Explosive Behaviors on Coupled Fractional-Order System

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Explosive behaviors on coupled fractional-order system

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Abstract Fractional derivatives provide a prominent platform for various chemical and physical system with memory and hereditary properties, while most of the previous differential systems used to describe dynamic phenomena including oscillation quenching are integer order. Here, effects of fractional derivative on the transition process from oscillatory state to stationary state are illustrated for the first time on mean-field coupled oscillators. It is found the fractional derivative could induce the emergence of a first-order discrete transition with hysteresis between oscillatory and stationary state. However, if the fractional derivative is smaller than the critical value, the transition will be invertible. Besides, the theoretical conditions for the steady state are calculated via Lyapunov indirect method which probe that, the backward transition point is unrelated to mean-field density. Our result is a step forward in enlightening the control mechanism of explosive phenomenon, which is of great importance to highlight the function of fractional-order derivative in the emergence of collective behaviors on coupled nonlinear model.

Keywords Explosive death · Fractional derivative · Mean-field coupling · Phase transition

1 Introduction

Spontaneous symmetry breaking in ensembles of interacting dynamic subsystems is an evergreen challenging problem on nonlinear science, which has widely applicability in diverse fields like physics, biology, and engineering [1–4]. In reality, spontaneous symmetry breaking issue in amplitude and phase variation on a coupled identical unit based on the breaking of the general translational and permutational symmetry of coupled oscillators, has induced several emergent collective phenomena such as chimera states [5–7], synchronization [8, 9], extreme events [10, 11] in natural and dynamic system. Recently, an ever-increasing interest on symmetry breaking phenomenon has been witnessed by the enormous burst study on explosive synchronization (ES) and fractional differential equations.

The emergence of explosive synchronization (ES) on nonlinear dynamic system, which is characterized by the sudden (discontinues) and irreversible transition between incoherence and coherence state as the variation of coupling strength, injects fresh blood to the research community to investigate the phase spontaneous symmetry breaking on coupled units. This kind of sharp and irreversible transition was first reported on phase model [12], where scale-free network and positive correlation between the frequencies of the oscillators and their degrees were expressed explicitly as the two key factors of ES. Later, an important work [13] that generated a more general framework, frequency-weighted coupling where the before two ingredients are included as specific cases, was established. It has made ES become a topic of great interest among researchers on nonlinear science and fascinated plenty of significant studies [14–17] and experimental evidence [18–21]. Besides, a new phenomenon of explosive oscillation quenching

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on a frequency-weighted networked system [22], named as Explosive death (ED), grabs the attention of researchers owing to its rich background on complex systems [23–25]. In contrast to the classical continuous oscillation quenching transitions [26, 27], ED implies the abrupt and irreversible process from oscillatory state to suppression of oscillation. Specifically, on account of the properties of the steady state, ED can be classified into explosive amplitude death and explosive oscillation death [28]. In fact, amplitude death (AD) appears when the coupled oscillators stabilize at a common homogeneous steady state (HSS), and when the stable HSS is associated with the coupling strength, nontrivial amplitude death (NAD) is observed. Whereas, oscillation death (OD) refers to that the system stays at the coupling related inhomogeneous steady state (IHSS), producing via the symmetry breaking, where all the oscillators occupy diverse stable branches of IHSS. Hence, when the system undergoes a first-order discontinuous transition from oscillatory to AD, explosive amplitude death occurs. While explosive oscillation death emerges when the stable state is OD. It is worth mentioning that, the rapid development of explosive behaviors has benefited a lot from the practical application on many complex networked systems and a great deal of research has been conducted for providing an effective control method on ED with different coupling schemes [29–35].

As we all know, fractional differential equations play a vital role in many fields of science and engineering owing to the fact that the dynamics of many system can be rendered better by the fractional derivative [36–38]. In detail, most processes in real world with special materials and chemical properties are more likely to show fractional behaviors, such as properties of memory, non-locality and history-dependence. In the past few decades, an increasing number of contributions have been done on fractional-order system, especially in viscoelastic materials [39], fractional control problems [40, 41], relaxation processes in sensory adaptation [42] and so on. In fact, the extensive studies focusing on explosive phenomenon are done in integer coupling oscillators, while the effect of fractional derivative on the spontaneous symmetry breaking including ED has fundamental importance to the analysis and control of coupled system. So far, the contribution of fractional derivative on explosive phenomenon has not been revealed. Thus, in order to give a mechanistic insight to the explosive behavior, we elucidate a strategy of fractional-order mean-field coupling system in this paper that is outlines as follows. In Sec. II, the fractional differential equations with Caputo derivative on mean-field coupling system is elaborated detailly. In Sec III, the transition processes from oscillatory to stable sta-

tionary state are presented numerically and theoretically. Furthermore, the roles of fractional derivative and mean-field density on explosive behaviors are revealed. Finally, the conclusion is summarized in Sec IV.

2 Mathematical description

Without loss of generality, we analyze the van der Pol oscillation equation that was first proposed by Balthasar van der Pol [43] in vacuum tube circuits. In fact, the classical van der Pol oscillator described by the second order differential equation: $\ddot{x}(t) - b(1 - x^2(t))\dot{x}(t) + x(t) = 0$ sever as a basic model of self-sustained oscillatory system, which is also known as unforced van der Pol equation. Recently, inspired by the development of fractional calculus, the fractional derivatives are applied in the van der Pol model in various forms. In particular, by introducing the capacitance via a fractance in the nonlinear RLC circuit, Pereira et al [44] considered the fractional-order van der Pol (FVDP) as $\frac{d^\lambda x(t)}{dt^\lambda} - b(1 - x^2(t))\frac{dx}{dt} + x(t) = 0$, $1 < \lambda < 2$. Meanwhile, another version of FVDP with a fractional damping term was obtained by Barbosa et al [45, 46] as $\frac{d^{1+\lambda}x(t)}{dt^{1+\lambda}} - b(1 - x^2(t))\frac{d^\lambda x(t)}{dt^\lambda} + x(t) = 0$, $0 < \lambda < 1$. Subsequently, Mishra et al [47] presented a general version that introduced two fractional order times derivatives in the van der Pol model, namely, $\frac{d^{2\alpha}x(t)}{dt^{2\alpha}} - b(1 - x^2(t))\frac{d^\alpha x}{dt^\alpha} + x(t) = 0$, $0 < \alpha < 1$. Further, the state space formulations are generated as,

$$\begin{cases} D_t^\alpha x_i = y_i \\ D_t^\alpha y_i = b(1 - x_i^2)y_i - x_i \end{cases} \quad (1)$$

In fact, the classical van der Pol model could be obtained in Eq. (1) with $\alpha = 1$. That is, fractional-order system response eventually converges to integer-order system. Since then, the fractional-order counterpart of van der Pol oscillator has drawn more and more attention, and many researchers in different branches had studied the dynamical properties of chaos and other complicated emergent collective phenomena.

Here, we intend to deal with the FVDP model in the form on Eq. (1). Owing to the fact that our main concern is whether explosive death could appear in the fractional-order dynamical system via a first-order discontinuous way, we consider a nonlinear model of two identical FVDP oscillators coupled mutually through a mean-field interaction. The dynamic is governed as follows,

$$\begin{cases} D_t^\alpha x_i = y_i + k(Q\bar{x} - x_i) \\ D_t^\alpha y_i = b(1 - x_i^2)y_i - x_i \end{cases} \quad (2)$$

Herein, $i = 1, 2$ represents the number of coupled oscillators. The value Q is the intensity of the mean filed with $0 \leq Q < 1$ and k ($k > 0$) indicates the coupling strength. The system parameter is b ($b > 0$) which shows the non-linearity and damping strength of the single van der Pol oscillators and expresses a limit cycle. $D_t^\alpha x_i$, $D_t^\alpha y_i$ are the Caputo derivatives of x_i and y_i , where $0 < \alpha \leq 1$ indicates α -order time derivative, respectively. $\bar{x} = \sum_{i=1}^2 x_i/2$ purposed the mean filed of the state variable x . In fact, the Caputo's definition of fractional derivative [48] is suitable to use in this letter on account of its superiority for the practice problem in engineering, which is ruled by,

$$D_t^\alpha x(t) = \frac{d^\alpha x(t)}{dt^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{x^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (3)$$

wherein, $\Gamma(\cdot)$ is the Gamma function and $n-1 < \alpha < n \in \mathbb{Z}^+$. Apparently, in the limit case $\alpha \rightarrow 1^-$, the first-order derivative $\dot{x}$ can be obtained from Eq. (3). That is, $\dot{x} = \lim_{\alpha \rightarrow 1^-} D_t^\alpha x$ and the Eq. (2) changes to the classical integer-order mean-field coupled van der Pol system, where oscillation quenching behaviors shows with the variation of the coupling intensity.

3 Discontinuous transitions on coupled fractional-order system

We are interested here in the transition process from oscillatory state to the stationary state. To quantify the state at each coupling strength, i.e., to distinguish the oscillatory state and steady state, we compute a quantify indicators $A(k) \in [0, 1]$ evaluated at a sample coupling strength k as the order parameter. In fact, $A(k)$ is related to the normalized average of $a(k) = \frac{\sum_{i=1}^N [(x_{i,\max})_t - (x_{i,\min})_t]}{N}$, $i = 1, 2$. The variable $a(k)$ indicates the difference between the global maximum and the minimum longtime average amplitude for a given coupling strength k when the system undergoes a sufficiently large transient time. The scaled average amplitude is given by,

$$A(k) = \frac{a(k)}{a(0)}, \quad (4)$$

Thus, $A(k) = 0$ terms to the 'death' state and $0 < A(k) \leq 1$ represents the oscillatory state. It is noted that the order parameter $A(k)$ is obtained numerically by the Runge-Kutta scheme. Owing to the fact that we are concentrating on investigating ED in fractional-order system, we simulate the system adiabatically in both forward and backward directions through a continuation increase and decrease of coupling strength with

step $\delta k = 0.02$. So far it has been reported [29] that in the limit case $\alpha \rightarrow 1^-$ for $N = 100$ coupled van der Pol oscillators, the order parameter $A(k)$ shows a sudden fall to zero with the increase of coupling strength. Still, as the coupling strength decrease, $A(k)$ performs an abrupt jump from zero to a value which is repainted in Fig. 1(a) in the case of $N = 2$, indicating the first-order transition appears in the integer-order mean-field coupling system is independent of the number of oscillators. On further observation of the time series near the critical transition points in Fig.1(b), this phenomenon is identified as explosive NAD.

Fig. 1 (a) The variation of order parameter $A(k)$ along with the variation of coupling strength k both in forward (orange) and backward (blue) directions to show explosive death. (b1-b2) Time transient of the coupled system near the critical forward and backward transition points. The other parameters are identified as $N = 2$, $b = 2$, $\alpha = 1$, $Q = 0.5$.

For the proposed consideration, we consider the numerical results on a system of $N = 2$ coupled fractional-order system with a fractional derivative $\alpha = 0.8$ in Fig.2. As expected, we observe that the order parameter $A(k)$ still exhibit a sharp, irreversible transition whenever we increase the coupling strength k from zero to the maximum values $k_{\max}$ or decrease the k from $k_{\max}$ to zero in Fig.2(a). Most certainly, these two sudden jumps emergent at different value of coupling strength k , showing a hysteresis area (HA) encircled by the forward and backward transition points k_f , k_b . However, when we reduce the coupling strength from the maximum k stacked in the forward direction, the transition path in the backward direction (blue lines) does not overlap the changing curve in the forward continuation (orange lines) after going through the backward transition point. That is, the oscillators cannot be restored to the initial state when the fractional derivative $\alpha = 0.8$ was introduced, which is definitely a far cry from the previous research. Note that both the critical forward (the yellow star) and backward (the green cross) transition points are moving advanced compared to the integer order system in Fig.1(a). To better characterize the properties of the emerging explosive death, we then portray the corresponding time transient of the coupled system near the critical forward and backward

transition points in Fig.2(b1) and 2(b2). We can infer that the oscillators synchronize for a smaller value of k . Then with the coupling strength getting larger, the synchronous behavior disappears immediately after the critical forward transition point $k_f = 1.0$. Interestingly, at the same time, the oscillators stabilized at two inhomogeneous steady branches. Namely, once the fractional derivative exists, the system will preferentially stabilize at the OD state rather than the homogenous equilibrium. In fact, when the coupling strength k is becoming smaller with the decrement, figure.2(b2) reveals that the OD state is lost as soon as the backward transition point $k_b = 0.72$ (less than k_f) reaches. It can be clearly seen that the system oscillates with two opposite amplitudes after this critical coupling strength, indicating that the system reaches an anti-synchrony state rather than initial synchronization. This is not surprising, since the initial state for the backward direction is inhomogeneous at $k = k_{\max}$, the anti-synchronized state is easier to obtain. Thus, not only the hysteresis area of OD and synchronized state is observed, but also a fresh region of synchronization and anti-synchronization appears. Moreover, the results above are independent of the number of coupled oscillators. In fact, in the case of $N = 100$, the explosive OD phenomenon could also appear, which has the same conclusion with the case $N = 2$.

3.1 Amplification of the hysteresis area

When the explosive discontinuous transition happens, there must exist divergent critical transition points in two directions owing to the adiabatic methods. The region (nondimensional area) enclosed by these two points is so called hysteresis area which is known as the coexistence of OD and synchronization in the previous section. To achieve more enhancement with the characteristics of HA in the fractional-order system, we perform the variation of forward transition point k_f , backward transition point k_b and HA width along with the decrease of fractional derivative α from 1 to 0 by decrement $\delta\alpha = 0.01$ in Fig.3 for a fixed $Q = 0.5$, and some interesting conclusions are drawn. Specifically, a tall hysteresis area is clearly found to pop up for $\alpha = 0.99$ in Fig.3(a), meaning that the explosive OD happens even for $\alpha = 0.99$. In other words, there is bound to exhibit explosive OD not explosive NAD once the fractional derivative is introduced. Then we decrease α and observe a uniform reduction of the hysteresis region. Until the threshold value $\alpha_{\min} = 0.72$ arrive, the forward and backward transition points coincide, indicating that the appearance of hysteresis loop made up of OD and oscillatory state is closely connected with

fractional derivatives. In fact, the critical forward and backward transition points in the current paper are described as the points after the state variation, thus, when $k_b - k_f = \delta k = 0.02$ is established, we believe the two critical points overlaps. It is tentatively suggested that, the hysteresis region is decreasing with the reduce of fractional derivative. More to the point, the fractional derivative $\alpha > 0.72$ is what we see as a qualifying condition to ensure the occurrence of explosive OD. Besides, we note that, with the increase of α , region of hysteresis area enlarges linearly. Thereby, the variation of HA width with the fractional derivative is well fitted by the linear function $HA\ width = 3.2699\alpha - 2.3277$ (dotted line in Fig.3(b)).

Encouraged by the interesting observation on the occurred condition of the explosive OD in fractional-order system, we now proceed to vary the intensity of the mean field Q and investigate the peculiarity of the explosive transition. In fact, we start with the value $Q = 0$ and increase it with increment $\delta Q = 0.1$ and go up to $Q = 0.9$ as described in Fig.4. We note that, the threshold value $\alpha_{\min}$ doesn't growth with the enlargement of the mean-field intensity Q . Further, the intensity of the mean field could implement as a regulator to achieve the reviving of explosive behavior. In other words, to take a small example, in the case $Q = 0.1$, the explosive OD will vanish if the fractional derivative fit with $\alpha < 0.8$. Then we increase Q to $Q = 0.4$, the fractional derivative α in the interval of $0.72 < \alpha \leq 0.8$ could induce explosive behavior. In this way, the value Q could adjust the occurrence of the first-order transition. Thus, it verified that the mean field intensity plays a vital role of controlling the qualifying condition of the explosive OD.

Moreover, for detail investigation, we then compute the critical transition points during the forward and backward continuation with the increase of the mean field intensity for different fractional derivative α in Fig.5(a-b). On the one hand, for a fixed value Q , a minor enhancement of the forward transition point k_f and backward transition point k_b when we enlarge α is observed. On the other hand, in the case of an established α , k_f perform a tiny enhancement at first, then presents a substantial increment while $Q > 0.4$. At the same time, k_b remain unchanged no matter how we vary Q . In fact, it is not surprising to note that the HA width in Fig.5 (c) performs the similar results with the forward transition points as it obtained by subtracting the value of backward transition point from the value of forward transition point. Meaning that the fractional derivative and the mean field intensity could influence the appearance of the discontinuous transition from synchronization to stationary state in the forward

Fig. 2 (a) The change of order parameter $A(k)$ along with the variation of coupling strength k both in forward (orange) and backward (blue) directions on the coupled fractional-order system with $\alpha = 0.8$. (b1-b2) Time transient of the system near the critical transition points to show the explosive OD. The other parameters are the same as fig.1.

Fig. 3 (a) The variation of forward (yellow stars) and backward (green triangle represents the numerical result and black line is theoretical result) transition points along with the coupling strength k on the coupled fractional-order system with $Q = 0.5$. (b) The change of HA width with the change of fractional derivative α . Besides, the $\alpha_{\min}$ is marked if $k_b - k_f = \delta k = 0.02$ first appears as we decrease α from 1. The other parameters are the same as the previous case above.

Fig. 4 The threshold value of fractional derivative $\alpha_{\min}$ of explosive OD is shown as a function of the mean field intensity Q for $N = 2$, $b = 2$.

direction. Besides, the fractional derivative also acts an important role to adjust the backward transition from

OD to anti-synchronized state. However, the backward transition point is entirely unrelated to the mean field intensity that is in good agreement with the theoretical results (see details in Sec 3.2), which indicates that the backward transition point is independent of the mean field intensity Q .

3.2 Analytical analysis of critical transition point

To further enunciate the transition in the explosive behavior, we move to some theoretical analysis via Lyapunov indirect method in the next step of our study. Specifically, the fixed points of the coupled fractional-order system can be divided into three categories: (a) trivial steady state $E_0 = (0, 0, 0, 0)$. (b) nontrivial steady states, namely, the homogeneous steady states (HSS)

Fig. 5 The change of (a) Forward transition points, (b) Backward transition points and (c) The HA width are shown along with the enhancement of mean field intensity Q when the fractional derivative α varies from 0.74 to 0.86 by increment 0.01 (Green triangle, blue diamond, yellow pentagon, orange asterisk, red cross, purple round, pink hexagram) on the coupled fractional-order system. The other parameters are the same as in the previous case.

$E' = (x', y', x', y')$, wherein, $x' = \pm \sqrt{1 - \frac{1}{bk(1-Q)}}$, $y' = k(1-Q)x'$ and they exist when the coupling strength fits the condition $k > 1/b(1-Q)$. (c) inhomogeneous steady states (IHSS) $E^* = (x^*, y^*, -x^*, -y^*)$ appearing with the symmetry breaking with $x^* = \sqrt{1 - \frac{1}{bk}}$ and $y^* = kx^*$. Note that the IHSS exist under the condition of $k > 1/b$. Moreover, the IHSS is independent of the mean-field density, which is showing detailly from the form of the inhomogeneous fixed point. From the results presented above, it is clear observed that the stationary state of mean field coupling fractional-order system is OD. Thus, we mainly adopt the Lyapunov indirect method on the coupling related IHSS E^* . Thereby, we perform the linear stability analysis in the inhomogeneous equilibrium E^* to obtain the stable boundary, in other words, the critical transition point. The characteristic equation is given by,

$$\left[\lambda^2 - \left(\frac{1}{k} - k(Q-1) \right) \lambda + 2bk - 2 + Q \right] \times \left[\lambda^2 - \left(\frac{1}{k} - k \right) \lambda + 2bk - 2 \right] = 0. \quad (5)$$

Then the corresponding eigenvalues are described as,

$$\lambda_{1,2}^* = \begin{cases} \frac{1}{2} \left(\frac{1}{k} - k(1-Q) \pm \sqrt{-\Delta_1} i \right), & \text{if } \Delta_1 < 0 \\ \frac{1}{2} \left(\frac{1}{k} - k(1-Q) \pm \sqrt{\Delta_1} \right), & \text{if } \Delta_1 > 0 \end{cases}, \quad \lambda_{3,4}^* = \begin{cases} \frac{1}{2} \left(\frac{1}{k} - k \pm \sqrt{-\Delta_2} i \right), & \text{if } \Delta_2 < 0 \\ \frac{1}{2} \left(\frac{1}{k} - k \pm \sqrt{\Delta_2} \right), & \text{if } \Delta_2 > 0 \end{cases}. \quad (6)$$

Herein, $\Delta_1 = \left(\frac{1}{k} + k(1-Q) \right)^2 - 8bk + 4$, $\Delta_2 = \left(\frac{1}{k} + k \right)^2 - 8bk + 4$. According to the stability theorem of fractional systems proposed by Matignon [49], the fixed points E^* is asymptotically stable if all the eigenvalues λ_m^* ($m = 1, 2, 3, 4$) corresponding to the linear matrix content the criterion,

$$|\arg(\lambda_m^*)| > \frac{\alpha\pi}{2}. \quad (7)$$

That is, if the absolute values of all the eigenvalues corresponding to the linear matrix are greater than $\alpha\pi/2$, the nonlinear system is asymptotically stable. Hence, the stable inhomogeneous equilibrium of cou-

pled fractional-order oscillators, i.e., OD state can be determined as following,

$$\left\{ \begin{array}{l} k_b = \frac{1}{b}, \text{ if } 0 < b \leq \sqrt{1-Q}, \\ \alpha < \min \left\{ \begin{array}{l} \frac{2}{\pi} \arctan \left| \frac{\sqrt{-\Delta'_1}}{\frac{1}{k_b} - (1-Q)k_b} \right|, \\ \frac{2}{\pi} \arctan \left| \frac{\sqrt{-\Delta'_2}}{\frac{1}{k_b} - k_b} \right| \end{array} \right\}, \\ \text{if } b > \sqrt{1-Q}. \end{array} \right. \quad (8)$$

Wherein, $\Delta'_1 = \left(\frac{1}{k_b} + k_b(1-Q) \right)^2 - 8bk_b + 4$, $\Delta'_2 = \left(\frac{1}{k_b} + k_b \right)^2 - 8bk_b + 4$. Therefore, an explicit illustration of the backward critical transition point is exhibited. In fact, when the damping coefficient b is smaller than $\sqrt{1-Q}$, the backward transition k_b is $1/b$. Otherwise if the damping coefficient b and the mean-field density Q meet with the situation $b > \sqrt{1-Q}$, a relationship between the fractional derivative α and the critical backward transition point k_b is drawing clearly by the expression in Eq. (8). One final thing to note here is that, on the condition of Fig.3 and Fig.5, we can examine that the backward critical transition point satisfies the criteria $\alpha = \frac{2}{\pi} \arctan \left| \frac{\sqrt{-\Delta'_2}}{\frac{1}{k_b} - k_b} \right|$ that is marked as black solid line in Fig.3(a). Besides, one can find that the criteria have no relationship to the mean-field density Q which is fitted with the numerical results in Fig.5(b). Therefore, it verified that the backward transition point is independent of the mean-field intensity.

4 Conclusion

The present study has dealt with the problem of explosive phenomenon on a system of mean-field coupling fractional-order nonlinear oscillators. We demonstrate the setting of a sudden and irreversible transition from oscillatory to stationary states, where it is observable even when the fractional derivative is proposed to describe the chemical properties and special materials in real world. It is uncovered that, once the fractional derivative is introduced, the system prefers to stabilize at the inhomogeneous steady state rather than the homogenous equilibrium. Meanwhile, it is indicated that fractional derivative exhibits an amplification of the hysteresis area enclosed by the critical forward and backward transition points of the explosive transition. Indeed, our result is not limit to the case of two coupled oscillators, which is fully robust against the large amount of the oscillators. Moreover, for further consideration on the role of the fractional derivative, it could

be revealed that, there is a minimum fractional derivative for inducing explosive behavior when the mean-field density is fixed. In other words, the appearance of the first-order discontinuous transition could be adjusted via the fractional derivative. Interestingly, the threshold value $\alpha_{\min}$ doesn't increase with the enhancement of the mean-field density Q . Thus, an appropriate mean-field density and sufficiently large fractional derivative is liable to induce the emergence of explosive behavior.

Besides, it is further verified that the backward transition point has no relationship with the mean-field density numerically and theoretically. The mean-field density could just regulate the forward direction to control the coexistence area of oscillatory and stationary state. Notably, our findings mainly focus on a new attempt in the fields of investigation and application of the explosive transition in natural systems showing fractional behaviors. Our results about understanding the role of fractional derivative on collective behaviors especially in oscillation quenching can be constructive in absorbing the underlying mechanism of many realistic chemical and physical systems with properties of memory, non-locality and history dependence.

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Declarations

Conflict of Interest: The authors declare that they have no conflict of interest, with respected to the research, authorship and publication.

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