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A Hybrid Space-Time Modelling Approach for Forecasting Monthly Temperature

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Abstract

Spatio-temporal forecasting has various applications in climate, transportation, geo-statistics, sociology, economics and in many other fields of study. The modelling of temperature and its forecasting is a challenging task due to spatial dependency of time series data and nonlinear in nature. To address these challenges, in this study we proposed hybrid Space-Time Autoregressive Moving Average-Generalized Autoregressive Conditional Heteroscedasticity (STARMA-GARCH) model in order to describe and identify the behaviour of monthly maximum temperature and temperature range in Bihar. At the modelling process of STARMA, spatial characteristics are incorporated into the model using a weight matrix based on great circle distance between the regions. The residuals from the fitted STARMA model have been tested by Brock, Dechert, and Scheinkman (BDS) and Autoregressive Conditional Heteroscedasticity-Lagrange Multiplier (ARCH-LM) test for the behaviour of nonlinearity and ARCH effect respectively. The test results revealed that presence of both nonlinearity and ARCH effect. Hence GARCH modelling is necessary. Therefore, the hybrid STARMA-GARCH model is used to capture the dynamics of monthly maximum temperature and temperature range. The results of the proposed hybrid STARMA (1₁, 0, 0) – GARCH (0, 1) model has better modelling efficiency and forecasting precision over STARMA (1₁, 0, 0) model.

Keywords: STARMA, ARCH/GARCH, temperature, nonlinearity, spatial weight matrix.

1. Introduction

Agricultural sectors are highly influenced by weather and climatic factors. It is sensitive to short-term changes in weather and long term variations in climate. Weather parameters variability in small regions, a few kilometres in size, and it is important in many hydrological, agricultural and energy contexts. Agricultural yields are directly and indirectly affected by changes in climatic factors such as, rainfall, temperature, moisture, solar radiation etc., and the severity of extreme events like, floods, droughts, wind storms etc. Due to the effect of global warming there is a significant increase in global greenhouse gases and as a result there is steady rise in temperature of the planet earth. Therefore, mitigating climate change is one of the biggest challenges before humankind.

Temperature is the most important environmental factors at the earth surface. Forecasting of air temperature has been a crucial environmental factor required in many different areas such as agriculture, industry, energy, environment, tourism, etc. Therefore, there is a need of systematic and scientific prediction of temperature for better planning, policy making, and disaster management in the developing countries like India. There are various statistical methods used in forecasting monthly temperature. One of the most popular statistical methods used for forecasting temperature time series is Box-Jenkins' Autoregressive Integrated Moving Average (ARIMA). This model is suitable for only univariate and linear time series data. If the time series data set has non-linear and spatio-temporal relationship, then ARIMA model is not suitable.

The extension of univariate ARIMA time series models into the spatio-temporal domain results in a general class of Space-Time Autoregressive Integrated Moving Average (STARIMA) models (Cliff and Ord, 1975a; Cliff et al. (1975)). It is used for the computation of linear dependencies between the variables in both time and space.

Application of spatial statistics mainly found in geo-statistics but in recent years the application of spatial statistics has been extended to sociology, economics, and environmental, ecological and agricultural sciences. Mention may be made of some recent works which show interest in spatio-temporal models, using both spatial and temporal information, for example; Kamarianakis and Prastacos (2005) work on traffic flow data set, Kurta and Tunay (2015) developed STARMA model using of regional bank deposits data set, Zhao et al. (2018) Modelling on wind power data set, Rathod et al. (2018) modelling and forecasting on temperature data set, and Saha et al. (2020) developed forecasting model on rainfall data set. STARMA models are special cases of Vector Autoregressive Moving Average (VARMA) models but VARMA models involve more number of parameters compared to STARMA models (Brynjarsdottir and Berliner 2014), and Gupta and Robinson 2015)). However, one main disadvantage of using STARMA model is that it cannot deal with nonlinear data series. To analyse data sets with nonlinearity there are many statistical techniques such as ARCH technique first use by Engle (1982), GARCH technique (Bollerslev 1986), Threshold Autoregressive (TAR) technique (Tong and Lim 1980) etc.

In this paper an attempt has been made to develop a hybrid STARMA-GARCH model in order to investigate and predict spatio-temporal behaviour of monthly maximum temperature and temperature range. There are various hybrid statistical techniques available in the literature for modelling the rainfall data (Yusof et al. (2013)), forecasting the exchange rate data (Pahlavani and Roshan 2015), forecasting oil price data (Chaido Dritsaki 2018) but they all deal with only temporal data series. In real applications, weather data (rainfall, temperature, humidity etc.) constitute of dynamics involving spatial, temporal, linear and nonlinear features. Very few literatures have been found on spatio-temporal time series data which deal with nonlinear phenomena. Hence, this paper has made an attempt to develop a hybrid STARMA-GARCH model that can capture the spatio-temporal behaviour with linear and nonlinear dynamics of the data series under consideration.

2. Material and Methods

2.1. Study data

The monthly maximum temperature and temperature range of Bihar have been used here. The state of Bihar is categorized into four comprehensive agro-climatic zones viz., Zone-I (North-West Alluvial Plains), Zone-II (North-East Alluvial Plains), and Zone-III (South Alluvial Plains). Further, Zone-III is divided into two parts Zone-III-A and Zone-III-B. For the present study, monthly maximum temperature and temperature range of eight districts of Zone-III-B, viz., Patna (PAT), Nalanda (NAL), Bhojpur (BHO), Rohtas (ROH), Gaya (GAY), Jahanabad (JAH), Aurangabad (AUR), and Kaimur (KAM) has been considered, which are presented in **Fig. 1**. The data set covers the period from January, 1981 to December, 2020. The data set on temperature has been collected from the NASA prediction of worldwide energy resources (<https://power.larc.nasa.gov/data-access-viewer/>). The data set from January, 1981 to December, 2019 and data set from January, 2020 to December, 2020 have been used for the purpose of model building and model validation respectively.

Fig. 1: Geographical map of Bihar

2.2. STARMA Model

It is to be noted that the space-time data series is a vector time series. The m -dimensional VARMA process of order (p, q) is given by:

$$\mathbf{Z}_t = \sum_{k=1}^p \phi_k \mathbf{Z}_{t-k} + \boldsymbol{\varepsilon}_t - \sum_{k=1}^q \theta_k \boldsymbol{\varepsilon}_{t-k} \quad (1)$$

where $\mathbf{Z}_t$ is the $N \times 1$ vector of observations at time t at N locations and $\boldsymbol{\varepsilon}_t$ is the random error vector with $\boldsymbol{\varepsilon}_t \sim N(0, \sigma^2 \mathbf{I}_N)$ independently distributed at each time t , and $\sigma^2 \mathbf{I}_N$ is an $N \times N$ positive definite variance matrix. Further, $\phi_1, \dots, \phi_p$ are the autoregressive coefficients and $\theta_1, \dots, \theta_q$ are the moving average coefficients at all $N \times N$ matrices with $\theta_0 \equiv \mathbf{I}$. VARMA is not capable of modelling the spatial relationships and no assumptions of spatial stationarity can be made. Thus, incorporation of spatial dependence into a general VARMA model, turns VARMA model into STARMA model. A STARMA model is expressed as:

$$\mathbf{Z}_t = \sum_{k=1}^p \sum_{l=0}^{\lambda_k} \phi_{kl} \mathbf{W}^l \mathbf{Z}_{t-k} - \sum_{k=1}^q \sum_{l=0}^{m_k} \theta_{kl} \mathbf{W}^l \boldsymbol{\varepsilon}_{t-k} + \boldsymbol{\varepsilon}_t \quad (2)$$

where $\mathbf{Z}_t$ is the $N \times 1$ vector of observations at time t at the N locations. p is the order of autoregressive, q is the order of moving-average, λ_k is the spatial order of the k^{th} autoregressive term, m_k is the spatial order of the k^{th} moving-average term, ϕ_{kl} is the autoregressive parameter at temporal lag k and spatial lag l (scalar), θ_{kl} is the moving average parameter at temporal lag k and spatial lag l (scalar), $\mathbf{W}^l$ is the $N \times N$ spatial weight matrix with spatial order l and $\boldsymbol{\varepsilon}_t$ is the random disturbances are assumed to be normally distributed with

$$E[\boldsymbol{\varepsilon}_t] = 0,$$

$$E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{(t+s)}] = \begin{cases} \sigma^2 \mathbf{I}_N, & s = 0 \\ 0, & s \neq 0 \end{cases}, \text{ and } E[\mathbf{Z}_t \boldsymbol{\varepsilon}'_{t+s}] = 0, \text{ for } s > 0$$

The STARMA $(p_{\lambda_1 \dots \lambda_p}, q_{m_1 \dots m_q})$ becomes a STAR model when $q = 0$, which is expressed as:

$$\mathbf{Z}_t = \sum_{k=1}^p \sum_{l=0}^{\lambda_k} \phi_{kl} \mathbf{W}^l \mathbf{Z}_{t-k} + \boldsymbol{\varepsilon}_t \quad (3)$$

when $p = 0$, it becomes a STMA model, which is expressed as:

$$\mathbf{Z}_t = \boldsymbol{\varepsilon}_t - \sum_{k=1}^q \sum_{l=0}^{m_k} \theta_{kl} \mathbf{W}^l \boldsymbol{\varepsilon}_{t-k} \quad (4)$$

The STARMA model-building procedure is similar to univariate ARIMA (Box and Jenkins 1976) model building procedure viz., identification of the model, estimation of the parameters, and diagnostic checking of the appropriateness of model for the observed data (Pfeifer and Deutsch 1980, 1980c, 1981, 1981a, b).

- 2.2.1. Preliminary Stage:** Similar to ARIMA model, the first step is always to check for stationarity of time series data. If the time series data is nonstationary, then differencing should be done.
- 2.2.2. Identification:** After making stationary of data series, next step is to examination of sample space-time autocorrelation (ST-ACF) and space-time partial autocorrelation functions (ST-PACF) with incorporate the space weight matrices in the analysis. The theoretical space-time auto variances can be shown in **Table 1**.

Table 1: The ST-ACF and ST-PACF for STAR, STMA, and STARMA models.

Model form	ST-ACF	ST-PACF
STAR ($p_{\lambda_1 \dots \lambda_p}$)	Tails off	Cuts off after p time lags, λ_p spatial lags
STAR ($q_{m_1 \dots m_q}$)	Cuts off after p time lags, λ_p spatial lags	Tails off
STARMA ($p_{\lambda_1 \dots \lambda_p}, q_{m_1 \dots m_q}$)	Tails off	Tails off

- 2.2.3. Estimation:** After identification of proper STAR and STMA orders, next step is to estimate the parameters (ϕ_{kl}, θ_{kl}) of the STARMA model. The parameters are estimated by Conditional maximum likelihood estimates (Pfeifer and Deutsch 1980b, c). To find the maximum likelihood estimators, the normality assumption of residuals should be satisfied i.e. $\boldsymbol{\varepsilon}_t \sim N(0, \sigma^2 \mathbf{I}_N)$. Estimation of the parameters by maximum likelihood is a matter of minimizing the sum of squared residuals,

$$S(\phi, \theta) = \sum_{t=1}^T \boldsymbol{\varepsilon}_t' \boldsymbol{\varepsilon}_t, \quad (5)$$

where

$$\boldsymbol{\varepsilon}_t = \mathbf{Z}_t - \sum_{k=1}^p \sum_{l=0}^{\lambda_k} \phi_{kl} \mathbf{W}^l \mathbf{Z}_{t-k} + \sum_{k=1}^q \sum_{l=0}^{m_k} \theta_{kl} \mathbf{W}^l \boldsymbol{\varepsilon}_{t-k}, \quad (6)$$

for $t = 1, 2, \dots, T$, while setting $\boldsymbol{\varepsilon}_t$ and $\mathbf{Z}_t$ equal to zero for $t < 1$.

- 2.2.4. Diagnostic checking:** The fitted model is verified by first checking if the residuals are white noise for the best fitted model, the residuals should be white noise, i.e., $\boldsymbol{\varepsilon}_t \sim N(0, \sigma^2 \mathbf{I}_N)$. Auto correlation of residuals are tested by the Multivariate Box-Pierce test.

2.3. Construction of Spatial Weight matrix:

Prior to STARMA modelling, there is a need to define suitable spatial weight matrix ($\mathbf{W}^l$). Choice of the spatial weight matrix is very important because different spatial weight matrices often lead to different results. There are different ways to construct spatial weight matrices, including shared border and distance between two locations. The simplest form of weights is the binary weight, where two areas shared a common boundary gets weight 1 and otherwise gets 0 (Griffith (1996) and (2009)). In a spatial weight matrix, row normalization is a

common practice (each row total is 1) and weights reflects a hierarchical order of spatial neighbours. The first order neighbours are closer than the second order ones, and the second order neighbours are closer than the third order neighbours and so on. In addition to that, each space unit is defined as its zero spatial order neighbour, and the zero order spatial weight matrix ($\mathbf{W}^0$) can be defined as unit matrix (**Table 2**). In this article, a spatial weight matrix ($\mathbf{W}^1$) has been formed by taking weights that are inverse the distance between the two regions.

$$w_{ij} = \begin{cases} 1/d_{ij} & i \neq j \\ 0 & i = j \end{cases} \quad (7)$$

The elements are scaled such that

$$\sum_{j=1}^N w_{ij} = 1 \quad (8)$$

for each i . The distance (d_{ij}) between the two geographical regions i and j is computed by using great circle distance formula of spherical law of cosines which is expressed as:

$$d_{ij} = \text{acos}(\sin\varphi_1 \cdot \sin\varphi_2 + \cos\varphi_1 \cdot \cos\varphi_2 \cdot \cos\varDelta\lambda) \times R, \quad (9)$$

where $\varphi_1 = \text{lat}_1 \times \pi/180$, $\varphi_2 = \text{lat}_2 \times \pi/180$, $\varDelta\lambda = (\text{lon}_2 - \text{lon}_1) \times \pi/180$, and R = radius of the earth (6,371 km).

For each location, Great circle distance (d_{ij}) between the locations has been calculated using longitude and latitude coordinate (**Table 3**) which is represented in **Table 4**. Finally, The inverse distance weight matrix ($\mathbf{W}^1$) has been created through row normalization processes as defined in **Table 5**.

Table 2: Spatial weight matrix of zero order ($\mathbf{W}^0$)

Location	AUR	BHO	GAY	JAH	NAL	KAM	PAT	ROH
AUR	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
BHO	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
GAY	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
JAH	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
NAL	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
KAM	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
PAT	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
ROH	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table 3: Latitude and Longitude for all locations

Location	Latitude	Longitude
Aurangabad	25.03651 N	84.40261 E
Bhojpur	25.47361 N	84.53691 E
Gaya	24.78501 N	84.99271 E
Jahanabad	25.18341 N	85.10731 E
Nalanda	25.20261 N	85.54631 E
Kaimur	25.04031 N	83.60651 E
Patna	25.60131 N	85.13751 E
Rohtas	24.69371 N	84.00201 E

Table 4: Great circle distance for each locations

Location	AUR	BHO	GAY	JAH	NAL	KAM	PAT	ROH
AUR	0.00	50.44	65.75	72.81	116.62	80.20	96.96	55.55
BHO	50.44	0.00	89.26	65.79	105.82	105.24	61.91	102.09
GAY	65.75	89.26	0.00	45.78	72.59	142.65	91.93	100.56
JAH	72.81	65.79	45.78	0.00	44.22	151.94	46.57	124.03
NAL	116.62	105.82	72.59	44.22	0.00	196.12	60.43	165.66
KAM	80.20	105.24	142.65	151.94	196.12	0.00	166.05	55.47
PAT	96.96	61.91	91.93	46.57	60.43	166.05	0.00	152.47
ROH	55.55	102.09	100.56	124.03	165.66	55.47	152.47	0.00

Table 5: Row normalized inverse distance spatial weight matrix (W^1)

Location	AUR	BHO	GAY	JAH	NAL	KAM	PAT	ROH
AUR	0.000	0.203	0.155	0.140	0.087	0.127	0.105	0.184
BHO	0.218	0.000	0.123	0.167	0.103	0.104	0.177	0.108
GAY	0.169	0.125	0.000	0.244	0.153	0.078	0.121	0.110
JAH	0.125	0.139	0.200	0.000	0.206	0.060	0.197	0.074
NAL	0.104	0.115	0.168	0.275	0.000	0.062	0.202	0.073
KAM	0.192	0.147	0.108	0.102	0.079	0.000	0.093	0.279
PAT	0.118	0.184	0.125	0.246	0.189	0.069	0.000	0.069
ROH	0.237	0.129	0.131	0.106	0.079	0.238	0.079	0.000

2.4. BDS Test:

The BDS test is a nonparametric test for serial independence based on the correlation integral of the scalar series and proposed by Brock, Dechert, and Scheinkman (1996). In this paper, this test will be used to check the non-linear dynamics in residuals for the fitted STARMA model. The test hypotheses are as follow:

Null hypothesis (H_0): Residuals are independently and identically distributed and linear.

Alternative hypothesis (H_1): Residuals are not independent and identical and non-linear.

Test statistic:

$$B_{e,\epsilon} = \sqrt{T} \frac{A_{e,\epsilon} - A_{1,\epsilon}}{s_{e,\epsilon}}, \quad (10)$$

$$A_{e,\epsilon} = \frac{2}{T_e(T_e-1)} \sum \sum_{e \leq s < t \leq T} I(z_t^e, z_s^e, \epsilon), \quad (11)$$

$$A_{1,\epsilon} = P(|z_t - z_s| < \epsilon)^e; \quad T_e = T - e + 1, \quad (12)$$

$$I(z_t^e, z_s^e, \epsilon) = \begin{cases} 1, & \text{if } |z_{t-i} - z_{s-i}| < \epsilon, i = 0, 1, 2, \dots, e-1 \\ 0, & \text{Otherwise} \end{cases} \quad (13)$$

where e = embedding dimension, z_t = residuals data of STARMA model at time t , $t = 1, 2, \dots, T$ and $B_{e,\epsilon}$ = converges to the standard normal distribution under the null hypothesis

Decision: If the value of $|B_{e,\epsilon}| > 1.96$, then the null hypothesis will be rejected, otherwise accepted at 5 % level of significance.

2.5. ARCH-LM Test:

GARCH modelling starts by identifying whether or not the space-time series data contains heteroscedasticity. The residuals obtained from the data are checked by using ARCH-LM test to know the ARCH effect (Engle 1982; Tsay 2005).

The steps for ARCH-LM test:

I. Define the linear regression,

$$\varepsilon_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + \dots + a_q \varepsilon_{t-q}^2 + \varepsilon_t \quad (14)$$

where ε_t denotes the error term, q is the pre specified positive integer.

II. The test statistic is:

$$LM = TR^2$$

where

$$R^2 = \frac{\sum_{i=1}^n (\hat{x}_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (15)$$

and T is total number of observation. Here R^2 follows a $\chi^2(q)$ distribution.

III. The null and alternative hypotheses are

$$H_0: a_1 = a_2 = \dots = a_q = 0$$

$$H_1: \text{all are not equal of zero}$$

2.6. GARCH Model:

If there is ARCH effect, then the next step involves in identifying the appropriate orders for the GARCH (p, q) model. The GARCH model is a generalized form of ARCH model. The GARCH (p, q) model in which conditional variance is modelled as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \quad (16)$$

where $\alpha_0 > 0$, $\alpha_i > 0, \beta_j \geq 0$ and $\sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 < 1$.

2.7. Propose Hybrid STARMA-GARCH Model:

There are two phases in the hybrid STARMA-GARCH modelling. In the first phase, a STARMA model is fitted on stationary and linear space-time data and in the second phase, residuals of the STARMA model are fed as an input to the GARCH model. This hybrid model, which combines STARMA and GARCH model containing non-linear residuals patterns is applied to forecast the air temperature data.

2.8. Diagnostic Checking of the Hybrid STARMA-GARCH Model:

The diagnostic tests of hybrid STARMA-GARCH models are based on residuals. Normality of the residuals is tested using Ljung and Box Q -statistics (Ljung and Box 1978) for the serial correlation test. Test hypotheses are given below:

$$H_0: \text{no serial correlation in the residuals}$$

$$H_1: \text{there is serial correlation}$$

The Q -statistic is given as:

$$Q = N(N+2) \sum_{j=1}^L \frac{\widehat{\rho_j^2}}{(N-j)}, \quad (17)$$

where N is the sample size, L is the autocorrelation lags, $\widehat{\rho_j^2}$ is the square of the estimated autocorrelation at lag j .

Decision: If the value of $Q > 1.96$, then the null hypothesis will be rejected at 5 % level of significance.

2.9. Forecast Evaluation:

Mean absolute percentage error (MAPE) have been used to evaluate the forecast efficiency for which the formula is given below:

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^n \left| \frac{z_i - \hat{z}_i}{z_i} \right| \times 100, \quad (18)$$

where n = total numbers of predicted value, z_i = actual value at time t , and $\hat{z}_i$ = corresponding predicted value.

3. Results and Discussion

3.1. Preliminary Stage

Before proceeding to use STARMA modelling, it is important to check the stationary of the time series data. The results of the Augmented Dickey –Fuller test showed that spatio-temporal data series of all locations satisfy stationarity. The time series plots of data set (Fig. 2 and 3) indicate the presence of seasonality which needs to be removed before analysis. Therefore, the original time series data are seasonally adjusted by seasonal differencing with period 12. Finally, seasonally adjusted data series of each location become stationary and have been used for analysis.

Fig. 2: Time series plots for monthly maximum temperature.

Fig. 3: Time series plots for monthly temperature range.

3.2. Identification:

The model identification is a very crucial step for spatio- temporal modelling. First the presence of correlation in space-time data is checked. The values of chi-square of Multivariate Box-Pierce test for maximum temperature and temperature range are 3346.149 and 2824.946 with p value < 0.001 respectively which tell that reject the null hypothesis (H_0 : non-correlation) at 1% level of significance. After checking the presence of space-time correlation, the suitable candidate STRAMA model has been selected with help of ST-ACF and ST-PACF plots. The ST-ACF and ST-PACF plots of original data series have shown the presence of seasonality. Therefore, seasonally adjusted data of ST-ACF and ST-PACF plots are used to determine STARMA model (**Fig. 4 and 5**). Here STAR (1_1) and STMA (0) have been selected for both the maximum temperature and temperature range. Therefore, the STARMA ($1_1, 0, 0$) model has been identified. This model is also called STAR (1_1) model.

Fig. 4: ST-ACF and ST-PACF plots for seasonality adjusted data series of temperature range

Fig. 5: ST-ACF and ST-PACF plots for seasonality adjusted data series of maximum temperature

3.3. Estimation:

Parameters of the fitted STARMA (1, 0, 0) model is estimated by maximum likelihood method. The estimated parameters and its standard error are presented in **Table 6**. All the estimated parameters are statistically significant at 5% level of significance. The STARMA (1, 0, 0) model can be expressed as:

$$\mathbf{Z}_t = 0.562821 (\mathbf{W}^0) \mathbf{Z}_{t-1} - 0.142759 (\mathbf{W}^1) \mathbf{Z}_{t-1} + \varepsilon_t, \quad (19)$$

$$\mathbf{Z}_t = 0.808697 (\mathbf{W}^0) \mathbf{Z}_{t-1} - 0.384389 (\mathbf{W}^1) \mathbf{Z}_{t-1} + \varepsilon_t, \quad (20)$$

where the equations (19) and (20) represent the STARMA models for temperature range and maximum temperature respectively.

Table 6: Parameters estimate for STARMA (1, 0, 0) model

Parameters	Estimated (Temp. Range)	Estimated (Max. Temp.)
ϕ_{10}	0.5628 ** (0.0647)	0.8086** (0.040)
ϕ_{11}	-0.1427* (0.0662)	-0.3843** (0.042)
σ^2	3.1850	4.5591
log likelihood	-4938.03	-4810.747
BIC	9892.517	9637.949

values in the parenthesis indicates the standard error, * & ** are the significant at 1 % & 5% level of significance respectively.

3.4. Diagnostic checking:

After parameters estimation, diagnostic checking of residuals is applied by using Multivariate Box-Pierce Non Correlation test. The test applied to the residuals give the chi-square statistics values as 390.25 and 455.12 with p values < 0.001 for maximum temperature and temperature range respectively, which tell that reject the null hypothesis (H_0 : non-correlation) at 1% level of significance. Presence of spatio-temporal correlation within residuals also be seen in the ST-ACF and ST-PACF plots of residuals (**Fig. 6 and 7**).

Fig. 6: ST-ACF and ST-PACF plots for residuals of temperature range

Fig. 7: ST-ACF and ST-PACF plots for residuals of maximum temperature

3.5. BDS and ARCH-LM Test for Residuals:

The BSD test results show the presence of both the linear and nonlinear patterns in the correlated residuals of maximum temperature and temperature range. The ARCH-LM test of square residuals are suffering from serial correlation in the most of the locations. Hence, it is necessary to develop a best fitted model for analysis of monthly temperature. A GARCH modelling is necessary to handle heteroscedasticity in the residuals series.

3.6. Hybrid STARMA-GARCH Model:

The STARMA-GARCH model is a combination of two models, first one is the STARMA model that take into account of mean behaviour of a space- time series and the second one is the GARCH model that used to model the variance of the series. In this hybrid model, residuals of the fitted STARMA model are used for suitable GARCH modelling. The results of minimum AIC and BIC values, the GARCH (0, 1) model is found to be the best fitted model for monthly maximum temperature and temperature range. The estimated model parameters are presented in **Tables 7** and **8**. For diagnostics of the hybrid model, Ljung-Box Q -test are applied to the residuals of the fitted hybrid STARMA (1₁, 0, 0) – GARCH (0, 1) model. The test results revealed that null hypothesis (H_0 : no serial correlation) is accepted and indicate the residuals of hybrid model have no autocorrelation. Therefore, the fitted hybrid model is specified correctly and suitable for forecasting for temperature.

Table 7: Estimated parameters of GARCH (0, 1) model for max. temperature

District	AUR	BHO	GAY	JAH	NAL	KAM	PAT	ROH
Parameter	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate
μ	0.0405	0.0130	-0.0160	-0.0249	-0.0390	0.0584	-0.0949	0.0392

	(0.0393)	(0.0390)	(0.0377)	(0.0384)	(0.0379)	(0.0395)	(0.0375)	(0.0386)
	0.0484	0.0459	0.0621	0.0392	0.0295	0.0489	0.0292	0.0569
ARI	(0.0464)	(0.0464)	(0.0464)	(0.0464)	(0.0463)	(0.0464)	(0.0465)	(0.0464)
	0.0049	0.0027	0.0023	0.0026	0.0029	0.0064	0.0010	0.0030
ω	(0.0008)	(0.0006)	(0.0004)	(0.0005)	(0.0016)	(0.0009)	(0.0001)	(0.0009)
	0.9934	0.9961	0.9965	0.9962	0.9955	0.9916	0.9983	0.9958
β	(0.0011)	(0.0009)	(0.0006)	(0.0008)	(0.0025)	(0.0012)	(0.0001)	(0.0013)

values in the parenthesis indicate standard error

Table 8: Estimated parameters of GARCH (0, 1) model for temperature range.

District	AUR	BHO	GAY	JAH	NAL	KAM	PAT	ROH
Parameter	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate	Estimate
	0.0315	0.0100	-0.0450	-0.0308	-0.0844	0.0685	0.00072	-0.0006
μ	(0.0396)	(0.0404)	(0.0393)	(0.0400)	(0.0395)	(0.039)	(0.0401)	(0.0391)
	-0.0655	-0.0457	-0.0588	-0.0371	-0.0361	-0.0879	-0.0394	0.0709
ARI	(0.0463)	(0.0463)	(0.0463)	(0.0463)	(0.0463)	0.0462	(0.0463)	(0.0463)
	0.0006	0.0015	0.0005	0.0034	0.0006	0.0059	0.0006	0.0005
ω	(0.000)	(0.0001)	(0.000)	(0.001)	(0.0000)	(0.0009)	(0.0000)	(0.000)
	0.999	0.9977	0.999	0.9954	0.999	0.9925	0.999	0.999
β	(0.000)	(0.0002)	(0.0000)	(0.0020)	(0.0000)	(0.001)	(0.0000)	(0.0000)

values in the parenthesis indicate standard error

3.7. Comparative Analysis

To evaluate the forecast efficiency, MAPE has been computed for the forecasting performance for all eight locations. The MAPE values under training data set for maximum temperature and temperature range are given in **Table 9**. According to the results of the MAPE, the hybrid model has lowest mean absolute percentage errors compared to STARMA model for both the maximum temperature and temperature range in all eight locations. Further, forecasts have been carried out for model validation. The forecast values along with the forecast mean absolute percentage errors are given in **Tables 10 to 17** for all eight locations. It can be seen from the table that the hybrid model is more reliable and that the error between the forecast values and validation data values (actual values) is smaller and forecast values are very close to validation data values for all eight locations. Hence, overall, it can be concluded that the fitted hybrid model performed better than the STARMA model for both the variables viz., maximum temperature and temperature range in all eight locations.

Table 9: Model performance under training for monthly temperature.

	Maximum temperature		Temperature range	
Location	STARMA model	Hybrid model	STARMA model	Hybrid model
AUR	2.0647	0.1520	6.1119	0.4884
BHO	2.0566	0.1019	6.1438	0.3011
GAY	2.0415	0.1359	6.1289	0.5127

JAH	2.0286	0.1052	8.6080	0.3382
NAL	1.9940	0.1279	6.1425	0.7804
KAM	2.0730	0.1963	6.1351	0.7771
PAT	2.0415	0.3076	6.1419	0.2505
ROH	2.0751	0.1571	6.1347	0.4445

Table 10: Model performance under testing data set for maximum temperature

		Aurangabad		Bhojpur		
Series	Actual	Forecast		Actual	Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	22.024	23.318	23.177	21.945	23.238	23.131
Feb-20	25.805	25.816	25.781	25.711	25.737	25.728
Mar-20	32.130	32.530	32.49	32.019	32.429	32.417
Apr-20	38.608	36.855	36.815	38.446	36.726	36.714
May-20	41.119	40.371	40.331	40.610	39.943	39.930
Jun-20	35.550	34.342	34.302	35.632	34.417	34.404
Jul-20	32.050	33.001	32.961	32.04	32.987	32.975
Aug-20	32.131	31.375	31.335	32.159	31.407	31.394
Sep-20	31.650	32.782	32.742	31.593	32.726	32.713
Oct-20	31.111	30.365	30.325	30.812	30.127	30.115
Nov-20	27.502	29.483	29.443	26.762	28.859	28.847
Dec-20	25.070	25.179	25.139	24.237	24.500	24.487
MAPE		2.988	2.929	MAPE	3.068	3.021

Table 11: Model performance under testing data set for maximum temperature

Series	Actual	Gaya		Actual	Jahanabad	
		Forecast			Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	21.695	22.987	22.882	21.766	23.067	23.014
Feb-20	25.506	25.546	25.570	25.497	25.551	25.580
Mar-20	31.138	31.691	31.707	31.786	32.228	32.253
Apr-20	37.884	36.261	36.277	38.114	36.452	36.478
May-20	40.495	39.850	39.867	39.977	39.415	39.440
Jun-20	35.103	33.981	33.998	35.645	34.432	34.458
Jul-20	31.952	32.913	32.930	31.984	32.935	32.961
Aug-20	31.743	31.064	31.080	32.104	31.367	31.392
Sep-20	31.181	32.384	32.401	31.516	32.657	32.683

Oct-20	30.375	29.765	29.781	30.688	30.030	30.056
Nov-20	26.516	28.65	28.672	26.353	28.514	28.540
Dec-20	24.185	24.457	24.473	23.473	23.873	23.898
MAPE		3.0982	3.069367	MAPE	3.1411	3.136

Table 12: Model performance under testing data set for maximum temperature

		Nalanda					Kaimur	
		Forecast					Forecast	
Series	Actual	STARMA	Hybrid	Actual	STARMA	Hybrid		
Jan-20	21.693	23.001	22.982	21.634	22.943	22.782		
Feb-20	25.361	25.440	25.482	25.585	25.606	25.553		
Mar-20	31.677	32.134	32.174	32.021	32.424	32.366		
Apr-20	37.654	36.075	36.114	38.534	36.793	36.735		
May-20	39.270	38.825	38.864	41.341	40.554	40.496		
Jun-20	35.549	34.357	34.396	35.279	34.121	34.063		
Jul-20	31.872	32.838	32.877	31.93	32.900	32.842		
Aug-20	31.970	31.261	31.300	32.065	31.324	31.266		
Sep-20	31.398	32.555	32.594	31.674	32.800	32.742		
Oct-20	30.684	30.033	30.072	31.501	30.693	30.635		
Nov-20	26.284	28.450	28.490	28.364	30.195	30.137		
Dec-20	23.084	23.557	23.597	25.532	25.567	25.509		
MAPE		3.151	3.167	MAPE	2.930	2.874		

Table 13: Model performance under testing data set for maximum temperature

		Patna					Rohtas	
		Forecast					Forecast	
Series	Actual	STARMA	Hybrid	Actual	STARMA	Hybrid		
Jan-20	21.050	22.412	22.447	21.929	23.205	23.057		
Feb-20	24.564	24.714	24.810	25.777	25.797	25.764		
Mar-20	30.403	31.085	31.179	31.385	31.898	31.858		
Apr-20	36.741	35.287	35.382	38.156	36.498	36.459		
May-20	38.627	38.322	38.417	41.061	40.310	40.271		
Jun-20	34.739	33.657	33.752	35.050	33.946	33.907		
Jul-20	31.840	32.848	32.943	31.862	32.827	32.788		
Aug-20	31.611	30.928	31.023	31.662	31.007	30.968		
Sep-20	30.941	32.216	32.311	31.185	32.374	32.335		
Oct-20	30.163	29.558	29.653	30.504	29.886	29.847		
Nov-20	25.897	28.176	28.271	27.013	29.052	29.013		
Dec-20	22.906	23.359	23.454	24.880	25.050	25.011		
MAPE		3.285	3.356	MAPE	3.002	2.934		

Table 14: Model performance under testing data set for temperature range

Series	Actual	Aurangabad		Actual	Bhojpur	
		Forecast			Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	13.160	13.003	13.136	13.020	12.874	12.952
Feb-20	15.326	14.619	14.644	15.226	14.540	14.547
Mar-20	14.650	14.167	14.199	14.560	14.089	14.099
Apr-20	15.882	15.066	15.098	15.836	15.033	15.043
May-20	14.490	14.048	14.079	14.217	13.835	13.845
Jun-20	7.903	9.018	9.050	7.949	9.055	9.065
Jul-20	5.672	7.351	7.383	5.613	7.303	7.313
Aug-20	5.954	7.543	7.575	5.942	7.536	7.546
Sep-20	6.415	7.913	7.944	6.226	7.767	7.777
Oct-20	9.207	10.016	10.048	8.765	9.682	9.692
Nov-20	12.890	12.826	12.857	12.312	12.386	12.396
Dec-20	14.572	14.091	14.122	14.060	13.703	13.713
MAPE		10.303	10.308	MAPE	10.486	10.478

Table 15: Model performance under testing data set for temperature range

Series	Actual	Gaya		Actual	Jahanabad	
		Forecast			Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	12.915	12.771	12.830	12.789	12.670	12.691
Feb-20	15.044	14.341	14.290	14.982	14.351	14.318
Mar-20	13.884	13.580	13.536	14.349	13.916	13.885
Apr-20	15.660	14.868	14.823	15.618	14.869	14.838
May-20	14.343	13.942	13.897	13.834	13.538	13.507
Jun-20	8.098	9.150	9.105	8.032	9.121	9.090
Jul-20	5.982	7.594	7.549	5.531	7.237	7.206
Aug-20	6.044	7.602	7.557	5.895	7.503	7.473
Sep-20	6.263	7.804	7.759	6.057	7.636	7.605
Oct-20	8.450	9.435	9.390	8.465	9.456	9.426
Nov-20	11.971	12.133	12.088	11.926	12.091	12.060
Dec-20	13.902	13.578	13.533	13.486	13.269	13.238
MAPE		10.125	9.911	MAPE	10.631	10.491

Table 16: Model performance under testing data set for temperature range

Series	Actual	Nalanda		Actual	Kaimur	
		Forecast			Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	12.592	12.497	12.458	13.029	12.877	13.091
Feb-20	14.742	14.174	14.087	15.201	14.503	14.559
Mar-20	14.223	13.807	13.723	14.616	14.134	14.203
Apr-20	15.305	14.637	14.553	15.780	14.980	15.049
May-20	13.420	13.216	13.131	14.595	14.126	14.195
Jun-20	8.089	9.170	9.085	7.845	8.971	9.039
Jul-20	5.424	7.151	7.066	5.679	7.357	7.426
Aug-20	5.792	7.429	7.345	6.022	7.593	7.662
Sep-20	5.885	7.502	7.417	6.650	8.092	8.160
Oct-20	8.260	9.304	9.219	9.763	10.437	10.506
Nov-20	11.714	11.927	11.843	13.542	13.321	13.390
Dec-20	13.050	12.940	12.856	14.841	14.294	14.363
MAPE		10.804	10.103	MAPE	10.477	10.231

Table 17: Model performance under testing data set for temperature range

Series	Actual	Patna		Actual	Rohtas	
		Forecast			Forecast	
		STARMA	Hybrid		STARMA	Hybrid
Jan-20	12.861	12.803	12.849	13.166	12.984	13.105
Feb-20	15.198	14.591	14.589	15.201	14.481	14.472
Mar-20	14.862	14.325	14.325	14.000	13.674	13.674
Apr-20	15.591	14.880	14.880	15.698	14.906	14.905
May-20	13.541	13.316	13.316	14.615	14.149	14.148
Jun-20	7.808	8.966	8.967	8.016	9.092	9.092
Jul-20	5.074	6.886	6.887	5.924	7.549	7.548
Aug-20	5.617	7.301	7.301	5.935	7.522	7.521
Sep-20	5.933	7.537	7.538	6.344	7.864	7.863
Oct-20	8.684	9.628	9.629	8.669	9.602	9.602
Nov-20	12.255	12.337	12.337	12.389	12.450	12.449
Dec-20	13.651	13.398	13.399	14.316	13.893	13.892
MAPE		11.270	11.245	MAPE	10.204	10.129

4. Conclusions

In this study, the problem is to forecast the monthly maximum temperature and temperature range as a spatio-temporal forecasting problem for III-B agro climatic zone of Bihar. In the real-world phenomena, weather parameters work in a very complex manner such they can vary over small regions and for short periods of time which lead to linear, as well as nonlinear patterns. In these variations there are various statistical techniques used in forecasting of weather parameters. One of the most popular statistical method used for forecasting temperature time series is Box-Jenkins ARIMA but this model is suitable for only univariate and linear time series data. If the multivariate spatio-temporal time series data set shows non-linear pattern, then ARIMA model is not suitable for its modelling. In order to overcome the drawback of traditionally used models, hybrid model combining the STARMA and GARCH have been investigated. Based on the results obtained by the STARMA model and hybrid model for both the variables viz., maximum temperature and temperature range, it can be seen that fitted hybrid model performed better as compared to STARMA model for the training data set. After validation of proposed hybrid model, the fitted hybrid model has lower forecast errors than the STARMA model. Hence, it can be concluded that the spatio-temporal hybrid model has better forecasting accuracy because of the inclusion of both spatial information and nonlinear patterns in the temperature data sets. The proposed hybrid model also be used to model variations in other weather parameters, which will be taken up in a forthcoming paper.

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Data availability: Data have been recorded from the NASA prediction of worldwide energy resources (<https://power.larc.nasa.gov>), which are freely available.

Code availability: The codes written for the current study are available on reasonable request from the corresponding author.

Declarations

Conflict Interests: The authors declare that they have no conflict of interest.

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