Heat Transfer Analysis For Oscillating Flow of Magnetized Fluid By Using The Modified Prabhakar-Like Fractional Derivatives

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Heat transfer analysis for oscillating flow of magnetized fluid by using the modified Prabhakar-like fractional derivatives

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Abstract:

In this analysis, an unsteady and incompressible flow of magnetized fluid in presence of heat transfer has been presented with fractional simulations. The oscillating plate with periodically variation has induced the flow. The model is formulated in terms of partial differential equations (PDE's). The traditional PDEs cannot analyze and examine the physical behavior of flow parameters with memory effects. On this end, the solution approach is followed with the efficient mathematical fractional technique namely Prabhakar fractional derivative. The non-dimensional leading equations are transformed into the fractional model and then solved with the help of the Laplace transformation scheme. The effects and behavior of significant physical and fractional parameters are analyzed graphically and numerically. As a result, we have concluded that the temperature and velocity profiles decrease with the enhancement of fractional parameters. Furthermore, with time both (temperature and velocity fields) decreasing away from the plate and asymptotically increases along y -direction, which also satisfies the corresponding conditions.

Keywords: Heat transfer, MHD flow, oscillating flow, Prabhakar-like fractional derivatives.

1. Introduction

The investigations regarding the heat transfer phenomenon is important because all the industrial mechanism and engineering processes involves this phenomenon. The key importance of heat transfer is inspected in the thermal systems, nuclear processes, gas processing, various chemical processes, refining industries, electronics etc. This chemical

processes and processing industries also involves the heat transfer significances like cooling and heating of different chemical equipment and production of chemical materials. The modeling of heat transfer is based on the famous Fourier's theory. Following to such novel implications in mind, a variety of research has been done by investigators. Turkyilmazoglu [1] inspected the heat transfer fluctuation for the viscoelastic fluid with distinct types of physical properties. Mishra and Bhatti [2] observed the influences of Ohmic heating for the convectively heated problem via numerical simulations. Nayak et al. [3] presented the heat transfer investigation for the magnetized material subject to the saturated porous space. Alamri et al. [4] presented the novel thermal contribution regarding the heat transfer phenomenon in stretched cylinder. The vertical flow determination for the heat transfer problem with modified Fourier's expressions has been worked out by Li et al. [5]. Waini et al. [6] focused on the participation of hybrid nanoparticles for improving the heat characteristics against the shear flow. Hayat et al. [7] utilized the melting heating applications for the variable thickness flow subject to the chemical aspect. Hassan et al. [8] observed the heat transfer rate for the shear thinning type nano-materials with hybrid nanoparticles with experimental justifications. The heat pattern in the horizontally moving space with radiative phenomenon was identified by Benos et al. [9]. Mahanthesh et al. [10] presented the heat transfer analysis for the isothermal wedge flow with characterization of Ag and MoS₂ nanoparticles. Shehzad et al. [11] performed a computation thermal problem focusing the Ohmic and dissipation features. Khan et al. [12] provided the improved heat transfer mechanism based on the slip flow with micropolar nanofluid and thermal radiative approach.

The progress in the fractional

The growing work in the fractional calculus provides modern definitions of various kinds of derivatives and integrals. Due to new characterization of operators, the interest is developed by scientists towards the different types of fractional derivatives. In the world real problems, many fractional techniques are implemented like diffusion problems, thermal modeling, biological problems, plasma physics, bio-chemistry, industrial problem and engineering. In the fractional derivatives, the descriptions of non-local and local definitions are quite interesting. Amongst both, the non-local derivatives are assumed to be more novel as these derivatives convey the history of function. The concept of fractional simulations is completed with the different types of functions and definitions. One of the most famous fractional research involving the fractional derivatives was reported by Caputo and Fabrizio [13]. This research successfully explained the aspects of non-local kernels and phenomenon. Harrouche et al. [14] implemented the approach of Caputo-Fabrizio (CF) derivatives for performing the simulations for the drug

pharmacokinetic problem. Moore et al. [15] used the Caputo and Fabrizio model for the treatment of HIV/AIDS disease problem. The implementations of Caputo-Fabrizio derivatives simulations regarding the diffusion flow problem were worked out by Wei et al. [16]. Alzahrani et al. [17] used the CF- definition for the coronavirus modeling problem. Momani et al. [18] focused on the solution of Bernoulli equation via CF model. The work of Shaikh et al. [19] explained the physical insight of reactive diffusion thermal model where the simulations were carried out by CF approach. The integral approach for the Hermite–Hadamard inequalities with new convex function with help of CF-model has been focused by Wang et al. [20]. The study of Covid-19 diseases with CF implementation was performed by Baleanu et al. [21]. In recent times, some modifications has been done by researchers regarding the CF definition. On this end, Atangana and Baleanu (AB) [22] defined new operators and definitions of fractional simulations known as AB-derivatives. The AB-derivatives is defined on the basis of non-singular kernels which terms as Mittag–Leffler functions. This definition is proved more effective as it does not allow any singularity. While inspecting the comparative task between CF and AB-fractional definitions, the AB derivatives are declared more precise. Sheikh et al. [23] used the AB- fractional technique for the solution of Casson fluid model problem with external chemical reaction assertive. Panda et al. [24] focused on the implementation of same approach for a fractional type Willis aneurysm problem. Moreover, the discussion for the perturbed boundary value system with nonlinear singularity was also provided. Kumar et al. [25] observed the infection and threats of mosaic disease via mathematical modeling and later on fractional approach with AB-deviations was followed for the simulation procedure. Raza et al. [26] used same definitions for assessing the nanoparticles characterization over vertically infinite plate. The AB-formulation for the integro-differential systems was intended by Arjunan et al. [27]. Kumar et al. [26] used the CF and AB-fractional work for the mathematical formulation of a mosaic disease problem. Some more recent work can be seen in refs. [27-32]. This fractional investigation deals with the determination of heat transfer applications for the viscous fluid due to periodically accelerating surface. The applications of inclined magnetic force are also implemented. The model problem is solved with the latest frame work of fractional derivatives namely Prabhakar fractional approach. The results are simulated at various time instant due to periodic oscillation. The physical insight parameters are addressed.

2. Problem description

Consider an unsteady compressible fluid is moving on an infinite inclined plane with the inclination of angle δ by variable temperature. It also supposed a magnetic field of strength B_0 is applied on the plate in the absence of any electric field. Initially at $t = 0$, the plate and fluid

both are in the rest position with constant temperature T_∞ (see Fig. 1). With time $t > 0^+$ the plate starts to oscillate with constant velocity $U_o H(t) \cos(\omega t)$, where ω representing the frequency of oscillation, and the fluid starts to flow on the plate due to oscillations. So with the help of Boussinesq's approximation, the partial differential leading equations that govern this problem can be stated as:

Fig 1: Flow geometry

$$\rho \frac{\partial w_{(\zeta,t)}}{\partial t} = \mu \frac{\partial^2 w_{(\zeta,t)}}{\partial \zeta^2} - \frac{\sigma B_0^2}{\rho} \sin(\theta) w_{(\zeta,t)} + g \beta_T (T_{(\zeta,t)} - T_\infty) \cos(\delta) \quad (1)$$

$$\rho C_p \frac{\partial T_{(\zeta,t)}}{\partial t} = - \frac{\partial q_{(\zeta,t)}}{\partial \zeta} - Q(T_{(\zeta,t)} - T_\infty) \quad (2)$$

$$q_{(\zeta,t)} = -k \frac{\partial T_{(\zeta,t)}}{\partial \zeta} \quad (3)$$

where $q_{(\zeta,t)}$ signifies the thermal flux rate by Fourier's law with its consistent boundary conditions, as follows

$$w_{(\zeta,0)} = 0, \quad T_{(\zeta,0)} = T_\infty, \quad ; \quad \zeta \geq 0 \quad (4)$$

$$w_{(0,t)} = U_o H(t) \cos(\omega t), \quad T_{(0,t)} = T_\infty + (T_w - T_\infty), \quad t > 0 \quad (5)$$

$$w_{(\zeta,t)} \rightarrow 0, \quad T_{(\zeta,t)} \rightarrow T_\infty, \quad as \quad \zeta \rightarrow \infty \quad (6)$$

For the non-dimensional form of the leading equations and consistent conditions, introducing the following non-dimensional values

$$w^* = \frac{w}{U_o}, \quad \zeta^* = \frac{U_o}{v_f} \zeta, \quad t^* = \frac{U_o^2}{v_f} t, \quad T^* = \frac{T_{(\zeta,t)} - T_\infty}{T_w - T_\infty}, \quad q^* = \frac{q}{q_o}$$

into the proceeding governing equations and conditions (1)-(7) and neglecting the steric notation. We obtain the subsequent non-dimensional forms as follows

$$\frac{\partial w_{(\zeta,t)}}{\partial t} = \Omega_1 b_1 \frac{\partial^2 w_{(\zeta,t)}}{\partial \zeta^2} - M \sin(\theta) w_{(\zeta,t)} + Gr T_{(\zeta,t)} \cos(\delta) \quad (7)$$

$$\frac{\partial T_{(\zeta,t)}}{\partial t} = -\Omega_2 \frac{\partial q_{(\zeta,t)}}{\partial \zeta} - F T_{(\zeta,t)} \quad (8)$$

$$q_{(\zeta,t)} = -c_1 \frac{\partial \vartheta_{(\zeta,t)}}{\partial y} \quad (9)$$

with the following non-dimensional conditions

$$w_{(\zeta,0)} = 0, \quad T_{(\zeta,0)} = 0; \quad \zeta \geq 0$$

$$w_{(0,t)} = H(t) \cos(\omega t), \quad T_{(0,t)} = 1, \quad t > 0$$

$$w_{(\zeta,t)} \rightarrow 0, \quad T_{(\zeta,t)} \rightarrow 0; \quad \zeta \rightarrow \infty$$

Where:

$$\Omega_1 = \frac{\tau_o}{\rho U_o^2}, \quad \Omega_2 = \frac{q_o}{\rho C_p (\vartheta_w - \vartheta_\infty) U_o}, \quad b_1 = \frac{\mu U_o^2}{v \tau_o}, \quad c_1 = \frac{k U_o (\vartheta_w - \vartheta_\infty)}{v q_o}$$

$$F = \frac{Qv}{\rho C_p U_o^2}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad Gr = \frac{g(v\beta_T)_f (\vartheta_w - \vartheta_\infty)}{U_o^3}$$

Here for the solution of momentum and energy equations we have exploited an efficient fractional mathematical model well known as Prabhakar fractional derivative, which can be defined mathematically as [33, 34]:

$$\begin{aligned} {}^c \mathcal{D}_{\alpha,\beta,\alpha}^\gamma h(t) &= E_{\alpha,n,-\beta,\alpha}^{-\gamma} h^n(t) = e_{\alpha,n,-\beta}^{-\gamma}(\alpha; t) * h^n(t) \\ &= \int_0^t (t-\tau)^{n-\beta-1} E_{\alpha,n,-\beta}^{-\gamma}(\alpha(t-\tau)^\alpha) h^n(\tau) d\tau \end{aligned}$$

Where ${}^c \mathcal{D}_{\alpha,\beta,\alpha}^\gamma$, $h^{(n)}$ represents the Prabhakar fractional operator and the n th derivative of $h(t) \in AC^n(0, b)$, $AC^n(0, b)$ respectively, and

$$E_{\alpha,\beta,\alpha}^\gamma h(t) = \int_0^t (t-\tau)^{\beta-1} E_{\alpha,\beta}^{-\gamma}(\alpha(t-\tau)^\alpha) h(\tau) d\tau$$

is known as Prabhakar integral with

$$E_{\alpha,\beta}^\gamma(z) = \sum_{m=0}^{\infty} \frac{\Gamma(\gamma+m) z^m}{m! \Gamma(\gamma) \Gamma(\alpha m + \beta)}$$

is of three-parametric Mittag-Leffer function, and $e_{\alpha,\beta}^{\gamma}(\alpha; t) = t^{\beta-1}E_{\alpha,\beta}^{\gamma}(\alpha t^{\alpha})$ is the Prabhakar kernel. The Laplace transform of Prabhakar fractional derivative operator ${}^C\mathfrak{D}_{\alpha,\beta,\alpha}^{\gamma}$ is derived as

$$\begin{aligned}\mathcal{L}\left[{}^C\mathfrak{D}_{\alpha,\beta,\alpha}^{\gamma}h(t)\right] &= \mathcal{L}\left[h^m(t) * e_{\alpha,m-\beta}^{-\gamma}(\alpha; t)\right] = \mathcal{L}\{h^m(t)\}\mathcal{L}\left\{e_{\alpha,m-\beta}^{-\gamma}(\alpha; t)\right\} \\ &= \mathcal{L}\{h^m(t)\}s^{\beta-m}(1-\alpha s^{-\alpha})^{\gamma}\end{aligned}\quad (10)$$

and by taking $\beta = \gamma = 0$, we can obtain the classical Fourier's law. And as the Prabhakar fractional derivative mostly depends on Fourier's law of thermal conductivity, so the Fourier's law in the sense of Prabhakar fractional derivative will become as,

$$q_{(\zeta,t)} = -c_1 {}^t\mathfrak{D}_{\alpha,\beta,\alpha}^{-\gamma} \frac{\partial T_{(\zeta,t)}}{\partial \zeta} \quad (11)$$

3. Solution of the problem

3.1. Solution of the energy field

As the energy equation involve Fourier's law of thermal flux so, by utilizing the LT scheme on Eqs. (8) and (9) for the solution of energy profile and on its corresponding conditions

$$s \bar{T}_{(\zeta,s)} = -\Omega_2 \frac{\partial \bar{q}_{(\zeta,s)}}{\partial \zeta} - F \bar{T}_{(\zeta,s)} \quad (12)$$

$$\bar{q}_{(\zeta,s)} = -c_1 (s^{\beta}(1-\alpha s^{-\alpha})^{\gamma}) \frac{\partial \bar{T}_{(\zeta,s)}}{\partial \zeta} \quad (13)$$

$$\bar{T}_{(0,s)} = \frac{1}{s}, \quad \bar{T}_{(\zeta,s)} \rightarrow 0; \quad \zeta \rightarrow \infty \quad (14)$$

Introducing Eq. (13) into (12), for the solution of temperature field

$$\frac{\partial^2 \bar{T}_{(\zeta,s)}}{\partial \zeta^2} - \frac{1}{\Omega_2 c_1} \frac{F+s}{s^{\beta}(1-\alpha s^{-\alpha})^{\gamma}} \bar{T}_{(\zeta,s)} = 0 \quad (15)$$

By using the above conditions and solving the ordinary differential Eq. (15), the solution of temperature distribution will be yield as

$$\bar{T}_{(\zeta,s)} = \frac{1}{s} e^{-\zeta \sqrt{\frac{1}{\Omega_2 c_1 s^{\beta}(1-\alpha s^{-\alpha})^{\gamma}} (F+s)}} \quad (16)$$

For the inverse of Laplace of Eq. (16), we will use numerical techniques namely as Stehfest and Tzou's algorithms in Table 1.

3.2. Classical solution of Temperature Field ($\beta = \gamma = 0$)

For classical thermal transport take $\beta = \gamma = 0$, so

$$\mathcal{L}[e_{\alpha,0}^0(\alpha; t)] = 1 = \delta(t)$$

Where $\delta(t)$ is the Dirac's Delta distribution. By using this, the generalized Fourier's law will be transformed into classical Fourier's law. And

$$\bar{T}_{(\zeta,s)} = \frac{1}{s} e^{-\zeta \sqrt{\frac{1}{\Omega_2 c_1} (F+s)}} \quad (17)$$

With its Laplace inverse

$$T_{(\zeta,t)} = \frac{1}{2} e^{-\zeta \sqrt{\frac{F}{\Omega_2 c_1}}} \left(2 - \operatorname{Erfc} \left[\frac{2\sqrt{\Omega_2 c_1 F t} - \zeta}{2\sqrt{\Omega_2 c_1 t}} \right] + e^{2\zeta \sqrt{\frac{F}{\Omega_2 c_1}}} \operatorname{Erfc} \left[\frac{2\sqrt{\Omega_2 c_1 F t} + \zeta}{2\sqrt{\Omega_2 c_1 t}} \right] \right)$$

3.3.Solution of the momentum field

In this section, the solution of the momentum equation will be derived with the same procedure as used for the energy equation solution. By utilizing the LT scheme on Eq. (7) and using its corresponding conditions, we obtain

$$s \bar{w}_{(\zeta,s)} = \Omega_1 b_1 \frac{\partial^2 w_{(\zeta,s)}}{\partial \zeta^2} - M \sin(\theta) \bar{w}_{(\zeta,s)} + Gr \bar{T}_{(\zeta,s)} \cos(\delta) \quad (18)$$

$$\bar{w}_{(0,s)} - \frac{s}{s^2 + \omega^2} = 0, \quad \bar{w}_{(\zeta,s)} \rightarrow 0; \quad \zeta \rightarrow \infty$$

Using these conditions, the solution of the momentum equation will become as

$$\begin{aligned} \bar{v}_{(y,s)} = & \frac{s}{s^2 + \omega^2} e^{-y \sqrt{\frac{s+M}{\Omega_1 b_1}}} \\ & + \frac{Gr \cos(\delta)}{\Omega_1 b_1} \frac{1}{\frac{1}{\Omega_2 c_1} \frac{F+s}{s^\beta (1-\alpha s^{-\alpha})^\gamma} - \frac{s+M}{\Omega_1 b_1}} \left(\frac{e^{-y \sqrt{\frac{s+M}{\Omega_1 b_1}}}}{s} \right. \\ & \left. - \frac{e^{-y \sqrt{\frac{1}{\Omega_2 c_1 s^\beta (1-\alpha s^{-\alpha})^\gamma} \frac{F+s}{s}}}}{s} \right) \end{aligned} \quad (24)$$

To find out the Laplace inverse, different authors have used different numerical inverse methods. Therefore, here we will also use the Stehfest algorithm to analyze the solution of temperature and velocity profile numerically. Gaver-Stehfest algorithm [35] mathematically can be defined as

$$w(\xi, t) = \frac{\ln(2)}{t} \sum_{n=1}^N v_n \bar{w} \left(\xi, n \frac{\ln(2)}{t} \right)$$

Where N is a positive integer, and

$$v_n = (-1)^{n+\frac{N}{2}} \sum_{r=\lceil \frac{q+1}{2} \rceil}^{\min(q, \frac{N}{2})} \frac{r^{\frac{N}{2}} (2r)!}{\left(\frac{N}{2} - r\right)! r! (r-1)! (q-r)! (2r-q)!}$$

However, we have also used another approximation for the solution of temperature and velocity field, Tzou's algorithm for the validity and comparison of our attained numerical results by the Stehfest method. Tzou's method can be defined as mathematically

$$w(\xi, t) = \frac{e^{4.7}}{t} \left[\frac{1}{2} \bar{w} \left(r, \frac{4.7}{t} \right) + Re \left\{ \sum_{j=1}^N (-1)^k \bar{w} \left(r, \frac{4.7 + k\pi i}{t} \right) \right\} \right]$$

Where i is the imaginary unit and $Re(\cdot)$ is the real part and $N > 1$ is a natural number.

Results and Discussion

In this article, an unsteady viscous and incompressible MHD fluid flow on an inclined infinite plate with temperature is considered. The non-dimensional leading equations are transformed into a fractional model with help of a new mathematical fractional model namely as Prabhakar fractional derivative. The solution of the transformed fractional model is obtained with the help of LT and different inverse numerical techniques are utilized for the Laplace inverse of governing equations. For the illustration of different parameters on velocity and temperature profiles, figures (2-7) are plotted. In Fig. 2(a-c) the effects of fractional constraint on the temperature profile are highlighted by changing the value of the fractional parameter at different values of the time. This can be seen that the variation in fractional constraint the temperature profile decreases, whereas temperature decreasing away from the plate and asymptotically increases along y -direction, which is also referring to the corresponding conditions. Similarly, the behavior of the fractional constraint on the velocity field is displayed in Fig 3(a-c) at different times. This can be seen that in the figure that the velocity profile also decreases with the enhancement in the fractional parameter where with the passage of time velocity field asymptotically increases. In Fig. 4(a-c), the effects of the Grashof number on velocity profile are observed by varying the values of the Grashof number and keeping other values constant. It is observed that enhancement in the values of Grashof number speeds up the motion of the fluid. Physically, an enhancement in Grashof number increases the bouncy forces which rises the thickness of the momentum boundary layer and speed up the fluid motion. Figs. 5 and 6(a-c) highlighting the influence of the angle of inclination of the Magnetic field and the effects of magnetic field parameter on the velocity profile. The velocity field was found to decreases by increasing the values of both parameters, the angle of inclination of magnetic field and magnetic parameter. Physically, an increase in the magnetic field results in to increase in Lorentz force which decreases the velocity of the fluid.

The comparison of the numerical techniques namely Stehfest and Tzou's for temperature and velocity field is plotted in Fig 7(a) and 7(b) respectively. As plotted in the figures, the curves of both numerical techniques overlap with each other for both temperature and velocity field, which represents the validation of our obtained results. Finally, in the end, the numerical

analysis of the temperature and velocity profile is analyzed in Table 1 different numerical schemes.

4. Conclusions

In this work, the solution of unsteady and incompressible fluid flowing on an inclined plate is solved by the mathematical fractional scheme namely as Prabhakar-like thermal fractional technique. The non-dimensional fractional leading equations are solved with help of the LT scheme and different numerical techniques are utilized for the Laplace inverse of temperature and velocity profiles. The influence of different constraints on leading equations is examined graphically and numerically. The main points confabulated from graphical and numerical representation can be bullets as

- The temperature field decreases by raising the values of the fractional parameters and asymptotically increases with time.
- The momentum field also decreases with the increase in the values of fractional constraint.
- Velocity profile speeds up by enhancing the value of Grashof number Gr due to increases in buoyancy effect.
- Temperature and velocity profiles both asymptotically increases with the passage of time.
- The solution curves of both numerical techniques overlap with each other which validates our obtained results.

Declaration Statements: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Availability of data and material: All data that support the findings of this study are included within the article (and any supplementary files).

Competing interests: The authors declared that they have no conflict of interest and the paper presents their own work which does not infringe any third-party rights, especially authorship of any part of the article is an original contribution, not published before and not being under consideration for publication elsewhere.

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Fig 2: Temperature profile with the variation in fractional parameters at different time

Fig 3: Velocity profile with the variation in fractional constraints $Pr = 6.5$, $Gr = 0.75$, $M = 0.5$, $\omega = 0.5$, $\delta = \frac{\pi}{4}$, $\theta = \frac{\pi}{4}$

Fig 4: Velocity profile with the variation in Grashof number $\alpha = \beta = \gamma = 0.5$, $Pr = 6.5$, $M = 0.5$, $\omega = 0.5$, $\delta = \frac{\pi}{4}$, $\theta = \frac{\pi}{4}$

Fig 5: Velocity profile with the variation in the angle of inclination of magnetic field $\alpha = \beta = \gamma = 0.5$, $Pr = 6.5$, $Gr = 0.75$, $M = 0.5$, $\omega = 0.5$, $\delta = \frac{\pi}{4}$, $\theta = \frac{\pi}{4}$

Fig 6: Velocity profile with the variation in Magnetic field $\alpha = \beta = \gamma = 0.5$, $Pr = 6.5$, $Gr = 0.75$, $\omega = 0.5$, $\delta = \frac{\pi}{4}$, $\theta = \frac{\pi}{4}$

Fig 7: Velocity and temperature profiles comparison for numerical techniques

Table 1: Numerical analysis of temperature and velocity profile by Stehfest and Tzou's method

ζ	$T_{(\zeta,t)}$ by Stehfest	$T_{(\zeta,t)}$ by Tzou's	$w_{(\zeta,t)}$ by Stehfest	$w_{(\zeta,t)}$ by Tzou's
0.1	0.9763	0.9862	0.6177	0.6172
0.3	0.6635	0.6871	0.7328	0.7312
0.5	0.4444	0.4629	0.7793	0.7770
0.7	0.2931	0.3072	0.7802	0.7775
0.9	0.1903	0.2009	0.7516	0.7487
1.1	0.1217	0.1294	0.7051	0.7022
1.3	0.0766	0.0821	0.6489	0.6460
1.5	0.0475	0.0513	0.5885	0.5858
1.7	0.0290	0.0316	0.5278	0.5253
1.9	0.0174	0.0192	0.4690	0.4667

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