

Determination of Equivalent Mohr-Coulomb Criterion from Generalised Hoek-Brown Criterion

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Research Article

Keywords: equivalent Mohr-Coulomb parameter, Hoek-Brown criterion, least squares method, tensile strength

Posted Date: November 15th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1073021/v1>

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1 **Determination of Equivalent Mohr-Coulomb Criterion from**
2 **Generalised Hoek-Brown Criterion**

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Abstract

A new analytical solution is presented for determining equivalent Mohr-Coulomb (MC) shear strength parameters over an arbitrary interval of minor principal stress σ_3 from the generalised Hoek-Brown (HB) criterion using least squares method. Comparison with several published examples demonstrates that the proposed solution had a capacity to accurately determine equivalent MC parameters over a given interval of σ_3 , as well as instantaneous MC parameters by using a very small interval of σ_3 . EMC parameters depended heavily on the interval of σ_3 , which highlighted the importance of intervals of σ_3 . A calculation case shows that the equivalent internal friction angle and cohesion over the interval of σ_3 from tension cut-off $\sigma_{\text{cut-off}}$ to maximum minor principal stress $\sigma_{3\text{max}}$ were approximately 12% smaller and 10.3% larger than those over an interval from tensile strength to $\sigma_{3\text{max}}$, respectively. The proposed solution offers great flexibility for the application of the HB criterion with existing methods based on the MC criterion for rock engineering practice.

Keywords: equivalent Mohr-Coulomb parameter; Hoek-Brown criterion; least squares method; tensile strength

43 **1 Introduction**

44 The linear Mohr-Coulomb (MC) and nonlinear Hoek-Brown (HB)
45 criteria are the most widely used failure criteria in slope stability analysis,
46 and most of the currently used geotechnical codes are based on the
47 application of the MC parameters. Consequently, it is necessary to convert
48 the HB criterion to the MC criterion without significantly affecting the
49 results of analysis. However, the HB criterion is expressed major and minor
50 principal stresses σ_1 and σ_3 , and its conversion to the relationship between
51 shear strength τ_f and normal stress σ_n is not straightforward.

52 Generally, the HB failure envelope is replaced by a tangent line and
53 instantaneous MC (IMC) parameters are determined by locating the
54 tangent of the HB envelope at a specified σ_n . The nonlinearity of the HB
55 criterion causes IMC parameters to vary with σ_3 , consequently, the method
56 theoretically yields the most accurate MC parameters but only for a specific
57 stress state. An exact solution for intact rock was proposed by Hoek (1983),
58 and IMC parameters can be obtained numerically for rock masses (Kumar
59 1998; Carranza-Torres 2004; Priest 2005; Yang and Yin 2006; Shen et al.
60 2012a; Lee and Pietruszczak 2017). Alternatively, another method provides
61 equivalent MC (EMC) parameters by best fitting an averaged line
62 equivalent to the HB criterion over an interval of σ_3 . EMC parameters are
63 more appropriate to evaluate the overall strength of rock mass, especially
64 for preliminary design purposes. EMC parameters were determined by

65 linear regression analysis for intact rock (Hoek and Brown, 1997) and rock
66 masses (Hoek et al., 2002). Note that the solution proposed by Hoek et al.
67 (2002) is valid only for the artificial interval from biaxial tensile strength
68 σ_{t2} to maximum minor principal stress σ_{3max} . No accurate analytical
69 solution for determining EMC parameters over an arbitrary interval of σ_3
70 is available yet.

71 A new analytical solution was proposed to determine EMC parameters
72 over an arbitrary interval of σ_3 using least squares method. The reliability
73 and performance of the proposed method were verified based on several
74 published examples. The importance of the intervals of σ_3 was highlighted
75 by comparing EMC parameters over different intervals of σ_3 .

76

77 **2 Overview of the Equivalent Hoek-Brown and Mohr-Coulomb** 78 **criteria**

79 The HB failure criterion for intact and fractured rock masses was first
80 proposed in 1980 to describe a nonlinear empirical relationship between σ_1
81 and σ_3 . Since then, it has been widely accepted by the international rock
82 mechanics, and updated several times in response to experience gained
83 with its applications (Hoek and Brown 2019). The HB criterion in the latest
84 version, often referred to as the generalised Hoek-Brown criterion, is
85 expressed as (Hoek and Brown 1997):

$$86 \quad \sigma_1 = \sigma_3 + \sigma_{ci} \left(\frac{m_b \sigma_3}{\sigma_{ci}} + s \right)^a \quad (1)$$

87 where, σ_1 and σ_3 are the major and minor principal stresses, and σ_{ci} is the
 88 uniaxial compressive strength of the intact rock mass. m_b , s , and a are the
 89 constants of the HB criterion. Hoek et al. (2002) proposed the following
 90 revised empirical expressions for m_b , s , and a .

$$91 \quad m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (2)$$

$$92 \quad s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (3)$$

$$93 \quad a = \frac{1}{2} + \frac{1}{6} \left[\exp\left(-\frac{GSI}{15}\right) - \exp\left(-\frac{20}{3}\right) \right] \quad (4)$$

94 where GSI, m_i , and D is the Geological Strength Index, rock type-
 95 dependent parameter, and the disturbance factor, respectively.

96 The equations provided by Hoek et al. (2002) for determining EMC
 97 parameters over an artificial interval $[\sigma_{t2}, \sigma_{3\max}]$ are:

$$98 \quad \varphi = \arcsin \left(\frac{6am_b \left(\frac{m_b \sigma_{3\max}}{\sigma_{ci}} + s \right)^{a-1}}{2(2+a)(1+a) + 6am_b \left(\frac{m_b \sigma_{3\max}}{\sigma_{ci}} + s \right)^{a-1}} \right) \quad (5)$$

$$99 \quad c = \frac{\sigma_{ci} \left[(1-a) \frac{m_b \sigma_{3\max}}{\sigma_{ci}} + (1+2a)s \right] \left(\frac{m_b \sigma_{3\max}}{\sigma_{ci}} + s \right)^{a-1}}{(2+a)(1+a) \sqrt{1 + \frac{6am_b \left(\frac{m_b \sigma_{3\max}}{\sigma_{ci}} + s \right)^{a-1}}{(2+a)(1+a)}}} \quad (6)$$

100 Note that the value of $\sigma_{3\max}$ has to be determined for each case. Hoek et
 101 al. (2002) suggested empirical formulae to determine the value of $\sigma_{3\max}$ for

102 deep tunnels and slopes, respectively.

103

104 **3 Proposed solution to determine EMC parameters**

105 A new analytical solution was formulated by least squares
106 approximation to determine EMC parameters over an arbitrary interval of
107 σ_3 , of which the lower limit is not necessary to be σ_{t2} . That is, for an
108 arbitrary interval of σ_3 defined by $[\sigma_{3a}, \sigma_{3b}]$, we search for a best linear
109 approximation expressed as the MC criterion of the generalised HB
110 criterion in the sense of least squares, as illustrated in **Fig. 1**.

111 The generalised HB is expressed as

$$112 \quad \sigma_{1HB} = \sigma_3 + \sigma_{ci} \left(\frac{m_b \sigma_3}{\sigma_{ci}} + s \right)^a \quad (7)$$

113 where σ_{1HB} is the major principal stress of the HB criterion.

114 The equivalent MC criterion can be expressed using σ_1 and σ_3

$$115 \quad \sigma_{1MC} = p + q\sigma_3 \quad (8)$$

116 where σ_{1MC} is the major principle stress of the equivalent MC criterion, and

$$117 \quad p = \frac{2c \cos \varphi}{1 - \sin \varphi} \quad (9)$$

$$118 \quad q = \frac{1 + \sin \varphi}{1 - \sin \varphi} \quad (10)$$

119 where φ is internal friction angle, and c is cohesion.

120 In the sense of least squares, choose p and q to minimize the integral

$$121 \quad Q = \int_{\sigma_{3a}}^{\sigma_{3b}} (\sigma_{1MC} - \sigma_{1HB})^2 d\sigma_3 \quad (11)$$

122 A minimum of Q gives

$$123 \quad p = \frac{4(\sigma_{3b}^2 + \sigma_{3b}\sigma_{3a} + \sigma_{3a}^2)S_1 - 6(\sigma_{3b} + \sigma_{3a})S_2}{(\sigma_{3b} - \sigma_{3a})^3} \quad (12)$$

$$124 \quad q = \frac{-6(\sigma_{3b} + \sigma_{3a})S_1 + 12S_2}{(\sigma_{3b} - \sigma_{3a})^3} \quad (13)$$

125 where,

$$126 \quad S_1 = \frac{1}{2}(\sigma_{3b}^2 - \sigma_{3a}^2) + \frac{\sigma_{ci}^2}{(1+a)m_b} \left[\left(\frac{m_b\sigma_{3b} + s}{\sigma_{ci}} \right)^{1+a} - \left(\frac{m_b\sigma_{3a} + s}{\sigma_{ci}} \right)^{1+a} \right] \quad (14)$$

$$127 \quad S_2 = \frac{1}{3}(\sigma_{3b}^3 - \sigma_{3a}^3) + \frac{\sigma_{ci}^3}{(2+a)(1+a)m_b^2} \left\{ \begin{array}{l} \left(\frac{m_b\sigma_{3b} + s}{\sigma_{ci}} \right)^{1+a} \left[(1+a)\frac{m_b\sigma_{3b} - s}{\sigma_{ci}} \right] \\ - \left(\frac{m_b\sigma_{3a} + s}{\sigma_{ci}} \right)^{1+a} \left[(1+a)\frac{m_b\sigma_{3a} - s}{\sigma_{ci}} \right] \end{array} \right\} \quad (15)$$

128 Using Eqs. (12) and (13), φ and c can be expressed as

$$129 \quad \varphi = \arcsin\left(\frac{q-1}{q+1}\right) \quad (16)$$

$$130 \quad c = \frac{p}{2\sqrt{q}} \quad (17)$$

131 Noted that, substituting the lower and upper limits $\sigma_{3a} = -s\sigma_{ci}/m_b$ and σ_{3b}
 132 $= \sigma_{3\max}$ into Eqs. (12) and (13), p and q can be obtained. After p and q
 133 obtained, using Eqs. (16) and (17), the expressions of φ and c exactly
 134 identical to Eqs. (5) and (6), can be obtained.

135 Although Hoek et al. (2002) declared that the fitting process involves
 136 balancing the areas above and below the MC envelope, the expressions of
 137 φ and c can also be obtained using least squares approximation. It should
 138 be pointed out that $-s\sigma_{ci}/m_b$ is biaxial tensile strength σ_{t2} because
 139 substituting $\sigma_3 = -s\sigma_{ci}/m_b$ into Eq. (1) gives $\sigma_1 = -s\sigma_{ci}/m_b$.

140

141 **4 Validation of the proposed solution**

142 **4.1 Example 1**

143 The first example is adopted from Hoek et al. (2002). This is a typical
144 example in which $\sigma_{ci} = 50000$ kPa, $GSI = 45$, and $m_i = 10$. For an
145 undisturbed in-situ rock mass ($D = 0$) surrounding a tunnel at a depth of
146 100 m, σ_{3max} was calculated by the formula suggested by Hoek et al. (2002)
147 for tunnels, assuming $\gamma = 27$ kN/m³, while the lower limit of σ_3 was
148 assumed to be $\sigma_{t2} = -s\sigma_{ci}/m_b$. For the rock mass with the same parameters
149 but in highly disturbed slope ($D = 1$) with 100 m height, σ_{3max} was
150 calculated by the formula suggested by Hoek et al. (2002) for slopes, while
151 the lower limit of σ_3 was also assumed equal to σ_{t2} . **Table 1**, an extract from
152 a basic Microsoft Excel spreadsheet, illustrates that the calculated EMC
153 parameters using the present solution agreed very well with those reported
154 by Hoek et al. (2002). This verified the reliability and performance of the
155 proposed solution.

156

157 **4.2 Example 2**

158 The proposed least squares solution can also be used to approximately
159 calculate IMC parameters using a small interval of σ_3 centered at the
160 specified σ_{30} . If σ_n was specified, a process of Kumar et al. (2020) should
161 be used to calculate σ_3 corresponding to a given σ_n .

162 The second example is chosen from the works of Hoek et al. (2002). The

163 associated parameters are as follows: $\sigma_{ci} = 30000$ kPa, $\sigma_n = 800$ kPa, GSI =
164 15, $D = 0.7$, and $m_i = 16$. At a given σ_n of 800 kPa, the specified σ_{30} of
165 480.5339 kPa was obtained using Newton-Raphson method. Thus, the
166 interval of σ_3 was assigned to $[0.999\sigma_{30}, 1.001\sigma_{30}]$. **Table 2** shows the IMC
167 parameters at the specified σ_n of 800 kPa, and the results obtained from this
168 study had a good consistence with those reported by others.

169 More examples published in the literature (e.g. Hoek and Brown 1997;
170 Priest 2005; Shen et al. 2012b) were also employed for comparison, which
171 is not presented here for space limit. The comparison demonstrates that
172 there was a very close agreement in the values of MC parameters with the
173 published results. This implies that the proposed solution had the capacity
174 to accurately determine EMC parameters for different rock types over a
175 wide range of stress levels, as well as IMC parameters at specified stress
176 level using a small interval of σ_3 .

177

178 **5. Discussions**

179 **5.1 EMC parameters over the interval from uniaxial tensile strength** 180 **to maximum minor principal stress**

181 As mentioned in Section 3, the method proposed by Hoek et al. (2002)
182 is valid only for the interval $[\sigma_{t2}, \sigma_{3max}]$. For practical slope stability
183 problems, uniaxial tensile strength σ_{t1} or tension cut-off $\sigma_{cut-off}$ is a more
184 realistic lower limit of σ_3 than σ_{t2} , although Hoek (1983) declared that σ_{t2}

185 is equal to σ_{t1} for brittle materials. Actually, the two tensile strength is not
186 necessarily equal to each other (Michalowski and Park 2020); this is
187 dependent on the curvature of initial segment of HB envelope.

188 Unfortunately, σ_{t1} cannot be represented by a convenient close-form
189 expression in Eq. (1); it can be obtained by solving the HB criterion under
190 the condition $\sigma_1 = 0$ using the Newton-Raphson method. The calculation
191 shows that a good initial guess for σ_{t1}/σ_{ci} is $0.999\sigma_{t2}/\sigma_{ci}$, and the Newton-
192 Raphson method can converge in 4-5 iterations with correct digits larger
193 than 8.

194 Example 1 was used to investigate the difference in the values between
195 σ_{t2} and σ_{t1} respected to HB constants. **Fig. 2** illustrates that the difference
196 in estimated values between σ_{t1} and σ_{t2} was remarkable at a small m_i . For
197 extremely small values of m_i , σ_{t2} was 10-20% larger than σ_{t1} . However, the
198 difference between the two became insignificantly at a large m_i . This
199 coincided with the conclusion drawn by Sari (2010). The EMC parameters
200 over the intervals $[\sigma_{t2}, \sigma_{3max}]$ and $[\sigma_{t1}, \sigma_{3max}]$ were also determined for
201 Example 1. Calculations demonstrate that the discrepancy in c and ϕ over
202 the intervals $[\sigma_{t2}, \sigma_{3max}]$ and $[\sigma_{t1}, \sigma_{3max}]$ was less than 2%, even the
203 maximum difference in values of σ_{t1} and σ_{t2} was up to 16.18% when GSI =
204 10, $m_i = 3$, and $D = 1$. The slight difference in EMC parameters over the
205 two intervals was attributed to the small values of σ_{t1} and σ_{t2} when GSI and
206 m_i was small; with the increasing GSI and m_i , the values of σ_{t1} and σ_{t2}

207 increased but the difference in the two values reduced and was negligible
208 in a practical sense.

209

210 **5.2 EMC parameters over the interval from tension cut-off to** 211 **maximum minor principal stress**

212 Triaxial extension test results of Ramsey and Chester (2004) were
213 reproduced in **Fig. 3**, combined with the HB envelope. The HB envelope
214 projects back in tensile regime to give an intercept of $\sigma_3 = \sigma_{t1} = -17.2$ MPa
215 for $\sigma_1 = 0$; the calculated $\sigma_{t2} = -17.5$ MPa; this does not correspond to the
216 tensile failure data which gives an average tensile strength $\sigma_3 = -7.75$ MPa.
217 In other words, the HB criterion has no provision for predicting the tensile
218 strength which was highlighted by the ‘tension cut-off’ (broken line in **Fig.**
219 **3**) (Hoek and Martin 2014). Hoek and Brown (2019) recommended that
220 the MC parameters, derived from the HB criterion, should not be used
221 without a tension cut-off. For practical slope stability problems, σ_{t1} or σ_{cut-}
222 $_{off}$ is a more realistic lower limit of σ_3 than σ_{t2} . For tunnel problems, zero
223 should be a better estimate for the lower limit of σ_3 .

224 The EMC parameters over different intervals of σ_3 were determined for
225 the rock mass tested by Ramsey and Chester (2004), for which the
226 associated parameters by back analysis are as follows: $\sigma_{ci} = 129300$ kPa,
227 $m_b = 7.37$, $s = 1$, and $a = 0.5$, respectively. **Table 3** lists the EMC parameters
228 over different intervals represented with tensile strength and slope height,

229 to which the corresponding value of $\sigma_{3\max}$ was calculated by using the
230 empirical formula given by Hoek et al. (2002). It is evident that the EMC
231 parameters for the interval of σ_3 from $\sigma_{\text{cut-off}}$ to $\sigma_{3\max}$, as well as those over
232 the interval $[0, \sigma_{3\max}]$, were markedly different from those over the interval
233 $[\sigma_{t2}, \sigma_{3\max}]$ and $[\sigma_{t1}, \sigma_{3\max}]$. In this case, φ over the intervals $[\sigma_{\text{cut-off}}, \sigma_{3\max}]$
234 and $[0, \sigma_{3\max}]$ was approximately 12% and 16.5% smaller than that over the
235 interval $[\sigma_{t2}, \sigma_{3\max}]$, respectively; c over the intervals $[\sigma_{\text{cut-off}}, \sigma_{3\max}]$ and $[0,$
236 $\sigma_{3\max}]$ was approximately 10.3% and 15.7% larger than that over the
237 interval $[\sigma_{t2}, \sigma_{3\max}]$, respectively. Thus, the practical interval of σ_3 should be
238 carefully determined when calculating EMC parameters.

239

240 **6 Conclusions**

241 The study provided a new analytical formulation of equivalent MC shear
242 strength parameters over an arbitrary interval of σ_3 for the HB criterion
243 based on least squares method. Comparison with published results
244 demonstrates that the proposed solution had the capacity to accurately
245 determine the EMC parameters over a broad range of values of GSI, D , m_i ,
246 and σ_{ci} . Some conclusions can be drawn as follows:

247 (1) The proposed solution was valid to determine EMC parameters over
248 an arbitrary interval of σ_3 . The solution for the EMC parameters provided
249 by Hoek et al. (2002) is a special case of the proposed solution in case that
250 the lower limit of σ_3 equals to the biaxial tensile strength σ_{t2} .

251 (2) The present solution can also be used to approximately calculate IMC
252 parameters at a specified σ_n using a very small interval of σ_3 . The
253 comparison demonstrates that there was a close agreement in the values of
254 IMC parameters published in the literature.

255 (3) The calculated EMC parameters heavily depended on the specified
256 interval of σ_3 . The discrepancy in φ and c over the intervals $[\sigma_{t2}, \sigma_{3\max}]$ and
257 $[\sigma_{t1}, \sigma_{3\max}]$ was negligible in a practical sense. A calculation case
258 demonstrates that the equivalent φ over the intervals $[\sigma_{\text{cut-off}}, \sigma_{3\max}]$ and $[0,$
259 $\sigma_{3\max}]$ was approximately 12% and 16.5% smaller than that over the
260 interval $[\sigma_{t2}, \sigma_{3\max}]$, respectively; the equivalent c over the intervals $[\sigma_{\text{cut-off}},$
261 $\sigma_{3\max}]$ and $[0, \sigma_{3\max}]$ was approximately 10.3% and 15.7% larger than that
262 over the interval $[\sigma_{t2}, \sigma_{3\max}]$, respectively.

263

264 **Acknowledgements**

265 This research was supported by the Second Tibetan Plateau Scientific
266 Expedition and Research (STEP) program (Grant No. 2019QZKK0905),
267 the National Natural Science Foundation of China (41402244, 41877226,
268 41877237).

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271

272

273 **Competing interests**

274 The authors declare that they have no known competing financial
275 interests or personal relationships that could have appeared to influence the
276 work reported in this note.

277

278 **List of Notations**

- 279 a Hoek-Brown material constant
- 280 c Cohesion of equivalent Mohr-Coulomb criterion (kPa)
- 281 D Disturbance factor
- 282 GSI Geological Strength Index of rock mass
- 283 H_s Height of slope (m)
- 284 H_t Depth of tunnel below surface (m)
- 285 m_b Hoek-Brown material constant of rock mass
- 286 m_i Hoek-Brown material constant of intact rock
- 287 p Auxiliary variable
- 288 q Auxiliary variable
- 289 Q Integral of squared residuals over an interval of minor principal stress
- 290 s Hoek-Brown material constant
- 291 S_1 Auxiliary variable
- 292 S_2 Auxiliary variable
- 293 φ Internal friction angle of equivalent Mohr-Coulomb criterion ($^\circ$)
- 294 σ_1 Major principal stress (kPa)
- 295 σ_{1HB} Major principal stress of Hoek-Brow criterion (kPa)
- 296 σ_{1MC} Major principal stress of equivalent Mohr-Coulomb criterion (kPa)
- 297 σ_3 Minor principal stress (kPa)
- 298 σ_{30} Specified minor principal stress (kPa)
- 299 σ_{3a} Lower limit of minor principal stress (kPa)

- 300 σ_{3b} Upper limit of minor principal stress (kPa)
- 301 σ_{3max} Maximum minor principal stress (kPa)
- 302 σ_{ci} Uniaxial compressive strength of intact rock (kPa)
- 303 σ_{cm} Rock mass strength (kPa)
- 304 $\sigma_{cut-off}$ Tension cut-off (kPa)
- 305 σ_n Normal stress (kPa)
- 306 σ_t Tensile strength (kPa)
- 307 σ_{t1} Uniaxial tensile strength (kPa)
- 308 σ_{t2} Biaxial tensile strength (kPa)
- 309 τ Shear strength (kPa)
- 310 γ Unit weight of rock mass (kN/m³)

311

312 **References:**

- 313 Carranza-Torres C (2004) Elasto-plastic solution of tunnel problems using
314 the generalised form of the Hoek-Brown failure criterion. *International*
315 *Journal of Rock Mechanics and Mining Science* 41: 629-639
- 316 Hoek E (1983) Strength of jointed rock masses. *Géotechnique* 33(3): 187-
317 223
- 318 Hoek E, Brown ET (1997) Practical estimates of rock mass strength.
319 *International Journal of Rock Mechanics and Mining Science* 34(8):
320 1165-1186
- 321 Hoek E, Brown ET (2019) The Hoek-Brown failure criterion and GSI –
322 2018 edition. *Journal of Rock Mechanics and Geotechnical Engineering*
323 11: 445-463
- 324 Hoek E, Carranza-Torres C, Corkum B (2002) Hoek-Brown failure
325 criterion -2002 Edition. In *Proceedings of 5th North American Rock*
326 *Mechanics Symposium and 17th Tunnelling Association of Canada*
327 *Conference* (Hammah R.; Bawden W.; Curran J.; Telesnicki M. (eds)).
328 Toronto: pp. 267-273
- 329 Hoek E, Martin CD (2014) Fracture initiation and propagation in intact
330 rock – A review. *Journal of Rock Mechanics and Geotechnical*
331 *Engineering* 6: 287-300
- 332 Kumar P (1998) Shear failure envelope of Hoek-Brown criterion for
333 rockmass. *Tunnelling and Underground Space Technology* 13(4): 453-

334 458

335 Kumar V, Burman A, Himanshu N, Barman B (2020) Solution of minor
336 principal stress for generalised Hoek-Brown rock material in two
337 dimension using Newton-Raphson method. *Geotechnical and*
338 *Geological Engineering* 38: 1817-1837

339 Lee YK, Pietruszczak S (2017) Analytical representation of Mohr failure
340 envelope approximating the generalized Hoek-Brown failure criterion.
341 *International Journal of Rock Mechanics and Mining Sciences* 100: 90-
342 99

343 Michalowski RL, Park D (2020) Stability assessment of slopes in rock
344 governed by the Hoek-Brown strength criterion. *International Journal*
345 *of Rock Mechanics and Mining Sciences* 127:104217

346 Priest (2005) Determination of shear strength and three-dimensional yield
347 strength for the Hoek-Brown criterion. *Rock Mechanics and Rock*
348 *Engineering* 38(4): 299-327

349 Ramsey J, Chester F (2004) Hybrid fracture and the transition from
350 extension fracture to shear fracture. *Nature* 428: 63-66

351 Sari M (2010) A simple approximation to estimate the Hoek-Brown
352 parameter ' m_i ' for intact rocks. In *ISRM International Symposium -*
353 *EUROCK 2010* (Zhao J.; Labiouse V.; Dudt JP.; Mathier JF. (eds)).
354 London: Taylor & Francis Group, pp. 169-172

355 Shen JY, Priest SD, Karakus M (2012a) Determination of Mohr-Coulomb

356 shear strength parameters from Generalized Hoek-Brown criterion for
357 slope stability analysis. *Rock Mechanics and Rock Engineering* 45: 123-
358 129

359 Shen JY, Karakus M, Xu CS (2012b) Direct expression for linearization of
360 shear strength envelopes given by the Generalized Hoek-Brown criterion
361 using genetic programming. *Computers and Geotechnics* 44: 139-146

362 Yang XL, Li L, Yin JH (2004) Seismic and static stability analysis for rock
363 slopes by a kinematical approach. *Géotechnique* 54(8): 543-549

364 Yang XL, Yin JH (2006) Linear Mohr-Coulomb strength parameters from
365 the non-linear Hoek-Brown rock masses. *International Journal of Non-
366 linear Mechanics* 41: 1000-1005

367

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370 (Single column, Color online only)

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373 (Single column, Color online only)

374 **Fig. 3** Combined plot of HB criterion with tension cut-off

375 (Single column, Color online only)

376 **The figures in this work were created by MSOffice.**

377

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380 Example 1

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382 for Example 2

383 **Table 3** Equivalent MC parameters over different intervals of σ_3
384 represented with slope height and tensile strength for the rock mass tested
385 by Ramsey and Chester (2004)

386 **The tables in this work were created by MSOffice.**

Figures

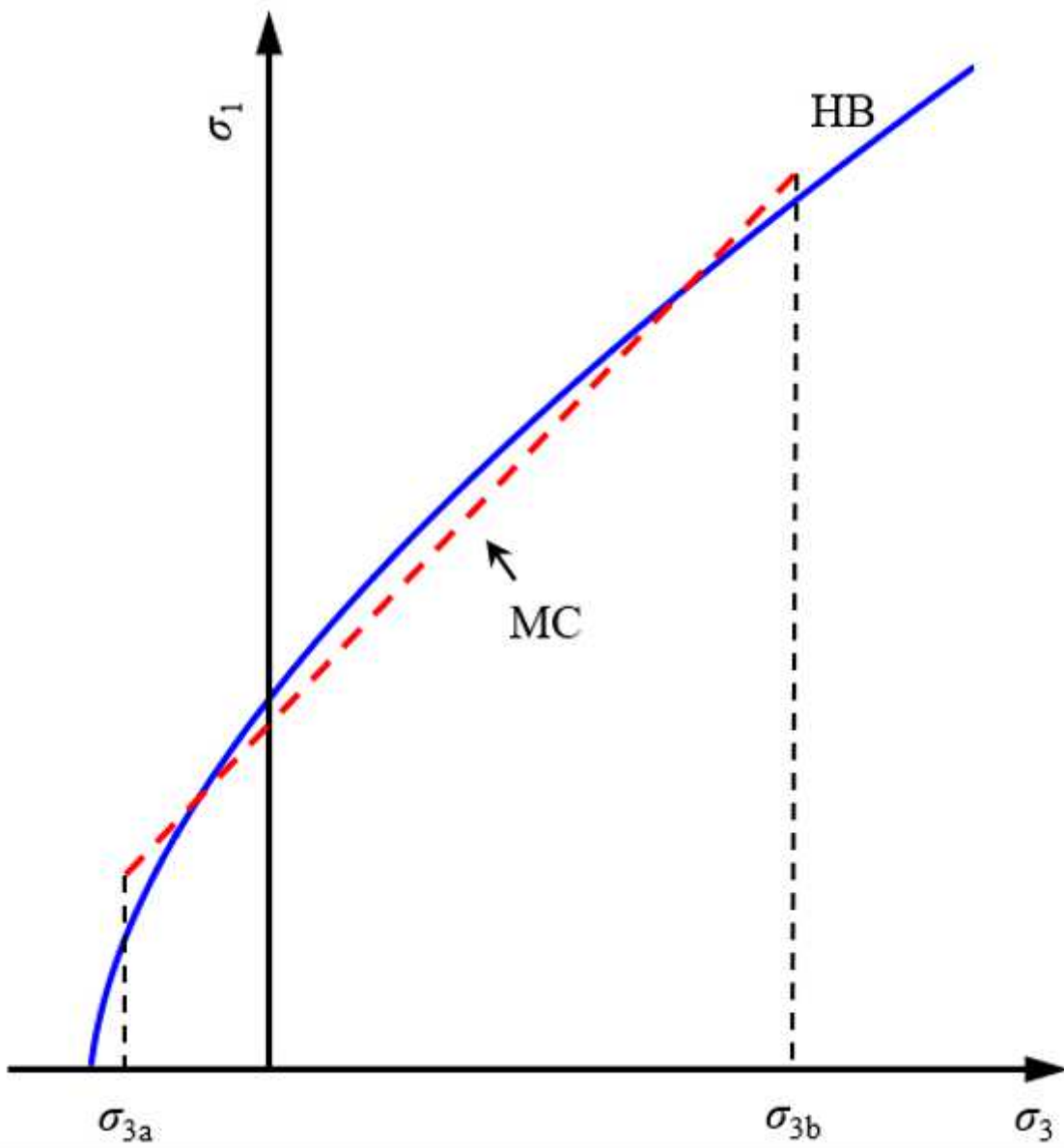


Figure 1

Equivalent MC criterion for HB criterion (Single column, Color online only)

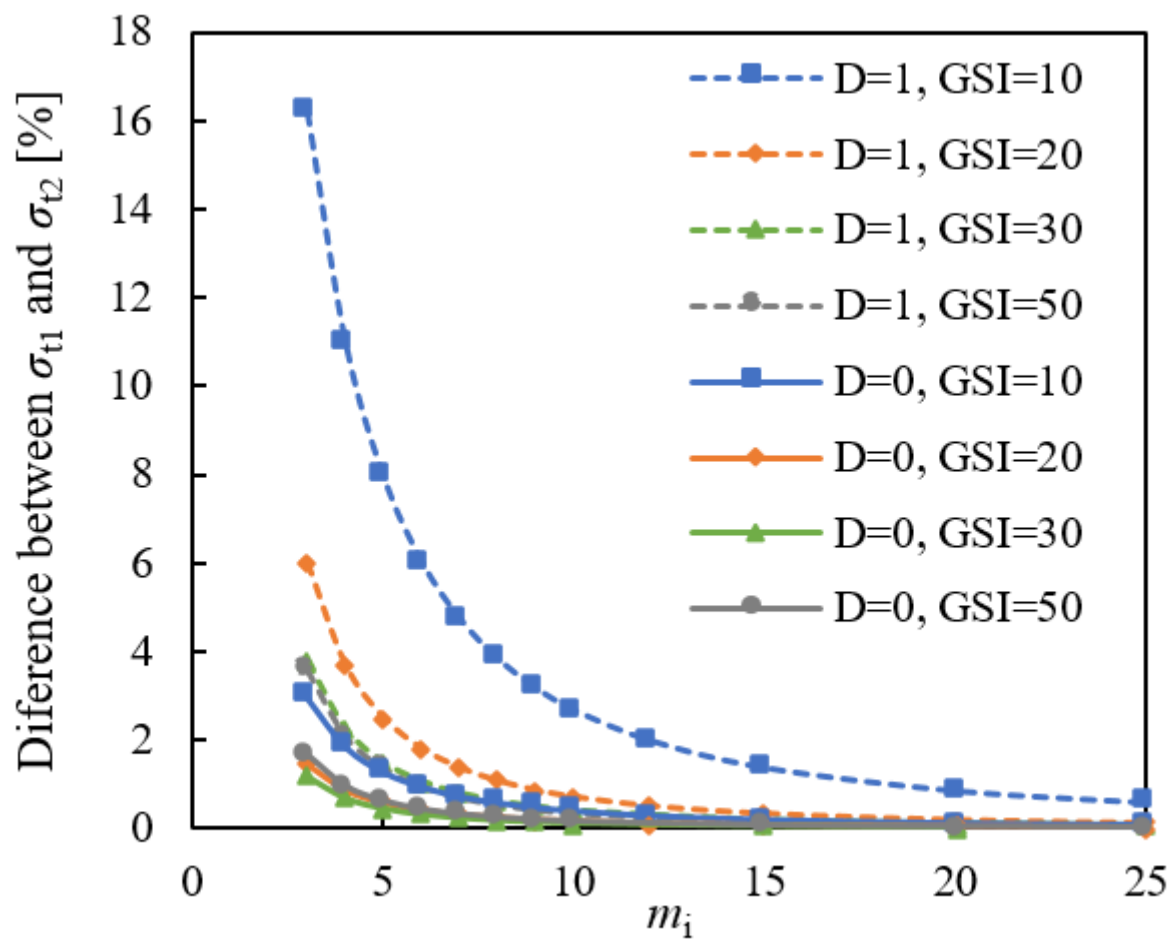


Figure 2

Difference in the values between the biaxial and uniaxial tensile strength (Single column, Color online only)

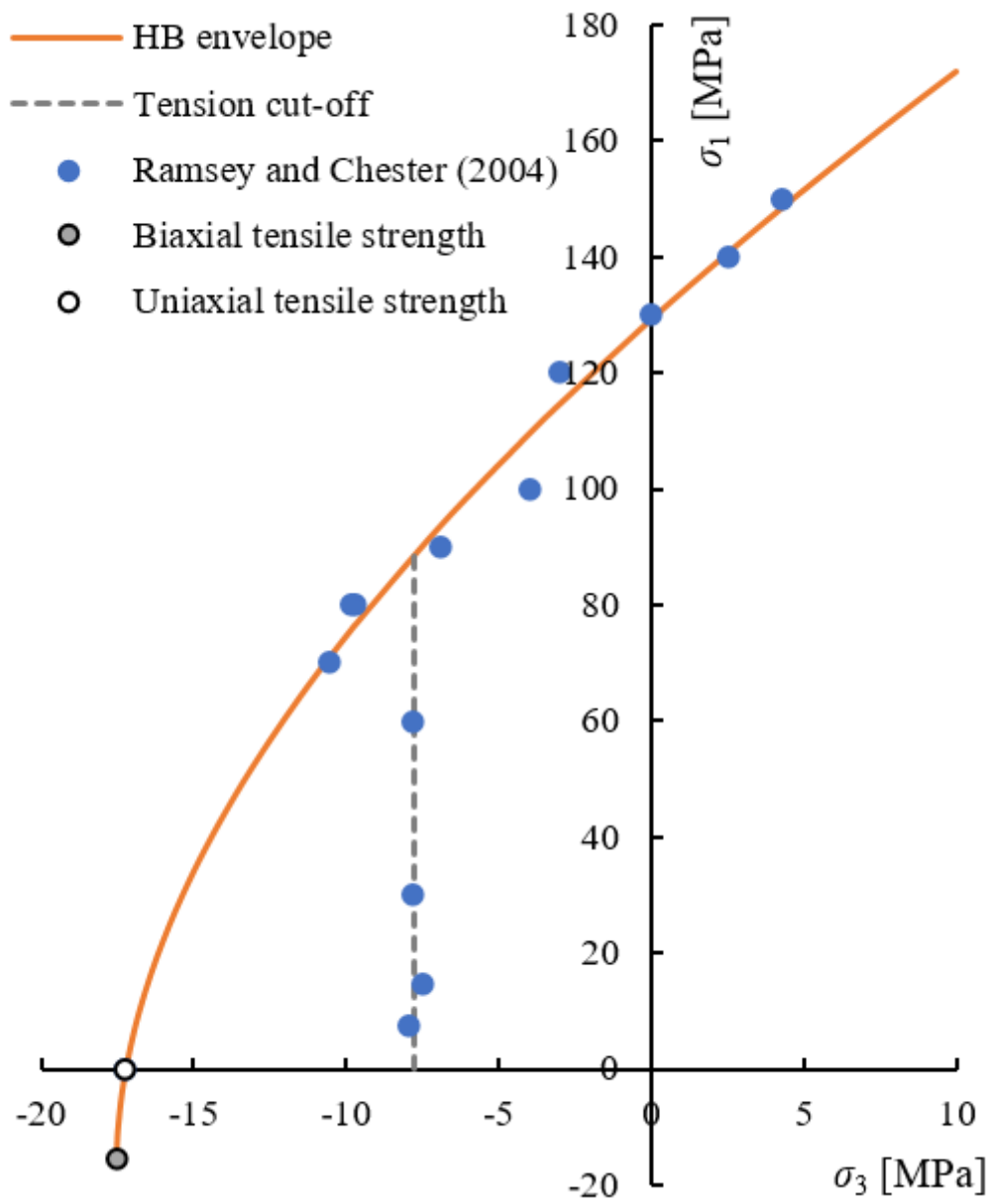


Figure 3

Combined plot of HB criterion with tension cut-off (Single column, Color online only)