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Sandeep Kumar

Lovely Professional University

Shivani Vij

DAV Institute of Engineering and Technology

Niti Kant

Lovely Professional University

Vishal Thakur (✉ vishal20india@yahoo.co.in)

Lovely Professional University <https://orcid.org/0000-0002-5347-6956>

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Efficient THz Generation by nonlinear interaction of Gaussian laser beam with the anharmonic and rippled CNTs aligned vertically in the array

Sandeep Kumar¹, Shivani Vij², Niti Kant¹, and Vishal Thakur^{1*}

^{1.} *Department of Physics, Lovely Professional University, G.T. Road, Phagwara - 144411, Punjab, India*

^{2.} *Department of Applied Sciences, DAV Institute of Engineering & Technology, Jalandhar, 144008, India*

Abstract

We purpose a theoretical analysis for the generation of efficient terahertz (THz) radiation by using the nonlinear interaction of Gaussian laser beam with vertically aligned anharmonic, and rippled carbon nanotubes (CNTs) array. This array of vertically aligned carbon nanotubes (VA-CNTs) is embedded on the base of the dielectric surface. The VA-CNTs have been magnetized by applying a static magnetic field mutually perpendicular to the direction of propagation of the Gaussian beam and length of CNTs. The Gaussian laser beam passing through the CNTs exerts a nonlinear ponderomotive force on the electrons of CNTs and provides them resonant nonlinear transverse velocity. This produces the nonlinear current which is further responsible for the generation of THz radiation. The anharmonicity plays a vital role in the efficient generation of THz radiation. The anharmonicity arises due to the nonlinear variation of restoration force on the various electrons of CNTs. This anharmonicity in the electrons of CNTs helps in broadening the resonance peak. We have observed that externally applied static magnetic field (110 kG to 330 kG) also paves the way for the enhancement of the normalized THz amplitude.

Keywords: Rippled CNTs, THz radiation, static magnetic field, dielectric surface, Gaussian laser, and nonlinear restoration force.

E-mail: vishal20india@yahoo.co.in

I. Introduction

One of the main tasks in the modern technical world is to develop compact and highly efficient terahertz generation sources. Such THz generation sources can bring revolutionary changes in many fields of science and technology, a few of these are THz-spectroscopy, security systems, medical and health sectors, etc. [1-5]. In the present century, researchers have provided variety of schemes for the efficient generation of THz radiations. In these schemes, THz radiations are generated using various methods, for example by employing laser coupling to anharmonic CNTs [6], by using a wiggler magnetic field on vertically aligned carbon nanotubes (VA-CNTs) [7], buy using anharmonic CNTs in the presence of static magnetic field [8], by using laser filaments in the presence of static electric and magnetic fields [9], etc. According to Batrakov et al. [10], THz radiations can be generated through CNTs by using a static electric field. Parashar and Sharma [11] have applied optical rectification in CNTs to generate THz radiations. The CNTs, when irradiated with laser beams provide one of the promising ways for the creation of compact and efficient sources of THz radiations. The extraordinary electrical conductivity, thermal conductivity, and compact dimensional characteristics of CNTs make them a front runner in the field of THz generation [12-16].

In the present paper, we develop an analytical model for the efficient THz generation by the interaction of Gaussian laser beam with the array of VA-CNTs in the presence of static magnetic field applied perpendicular to the direction of propagation of laser and length of CNTs. In 2nd segment of the paper, we have derived the relation for the nonlinear current density, which is further responsible for the generation of THz radiation. In 3rd segment of the paper, we have provided the THz wave dynamics to

calculate the normalized THz electric field. The discussion of results and conclusion has been provided in the last segment of this paper.

II. Evaluation of nonlinear current density

Consider a vertical array of single-walled anharmonic CNTs nested in dielectric surface (glass) as shown in Fig. 1. To magnetize these CNTs, a static magnetic field is applied along the y-direction transverse to the direction of laser propagation (z-direction) and the longitudinal axis of CNTs (x-direction). The microwave plasma-enhanced chemical vapor deposition synthesizing (MPECVD) technique can be engineered to obtain the deformed VA-CNTs [17-20]. The CNTs obtained by the above mentioned technique are known as rippled VA-CNTs. These are very softer and more prone to ripples as compared to crystalline CNTs obtained by other available methods like the arc discharge technique [21-24]. In the above mentioned MPECVD technique, the dimensions and alignment of SWCNTs can be easily controlled [25, 26]. In this way, we can produce density ripples of desired period and size in the CNTs. The number of CNTs per unit area in the array is n_q and corresponding modulated density of CNTs is $n_q = n_{0q} e^{iqx}$, here the term n_{0q} is the amplitude and q is the wavenumber of density ripples produced in CNTs. The free electron density of each CNT is n_0 . Each CNT is characterized by the inner radius a , outer radius b , and length L . Each SWCNT is normally shaped as a hollow cylinder of compact dimensions to determine the electrical conductivity [27]. As far as we are concerned with the response of SWCNTs to the transverse electric and magnetic fields of the laser beam, these nanotubes as solid cylindrical tubes [28, 29]. The amplitude modulated Gaussian laser beam of angular frequency ω and wavenumber k , having non-uniform intensity distribution propagates through VA-CNTs with electric field profile

$$\vec{E} = \hat{x}E_0[1 + \mu \cos \Omega(t - z/c)]e^{-i(\omega t - kz)}, \quad (1)$$

where μ is the modulation depth, Ω is the modulation frequency in the THz range, c is the speed of light and $\hat{x}$ is the unit vector. The intensity profile of the incident Gaussian laser beam is represented by the relation $E_0^2 = E_{00}^2 e^{-x^2/r_0^2}$, here r_0 is initial beam radius.

Fig. 1 Schematic representation of THz radiation generation from a rectangular array of horizontally aligned hollow CNTs nested in the glass plates under the effect of external magnetic and electric field

When the Gaussian laser beam interacts with the electrons of CNTs, provides oscillatory velocity to the electrons of CNTs. This oscillatory velocity of the electrons of CNTs is given as $\vec{v} = e\vec{E}/m i\omega$, here, e and m denote the electronic charge and mass respectively. The incident Gaussian beam also exerts a ponderomotive force on the electrons of CNTs as $\vec{F}_{PM} = -e\vec{\nabla}\phi_{PM}$, where ϕ_{PM} is the nonlinear ponderomotive potential. This nonlinear ponderomotive potential can be calculated by using the relation given $\phi_{PM} = (-mvv^*/2e)$

$$\phi_{PM} = \frac{-e}{4m\omega^2} E_{00}^2 \left[1 + 2\mu \cos \Omega \left(t - \frac{z}{c} \right) \right] e^{-x^2/r_0^2} . \quad (2)$$

By using the above equation (2), one can calculate x and z components of the nonlinear ponderomotive force in the exponential form. These components are acting perpendicular to the direction of the applied magnetic field (y-direction). These components are responsible for THz generation at resonance.

$$F_x = \frac{\mu e^2 E_{00}^2 x}{2m\omega^2 r_0^2} e^{-x^2/r_0^2} e^{-i\Omega(t-z/c)}. \quad (3)$$

$$F_z = \frac{-i\mu e^2 E_{00}^2 \Omega}{4m\omega^2 c} e^{-x^2/r_0^2} e^{-i\Omega(t-z/c)}. \quad (4)$$

The x-component of the ponderomotive force F_x is responsible for the oscillatory motion of the electrons of CNTs in a direction transverse to the direction of propagation of the laser, at the modulation frequency. This results in the shifting of the electron cylinder by the displacement $\vec{\Delta}$ from the ion cylinder along the x-direction as shown in Fig. 2. The overlapped region of the electron cylinder and the ion cylinder forms the space-charge field. The space charge field is not uniform. Hence, the restoration force experienced by the various electrons of the CNTs is not the same which further induces anharmonicity in the system of CNTs. Because of this CNTs become anharmonic. The space-charge electric field produced by ion and electron cylinders is represented by $\vec{E}_+$ and $\vec{E}_-$ respectively. Thus the net electric field at the point (r, ϕ, z) can be written as $\vec{E} = \vec{E}_+ + \vec{E}_-$.

$$\vec{E} = \frac{n_0 e (r^2 - a^2)}{2\epsilon} \frac{\vec{r}}{r^2} - \frac{n_0 e}{2\epsilon_0} \left[|\vec{r} - \vec{\Delta}|^2 - a^2 \right] \left[\frac{\vec{r} - \vec{\Delta}}{|\vec{r} - \vec{\Delta}|^2} \right]. \quad (5)$$

where, $\epsilon = \epsilon_0 \epsilon_r$ is known as the electric permittivity of the medium. As explained above, the x-component of the ponderomotive force F_x is responsible for the oscillatory motion of the electrons of CNTs along the x-direction, thus the expression for the corresponding x-component of the space-charge electric field can be derived from Eq. (5).

$$E_x = \frac{n_0 e}{2\epsilon} \left[\left(1 + \frac{a^2}{r^2}\right) \Delta_x + \left(\frac{5\cos\varphi}{r} + \frac{4\cos^2\varphi}{r} - \frac{(r^2-a^2)}{r^3} \cos\varphi\right) \Delta_x^2 \right]. \quad (6)$$

The restoring force for the electrons of CNTs along can be obtained by using the relation, $F_{Rx} = -eE_x$,

$$F_{Rx} = \frac{-n_0 e^2}{2\epsilon} \left[\left(1 + \frac{a^2}{r^2}\right) \Delta_x + \left(\frac{5\cos\varphi}{r} + \frac{4\cos^2\varphi}{r} - \frac{(r^2-a^2)}{r^3} \cos\varphi\right) \Delta_x^2 \right]. \quad (7) \quad \text{This}$$

restoration force is not the same for all the electrons of CNTs instead some of the electrons of CNTs experience a weak restoration force, whereas others experience a strong restoration force. This nonlinear behavior of restoration force is responsible for the anharmonicity in the CNTs. Hence, we need to calculate the φ (average) and the r (average) of restoration force to obtain it's linear (F_{LRx}) and nonlinear components(F_{NLRx}). These linear and nonlinear components of the restoration force are given as

$$\langle F_{LRx} \rangle = -\frac{n_0 e^2}{2\epsilon} \frac{\int_a^b \left(1 + \frac{a^2}{r^2}\right) 2\pi r \cdot dr}{\int_a^b 2\pi r \cdot dr} \Delta_x \quad \text{and} \quad \langle F_{NLRx} \rangle = -\frac{n_0 e^2}{2\epsilon} \left(\frac{1}{2\pi}\right) \int_0^{2\pi} \left(\frac{5\cos\varphi}{r} + \frac{4\cos^2\varphi}{r} - \frac{(r^2-a^2)}{r^3} \cos\varphi\right) d\varphi \Delta_x^2.$$

By using standard integrals, the above linear and nonlinear components can be simplified to get net average restoration force

$$\langle F_{LRx} \rangle + \langle F_{NLRx} \rangle = \frac{-m\omega_p^2}{2\epsilon_r} \Delta_x [1 + \beta + \alpha \Delta_x], \quad (8)$$

where, $\omega_p = [n_0 e^2 / m\epsilon_0]^{1/2}$ is the plasma frequency. The terms α and β are known as nonlinear anharmonic and characteristic parameters. The values of both parameters depend on the dimensions of CNTs. Both nonlinear parameters are represented by the relations $\beta = 2 \log_e(b/a) / (b^2/a^2 - 1)$ and $\alpha = 4/(b+a)$. Terms α and β are also responsible for nonlinear mixing in the response of the electrons of CNTs in the array.

As the static magnetic field $\vec{B}$ is applied along the y-axis, therefore, magnetic force can be resolved into their x and z components, $F_{Bx} = -ev_z B/c$ and $F_{Bz} = ev_x B/c$ respectively. Under the influence of electric fields of the lasers, an external static magnetic field $\vec{B}$ and space charge electric field, the displacement of electrons in CNTs can be controlled by the following set of equations

$$\frac{d^2\Delta_x}{dt^2} + \frac{\omega_p^2}{2\epsilon_r}(1 + \beta + \alpha\Delta_x)\Delta_x + \frac{F_{Bx}}{m} + \nu \frac{d\Delta_x}{dt} = \frac{-eE_x}{m}, \quad (9)$$

$$\frac{d^2\Delta_z}{dt^2} + \frac{F_{Bz}}{m} + \nu \frac{d\Delta_z}{dt} = \frac{-eE_z}{m}, \quad (10)$$

where, ν represents electron-neutral collision frequency, which is lesser than ω .

Fig. 2 shifting of the electron cylinder by displacement $\vec{\Delta}$ concerning the ion cylinder along the x-direction

On solving the equations (9) and (10), we obtain the x and z components for the displacement of the electrons of CNTs in the array

$$\Delta_x = \frac{-[(1+i\nu/\Omega)F_x + (i\omega_c/\Omega)F_z]}{m(1+i\nu/\Omega)\Omega^2[1-\omega_p^2(1+\beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1+i\nu/\Omega) + i\nu/\Omega]}, \quad (11)$$

$$\Delta_z = \frac{-[(1-\omega_p^2(1+\beta))/2\epsilon_r\Omega^2 + i\nu/\Omega]F_z - (i\omega_c/\Omega)F_x}{m(1+i\nu/\Omega)\Omega^2[1-\omega_p^2(1+\beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1+i\nu/\Omega) + i\nu/\Omega]}, \quad (12)$$

where, $\omega_c = eB/m$ is known as cyclotron frequency of the electrons in CNTs.

With the help of the above equations (11) & (12) and by using the fundamental relation of the velocity $v = d\Delta/dt$ we can determine the corresponding nonlinear velocity components of the electrons of CNTs.

$$V_x = \frac{i[(1+i\nu/\Omega)F_x + (i\omega_c/\Omega)F_z]}{m(1+i\nu/\Omega)\Omega[1-\omega_p^2(1+\beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1+i\nu/\Omega) + i\nu/\Omega]}. \quad (13)$$

$$V_z = \frac{i[(1-\omega_p^2(1+\beta))/2\epsilon_r\Omega^2 + i\nu/\Omega]F_z - (i\omega_c/\Omega)F_x}{m(1+i\nu/\Omega)\Omega[1-\omega_p^2(1+\beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1+i\nu/\Omega) + i\nu/\Omega]}. \quad (14)$$

One can use the relation $J_\omega^{NL} = -e n_0 v$ to calculate the nonlinear current density of the electrons of CNTs. The value of nonlinear current density is non-zero over the cross-sectional area of CNTs in the array. At the same time, its value is zero for the area lying in between the CNTs. Therefore we have calculated the average nonlinear current density of the array of CNTs by using the equation (3) & (4) and the relation $J_{av.\omega}^{NL} = -e n_0 v (n_q^* \pi(b^2 - a^2))$.

$$J_{av.\omega}^{NL} = \frac{-in_0 n_{0q} \pi(b^2 - a^2) \mu e^3 E_{00}^2 [x(1+i\nu/\Omega)/r_0^2 + \Omega/2c]}{2m^2 \Omega \omega^2 (1+i\nu/\Omega) [1-\omega_p^2(1+\beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1+i\nu/\Omega) + i\nu/\Omega]} e^{-x^2/r_0^2} e^{-i(\Omega t - k'z)} \quad (15)$$

$$J_{av.\Omega z}^{NL} = \frac{-n_0 n_{0q} \pi(b^2 - a^2) \mu e^3 E_{00}^2 [\Omega(1-\omega_p^2(1+\beta))/2\epsilon_r\Omega^2 + i\nu/\Omega] / + x\omega c/\omega r_0^2]}{2m^2 \Omega \omega^2 (1+i\nu/\omega) [1-\omega_p^2(1+\beta)/2\epsilon_r\omega^2 - \omega_c^2/\omega^2(1+i\nu/\omega) + i\nu/\omega]} e^{-x^2/r_0^2} e^{-i(\Omega t - k'z)} \quad (16)$$

The above nonlinear current density components represented by the equations (15) and (16) oscillates at frequency Ω and wavenumber $k' = \Omega/c - q$. This is different from the nonlinear ponderomotive force. However, with the ripples produced in CNTs have wave number q , there occurs matching of phase.

Under this condition, resonant excitation of THz radiation can be realized. The nonlinear current density terms make a significant contribution to the THz field (E_{TH}) and one can observe this contribution from the THz wave propagation equation.

III. THz wave dynamics

The wave equation is derived by using Maxwell's equations and describes the propagation of terahertz waves through the array of VA-CNTs.

$$-\nabla^2 \vec{E}_{\text{TH}} + \vec{\nabla}(\vec{\nabla} \cdot \vec{E}_{\text{TH}}) = \frac{-4\pi i \Omega}{c^2} \vec{J}_{\text{av}, \Omega}^{\text{NL}} + \frac{\Omega^2}{c^2} \vec{\epsilon} \vec{E}_{\text{TH}}. \quad (17)$$

In the presence of an external static magnetic field applied along the y-axis in the collisional plasma of CNTs, the electric permittivity assumes the form of an anisotropic tensor.

$$\vec{\epsilon} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}. \quad (18)$$

The components of this anisotropic dielectric tensor have the following values

$$\epsilon_{yz} = \epsilon_{zy} = \epsilon_{yx} = \epsilon_{xy} = 0, \quad \epsilon_{yy} = 1 - \omega_p^2 / i\Omega(\nu - i\Omega), \quad \epsilon_{xx} = \epsilon_{zz} = 1 - \omega_p^2(\nu - i\Omega) / i\Omega[(\nu - i\Omega)^2 + \omega_c^2]$$

$$\text{and } \epsilon_{zx} = -\epsilon_{xz} = -i\omega_c \omega_p^2 / \Omega[(\nu - i\Omega)^2 + \omega_c^2].$$

With the use of the above components of dielectric tensor, Eq. (17) can be modified as:

$$\frac{\Omega^2}{c^2} \epsilon_{zz} E_z - \frac{\Omega^2}{c^2} \epsilon_{zx} E_x = \frac{4\pi i \Omega}{c^2} J_{\text{av}, \Omega z}^{\text{NL}}. \quad (19)$$

$$-2ik' \frac{\partial E_x}{\partial z} + \left[k'^2 - \frac{\Omega^2}{c^2} \left(\epsilon_{xx} + \frac{\epsilon_{zx}^2}{\epsilon_{zz}} \right) \right] E_x = \frac{4\pi i \Omega}{c^2} J_{\text{av}, \omega x}^{\text{NL}} + \frac{\epsilon_{zx}}{\epsilon_{zz}} J_{\text{av}, \Omega z}^{\text{NL}}. \quad (20)$$

By using the phase-matching condition one can write

$$k'^2 - \frac{\Omega^2}{c^2} \left(\epsilon_{xx} + \frac{\epsilon_{zx}^2}{\epsilon_{zz}} \right) = 0. \quad (21)$$

With the help of the above equations (20) and (21) we can find the x-component of THz electric field

$$E_x = -\frac{-2\pi\Omega}{k'c^2} \left[J_{av,\Omega x}^{NL} + \frac{\epsilon_{zx}}{\epsilon_{zz}} J_{av,\Omega z}^{NL} \right] x. \quad (22)$$

One can simplify the above equation of THz electric field by substituting the values of $J_{av,\Omega x}^{NL}$ and

$J_{av,\Omega z}^{NL}$ from equations (15) and (16), to get normalized THz electric field expression

$$\begin{aligned} \frac{eE_x}{m\omega c} = \frac{\pi\mu}{\epsilon_r} [n_q \pi (b^2 - a^2)] \left(\frac{\Omega}{k'c} \right) \left(\frac{n_{0q}}{n_q} \right) \left(\frac{eE_{00}}{m\omega c} \right)^2 & \left[(1 + i\nu/\Omega) \left[1 + \omega_p^2(1 + \beta)/2\epsilon_r\Omega^2 - \omega_c^2/\Omega^2(1 + i\nu/\Omega) + i\nu/\Omega \right] \right]^{-1} \left[\frac{ix^2}{r_0^2} \left(1 + \right. \right. \\ & \left. \left. i\nu/\Omega \right) - \frac{(\omega_c^2/\Omega^2)(\omega_p^2/\Omega^2)}{(\omega_c^2/\Omega^2) - (1 + i\nu/\Omega)^2 + (\omega_p^2/\Omega^2)(1 + i\nu/\Omega)} \right] + \frac{ix\omega_c}{2c} \left(1 - \frac{(1 + \omega_p^2(1 + \beta)/2\epsilon_r\Omega^2 + i\nu/\Omega)\omega_p^2/\Omega^2}{(\omega_c^2/\Omega^2) - (1 + i\nu/\Omega)^2 + (\omega_p^2/\Omega^2)(1 + i\nu/\Omega)} \right) \right] e^{-x^2/r_0^2}. \end{aligned} \quad (23)$$

IV. Results & Discussion

To perform numerical calculations, we have used a carbon dioxide laser beam with the following specified parameters. The angular frequency and wavelength of the laser beam is $\omega = 1.78 \times 10^{14}$ rad/s and $\lambda = 10.5 \mu\text{m}$ respectively. The intensity of the laser beam is $I = 1.4 \times 10^{14}$ W/cm² with initial beam radius $r_0 = 40.0 \mu\text{m}$. For this laser beam, the value of $(eE_{00}/m\omega c)$ is of the order of 0.06. The length of each CNT in the array is $L = 1.0 \mu\text{m}$. In the array, each constituent CNT has an inner radius $a = 20$ nm and outer radius $b = 40$ nm. The inter-tube separation in the array of VA-CNTs is $d = 20$ nm. Corresponding to these specific dimensions of CNTs, the characteristic parameter β has a value of 0.4631. The relative permittivity of the glass substrate on which CNTs are grown is 3.5. The externally applied transverse static magnetic field lies in the range of 110 kG to 330 kG. In figure 3, we have plotted normalized THz field amplitude as a function of normalized THz frequency for modulation

index $\mu = 0.1$, the inner radius of CNT $a = 20$ nm, the outer radius of CNT $b = 40$ nm, and inter-tube separation $d = 20$ nm at various values of external static magnetic field $B = 110$ kG, 220 kG, and 330 kG. From this figure, it is evident that normalized THz amplitude increases with the increase of normalized THz frequency and reaches its peak value. If we still increase the normalized THz frequency, then normalized THz amplitude shows a decrease in its value from its peak value. The normalized THz amplitude attains its peak value at a particular normalized THz frequency, where surface plasmon resonance occurs. This resonance frequency condition is given by the relation $\omega = \omega_p[(1 + \beta)/2\epsilon_r + \omega_\alpha^2]^{1/2}$, where $\omega_\alpha = \omega_c/\omega_p$ is the ratio of cyclotron frequency to the plasma frequency of the electrons of CNTs. If one moves away from this resonance condition either on the left side or right side, then normalized THz amplitude shows a sharp decrease. This is because, at the resonance frequency, absorption of the laser beam by the CNTs becomes maximum. The external static magnetic field plays a key role in the enhancement of the normalized THz amplitude by increasing the nonlinearities in the array of VA-CNTs. Jain *et al.* [30] presented similar results, in the THz generation by the array of CNTs embedded on the metal surface under the effect of the external magnetic field. The surface plasmon resonance condition as mentioned above depends upon the value of the externally applied static magnetic field. This dependence shows that the surface plasmon resonance point slips towards the right of the normalized frequency with the increase of the static magnetic field.

Fig. 3 Variation of normalized THz field amplitude with normalized THz frequency at various values of external static magnetic field $B = 110$ kG, 220 kG and 330 kG for $\beta = 0.4631$, $a = 20$ nm, $b = 40$ nm, and $\mu = 0.1$.

To analyze the impact of dimensions of CNTs, we have plotted normalized THz field amplitude as a function of normalized THz frequency for modulation index $\mu = 0.1$, the inner radius of CNT $a = 20$ nm, the outer radius of CNT $b = 40$ nm, 30 nm, 25 nm, and inter-tube separation $d = 20$ nm at the optimized value of external static magnetic field $B = 330$ kG as shown in figure 4. Corresponding to the above values, we have three sets of (a, b) : $(20$ nm, 40 nm), $(20$ nm, 30 nm), and $(20$ nm, 25 nm) with characteristic parameters $\beta = 0.4631, 0.6483$, and 0.7928 respectively. From the figure, it is evident that the normalized THz amplitude increases with the decrease in the value of characteristic parameter β . Out of three sets of (a, b) , the THz amplitude is maximum for $(20$ nm, 40 nm) as compared to the other two sets. With the decrease in the value of the characteristic parameter, there occurs an increase in the size of CNTs. Larger size of CNTs means more absorption of the laser beam by the CNTs and hence results in the increase in the nonlinearities of the array of VA-CNTs. In the study of THz radiation by CNTs, Nemilentsau *et al.* [31] have explained the role played by external magnetic field and radii of

CNTs for the efficient generation. Our results are showing the dependence of external magnetic field and radii of CNTs as presented in the figures 3 & 4. Our results are in consistent with the results presented by Nemilentsau *et al.* [31]. Watanabe *et al.* [32] have also shown THz electric field variation with radii of CNTs in their experimental work. From figure 4, it is clear that in each curve normalized THz amplitude has its peak value at the surface plasmon resonance point. Moreover, as the surface plasmon resonance condition depends upon the characteristic parameter β , therefore with the increase in the value of β , the surface plasmon resonance point slips towards the right side of the normalized THz frequency.

Fig. 4 Variation of normalized THz field amplitude with normalized THz frequency for different values of characteristic parameter β at the optimized value of static magnetic field $B = 330$ kG.

In figure 5, we have shown a variation of normalized THz field amplitude with normalized THz frequency at various values of inter-tube separation $d = 20$ nm, 25 nm, and 30 nm. The values of other parameters are kept the same as that of figure 3. From figure 5, one can observe a fall in the normalized THz amplitude with the increase in the inter-tube separation distance of CNTs in the array. The decrease in

the inter-tube separation is responsible for the increase in the number density of CNTs in the array. This increase in the number density of CNTs results in the increase of the nonlinearities of the array of VA-CNTs. Vij *et al.* [7] has shown the similar results in their theoretical study of THz generation by using CNTs under the effect of a wiggler magnetic field. Also, the surface plasmon resonance condition $\omega = \omega_p[(1 + \beta)/2\epsilon_r + \omega_\alpha^2]^{1/2}$ is independent of the inter-tube separation distance d , therefore in this graph surface plasmon resonance point remains same for the three curves.

Fig. 5 Variation of normalized THz field amplitude with normalized THz frequency for different values of inter-tube separation at the optimized value of static magnetic field $B = 330$ kG and characteristic parameter $\beta = 0.4631$.

In figure 6, we have shown the variation of normalized THz amplitude with the normalized plasma frequency at different values of modulation indices of Gaussian laser beam $\mu = 0.05, 0.08$, and 0.1 and static magnetic field $B = 110$ kG, 220 kG, 330 kG, where as the other parameters are kept same as

Fig. 6 Variation of normalized THz field amplitude with normalized plasma frequency for different values of modulation index and static magnetic field for the characteristic parameter $\beta = 0.4631$.

that of figure 3. This graph shows that normalized THz amplitude increases with the increase of normalized plasma frequency and becomes maximum at the surface plasmon resonance point. For all the curves shown in figure 6, the surface plasmon resonance point shifts towards a higher value. This is because of the increase in the value of the static magnetic field as explained above. Figure 6, also explains the importance of modulation index, for the enhancement of normalized THz amplitude. Such dependence has also been explained by Kumar *et al.* [33] in their theoretical study THz generation by amplitude-modulated laser beam in ripple density plasma. So, the output of THz wave generation can be tuned by using appropriate values of the applied static magnetic field strength and modulation index.

Fig. 7 Variation of normalized THz field amplitude with normalized THz frequency for different values of normalized ripple amplitude at the optimized value of the static magnetic field $B = 330$ kG and the characteristic parameter $\beta = 0.4631$.

To study the significance of density ripples, we plot the graph between normalized THz amplitude and normalized THz frequency for different values of normalized ripple amplitude ($n = n_{0q}/n_0$) 0.1, 0.2, and 0.3 at the optimized value of the static magnetic field as shown in figure 7. The values of other parameters are kept the same as that of figure 3. From the figure 7, it is evident that there is a significant enhancement in the normalized THz amplitude with the increasing normalized ripple amplitude. The normalized THz amplitude becomes maximum at the optimized value of static magnetic field $B = 330$ kG and normalized ripple amplitude $n = 0.3$. The role of normalized ripple amplitude in enhancement of the normalized THz amplitude is quite reasonable as more and more electrons of

CNTs become responsible for the generation of nonlinear current. A similar result has been observed by Malik *et al.* [34] in their theoretical work of generating the THz radiation by beating the Gaussian lasers in the presence of the static magnetic field.

Figure 8 shows the variation of normalized THz amplitude with normalized collision frequency for different values of static magnetic field $B = 110$ kG, 220 kG, and 330 kG. The values of other parameters are kept the same as that of figure 3. This graph unveils the adverse impact of normalized collision frequency on the normalized THz field amplitude. From this graph, it is evident that collisions between electrons and neutrals reduce the normalized THz amplitude. This is because the collisions between the electrons present in the CNTs result in the reduction of ponderomotive force and nonlinear current. However by applying a suitable magnetic field, one can overcome this loss up to some extent.

Fig. 8 Variation of normalized THz amplitude with normalized collision frequency for different optimized values of static magnetic field $B = 110$ kG, 220 kG and 330 kG at characteristic parameter $\beta = 0.4631$.

Similar results have been shown by Singh and Malik [35] in their theoretical study of enhanced THz generation in magnetized collisional plasma.

Conclusion

The Gaussian laser beam makes a nonlinear interaction with VA-CNTs to resonantly excite THz radiation at the modulation frequency under the influence of an external magnetic field. This is responsible for the production of the nonlinear current which results in THz generation. The anharmonicity in the electrons of VA-CNTs helps in broadening the surface resonance peak. This also results in the further enhancement of THz generation. The surface plasmon resonance condition $\omega = \omega_p[(1 + \beta)/2\epsilon_r + \omega_\alpha^2]^{1/2}$ can be altered by varying the dimensions of CNTs and the strength of the applied static magnetic field.

Author Contribution

Sandeep Kumar: derivation, methodology and analytical modeling

Shivani Vij: graph plotting and writing

Niti Kant : numerical analysis and result discussion

Vishal Thakur: supervision, reviewing and editing

Data Availability

The data that supports the findings of this study are available inside the manuscript

Declarations

Consent to participate

Not applicable

Consent for Publication

Not applicable

Conflict of Interest

The authors declare no competing interest

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