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Research on Roundness Error Consistency Model for Crank Journal Cylindrical grinding

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Abstract: Cylindrical grinding is an important way to form the external shape error of the crank journal, and the accuracy consistency directly affects the interchangeability of products. To study the accuracy consistency of crank journal, a dynamic model of the grinding wheel-crankshaft grinding system based on Timoshenko beam is established, and the grinding transition process simulation algorithm with iterative convergence of grinding force-transient grinding amount cycle adapted to the model is proposed, which realizes the simulation of the roundness of the crank journal coupled with the process parameters of the grinding system. Aiming at the grinding position of each crank journal, the grinding roundness of five crank journals is simulated respectively. On this basis, the crank journal roundness consistency prediction model is established, and the effectiveness of the prediction model is verified by field experiments. Finally, the influence of grinding parameters on the consistency of the roundness of crank journal is studied. The research conclusion can provide a reference for the grinding accuracy consistency design of this type of crank journal.

Keywords: Crankshaft grinding; Grinding dynamics; Grinding simulation; Roundness; Consistency

1 Introduction

The consistency of crank journal outer circle accuracy has an important impact on the stable operation performance of crankshaft [1,2]. Grinding factors such as the dynamic unbalance, the rotation accuracy and the clamping error directly affect the accuracy consistency [3,4]. How to maximize the accuracy consistency of the outer circle of the crank journal is the key to the high motion accuracy and good interchangeability of the crankshaft.

Researchers have explored the accuracy consistency from different manufacturing fields. Yuan Julong et al. [5~7] established a grinding model for batch balls, and simulated the grinding trajectory of the cylindrical surface. On this basis, the standard deviation of the trajectory density was used as a quantitative evaluation index for processing consistency. Lv Binghai et al. [8~9] studied the distribution of spherical grinding tracks during the grinding process of ceramic balls through a combination of experiment and simulation, and used the uniformity of grinding tracks as a method to evaluate the consistency of the grinding accuracy of ceramic balls. Wen Donghui et al. [10~11] established a dynamic model for single-sided grinding processing, simulated the grinding processing path, and used the standard deviation of the distribution density of machining path points as the evaluation index to analyze the consistency of machining path on the workpiece surface. S Chen et al. [12] established a grinding model considering the vibration of the grinding wheel for the surface grinding of high-precision optical components. Through simulation and experiment comparison, the effect of curvature change on the uniformity of the grinding was studied. Liu Z et al. [13] established a track point density distribution model for the uneven polishing caused by the different

arrangements of the holes on the polishing pad in the thin copper substrate, and studied the effect of the hole arrangement on the polishing uniformity through simulation and experiments. Wei, H., et al. [14] established a prediction model of grinding material removal based on single abrasive and statistics, and verified the validity of the model through experiments. On this basis, the influence law of workpiece surface contour height consistency was analyzed. Zhou Weihua et al. [15] established the surface generation mechanism model of gear grinding, and studied the influence of process parameters on the surface quality uniformity of gear grinding.

The dynamics of the grinding system is the basis of modeling research on the crankshaft grinding system, and the research on grinding force has always been a hot topic [16~17]. The research on grinding force is the basis for the establishment of grinding system, and the grinding system established on the basis of grinding force provides important reference value for the research on grinding mechanism [18~21]. Xiong Wanli et al. [22~25] established a nonlinear vibration coupling model of double rotor grinding based on hydrostatic bearing on the basis of grinding force, this model reflects the law of material removal in grinding process and the coupling effect between grinding wheel and workpiece, and realizes the quantitative simulation of grinding rounding transition process.

In general, due to the great commercial value of the research on the grinding accuracy consistency of crank journal, the open literature dedicated to the research on the grinding accuracy consistency of crank journal is very limited. On the basis of absorbing references [22~23], this paper further study the roundness consistency problem after grinding the crank journal, which is based on Timoshenko beam theory. According

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to the different positions of each crank journal in the crankshaft structure, the roundness of grinding five crank journals of the crankshaft are quantitatively simulated. On this basis, the roundness consistency of each crank journal is quantitatively evaluated, and the effectiveness of roundness consistency analysis method of crank journal is verified by experiments. Finally, the influence law of roundness consistency after grinding of each crank journal is analyzed.

2 Dynamic Model of Crank Journal grinding

2.1 Element matrix of Timoshenko beam finite element grinding system

The grinding of crank journal can be equivalent to cylindrical grinding, but the crank journal grinding is different from the general cylindrical grinding, this is because the existence of the crank makes the rigidity of the crankshaft particularly poor, the stiffness along the crankshaft axis is not equal, and the stiffness of the crankshaft at different corners is also different, so that the crankshaft produces unequal elastic deformation in the machining process, which will affect the consistency of machining accuracy of crank journal. Since the shear effect and gyro effect are considered in Timoshenko beam finite element model [26], it has strong advantages in analyzing rotor dynamic characteristics.

Fig. 1 Timoshenko beam element

According to the Timoshenko beam theory, the crankshaft shaft system is discretized, and the classical 2-node 8-DOF Timoshenko beam element is used to create the element stiffness matrix, element mass matrix and element gyro matrix. In this model, each element contains 2 nodes, and each node contains 4 degrees of freedom.

Figure 1 shows the schematic diagram of the Timoshenko beam element. The tiny element has a node A and B at each end, which the length of the Timoshenko beam element is L . Each node has four generalized degrees of freedom, U , W , Θ , and Γ . Among them, U represents the displacement in the Y direction, W represents the displacement in the Z direction, Θ represents the offset angle around the Y axis, and Γ represents the offset angle around the Z axis, and the generalized displacement vector of nodes at both ends is shown in equation (1):

$$u_s = [U_A \ W_A \ \Theta_A \ \Gamma_A \ U_B \ W_B \ \Theta_B \ \Gamma_B]^T \quad (1)$$

The displacement on the section with a distance of s

from the leftmost node in the X-axis direction can be obtained by multiplying the interpolation function and the displacement vector, as shown in equation (2):

$$\begin{cases} U = [\varphi_1 \ 0 \ 0 \ \varphi_2 \ \varphi_3 \ 0 \ 0 \ \varphi_4] u_s = N_u u_s \\ W = [0 \ \varphi_1 \ -\varphi_2 \ 0 \ 0 \ \varphi_3 \ \varphi_4] u_s = N_w u_s \\ \Theta = [0 \ -\theta_1 \ \theta_2 \ 0 \ 0 \ -\theta_3 \ \theta_4 \ 0] u_s = N_\Theta u_s \\ \Gamma = [\theta_1 \ 0 \ 0 \ \theta_2 \ \theta_3 \ 0 \ 0 \ \theta_4] u_s = N_\Gamma u_s \end{cases} \quad (2)$$

In the equation (2), N_u , N_w , N_Θ and N_Γ represent the Lagrange displacement shape functions, and the expressions of $\varphi_1, \varphi_2, \varphi_3$ and φ_4 are shown in equation (3), the expressions of $\theta_1, \theta_2, \theta_3$ and θ_4 are shown in equation (4):

$$\begin{cases} \varphi_1 = \frac{1}{1+\Phi} [2a^3 - 3a^2 + 1 + \Phi(1-a)] \\ \varphi_2 = \frac{L}{1+\Phi} [a^3 - 2a^2 + a + \frac{1}{2}\Phi(1-a)a] \\ \varphi_3 = \frac{1}{1+\Phi} [-2a^3 + 3a^2 + 1 + \Phi a] \\ \varphi_4 = \frac{L}{1+\Phi} [a^3 - a^2 - \frac{1}{2}\Phi(1-a)a] \end{cases} \quad (3)$$

$$\begin{cases} \theta_1 = \frac{6a(a-1)}{L(1+\Phi)} \\ \theta_2 = \frac{1}{1+\Phi} [3a^2 - 4a + 1 + \Phi(1-a)] \\ \theta_3 = \frac{6a(1-a)}{L(1+\Phi)} \\ \theta_4 = \frac{1}{1+\Phi} [3a^2 - 2a + \Phi a] \end{cases} \quad (4)$$

$$\begin{cases} a = \frac{s}{L} \\ \Phi = \frac{12EI}{K'GAL^2} \end{cases} \quad (5)$$

In formula (5), E represents young's modulus, G represents material shear modulus, I represents the moment of inertia of the cross section, A represents the cross-sectional area, and K' represents the cross-sectional shape coefficient. By deriving the displacement vector from time, the linear velocities $\dot{U}$, $\dot{W}$ and angular velocities $\dot{\Theta}$, $\dot{\Gamma}$ in two directions on the corresponding section can be obtained. The expressions of these four generalized velocities are shown in equation (6):

$$\begin{cases} \dot{U} = N_u \dot{u}_s \\ \dot{W} = N_w \dot{u}_s \\ \dot{\Theta} = N_\Theta \dot{u}_s \\ \dot{\Gamma} = N_\Gamma \dot{u}_s \end{cases} \quad (6)$$

The expression of kinetic energy value of micro unit at constant angular velocity is shown in equation (7).

$$T_s = \frac{1}{2} \int_0^L \rho A \left\{ \begin{matrix} \dot{U} \\ \dot{W} \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{U} \\ \dot{W} \end{matrix} \right\} + J_d \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\} ds - \frac{1}{2} \int_0^L J_d \Omega (\dot{\Theta} \dot{\Gamma} - \dot{\Gamma} \dot{\Theta}) ds \quad (7)$$

The first term at the right end of equation (7) represents the translational energy and rotational energy, and the second term represents the kinetic energy under the action of gyro torque. Where, J_d represents the moment of inertia of the unit body around the diameter, and J_p represents the polar moment of inertia of the unit body. The strain energy of the micro unit can be expressed by equation (8):

$$V_s = \frac{1}{2} u_s^T \left(\int_0^L EI \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\} + K' GA \left\{ \begin{matrix} \dot{U} - \Gamma \\ \dot{W} + \Theta \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{U} - \Gamma \\ \dot{W} + \Theta \end{matrix} \right\} ds \right) u_s \quad (8)$$

From the application of Lagrange's theorem and finite element in rotor characteristic analysis, the element mass matrix of equation (9), the element stiffness matrix of equation (10) and the gyro torque element matrix of equation (11) are obtained from equation (7) and equation (8):

$$M_e = \int_0^L \rho A \left\{ \begin{matrix} N_u \\ N_w \end{matrix} \right\}^T \left\{ \begin{matrix} N_u \\ N_w \end{matrix} \right\} + J_d \left\{ \begin{matrix} N_\Theta \\ N_\Gamma \end{matrix} \right\}^T \left\{ \begin{matrix} N_\Theta \\ N_\Gamma \end{matrix} \right\} ds \quad (9)$$

$$K_e = \int_0^L EI \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{\Theta} \\ \dot{\Gamma} \end{matrix} \right\} + K' GA \left\{ \begin{matrix} \dot{U} - \Gamma \\ \dot{W} + \Theta \end{matrix} \right\}^T \left\{ \begin{matrix} \dot{U} - \Gamma \\ \dot{W} + \Theta \end{matrix} \right\} ds \quad (10)$$

$$G_e = \int_0^L J_p (N_\Gamma^T N_\Theta - N_\Theta^T N_\Gamma) ds \quad (11)$$

According to equations (9) (10) and (11), the element calculation matrix of the spindle unit body is obtained by using the mathematical calculation method. In this work, the 2-node 8-DOF Timoshenko beam given in reference [27] is used to calculate the above matrices.

2.2 Dynamic model of crank journal grinding

Figure 2 shows the simplified schematic diagram of crank journal grinding. This paper mainly studies the roundness of each crank journal of crankshaft after cylindrical cut in grinding, ignores the effects of grinding chatter, torsional deformation and thermal factors [28], and the grinding wheel system and the crankshaft system are simplified into two single-degree-of-freedom shaft systems. The radial feed process regards the two single-degree-of-freedom shaft systems as an interactive coupling process.

Fig. 2 Simplified diagram of crank journal grinding

According to the vibration mechanics, the differential dynamic equation in the time t can be established:

$$[M_1] \ddot{x}_1(t) + [C_1] \dot{x}_1(t) + [K_1] x_1(t) = F_{ub1}(t) + F_n(t) \quad (12)$$

$$[M_2] \ddot{x}_2(t) + [C_2] \dot{x}_2(t) + [K_2] x_2(t) = F_{ub2}(t) + F_n(t) \quad (13)$$

Where, $[M_1]$ represents the equivalent mass matrix of grinding wheel system, $[M_2]$ represents the equivalent mass matrix of crankshaft system, $[C_1] = -\omega_1[G_1]$ and $[C_2] = -\omega_2[G_2]$ represent the equivalent damping matrix of grinding wheel support system and crankshaft support system respectively, ω_1 and ω_2 represent the angular velocity of grinding wheel and crank journal respectively. $[K_1]$ and $[K_2]$ represent the equivalent stiffness matrices of grinding wheel spindle system and crankshaft system respectively. $x_1(t)$ represents the vibration displacement of the grinding wheel, and $x_2(t)$ represents the vibration displacement of the crank journal. $F_{ub1}(t)$ and $F_{ub2}(t)$ represent the dynamic unbalance forces of grinding wheel and crank journal, respectively; $F_n(t)$ represents the normal grinding force.

2.3 Coordination relationship between grinding deformation of crank journal

The dynamic unbalance force of the rotor system can be expressed as [29]:

$$F_{ub}(t) = Gm\omega \cdot 10^{-3} \cos(\omega t) \quad (14)$$

Where, G represents the dynamic balance accuracy grade of the rotor, and m represents the mass of the rotor, ω represents the rotational angular speed of the rotor (rad/s), so the $F_{ub1}(t)$ and $F_{ub2}(t)$ can be obtained.

In this paper, the grinding force calculation formula proposed by Chiu and Malkin is used to solve the grinding force value, and the plough and sliding effects are ignored, $F_n(t)$ can be expressed as [30]:

$$F_n(t) = \frac{\alpha u_{ch} v_w \delta_w(t) b}{v_g} = k_w \delta_w(t) \quad (15)$$

Where, α represents the ratio of normal grinding force to tangential grinding force, u_{ch} represents the specific cutting energy, b represents the grinding width, $\delta_w(t)$ represents the instantaneous grinding depth of the crank journal, v_w and v_g represent the linear speed of crank journal and grinding wheel respectively, and k_w represents the grinding stiffness.

Figure.3 shows the equivalent simplified schematic diagram of the crank journal grinding system. Point P' and point O' represent the theoretical position of grinding wheel and crankshaft axis respectively. Point P and point O respectively represent the actual positions of the grinding wheel axis and the crank journal axis at time t in the grinding process.

Fig.3 Simplified equivalent diagram of crank journal grinding

There is the following deformation coordination relationship:

$$L_1(t) + L_2(t) = x_1(t) + x_2(t) + L(t) \quad (16)$$

Where, $L_1(t)$ represents the distance between the grinding wheel axis and the contact point, which consider the grinding wheel radius, wear depth and grinding wheel contact deformation; $L_2(t)$ represents the distance between the crank journal axis and the contact point, which consider the radius of the crank journal, the wear amount and the crank journal contact deformation; $L(t)$ represents the distance between the grinding wheel and the crank journal.

The specific expressions of $L_1(t)$, $L_2(t)$ and $L(t)$ are:

$$L_1(t) = \Delta_s(t) - y_{k1}(t) \quad (17)$$

$$L_2(t) = r(t) - \Delta_w(t) - y_{k2}(t) \quad (18)$$

$$L(t) = L_0 - e(t) - p(t) - u_s(t) \quad (19)$$

Where, $\Delta_s(t)$ represents the wear depth of the grinding wheel, $y_{k1}(t)$ and $y_{k2}(t)$ represent the contact deformation of the grinding wheel and the crank journal, respectively; $r(t)$ represents the cylindrical contour of the crank journal before grinding, $\Delta_w(t)$ represents the grinding depth of the crank journal; L_0 represents the distance between the grinding wheel axis and the crank journal axis in the initial grinding state; $e(t)$ represents the radial runout error of the grinding wheel, $p(t)$ represents the radial runout error of the crankshaft, and $u_s(t)$ represents the nominal feeding amount.

$$\begin{cases} \Delta_s(t) = \Delta_s(t - \Delta t) + \delta_s(t) \\ \Delta_w(t) = \Delta_w(t - \Delta t) + \delta_w(t) \end{cases} \quad (20)$$

Where, $\Delta(t)$ represents the time step, $\delta_s(t)$ represents the instantaneous grinding amount of grinding wheel.

$$\delta_s(t) = \begin{cases} 0 & (F_n(t) \leq F_{k1}) \\ \frac{F_n(t) - F_{k1}}{k_s} & (F_n(t) > F_{k1}) \end{cases} \quad (21)$$

$$\delta_w(t) = \begin{cases} 0 & (F_n(t) \leq F_{k2}) \\ \frac{F_n(t) - F_{k2}}{k_w} & (F_n(t) > F_{k2}) \end{cases} \quad (22)$$

Where, F_{k1} represents the critical grinding force of the grinding wheel. If $F_n(t) \leq F_{k1}$, only elastic or plastic deformation occurs. If $F_n(t) > F_{k1}$, the wear begins, and k_s represents the wear rigidity of the grinding wheel. F_{k2} represents the critical grinding force of the crank journal. If $F_n(t) > F_{k2}$, the material of the crank journal will be removed. In the grinding process, the following relationship exists:

$$y_{k1}(t) + y_{k2}(t) = y_k(t) = \frac{F_n(t)}{k_a} \quad (23)$$

Where, $y_k(t)$ represents contact deformation during grinding, and k_a represents contact stiffness. Before grinding starts, the following relationship exists:

$$L_0 = r(0) + e(0) + p(0) \quad (24)$$

There are four deformation coordination relationships during grinding:

(1) If $F_n(t) = 0$, then $y_k(t) = 0$, $\delta_w(t) = 0$, $\delta_s(t) = 0$, it means that the grinding wheel and crank journal have just entered the contact state without grinding force. At this time, according to formula (16)~(24), the grinding coordination deformation equation can be expressed as:

$$u_s(t) \leq \Delta_s(t - \Delta t) + \Delta_w(t - \Delta t) + x_1(t) + x_2(t) + g(t) \quad (25)$$

Where, $g(t)$ can be expressed as follow:

$$g(t) = r(0) - r(t) + e(0) - e(t) + p(0) - p(t) \quad (26)$$

(2) If $0 < F_n(t) \leq F_{k2}$, then $\delta_w(t) = 0$, $\delta_s(t) = 0$, according to formula (16)~(24), the grinding coordination deformation equation can be expressed as:

$$u_s(t) = \Delta_s(t - \Delta t) + \Delta_w(t - \Delta t) + F_n(t) \cdot \frac{1}{k_a} + x_1(t) + x_2(t) + g(t) \quad (27)$$

(3) If $F_{k2} < F_n(t) \leq F_{k1}$, then $\delta_s(t) = 0$, according to formula (16)~(24), the grinding coordination deformation equation can be expressed as:

$$u_s(t) = \Delta_s(t - \Delta t) + \Delta_w(t - \Delta t) + F_n(t) \cdot \left(\frac{1}{k_w} + \frac{1}{k_a} \right) - \frac{F_{k2}}{k_w} + x_1(t) + x_2(t) + g(t) \quad (28)$$

(4) If $F_{k1} < F_n(t)$, according to formula (16)~(24), the grinding coordination deformation equation can be expressed as:

$$u_s(t) = \Delta_s(t - \Delta t) + \Delta_w(t - \Delta t) + F_n(t) \cdot \left(\frac{1}{k_s} + \frac{1}{k_w} + \frac{1}{k_a} \right) - \left(\frac{F_{k1}}{k_s} + \frac{F_{k2}}{k_w} \right) + x_1(t) + x_2(t) + g(t) \quad (29)$$

Then, the mathematical expressions of the nonlinear

vibration response coupling model of the grinding system are (12),(13),(25),(27~29). In this work, the *Newmark-β* numerical step-by-step integration method is used to solve $x_1(t)$ and $x_2(t)$ [31], and then obtains the total grinding depth $\Delta_w(t)$. The roundness after grinding of the crank journal can be obtained by using the least squares method [32].

3 Roundness consistency prediction method

3.1 Consistency evaluation method

The accuracy consistency of crank journal reflects the dispersion of machining accuracy, which is a statistical indicator for evaluating the uniformity of crankshaft manufacturing quality. The position of the crankshaft where the crank journal is located in this study is shown in Figure 4. The names of each crank journal are defined as J1, J2, J3, J4, J5, the horizontal distance between the center of each crank journal and the left end of the crankshaft is s_1, s_2, s_3, s_4 and s_5 respectively.

Fig. 4 The location and name of the crank journal

After crankshaft grinding, the roundness of each crank journal (J1, J2, J3, J4 and J5) is obtained to form an array $\{y_i\} (i=1,2,L 5)$. Set the consistency evaluation equation of crank journal outer roundness as follows:

$$P = 1 - \frac{\max(y_i) - \min(y_i)}{\bar{y}} \quad (30)$$

Where, $\bar{y}$ represents the mean value of the roundness array $\{y_i\} (i=1,2,L 5)$, $\max(y_i)$ represents the maximum roundness among the five crank journals of the crankshaft, and $\min(y_i)$ represents the minimum roundness among the five crank journals of the crankshaft. The larger the value of the consistency rate P , the better the consistency of the machining quality of the five crank journals, and vice versa.

3.2 Crank journal grinding consistency simulation algorithm

Based on the nonlinear vibration coupling calculation model of the grinding system, the consistency of roundness of crank journal is analyzed in this section. Figure 5 shows the flow of the consistency analysis algorithm proposed in this article, and each step is briefly described as follows:

(1) Determine grinding parameters, including crank

journal grinding structure parameters, grinding wheel grinding structure parameters, and grinding process parameters. Among them, the crank journal structure parameters include crank journal diameter, crankshaft length, crank journal number, crank journal density, crankshaft dynamic balance accuracy grade and other parameters; The structure parameters of the grinding wheel include the outer diameter, width, density, the length of the extended end and the dynamic balance accuracy grade; The grinding process parameters include the feed speed, the feed time of each grinding stage and the rotating speed.

(2) The initial error of the grinding system is determined according to engineering inspection error, including initial outer circle contour error $r(t)$ of crank journal, the radial runout error $e(t)$ of the grinding wheel and the radial runout error $p(t)$ of the crankshaft rotation. The error processing can be described in reference [23,33].

(3) According to the grinding position and grinding parameters of crank journal J1, the equivalent mass matrix, equivalent stiffness matrix and equivalent damping matrix in the grinding process of crank journal J1 are calculated by Timoshenko beam finite element.

(4) The nonlinear vibration coupling calculation model of grinding system is established by equation (12),(13),(25),(27~29). Introduce the value of λ , and $\lambda \in [0,1]$, divide $F_n(t)$ into n parts, and substitute the equally divided $F_n(t)$ into the nonlinear vibration coupling calculation model of grinding system, using the *Newmark-β* method to solve the $x_1(t)$ and $x_2(t)$. Set the calculation accuracy ε of equation (31):

$$\varepsilon = |u_s(t) - [\Delta_w(t - \Delta t) + \Delta_s(t - \Delta t) + F_n(t) \left(\frac{1}{k_w} + \frac{1}{k_s} + \frac{1}{k_a} \right) + r(t) - e(t) + p(t) + x_1(t) + x_2(t)]| \quad (31)$$

By changing the value of λ , it is possible to compare the ε values corresponding to all assumed normal grinding forces, and take the $F_n(t)$ corresponding to the minimum value of ε as the simulated value. At this time, when the calculation accuracy is satisfied, the simulation values such as $x_1(t)$, $x_2(t)$ and $\Delta_w(t)$ are calculated. On this basis, the difference between the initial contour error $r(t)$ and $\Delta_w(t)$ is calculated to obtain the contour point error of each area of the crank journal surface. The roundness of each contour error point is evaluated by the least squares method to obtain the final roundness y_1 after grinding of the crank journal J1.

(5) Repeat steps (D~H) to simulate the grinding roundness of crank journals J2~J5, and obtain the grinding roundness $\{y_i\} (i=1,2,L 5)$ of five crank journals of crankshaft.

(6) Substituting the calculated roundness of the crank journal of the crankshaft into the roundness consistency evaluation equation of equation (30), the value of the consistency rate P is calculated.

Fig. 5 Flow chart of consistency analysis of roundness of crank journal

4. Simulation and verification of roundness consistency of crank journal

4.1 Simulation analysis

In this section, taking grinding the outer circle of a certain batch of crank journal shown in Figure 4 as an example, the simulation analysis of the roundness consistency of crank journal outer circle grinding is programmed in Matlab environment. The grinding process of each crank journal adopts the same process parameters, as shown in Table 1 and Table 2, the grinding system error is shown in Table 3.

The following takes the crank journal J3 as an example to introduce the simulation process of grinding the external roundness. According to the grinding position and grinding parameters of J3, it is substituted into the simulation calculation program. Figure 6 shows the contour trajectory of each circle of the crank journal J3 during the grinding process. It can be seen that it gradually decreases in spiral shape and finally converges to circular shape.

Tab.1 The parameters of Grinding wheel and crank journal

Parameter	Value
Grinding wheel material	CBN
Wheel diameter (mm)	550
Crankshaft length (mm)	426
Material	42CrMoA
Crank journal diameter (mm)	46
s_1 (mm)	52
s_2 (mm)	136
s_3 (mm)	218
s_4 (mm)	301
s_5 (mm)	384
Crank journal speed (r/min)	46
Grinding wheel linear speed (m/s)	90
Wheel width (mm)	23
Grain size	100

Tab.2 Crank journal grinding process parameters

Process	Feed time (s)	Feed rate ($\mu\text{m/s}$)
Rough grinding	8	20
Fine grinding	5	8

Spark-out grinding	5	0
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Tab.3 Grinding system error amplitude setting

Error type	Error amplitude (μm)
$r(t)$	5
$e(t)$	0.5
$p(t)$	0.6

The change of roundness of crank journal J3 during grinding is shown in Figure.7. The roundness is evaluated by the least squares method for the contour point error after each cycle of grinding. During rough grinding, the roundness of the crank journal J3 gradually increases due to the large feed of the grinding wheel. With the decrease of the feed, the roundness of the crank journal J3 gradually decreases during the fine grinding. Finally, the roundness of the crank journal J3 reaches stability during the spark-out grinding. The roundness after grinding is evaluated by the least squares method as $1.62 \mu\text{m}$.

Fig.6 Grinding contour shape of crank journal J3

Fig. 7 Roundness change of crank journal J3

Using the consistency analysis method, the roundness of other crank journals are simulated and calculated respectively, as shown in Figure 8. Substituting the roundness into the consistency evaluation equation (30), the consistency rate of the crank journal roundness under the grinding conditions is calculated as $P=0.9948$. The P value of consistency

rate reflects the consistency of the crank journal roundness of the crankshaft, and provides a quantitative index for the subsequent analysis of the influence law of consistency.

Fig.8 Simulation value of roundness of crank journal

4.2 Experimental verification

According to the simulation example under the above grinding parameters, the consistency of crank journal grinding roundness is experimentally verified on the Junker grinder of Neijiang JinHong crankshaft Co., Ltd., as shown in Figure 9. In order to maintain the validity of the verification experiment, 10 crank journals in the same batch were externally ground in accordance with the grinding conditions shown in Tables 1 to 3 during the experiment.

Fig.9 Experimental drawing of cylindrical grinding of crank journal

ADCOLE MODELL1200 is used to detect 10 crankshafts after grinding, as shown in Figure 10, the crankshaft is vertically installed on the turntable, the probe is fixed on the support and contacts with the crank journal at a certain pressure. After adjusting the concentricity of the crankshaft and the turntable, the crankshaft rotates with the high-precision turntable, and the outer circle contour information of the crank journal is "copied" by the probe to the computer for data processing, then the roundness of the crank journal is obtained. The upper, middle and lower sections of each crank journal are detected once respectively, that is, there are 3 roundness for each crank journal and 15 roundness for each crankshaft.

Fig.10 Detection of roundness

According to the name of the crank journal, the detection data of the roundness of each crank journal of the 10 crankshafts are counted, Figure 11 shows the comparison between crank journal roundness detection data and simulation data. Due to certain differences in the detection process, there are maximum and minimum values in the roundness detection data, but the detected mean value of each crank journal roundness is highly consistent with the simulation value, which also shows that the proposed simulation algorithm model is reasonable.

Fig. 11 Comparison between crank journal roundness detection data and simulation data

According to the established consistency analysis equation, the consistency rate P of roundness of 10 crank journals detected in the experimental process is obtained. The specific value of consistency rate P of each crankshaft is shown in Figure.12, and the value range of P is $[0.9938, 0.9952]$. It can be seen that it is basically close to the consistency rate $P=0.9948$ simulated in Section 4.1, which shows that the roundness consistency analysis model proposed in this paper is effective.

Fig. 12 Comparison between crankshaft consistency detection data and simulation data

5 Influence law of roundness consistency of crank journal

Different grinding parameters affect the roundness consistency of crank journals. Using the constructed consistency analysis model, the influence law of different grinding parameters on the roundness consistency of crank journals can be analyzed to realize the optimal control of consistency.

5.1 Influence law of speed ratio on roundness consistency

The ratio of grinding wheel speed n_s to crankshaft speed n_w affects the roundness of cylindrical grinding. Under the condition of keeping other parameters unchanged, the influence of different speed ratios $\eta = n_s/n_w$ on the grinding roundness of crank journal is shown in Figure 13. The roundness of crank journal becomes larger as the η is close to the integer ratio. This is mainly because when the η is close to the integer ratio, the micro edge of the grinding wheel and the crank journal contact each other more times at the same position, resulting in the deterioration of the grinding contour uniformity and the increase of the roundness. The influence law is basically consistent with the conclusions of literature [34-35], and indirectly proves that the grinding dynamic coupling model and iterative simulation algorithm proposed in this paper are effective.

According to the established consistency analysis method, the change trend of the consistency rate P of the crank journal is obtained, as shown in Figure 14. When the speed ratio is close to an integer, the roundness consistency of each crank journal of the crankshaft is relatively good, mainly because when the speed ratio is close to an integer, the roundness of each crank journal is relatively large, which makes the value P of consistency rate larger and the consistency better. When the speed ratio is non integer, the roundness of each crank journal is relatively small, which makes the value P of consistency rate smaller and the consistency worse.

Fig.13 The influence of speed ratio η on roundness

Fig.14 The influence of speed ratio η on the consistency of roundness

5.2 Influence law of radial runout error of grinding wheel on roundness consistency

Based on the established model, under the condition of keeping other parameters unchanged, the influence law of the runout error $e(t)$ of the grinding wheel on the roundness and consistency of each crank journal is analyzed, as shown in Figure 15.

Fig. 15 Influence law of radial runout of grinding wheel axis on roundness and consistency of each crank journal

When the amplitude of the grinding wheel radial runout error $e(t)$ increases from $0.3\mu\text{m}$ to $0.6\mu\text{m}$, the

average roundness of the crank journal increases from $1.61\mu\text{m}$ to $2.07\mu\text{m}$, and the P value of the consistency rate increases from 99.1% to 99.65%. It can be concluded that within a certain range, as the amplitude of $e(t)$ increases, the mean value of the simulated roundness gradually increases, that is, the radial runout error of the grinding wheel axis reduces the grinding accuracy, which is consistent with the engineering practice. According to the established consistency evaluation equation, the calculated roundness consistency of each crank journal becomes better with the increase of axial radial runout error. The main reason is that the allowable error range becomes larger when the mean roundness becomes larger, resulting in the increase of consistency rate P in the consistency evaluation equation.

5.3 Influence law of radial runout error of crank journal on roundness consistency

In the process of crank journal rotation, the radial runout error $p(t)$ of crank journal also affects the surface roundness. Under the condition of keeping other parameters unchanged, the influence law of roundness and consistency of each crank journal under the grinding conditions of different radial runout error $p(t)$ of crank journal is analyzed, as shown in Figure 16.

Fig.16 Influence law of crank journal radial runout error on roundness and consistency of each crank journal

When the amplitude of the crank journal rotation radial runout error $p(t)$ increases from $1.0\mu\text{m}$ to $1.6\mu\text{m}$, the average roundness of the crank journal increases from $1.61\mu\text{m}$ to $2.35\mu\text{m}$, and the P value of the consistency rate increases from 99.05% to 99.63%. The influence law of radial runout error of the crank journal on the roundness and consistency of each crank journal is similar to the influence relationship of the grinding wheel runout error. Reducing the crank journal radial runout error will gradually reduce the mean roundness of the crank journal, which is also consistent with the engineering practice. When the roundness of each crank journal becomes smaller, the consistency of the roundness gradually deteriorates.

It can be seen that reducing the runout error of the two has the opposite effect on the roundness and consistency of the crank journal, that is, reducing the

radial runout error of the grinding wheel or the radial runout error of the crank journal can reduce the roundness of each crank journal, but the consistency of roundness becomes worse. Therefore, the design of grinding parameters should be balanced according to the influence law of these two factors on grinding quality, so as to make the grinding roundness and consistency index reach the best index level as far as possible.

5.4 Influence law of the initial roundness of crank journal on roundness consistency

There are some differences in the initial contour of the outer circle of the crank journal before grinding. Under the condition of keeping other parameters unchanged, the influence law of roundness and consistency of each crank journal under the grinding conditions of different initial roundness $r(t)$ of crank journal is analyzed, as shown in Figure 17.

Fig. 17 Influence law of initial roundness of crank journal on roundness and consistency of crank journals

When the amplitude of $r(t)$ of the crank journal increases from $5.0\mu\text{m}$ to $6.5\mu\text{m}$, the average roundness of the crank journal and the P value of the consistency rate change little. That is to say, the analysis based on simulation grinding shows that within a certain range, the roundness of the initial contour of the outer circle has little effect on the roundness and consistency of the crank journal, and it can almost be ignored. For cylindrical grinding under different initial contours, there are some differences in the grinding amount at the initial stage of grinding, but under the same grinding conditions, with the grinding feed, the grinding roundness will eventually tend to be consistent, which is basically consistent with the actual engineering experience.

6 Conclusion

(1) A dynamic model of the grinding wheel-crankshaft rotor grinding system based on Timoshenko beam is established, and the simulation algorithm for the iterative convergence of the grinding force-transient grinding amount that is compatible with the model is proposed, which realizes the simulation of crank journal grinding rounding transition process based on the coupling of grinding process parameters.

(2) According to the grinding position of each crank journal, the roundness of five crank journals is quantitatively simulated respectively. On this basis, the prediction method of roundness consistency of crank journal is established.

(3) The external cylindrical grinding of each crank journals was simulated and verified by multiple sets of field experiments under the same grinding parameters. The results show that the detected mean value of each crank journal roundness is highly consistent with the simulation value, and the consistency rate of the detected roundness is basically close to the simulation value, which shows that the roundness consistency analysis model proposed in this paper is effective.

(4) It is found that in a certain range, when the speed ratio is close to the integer ratio, the roundness of crank journal grinding becomes larger and the consistency of crank journal roundness becomes better; Reducing the radial runout error of grinding wheel and crank journal rotation can reduce the roundness of each crank journal, but the consistency of roundness becomes worse; The influence of initial roundness of the crank journal can almost be ignored. Therefore, these influence laws can provide references for the optimization design of grinding accuracy.

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