Integration of Information Theory, Fractal Theory and Statistical Analyses for Landslide Susceptibility Mapping at Regional Scales

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Research Article

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1   **Integration of information theory, fractal theory and statistical analyses for landslide susceptibility**

2   **mapping at regional scales**

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7   **Abstract** A new model, integrating information theory, fractal theory and statistical model for accurate landslide susceptibility
8   mapping (LSM) at regional scales, has been proposed. In this model, landslide conditional factors are firstly classified with an
9   optimal number of classes, which is determined by maximizing their information coefficients estimated from Shannon's entropy
10   model. The spatial association between influencing factors and induced landslides has been measured by introducing the variable
11   fractal dimension method (VFDM). The VFDM approach fully considers the characteristics of landslide fractal distribution. Then
12   the fractal dimensions (D) are calculated to provide multiple factors with various numerical weights. The proposed model
13   eventually combines the landslide frequency ratio (fr) of each factor with corresponding weight to achieve spatial prediction of
14   landslides, illustrated by an example area in China. In the study area, 500 landslides have been identified by aerial photograph
15   interpretation, extensive field investigations, historical and bibliographical landslide data. In the model, these landslides are
16   randomly split into a training dataset (70 %) and a validating dataset (30 %). Seven factors are recognized and analyzed by
17   frequency ratio (FR) method, including lithology, distance to fault, altitude, slope, aspect, distance to stream and distance to the
18   road. The receiver operating characteristic curve (AUROC) has been adopted to compare and validate the model results. Results
19   show that the proposed landslide model achieved a more accurate prediction with AUROC equal to 0.8467, over-performing than
20   the conventional frequency ratio method (AUROC=0.8088). According to the final prognostic landslide susceptibility map, 16.37 %
21   of the study area shows very high and high susceptibility, accounting for 63.55 % of the entire landslides. Evaluation of relative
22   factor importance based on a one-by-one factor removal test indicates that the lithology factor contributes unique information for
23   landslides. In conclusion, the example demonstrates that the proposed framework is promising for further improvement of LSM.

24   **Keywords** landslide susceptibility · Shannon's entropy index · variable fractal dimension method · frequency ratio · comparison
25   and validation

Introduction

Landslide describes the downslope transport of soil/rock, resulting from materials different geodynamic processes (Guzzetti et al. 2005). The landslide also represents a potentially significant natural hazard, contributing to the tremendous economic and social loss. Much research efforts have been devoted to denominating potential landslide hazard zones by evaluating responsible factors (Shahabi and Hashim 2015). Various methods being successfully applied in susceptibility assessment can be classified into qualitative (e.g., Regmi et al. 2010; Yalcin et al. 2011) and quantitative (e.g., Pradhan et al. 2011; Goetz et al. 2015) groups. Among quantitative approaches, statistical methods are favored in most publications (Chung and Fabbri 1999; Lee and Pradhan 2006). They were proven to be better options for comparatively large and complex areas (Cardinali et al. 2002). In addition, the recent development in machine learning and data mining have introduced new approaches for landslide susceptibility modeling such as support vector machine (Tien Bui et al. 2012), random forests (Catani et al. 2013) and decision tree (Pradhan 2013), some of these methods were reported to outperform conventional ones (Tien Bui et al. 2013). These approaches work well when handling large amounts of data. However, the model may not work well if considering all possible influencing factors, since the problem of over-fitting may occur. Screening these factors could decrease this phenomenon, but this could result in high computational cost and would be a time-consuming process (Wang et al. 2017).

Except for analyzing the physical properties of slope materials, the basic idea of landslide susceptibility assessment is to identify possible influencing factors that control landslide spatial distributions (Liucci et al. 2015). The highly heterogeneous character of landslide spatial distribution has been realized (Li et al. 2012). Literature reviews indicate that many landslides exhibit fractal distribution (Sezer 2010). Both the frequency-area statistics and frequency-size statistics satisfy the power-law

(fractal) function ([Guzzetti et al. 2002](#); [Malamud et al. 2004](#); [Guthrie et al. 2007](#); [Trigila et al. 2010](#); [Ghosh et al. 2012](#)). Also, geoscientists explored the spatial association between landslides and corresponding influencing variables ([Brunetti et al. 2009](#)). Some related and significant conclusions have been drawn include: 1) fine geological features leading to a high fractal dimension of landslide distribution ([Iwahashi et al. 2003](#)), 2) demonstrated fractal relation between the cumulative frequency of landslides and their triggering rainfall level ([Li et al. 2011](#)), and 3) fractal relation between landslide density and distances to conditional factors ([Zou and Carranza 2017](#)). Thus, the landslide is a nonlinear system and its spatial distribution is heterogeneous ([Rouai and Jaaidi 2003](#)). However, a retrospective of the previous studies showed that the fractal analyses used to characterize landslide spatial distribution are mostly based on the box-counting fractal dimension (BCFD) algorithms (e.g., [Zou et al. 2009](#); [Li et al. 2012](#)), which does not consider the variable-dimension fractal characteristics of natural hazards. Remarkable work from [Liu et al. \(2019\)](#) introduced a variable fractal dimension method (VFDM) to measure the spatial association between influencing factors and landslides. It concluded that the calculated fractal dimension values could be used as weights of their corresponding factors in LSM. The fractal dimension values/weights vary with different class numbers of the factors.

The objective of this study is to propose an alternative approach for landslide susceptibility mapping (LSM). The proposed new method integrates information theory, fractal theory and statistical analyses. Firstly, the optimal number of classes for the identified landslide conditional factors are determined by Shannon's entropy index. Next, the variable fractal dimension method (VFDM) is introduced into the model, which considers the heterogeneous fractal structure of landslide spatial distribution. A fractal measure of spatial association between landslide occurrences and their conditional variables has been adduced to assign the conditional factors with numerical weights. Finally, the fractal-based weights are

70 developed into the frequency ratio model. The proposed model for accurate LSM will be illustrated and
71 validated in an example area in China.

72 Data and materials

73 Study area

74 Located in northeast China, the study area, covering an area of about $1.17 \times 10^5 \text{ km}^2$, is between longitudes
75 $85^\circ 31' 37'' \text{E} \sim 91^\circ 01' 15'' \text{E}$ and latitudes $45^\circ 59' 35'' \text{N} \sim 49^\circ 10' 45'' \text{N}$. Altitudes in the study area range from
76 365 m to 4330 m above sea level (Fig. 1).

77
78 **Fig. 1** Location of the study area with landslide locations (Krasovsky 1940 as the reference coordinate system)

79 The topographic inclination follows the NE~SW direction. The terrain surface is relatively deep-
80 inclined with slope angles ranging from 0° to 83.97° . Areas with slopes less than 15° cover 82.64 % of
81 the total area, while areas with slopes larger than 45° account for only 2.06 %. Besides, areas with slopes
82 between 15° and 45° account for 15.30 %.

83 The climate belongs to type B (Arid), as defined in the Köppen climatic classification (Köppen 1884).

84 According to the National Meteorological Information Center (<http://data.cma.cn>). The average
85 temperature is 10.6 °C. Mean annual rainfall for the period 1998~2015 ranges between 83.9 mm and
86 430.3 mm. During the rainy seasons (May~August), the average rainfall varies between 224.5 mm and
87 418.4 mm per month.

88 The lithology map used in this study is constructed based on the Geological and Mineral Resources
89 maps at 1:200,000 scale. Rock types observed include volcanic, sedimentary and metamorphic varying
90 in age from Sinian to Quaternary. Data collected from the China Geology Survey (CGS)
91 (<http://www.cgs.gov.cn>) indicate that More than 50 geologic groups and units have been recognized. All
92 the lithology types have been classified into eight classes based on clay composition, weathering degree,
93 estimated physical (density) and strength parameters (Fig. 3a, Table 3). Around 53.9 % of the study area
94 is covered by metamorphic rocks (schist, gneiss, and metamorphic limestone).

95 **Data acquisition and processing**

96 **Landslide inventory map**

97 The unique mountain-basin landscape, complex geologic condition and frequent extreme weather are
98 responsible for the widespread landslide distribution in this area. Landslide inventory has been performed
99 to analyze the homogeneous population. According to statistics, over 85 % of slope movements in the
100 study area are rockfalls. Thus, only rockfalls have been selected. A total of 600 landslides were registered.
101 Exact occurrence dates of them are mostly unknown. According to documentary reports and publications,
102 landslides mainly occurred during and after the heavy rainfalls. Statistics of these landslides show that
103 the largest landslide size is approximately 39,420 m², the smallest around 792 m². The average landslide
104 size is estimated to be 10,733 m². Fig. 2 illustrates the cumulative frequency distribution of landslides
105 areas.

Fig. 2 Main characteristics of landslide distribution

For preparing a landslide inventory map, landslides are usually included as singular points or as polygons (Dou et al. 2015). The choice of points or polygons is comprehensively determined by the map's scale, data availability and research objectives (Wang et al. 2014). Comparisons of these sampling strategies can be found in some published articles (e.g., Oliveira et al. 2015). However, this issue is beyond the scope of this study. In this study, the 600 landslides are depicted as points in GIS shapefile format. The landslide inventory map has been then produced as a point map (Fig. 1).

Landslide conditional factors

Seven landslide conditional factors are recognized in the study area, including lithology, distance to fault, altitude, slope, aspect, distance to stream and distance to the road. The inclusion of these factors draws heavily on extensive literature reviews, experience gained from studying landslide phenomena, and sufficient data availability. Some ancillary data have been used to extract these conditional factors, as listed in Table 1 below. These factors and the landslide inventory map have been extracted with a grid size of 30 m×30 m to match the digital elevation model (DEM).

Table 1 Data source for the study area

No.	Data	Source

		Geological and Mineral Resources map at 1:200,000 scale
1	Lithology	http://www.cgs.gov.cn
		Geological and Mineral Resources map at 1:200,000 scale
2	Distance to fault	http://www.cgs.gov.cn
	DEM and derived factors	
3	(altitude, slope, aspect)	ASTER GDEM data (http://www.gscloud.cn)
4	Distance to stream	River and stream network map
5	Distance to road	Road network map

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122 Several classification methods are available to classify the continuous factors into different intervals,
123 including Equal Interval, Quantile, Natural Breaks (Jenks 1963), Geometrical Interval and Standard
124 Deviation available. Among them, the Natural Breaks (NB) and Geometrical Intervals (GI) are most
125 widely used in LSM. Choosing which method to adopt is based on the distribution type of the data. In
126 this study, both the NB and GI classification methods have been adopted to classify the continuous
127 conditional factors (Table 2).

128 **Methodology**

129 **Preparation of training and validating datasets**

130 Landslide observations are usually divided into a training dataset for model building and a validating
131 dataset for model evaluation. Different partition techniques, including time-partition, space-partition and
132 random-partition, might be utilized to obtain the independent dataset (Chung and Fabbri 2003). Since the
133 specific dates of landslides are mostly unknown in this study, the landslides have been randomly split
134 into two datasets with a 3:1 ratio. The first 450 landslide points are used for model construction, and the
135 left 150 landslide points are used for model validation. Also, the same number of non-landslide points

are randomly sampled from the landslide-free areas.

Determination of class numbers

In the treatment of classifying continuous landslide conditional factors, Shannon's entropy index has been adopted to determine optimal class numbers by maximizing the information coefficient of each factor. Shannon's entropy index, which was initially proposed by Claude Shannon (1948) for the entropy model, has been widely used to quantify the uncertainty and disorder in Information Theory. The earth science literature also contains a number of studies using the entropy method as a tool for integrated environmental assessments of natural processes (e.g., debris flows, sandstorms) (Yang and Qiao 2009; Yang et al. 2010).

Landslides are the result of the interaction processes of materials and energies with the environment. Thus, they can be measured and characterized by the entropy method (Pourghasemi et al. 2012; Shi and Jin 2009). Shannon's entropy index determining the optimal class numbers involves the computation of landslide density within each class per factor. The information coefficients (I_j) can be estimated each time for a different number of classes by selecting the maximum information coefficient (Tsangaratos et al. 2017). The calculation can be implemented using a series of formulas given below (cf., Bednarik et al. 2010; Constantin et al. 2011):

$$p_{ij} = \frac{N_{ij}}{A_{ij}} \quad (1)$$

$$(p_{ij}) = \frac{p_{ij}}{\sum_{j=1}^{n_j} p_{ij}} \quad (2)$$

$$H_j = -\sum_{j=1}^{n_j} (p_{ij}) \log_2(p_{ij}) \quad (3)$$

$$H_{jmax} = \log_2 n_j \quad (4)$$

$$I_j = \frac{H_{jmax} - H_j}{H_{jmax}} \quad (5)$$

where N_{ij} and A_{ij} denote the landslide percentage and area percentage for the j -th class of i -th factor,

n_j represents the total the number of classes, p_{ij} represents the probability density, H_j and H_{jmax} represent entropy values. Since the entropy values are above or equal to 0, the information coefficient is restricted to the domain of $[0, 1]$, with values closer to 0 indicating less information and values to 1 indicating more information extracted.

Spatial prediction modeling of landslides

Frequency ratio

The frequency ratio (FR) method has been commonly used to predict landslide spatial distribution for its simple input process, calculations, output process and intelligible principle (Lee and Pradhan 2007; Yilmaz 2009). Specifically, FR is the ratio of the area where landslides occurred to the total studied area (Lee and Talib 2005). It can be concluded that the item of “ p_{ij} ” in Eq. (1) represents the concept of “frequency ratio” (“ fr_{ij} ”) when recalling the entropy model mentioned above. In bivariate statistical analysis, larger frequency ratio values indicate a stronger correlation between landslide occurrence and the factor’s class and vice versa (Pradhan 2010).

Fractal dimension-weight of factors

The process of weighting followed after the interpretation of results obtained from the frequency ratio. This phase involves estimating landslide spatial distribution within the defined class number of each conditional factor based on fractal dimension. The fractal dimension will be used in the Fractal Theory as a measure, which is initially proposed by Benoit Mandelbrot to quantify the complication degree of seacoast shapes with statistical self-similarity (Mandelbrot 1967). The fractal distribution gives the formulas as:

$$N(r) \propto r^{-D} \quad (6)$$

where r represents a typical linear scaling such as distance, time, etc., the parameter D represents the

fractal dimension. Expression $N(r)$ is the resulting increase connected with r , such as stock index, price, monitoring data, etc. (Lu et al. 2012).

A constant-dimension fractal object would be obtained if the fractal dimension D kept constant with various linear scaling r . However, such a fractal relationship does not strictly exist in nature objects (Davis 2002; Shen 2002). Thus, some sophisticated phenomena such as the geological point process (e.g., mineral deposits, geohazards) cannot be well interpreted by traditional fractal methods (Agterberg 2013; Zou 2016).

Given that landslide distribution with its conditional factors is not strictly self-similar, the fractal analysis used here is the variable fractal dimension method (VFDM) (Liu et al. 2019). Concerning the i -th factor, let the linear scaling r_j denote the j -th class, N_j be landslide points within the j -th class, n_i be the number of classes determined by Shannon's entropy index. The fractal dimension D_i derived by the VFDM can be defined by four steps as follows (cf., Lu et al. 2011; Sun et al. 2016):

- Draw the original data series (r_j, N_j) ($j = 1, 2, \dots, n_i$) in double logarithmic coordinate. Where

$\{N_j\} = \{N_1, N_2, \dots, N_{n_i}\}$ is the basic data sequence.

- Construct cumulative-sum sequences with the basic data sequence $\{N_j\}$. We define the 1-order cumulative-sum sequence $\{S1_j\}$, the 2-order cumulative-sum sequence $\{S2_j\}$, and so on, as follows:

$$\begin{cases} \{S1_j\} = \{N_1, N_1 + N_2, \dots, N_1 + N_2 + \dots + N_{n_i}\} \\ \{S2_j\} = \{S1_1, S1_1 + S1_2, \dots, S1_1 + S1_2 + \dots + S1_{n_i}\} \\ \dots \\ \{S(k+1)_j\} = \{Sk_1, Sk_1 + Sk_2, \dots, Sk_1 + Sk_2 + \dots + Sk_{n_i}\} \end{cases} \quad (7)$$

where $k = 1, 2, \dots, n$.

- Build the n -order variable-dimension fractal sequence. Taking the 1-order cumulative-sum sequence for example, we can obtain the segmental fractal dimension $D1_{j,j+1}$ from the slope of double logarithmic coordinate by data points $(r_j, S1_j)$ and $(r_{j+1}, S1_{j+1})$. For n_i data pairs, $n_i - 1$

segmental fractal dimension can be obtained. Therefore, the 1-order segmental fractal dimension sequence can be arranged as:

$$\{D1_{j,j+1}\} = \{D1_{1,2}, D_{2,3}, \dots, D_{n_i-1,n_i}\} \quad (8)$$

while $Dn_{j,j+1}$ ($j = 1, 2, \dots, n_i$) represents the segmental fractal dimension for n -order cumulative-sum sequence.

- Select the optimal transform and determine the corresponding fractal dimension. By comparing every order cumulative-sum sequence, the one with the best linear fit effect is chosen. Thus, the fractal dimension D_i can be calculated as the slope of the linearly fitted line.

The fractal dimension measures the spatial associations between landslide conditional factors and landslide occurrence (Liu 2019), while the relative weights of the factors control the spatial pattern of landslide phenomenon (Liucci et al. 2015). Therefore, the fractal dimension values can be used as proxies for relative weights of the factors.

The above steps are repeated to calculate the fractal dimension D for all the conditional factors, then the weight of the i -th factor (W_i) can be defined by:

$$W_i = D_i / \sum_i D_i \quad (9)$$

The calculated fractal dimension D_i form the W_i value in landslide susceptibility assessment. Then the fractal-based W_i is embedded into the conventional frequency ratio model. We express landslide susceptibility by employing a Calibrated Landslide Susceptibility Index (*CLSI*), with its equation shown as:

$$CLSI = \sum_i W_i \times fr_i \quad (10)$$

Validation and comparison strategy

Receiver operating characteristic curve

The receiver operating characteristic (ROC) curve can evaluate landslide susceptibility models (Zweig and Campbell 1993). Specifically, in each mapping unit, the stochastic methods produce an estimate of the probability (between 0 and 1) of an unstable condition. Once an optimal cutoff value has been derived, the positive prediction and the negative would be given. Correspondingly, the positive prediction refers to landslide prediction, and the negative prediction refers to stable slope prediction. Comparing the predicted with the observed status, estimating the accuracy of the model can be achieved by counting the true and false positive/negative prediction.

The ROC curve depicts the relation between true positive (TP) and false positive (FP), which is a plot of 1-Specificity (X-axis; false positive rate (FPR)) versus Sensitivity (Y-axis; true positive rate (TPR)). These statistical quality parameters are measured as below:

$$1 - \text{Specificity} = \text{FPR} = \frac{FP}{FP+TN} \quad (11)$$

$$\text{Sensitivity} = \text{TPR} = \frac{TP}{TP+FN} \quad (12)$$

where TP and TN represent the number of correctly and incorrectly classified landslides, respectively. Similarly, the parameter FP and FN represent the number of correctly and incorrectly classified non-landslides. Specificity denotes the proportion of non-landslides that are correctly classified as non-landslides. Sensitivity denotes the proportion of landslides that are correctly classified as landslides (Shirzadi et al. 2017; Tien Bui et al. 2016).

The area under the ROC curve (AUROC) is used for quantitative comparison. The AUROC can be interpreted as the probability that a randomly chosen positive example (landslide) is ranked higher than a randomly chosen negative example (non-landslide) (Fawcett 2006). The AUROC can be calculated as:

$$\text{AUROC} = \frac{\sum TP + \sum TN}{P+N} \quad (13)$$

where P/N is the total number of landslides/non-landslides. A non-informative model has an AUROC

value close to 0.5, so that the AUROC values ranges from 0.5 to 1.0 commonly. An ideal model performs an AUROC value of 1.0 (perfect prediction) (Swets 1988).

Relative importance of conditional factors

The relative importance of landslide conditional factors to landslide models is strongly connected with the modeling approaches and data sampling methods. Factors with a high contribution in a specific model might be useless for another one (Tien Bui et al. 2016). Besides, if we do this with the same dataset that has already being used in model construction (i.e., training dataset), the contribution cannot be considered as indicators for factor importance. Instead, it implies the percent of a specific factor being engaged in the modeling process (Kornejady et al. 2017). Therefore, each conditional factors' relative importance and contribution are evaluated with one-by-one factor removal. The overall accuracy (AUROC) with the validating dataset can then be calculated. In such a context, the difference between the AUROC between these factors indicates their relative importance.

Results

Number of classes for landslide conditional factors

Following the procedure described above, the optimal number of classes for the five continuous landslide conditional factors is determined by maximizing their corresponding information coefficients, except for the two factors of lithology and aspect which were considered categorical factors. The analysis was performed from four to eight classes, using both the Geometrical Intervals (GI) and Natural Breaks (NB) classification methods. The information coefficients were calculated and listed in Table 2.

It can be observed from the table that for the distance to the fault factor, the information coefficient has the highest value of 0.1756 when classified into four classes by the GI method. Divided into five classes by the GI method, the three conditional factors of altitude, distance to stream and distance to road

maximize their information coefficients to 0.2804, 0.1985 and 0.2296. Finally, the slope factor presents the highest information coefficient value (0.2467) with five classes classified by the GI method. The most informative appears to be the altitude, followed by slope and distance to road factors when comparing the information coefficients among these conditional factors. The distance to the fault factor gives the least information.

Optimal class numbers of five continuous factors are given in Fig. 3. The two categorical factors of lithology and aspect were also illustrated in this figure.

Table 2 Information coefficients of each conditional factors

Conditional factors	Information Coefficient									
	4 Classes		5 Classes		6 Classes		7 Classes		8 Classes	
	GI	NB	GI	NB	GI	NB	GI	NB	GI	NB
Distance to fault	0.1756	Null	Null	Null	Null	Null	Null	Null	Null	Null
Altitude	0.2443	0.1580	0.2804	0.1695	0.2043	0.2116	0.2153	0.1731	0.2021	0.1733
Slope	0.1986	0.1061	0.2302	0.0851	0.2467	0.0788	0.2232	0.0744	0.1942	0.0623
Distance to stream	0.1614	Null	0.1985	Null	0.1940	Null	Null	Null	Null	Null
Distance to road	0.2241	0.2010	0.2296	Null	0.1334	Null	0.1437	Null	Null	Null

Note: The highest information coefficient value for each factor was indicated in bold text.

It can be concluded that the GI serves as a more suitable classification method for the current case, for the information coefficient values derived from this method are generally higher than those derived from

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287

Fig. 3 The classified landslide conditional factor maps

Landslide frequency ratio

After the determination of the optimal class number, the calculation of fr_{ij} followed. Table 3 sets out the complete calculation of fr_{ij} for each class of factors.

Table 3 The process of frequency ratio and information coefficient calculation

Conditional factor	Number of class (r_j)	Class description	Landslide points (N_j)	N_{ij}	A_{ij}	fr_{ij}/p_{ij}	I_j
Lithology	0	Waters and snow areas	0	0.0000	0.0138	0.0000	0.0722
	1	ophiolite	2	0.0044	0.0032	1.3958	
		Metamorphic rocks (schist, gneiss, metamorphosed sandstone)	56	0.1244	0.0381	3.2668	
	3	Intrusive volcanic rocks (granite, diorite, gabbro)	142	0.3156	0.1796	1.7575	

Conditional factor	Number of class (r_j)	Class description	Landslide points (N_j)	Landslide			
				N_{ij}	A_{ij}	fr_{ij}/p_{ij}	I_j
Distance to fault	4	Extrusive volcanic rocks (tuff, breccia, basalt)	47	0.1044	0.0744	1.4034	0.1756
	5	limestone	5	0.0111	0.0105	1.0582	
	6	Sedimentary rocks (sandstone, thinly bedded fine sandstone)	105	0.2333	0.1823	1.2798	
	7	Soft sedimentary rocks (mudstone, shale, coal line)	19	0.0422	0.1416	0.2982	
	8	Quaternary deposits (sand, soil, mud, gravel)	74	0.1644	0.3565	0.4613	
	1	<1,895 m	93	0.2067	0.1709	1.2093	
	2	1,895 m~8,571 m	176	0.3911	0.3531	1.1078	
	3	8,571 m~32,088 m	179	0.3978	0.3482	1.1423	
Altitude	4	>32,088 m	2	0.0044	0.1278	0.0348	0.2804
	1	<687 m	35	0.0778	0.2734	0.2845	
	2	687 m~807 m	8	0.0178	0.1242	0.1431	
	3	807 m~1,130 m	75	0.1667	0.2290	0.7277	
	4	1,130 m~1,997 m	284	0.6311	0.2239	2.8190	
Slope	5	>1,997 m	48	0.1067	0.1495	0.7136	0.2467
	1	<0.15°	47	0.1044	0.1450	0.7204	

Conditional factor	Number of class (r_j)	Class description	Landslide points (N_j)	Landslide			
				N_{ij}	A_{ij}	fr_{ij}/p_{ij}	I_j
Aspect	2	0.15°~0.66°	1	0.0022	0.0218	0.1022	
	3	0.66°~2.32°	31	0.0689	0.3205	0.2150	
	4	2.32°~7.75°	66	0.1467	0.2489	0.5893	
	5	7.75°~25.57°	179	0.3978	0.1660	2.3968	
	6	>25.57°	126	0.2800	0.0979	2.8589	
	0	Flat	0	0.0000	0.2119	0.0000	
	1	North	19	0.0711	0.0729	0.9754	
	2	Northeast	36	0.0800	0.0715	1.1191	
	3	East	86	0.1911	0.1221	1.5647	
	4	Southeast	49	0.1089	0.0671	1.6238	0.0646
	5	South	102	0.2267	0.1474	1.5379	
	6	Southwest	45	0.1000	0.0861	1.1608	
	7	West	71	0.1578	0.1467	1.0757	
	8	Northwest	29	0.0644	0.0743	0.8669	
	1	<4,700 m	169	0.3756	0.1705	2.2029	
Distance to stream	2	4,700 m~14,797 m	123	0.2733	0.2628	1.0400	
	3	14,797 m~36,483 m	121	0.2689	0.2694	0.9981	0.1985
	4	36,483 m~83,066 m	34	0.0756	0.1663	0.4543	
	5	>83,066 m	3	0.0067	0.1310	0.0509	
	1	<1,118 m	190	0.4222	0.1663	2.5391	0.2296

Conditional factor	Number of class (r_j)	Class description	Landslide points (N_j)				I_j
				N_{ij}	A_{ij}	fr_{ij}/p_{ij}	
Distance to road	2	1,118 m~3,414 m	92	0.2044	0.2736	0.7473	
	3	3,414 m~8,128 m	124	0.2756	0.3139	0.8779	
	4	8,128 m~17,808 m	42	0.0933	0.1823	0.5119	
	5	>1,7808 m	2	0.0044	0.0639	0.0695	

294 Note: For categorical factors of lithology and aspect, the “ p_{ij} ” values of class number “0” were not involved in the calculation of
295 information coefficients.

296 The calculated fr values for each class/interval of the conditional factors were plotted in Fig. 4.

297 According to the thematic map of lithological units with landslides, the most affected areas are built of
298 ophiolite and metamorphic rocks. Although they represent only 4 % of the total area of the studied
299 territory, more than 12 % of the total landslides occur in these two categories. For the altitude factor,
300 slope failures mainly occurred in class 4 (1,130 m~1,997 m), reaching 63 % of the total landslides. As
301 for aspect factor, slope deformations exhibit an approximating normal distribution. The landslide
302 inventory well represents the link between landslides and slope. It can be seen in Fig. 4 that landslides
303 are more likely to occur with an increasing slope gradient. The most critical slopes range between 7.75°-
304 25.57° (class 5), with more than 39 % landslides. Additionally, unlike distance to stream and distance to
305 road factors, which landslide occurrence decreases with increasing distances, landslide occurrences are
306 distributed evenly with varying distances to the fault.

Fig. 4 Variations of the computed fr values

Fractal-based weights

The measurement of spatial associations between landslides and the conditional factors using fractal dimension has been performed by the VFDM. Fig. 5 presents the linear fitting results of the relationships between the conditional factors and the transformed landslide cumulative-sum sequences. The figure shows that the paired datasets $(r_j, S2_j)$ of six factors can be well linear fitted after the two-order cumulative-sum transform, except for the slope factor.

Fig. 5 Relationship between landslide cumulative-sum sequences and conditional factors

From the slopes of linearly fitted lines, the fractal dimension values can be derived. Table 4 provides the D_i values that are used to weight these factors. The highest W_i (0.2071) of lithology factor indicates that the dominant effect upon landslide occurrence and distribution is posed by lithology formation, followed by altitude (0.1620), slope (0.1405) and aspect (0.1324) factors. Landslide occurrence is least susceptible to the distance to stream with the lowest W_i (0.0960).

Table 4 Weights of the factors calculated from fractal dimension

Conditional factors	Linear regression formulation	Correlation coefficient (R^2)	D_i	W_i
Lithology	$lgS2 = 3.1770lgr + 0.6197$	0.9549	3.1770	0.2071
Distance to fault	$lgS2 = 1.9026lgr + 1.9775$	0.9984	1.9026	0.1240
Altitude	$lgS2 = 2.4858lgr + 1.4335$	0.9687	2.4858	0.1620
Slope	$lgS3 = 2.1554lgr + 1.5749$	0.9767	2.1554	0.1405
Aspect	$lgS2 = 2.0315lgr + 1.4531$	0.9970	2.0315	0.1324
Distance to stream	$lgS2 = 1.4732lgr + 2.2278$	0.9995	1.4732	0.0960
Distance to road	$lgS2 = 2.1156lgr + 1.5640$	0.9969	2.1156	0.1379

Different from commonly used BCFD algorithm to measure the spatial association between landslides and conditional factors (e.g., Li et al. 2012; Liucci et al. 2015; Zou and Carranza 2017; Liu et al. 2019), this paper first defines the optimal class number of landslide conditional factor by Shannon's entropy model, measures the heterogeneous fractal distribution of landslides among the each factor's class by the VFDM, and uses the derived fractal dimensions to provide the conditional factors with numerical weights.

Model results and analysis

The final landslide susceptibility map can be created by summing weighted multiplications of the classified factor maps. Based on Eq. (10), the equation used to create a landslide susceptibility map can be described as:

$$CLSI = 0.2071 \times fr_{Lithology} + 0.1240 \times fr_{Distance\ to\ fault} + 0.1620 \times fr_{Altitude} + 0.1405 \times fr_{Slope} + 0.1324 \times fr_{Aspect} + 0.9060 \times fr_{Distance\ to\ stream} + 0.1379 \times fr_{Distance\ to\ road} \quad (14)$$

The result of this summation shows continuous values ranging from 0.1361 to 2.5896. Typically, the values should be divided into three or five classes expressing the different levels of landslide susceptibilities. In this study, the Natural Breaks classification method has been adopted to divide the calculate *CLSI* values into five susceptibility levels, according to Eq. (11) as:

- Very Low (VL) [0.0361~0.7038]
- Low (L) [0.7038~1.0983]
- Moderate (M) [1.0983~1.4831]
- High (H) [1.4831~1.8776]
- Very High (VH) [1.8776~2.5896]

For visualization, the final landslide susceptibility map was illustrated in Fig. 6a.

Fig. 6 The landslide susceptibility maps produced by the proposed method (a) and the FR method (b)

According to the statistical result of the landslide susceptibility map (Table 5), the VL susceptibility level accounts for 34.80 % of the total studied region, but only 0.89 % of landslides falls in this level. The L, M and H susceptibility zones donate at a value of 26.71 %, 22.12 %, 12.52 %, respectively. Only 3.85 % of the study region is classified as presenting VH susceptibility, yet it includes 24.00 % of the total number of landslides. It can also be observed that when moving from the very low susceptibility class successively to very high susceptibility class, the corresponding landslide density showed an increasing trend.

Table 5 Statistics of landslide susceptibility map

Susceptibility level	Areas (cell numbers)	Percentage of domain (%)	Percentage of Number of landslides	Percentage of landslides (%)
VL	45,158,011	34.80	4	0.89
L	34,657,849	26.71	54	12.00
M	28,704,258	22.12	106	23.56

H	16,245,339	12.52	108	24.00
VH	4,990,789	3.85	178	39.55

Validation and comparison

The prediction performance of the landslide model has been evaluated using the ROC curve. The ROC analysis shows that the proposed method (red line in Fig. 7) provides a satisfying prediction accuracy, with an AUROC value of 0.8467.

Fig. 7 ROC curves based on the validating dataset

The second landslide susceptibility map (Fig. 6b), together with the related ROC curve (blue line in Fig. 7), has been added using the same weight for all the conditional factors. Its ROC shows relatively lower prediction performance (AUROC=0.8088). Applying the same weight values is the same as merely summing the *fr* values. In this study, applying the fractal-based weights on frequent ratio gives a more accurate prediction for landslide susceptibility. The improved FR model shows higher prediction than the conventional FR model. This finding is consistent with similar comparative results (e.g., Yilmaz 2009; Wang et al. 2014; Kavzoglu et al. 2015; Liu et al. 2019).

Discussions

Factor relative importance

Several influencing factors of landslides have been identified and used to construct the LSM model. The evaluation of factor importance needs to be further discussed, presented in Fig. 8. Assessment of the AUROC values shows that when using all the seven conditional factors, the AUROC gives the highest value of 0.8467. Removing the lithology factor, the most significant reduction in AUROC occurs, indicating that lithology formation as a natural factor provides more information than others. The AUROC, by excluding the altitude factor, also exhibits considerable reduction, followed by the slope. These two factors also act as significant roles pointing out the geomorphology configuration on the propensity to landslides. Distance to stream produces the lowest reduction in AUROC when excluding it in the model, meaning that landslides in the study region do not respond much to this factor.

Since the validating dataset is obtained from the random-partition technique, the AUROC from excluding a factor indicates its more transferability when using future data or employing the model for another region (Kornejady et al. 2017).

Fig. 8 AUROC values for the proposed model using validating dataset

Comparison of typical BCFD and proposed VFDM method

By following the conclusion drawn by Liu (2019), the spatial distribution of landslides is usually a

heterogeneous fractal structure. We believe that the landslide spatial distribution in the study area also possesses heterogeneous fractal properties, characterized by the VFDM, instead of the most commonly used BCFD algorithm. Below we discuss this hypothesis.

First, landslides in the study area have been analyzed using the BCFD algorithm (cf., Li et al. 2012). Table 5 sets out the computed number of grid cells occupied by landslides N_j (Row 3), with different grid size r_j . The parameters N_j versus r_j have been plotted in a double logarithmic coordinate (Fig. 9a). It can be seen that these points are best fitted by a polynomial curve rather than a straight line. This result indicates that the spatial distribution of landslides presents variable dimension fractal characteristics, meaning the fractal dimension values change with the linear scaling r . In other words, the fractal structure of landslide distribution is a heterogeneous one.

Furthermore, the VFDM has been introduced to process the calculated data in Table 5. The landslide grid number N_j was transformed to obtain the 1-order cumulative-sum sequence $S1_j$ (Row 4), then the data series (r_j , $S1_j$) were plotted in double logarithmic coordinate (Fig. 9b). For comparison, these data series can be well fitted by a single straight line, from which we can obtain a constant fractal dimension value.

These outcomes comparisons support our hypothesis and prove the feasibility of VFDM for the current research.

Table 5 Results of the BCFD algorithm

Grid size (km)/ r_j	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
Total Grid number	11,798,635	469,796	117,986	29,763	13,331	7,564	4,871	3,416	2,524	1,955	1,552	1,265

Landslide grid												
	597	560	521	457	417	377	336	307	276	258	241	218
number/ N_j												
$S1_j$	597	1,157	1,678	2,135	2,552	2,929	3,265	3,572	3,848	4,106	4,347	4,565

Fig. 9 Variation in $N_j/S1_j$ with r_j

Conclusions

This paper presented a novel framework for landslide susceptibility mapping by integrating information theory, fractal theory and statistical analysis. Seven conditional factors were selected to perform statistical analyses, including lithology, distance to fault, altitude, slope, aspect, distance to stream and distance to the road. Significant steps of the proposed framework could be achieved by determining the optimal class number of landslide conditional factors by Shannon's entropy model and identifying the weights of landslide conditional factors via the variable fractal dimension method (VDFM). The proposed framework was illustrated using an example area in China.

Experimental results from this research showed that the proposed framework had a relatively high prediction performance. Specifically, the proposed method in this work outperformed the conventional FR method, showing a higher prediction accuracy in terms of AUROC values (0.8467) than that estimated for the FR method (0.8088). According to the final prognostic landslide susceptibility map, 16.37 % of the study area showed very high and high susceptibility, including the 63.55 % of the total number of landslides. The most susceptible areas were located in the northeast mountainous areas, while

low and very low susceptible zones characterized the central-south areas. Evaluation of relative factor importance based on a one-by-one factor removal test indicated that the lithology factor provides unique information compared to the rest.

Furthermore, for many regions where the landslides distribution presents fractal characteristics with heterogeneous structure, using the framework described here can make a promising landslide susceptibility assessment.

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Declarations

Conflict of interest The author declare no competing interest.

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