional insight into α that might prove useful in
537 further analysis of the constant, both from a mathematical and physical perspective.

2.3.1. Alternate Perturbative Series and Series Expansion with Non-Integer Exponents

In addition to the above, the full value of S_e can be formulated as follows:

$$\frac{S_e}{2} = C_0 \alpha_R^0 + C_1 \alpha_R + C_2 \alpha_R^2 + C_3 \alpha_R^3 + C_4 \alpha_R^4 + C_5 \alpha_R^5 + \dots, \quad (15)$$

where again, α_R is the reduced fine-structure constant ($1/16\pi$). Given the currently accepted value of α , the following values appear appropriate for the coefficients:

$$\begin{aligned} C_0 &= 4 + \frac{\sqrt{\frac{\pi}{3}} - 1}{2} & C_3 &= \frac{\pi}{3} \\ C_1 &= 0 & C_4 &= \frac{\pi}{7} \\ C_2 &= 1 & C_5 &= \frac{\pi}{11}. \end{aligned}$$

Unlike the expression in the previous section, the exactly solvable portion of this expression — that is, the leading term represented by coefficient, C_0 — is 4 plus a small amount from an expression involving pi. Such a formula could be important if there is a geometrical aspect to the *anomalous value* (which the expression after 4 in C_0 would likely account for) in addition to quantum activity (which the first through the fifth terms would account for).

From C_3 onward, the coefficients appear to be a fraction where each numerator is pi and each denominator falls within a linear sequence starting with 3 and then every fourth number thereafter (i.e., 3, 7, 11, ...). Increased precision on the 2018 CODATA value would be needed to determine if the pattern remained from C_5 onward. When applied to equation (5), the expression leads to a value for α^{-1} of 137.035999084, an exact, 12-significant-digit match with the 2018 CODATA value.

Equation (15) can also be written as a series expansion with several non-integer exponents:

$$\begin{aligned} \frac{S_e}{2} = & C_0 \alpha_R^0 + C_1 \alpha_R + C_2 \alpha_R^2 + C_3 \alpha_R^3 + C_4 \alpha_R^4 + \\ & C_5 \alpha_R^{4.001} + C_6 \alpha_R^{4.002} + C_7 \alpha_R^{4.003} + \\ & C_8 \alpha_R^{4.004} + C_9 \alpha_R^{4.005} + C_{10} \alpha_R^{4.006} \dots, \end{aligned} \quad (16)$$

where the coefficients are as follows:

$$\begin{aligned} C_0 &= 4 + \frac{\sqrt{\frac{\pi}{3}} - 1}{2} & C_3 &= 1 & C_6 &= 1 & C_9 &= 1 \\ C_1 &= 0 & C_4 &= 1 & C_7 &= 1 & C_{10} &= 1. \\ C_2 &= 1 & C_5 &= 1 & C_8 &= 1 \end{aligned}$$

There is a certain “elegant simplicity” to this formula, where all of the coefficients from C_2 onward are 1. The 5th through the 10th terms, with exponents of 4.001 through 4.006, appear as a subset of the 4th term, with the leading term to the 4th term being the principal portion of the equation.

2.3.2. Generalized Continued Fraction

Fig. 6 shows the full value of S_e modeled as a generalized continued fraction. The number 8 (for the number of principal electric field sectors involved in the coupling) is the leading term of the fraction. The remaining component represents the anomalous value. The partial numerators of the fraction begin with the number 3 and are 1 thereafter. The partial denominators comprise two distinct areas within the fraction: one representing a major set of corrections, with the values of 2^7 and 2^6 , in that order, and one representing a minor set, with the values of 2, 4, and 6, also in that order. The

major set of corrections alone is associated with an α^{-1} value of 137.035998746, a 99.999999% (8-significant-digit) match with the currently accepted value. The full fraction, of course, leads to the exact value of α .

$$S_e = 8 + \frac{3}{2^7 + \frac{1}{2^6 + \frac{1}{2 + \frac{1}{4 + \frac{1}{6 + \ddots}}}}}$$

Major Set of Corrections

Minor Set of Corrections

$$\alpha^{-1} = 2 \left| S_e \right| e\pi$$

$$\alpha^{-1} = 137.035999084 \text{ (via formula)}$$

$$\alpha^{-1} = 137.035999084(21) \text{ (CODATA, 2018)}$$

Exact match.

Fig. 6 | Shown is the value of S_e , including its anomalous component, modeled as a generalized continued fraction. The partial denominators comprise two distinct areas within the fraction (which is truncated in the figure): one representing a major set of corrections, with the values of 2^7 and 2^6 , in that order, and one representing a minor set, with the values of 2, 4, and 6, also in that order. The fraction, in conjunction with equation (5), fully models the currently accepted value of α mathematically to an exact match.

597 There are several noteworthy issues concerning the fraction, the first of which is its
 598 regular structure, providing a straightforward representation of the full value of S_e .
 599 Indeed, the possibility exists that the fraction has a regular structure that extends
 600 indefinitely, as the generalized continued fraction of pi does, although a more precise
 601 experimental value of α would be needed to be sure.

602 For example, the partial denominators could continue as multiples of 2 or even powers
 603 of 2. Using multiples of 2 and extending the fraction after the partial denominator of 6
 604 through a partial denominator of 12 results in an α^{-1} value of 137.035999084059.
 605 Extending the partial denominators as powers of 2, going from 2 through 2^6 , results in
 606 an α^{-1} value of 137.035999083716. Both are consistent with 2018 CODATA value of
 607 137.035999084(21) (Fig. 7).

$$\begin{array}{l}
 S_e = 8 + \cfrac{3}{2^7 + \cfrac{1}{2^6 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{8 + \cfrac{1}{10 + \cfrac{1}{12 + \ddots}}}}}}}}
 \end{array}$$

608

609 Extending partial denominators as multiples of 2 leads to an α^{-1} value of
 610 137.035999084059...
 611

$$S_e = 8 + \cfrac{3}{2^7 + \cfrac{1}{2^6 + \cfrac{1}{2 + \cfrac{1}{2^2 + \cfrac{1}{2^3 + \cfrac{1}{2^4 + \cfrac{1}{2^5 + \cfrac{1}{2^6 + \ddots}}}}}}}}$$

612

613 Extending partial denominators as powers of 2 leads to an α^{-1} value of
 614 137.035999083716...

615

$$\pi = 3 + \cfrac{1^2}{6 + \cfrac{3^2}{6 + \cfrac{5^2}{6 + \cfrac{7^2}{6 + \cfrac{9^2}{6 + \ddots}}}}}$$

616

617 Pi shown as a generalized continued fraction with a regular structure that extends
 618 indefinitely, for comparison.

619 **Fig. 7 | Possible generalized continued fractions for S_e , each with a regular structure.**

620 **Two examples are shown: A) one where the lower partial denominators continue as**

621 **multiples of 2, and B) one where they continue as powers of 2. Each example leads to**

622 a value for α that is consistent with the currently accepted value of the constant. C)

623 The regular structure of a generalized continued fraction for π is shown for

624 comparison.

625 The generalized continued fraction for S_e would be a particularly powerful tool if future

626 evidence suggests that it does indeed have a regular structure throughout. If so, the

627 fraction could be used to determine the value of α to as many decimal places as desired,

628 similar to the potential of the perturbative formulas above. Of course, the possibility

629 also exists that there is no regular structure, or not one beyond the partial

630 denominators of 2^7 and 2^6 . There is the further possibility that the fraction is finite, with

631 or without a regular structure.

632 As noted above, a recent experiment suggests α^{-1} might have a value closer to

633 137.035999206(11) [14]. This would be consistent with a generalized continued fraction

634 where all partial numerators and denominators following the partial denominator of 2^6

635 were a 1, or simply where the remaining fraction after 2^6 were the inverse of the Golden

636 Ratio, 1.6180339887498948482. In this case, α^{-1} calculates to a value of approximately

637 137.035999213, which falls within the margin of error of the experimental result (Fig. 8).

$$S_e = 8 + \frac{3}{2^7 + \frac{1}{2^6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}}}}}$$

Golden Ratio

638

639 Fraction leads to an α^{-1} value of 137.035999213, consistent with the recent
 640 experimental result of 137.035999206(11)[14].

641 **Fig. 8 | A generalized continued fraction for S_e , where the partial numerators and**
 642 **denominators following the partial denominator of 2^6 all have the value of 1, leading**
 643 **to an α^{-1} value of approximately 137.035999213, consistent with the recent**
 644 **experimental result of 137.035999206(11) [14]. In this case, the remaining fraction**
 645 **after the partial denominator of 2^6 would simply be the inverse of the Golden Ratio.**

646 Another noteworthy aspect of the expression is the fact that there are large terms early
 647 in the fraction (again, 2^7 and 2^6), making the anomalous value easy to approximate
 648 through rational values and leading to a substantial match with the established value of
 649 α by way of only a few mathematical steps. Indeed, normal rounding procedures lead to
 650 a 9-significant-digit match (137.035998746 to 137.035999). As such, these two partial
 651 denominators are possibly a firm, universal truth concerning the (zero energy) value of

the constant. That is, as noted above, experimental results have a tendency to differ slightly. As such, the terms of the fraction that would follow 2^6 cannot yet be definitively stated. However, many, if not all, agree on at least the first 9 significant digits of the constant's value, which the first two partial denominators of the generalized continued fraction well lead to.

The portion after 2^6 could be an irrational value, such as the following:

$$S_e = 8 + \frac{3}{2^7 + \frac{1}{2^6 + \sqrt{1/5}}}$$
(17)

Although not a traditional generalized continued fraction, this expression also leads to a value for α that is a 12-significant-digit (exact) match with the current CODATA value of the constant. The point here is simply that something “nonclassical” might be happening at a physical level in relation to the area of the fraction following 2^6 (or a lower point), likely quantum mechanical activity. This would be consistent with the fact that many experiments agree on the first 9 significant digits of α , which again, the portion of the fraction down to 2^6 will lead to. However, they tend to differ slightly beyond this point. A robust quantum mechanical analysis, if possible, might help to definitively home in on the value of α beyond the first 9 significant digits, and thereby this area of the fraction and which experimental results are likely the most accurate.

3. Conclusion

672 The nature of α has been a mystery since its discovery more than 100 years ago, and
673 there have been numerous attempts to identify the mathematical basis of the constant.
674 This study presents a full mathematical formula for α that leads to an exact match with
675 the 2018 CODATA value of the constant, and that, importantly, is connected to a
676 physical aspect of the electric field. As such, it is likely connected to quantum
677 mechanical activity also, as the overall electromagnetic field is quantized in nature.

678 At the heart of the mathematics is the idea of a dimensionless *anomalous electric field*
679 value of about 0.023 associated with the electron. This is particularly notable, as the
680 electron is also associated with a dimensionless *anomalous magnetic field* value of
681 about 0.023/10, which has been well established through perturbative methods applied
682 to QED and through experimentation. In fact, the anomalous value for the electric field
683 is mathematically linked to the anomalous values of the magnetic fields of each of the
684 charged leptons by way of a simple expression, suggesting that it is a real feature of the
685 electric field, and further suggesting yet another link between electric and magnetic
686 phenomena.

687 Altogether, the concepts introduced here suggest that α is not a random value in the
688 universe, nor does it represent an impenetrable box. Instead, there appear to be
689 accessible mathematics and physics (i.e., quantum activity) inside this box governing the
690 constant and thereby all of the physical phenomena associated with it. This suggests
691 that deeper levels of understanding and discovery might be possible in the setting of the
692 constant. And as α is one of about two dozen dimensionless constants upon which the
693 universe is built, it also suggests that it might be possible to simplify the universe into a

694 small set of mathematical constants. The constants might simply need to be arranged in
695 different ways in equations in association with certain physical (perhaps geometrical)
696 conditions of particles and fields, with quantum mechanical activity “filling in the gaps”
697 in terms of attaining accurate and precise results.

698 Of particular note, the transformation of α from an *ad hoc* value in the Standard Model
699 would serve as a sizeable crack in the ceiling holding the Standard Model back from
700 being a more complete theory of the elementary particles and their interactions. And as
701 α unites fundamental aspects of electromagnetism, quantum physics, and relativity, a
702 deeper understanding of its nature might also assist in efforts to unite the seemingly
703 incompatible physical theories of general relativity and quantum mechanics.

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6. Author Contribution Statement

The author provided all written content in the manuscript.

7. Additional Information

The author declares no competing interests.

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The author certifies that he has no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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The data that support the findings of this study are openly available in the 2018 CODATA Recommended Values of the Fundamental Physical Constants, reference number 1.

8. Figure Legends

Fig. 1 | The electric interaction between an electron and positron (before annihilation), as depicted using classical field lines. On the far side of the particles, there are field lines that do not take part in the interaction, with the field lines simply extending into space, representing a blind spot in the field.

Fig. 2 Shown is a depiction of the magnetic field analogue of the electric field. The drawing shows how iron filings arrange themselves around a magnetic due to the influence of the magnetic field lines. The iron filings effectively map the field lines, showing the blind spot in the field at a macroscopic level, where the field lines in the middle of the far sides of the magnet extend into space with no involvement in the interaction between the magnetic poles.

Fig. 3 | Division of the electric field into an odd number of sectors, to relegate the blind spot in the field to one sector. Starting on the far side at the 180-degree mark of the field, in the center of the blind spot, and moving incrementally in 1-degree steps on each side of that mark, fractions of 360 degrees were identified that resulted in a whole odd number in the denominator. Five numbers (3, 5, 9, 15, 45) were identified up to a span of 120 degrees for the blind spot (60 degrees on either side of the center).

Fig. 4 | Division of the electron and positron's electric fields into 9 sectors each. Only about 8 sectors per particle would be involved in the coupling. The sector on the far side of each particle represents a blind spot in the field, where the field lines in that sector largely extend into space with no involvement in the interaction.

Fig. 5 | Shown are the posited 9 sectors of the electric field surrounding an electron. Eight sectors would be involved in an interaction plus a portion of the ninth — a small amount above the blind spot and a small amount below it, corresponding to 2 subsections, each here referred to as $(S_e)_\alpha$, for the anomalous electric field sector value involved in the interaction. Each subsection would be 0.0117173295675. Together, they constitute the full correction of 0.0234346591350 needed on the value 8 in the calculation of α .

Fig. 6 | Shown is the value of S_e , including its anomalous component, modeled as a generalized continued fraction. The partial denominators comprise two distinct areas within the fraction (which is truncated in the figure): one representing a major set of corrections, with the values of 2^7 and 2^6 , in that order, and one representing a minor set, with the values of 2, 4, and 6, also in that order. The fraction, in conjunction with equation (5), fully models the currently accepted value of α mathematically to an exact match.

Fig. 7 | Possible generalized continued fractions for S_e , each with a regular structure. Two examples are shown: A) one where the lower partial denominators continue as multiples of 2, and B) one where they continue as powers of 2. Each example leads to a value for α that is consistent with the currently accepted value of the constant. C) The regular structure of a generalized continued fraction for pi is shown for comparison.

Fig. 8 | A generalized continued fraction for S_e , where the partial numerators and denominators following the partial denominator of 2^6 all have the value of 1, leading to

an α^{-1} value of approximately 137.035999213, consistent with the recent experimental result of 137.035999206(11) [14]. In this case, the remaining fraction after the partial denominator of 2^6 would simply be the inverse of the Golden Ratio.

9. Tables

Table 1. The increase in the precision of the value of α_r through Standard Model calculations is correlated with an increasingly closer match between the calculated value of α using the study equations and the accepted value of the constant

Reference	Value of α_r Through Standard Model Calculations	Calculated Value of α^{-1} by Inputting the α_r Value into Equations (5), (6), (7) in This Study*
Samuel & Li, 1991, ref. 9	0.0011773(3)	<u>137.03</u> 6013882 (5 significant digits)
Eidelman & Passera, 2007, ref. 10	0.00117721(5)	<u>137.036</u> 003561 (6 significant digits)
Keshavarzi, Nomura & Teubner, 2020, ref. 8	0.001177171(39)	<u>137.0359</u> 99089 (7 significant digits)

* Significant digits in bold/underline, corresponding to precision of α_r value.

Note: 2018 CODATA value of $\alpha^{-1} = 137.035999084(21)$.

Table 2. Suggested types of the fine-structure constant, α , presented as inverse values

Type	Symbol	Equation	Value
Basic value	α_B^{-1}	18π	153.715216008
True (Corrected) value	α^{-1}	16.046869325π	137.035999084*
Reduced (Uncorrected) value	α_R^{-1}	16π	136.635747562

*2018 CODATA value.

812 **Table 3. Coefficients for Equation (13), Four Possible Sets***

Coefficient	Set 1	Set 2	Set 3	Set 4
C_1	0.5	0.5	0.5	0.5
C_2	$4/\pi$	$4/\pi$	$4/\pi$	$4/\pi$
C_3	$(3/\pi)^3$	$(3/\pi)^3$	$(3/\pi)^3$	$(3/\pi)^3$
C_4	$(2/\pi)^4$	$(2/\pi)^5$	$(2/\pi)^6$	$(2/\pi)^7$
C_5	$(1/\pi)^5$	$(1/\pi)^7$	$(1/\pi)^9$	$(1/\pi)^{11}$
Calculated α^{-1} value	137.035999085306...	137.035999084705...	137.035999084323...	137.035999084080...

813 CODATA α^{-1} value: 137.035999084(21).

814 *Each coefficient in the table must be multiplied by a factor of 10 before applying to
815 equation (13).

816