The case of a repulsive force in an inhomogeneous gravitational field without shedding momentum

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Article

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Abstract

This paper considers the case of generating a repulsive force that acts on a body in an inhomogeneous gravitational field in the direction opposite to the force of gravitational attraction. This force differs from the known reactive force in that shedding momentum is not required to generate it, that is, it is not necessary, for example, to eject any mass. At the same time, a necessary condition for generating this type of force is the presence of an inhomogeneous gravitational field. Being able to generate force in this fashion is especially relevant for solving problems related to movement in empty space. In a sense, we could dub this force "quasi antigravitational".

1. Introduction

The idea of generating a force that can be used to change the momentum of a body as it moves in empty space has long been the subject of various fundamental and applied studies. These research results led to development of various technical devices operating on the principle of generating a reactive force, that is, the force that appears as a result of the body shedding some of its momentum. As a rule, a certain mass of matter is ejected somehow, moving away from the body at a non-zero relative velocity. As a result, the velocity of the body changes in accordance with the law of conservation of momentum.

A significant disadvantage of such devices is the fact that, when moving in an empty space, generation of a reactive force requires ejecting a certain mass, while this substance itself must be somehow stored in advance in the device. Consequently, when, being obviously limited, the reserves of this substance run out, the device will cease to generate a reactive force.

Some studies also consider cases of plausible technical devices operating on the principle of exploiting electromagnetic radiation pressure. Two examples of these are the so-called VEM drive [1] and the solar sail [2]. In the VEM drive, the source of electromagnetic radiation is a part of it, while in the solar sail, the source of radiation is an external body. The disadvantage of such projects is that the force that can be obtained by implementing them in practice is very small.

This paper aims to answer the following fundamental question: is it possible to generate a mechanical repulsive force directed away from the centre of gravity in empty space without shedding momentum? In other words, this should be neither a reactive force nor a force obtained in some way through electromagnetic interaction with the environment. The question under consideration can be rephrased as follows: is it possible in this case to use an external gravitational field to generate a repulsive force directed away from a massive body representing the centre of attraction?

There exist studies substantiating this possibility via the properties of certain solutions to the three-body problem (see, for example, [3]).

Theoretical investigations also show that generating additional space-time curvature via a certain technique may in principle become a source of such a force. This is the principle behind the so-called

WARP drives. At present, a fairly large number of works deal with similar ideas, for example, [4–14]. However, preliminary assessments of this force generation method indicate that the energy costs for increasing the space-time curvature would be too large for this method to be feasible at the current level of development of science and technology. This makes such projects a very distant applied research prospect.

Another possible investigation direction is seeking the repulsive force within the framework of the general theory of relativity. This force is commonly believed to arise as a consequence of the solutions to the equations of the general theory of relativity [15–20].

There also exist papers that discuss the possibility of generating a repulsive force due to general relativity postulating possible existence of negative mass (see [21–24]) or certain hypothetical properties of vacuum [25].

It should be noted that these methods of generating a repulsive force without shedding material momentum are in general at the initial stages of theoretical investigation. At the moment, their practical implementation is out of the question.

The case presented in this paper provides a positive answer to the question stated above. We specifically consider a method of generating a repulsive force without shedding momentum that can be readily implemented at the current stage of technological development. The necessary condition for generating this force turns out to be the inhomogeneity of the external gravitational field in which the body is located. We consider a field created by a massive spherical body as an example. Such fields are known to exist around planets or stars.

2. Methods

2.1 Describing the motion of a small body in the gravitational field of a larger body

Further reasoning requires describing the trajectory of a small body in the gravitational field of a massive spherical body. This paper will use a nonrelativistic approximation to describe this type of motion. We can confidently state that the problem of determining the characteristics of the motion of a small body in the gravitational field of a larger body is a classical one. As a rule, numerous textbooks on classical mechanics provide example solutions describing the trajectory of a small body when moving in the gravitational field of a massive body. Consequently, there exist various methods for deriving respective motion trajectory equations. For example, a derivation can be found in [26].

Here we will consider a massive spherical body creating a centrosymmetric gravitational field in the space surrounding it. This means that any other body located relatively close to the massive body is affected by a force that is commonly called Newton’s force of gravitational attraction.

Our further calculations will assume M to be the mass of the spherical body that creates the gravitational field (we will call it the larger body), it being much greater than m, the mass of the small body in its gravitational field, i.e., $M \gg m$. We will consider the small body as a material point in comparison with the larger body. This approximation makes it possible not to take into account the motion of the larger body, but to consider only the small body moving relative to the larger one.

The type of trajectory for a small body moving relative to a larger one is determined by the magnitude of the body velocity and its direction at a certain point in space where the motion takes place. Excluding those cases when the small body falls on the surface of the larger body, the possible trajectories of the small body are conic section curves: a circle, an ellipse, a parabola, a hyperbola. They are usually called orbits.

It should also be taken into account that when a small body moves in the gravitational field of a larger body, the laws of conservation of energy and angular momentum hold.

So, let r be the distance between the small body and the centre of the larger body at a point in time. Then the magnitude of Newton's gravitational attraction force F_G between these bodies is determined by the expression:

$$\boxed{F_G = G \bullet \frac{m \bullet M}{r^2}, \qquad (1)}$$

where the gravitational constant is $G \approx 6{,}67 \bullet 10^{-11}\ m^3/(kg\ s^2)$.

In order to describe the trajectory of the small body, it is convenient to introduce a Cartesian coordinate system (X, Y, Z), the origin of which (point O) is the centre of the larger body (Fig. 1). Assuming that the motion is planar, let us restrict ourselves to the case when the velocity vector of the small body lies in the plane (X, Y, 0). Then the trajectory of the small body will lie in the same plane. Therefore, in this case the Z axis is not required in practice to describe the motion (Fig. 1 does not show the Z axis, but we may assume that it is directed towards the observer, perpendicular to the figure plane at point O). In the case under consideration, the radius vector of the small body relative to the origin can be written in the coordinate form:

$$\boxed{\vec{r} = (x, y, 0). \qquad (2)}$$

The length of the radius vector is the distance from the small body to the centre of the larger body:

$$r = \sqrt{x^2 + y^2}. \quad (2')$$

If we introduce the angle φ between the *Y* axis and the radius vector of the small body (Fig. 1), then the coordinates of the radius vector can be expressed as follows:

$$\begin{cases} x = r \bullet sin\phi \\ y = r \bullet cos\phi \end{cases}. \quad (3)$$

The velocity vector of the body and the radius vector are linked by the equation:

$$\vec{V} = \frac{dr}{dt}. \quad (4)$$

The velocity vector for the small body can be written in the coordinate form:

$$\vec{V} = \left(V_X, V_Y, 0\right),. \quad (4')$$

therefore, the velocity magnitude of the small body relative to the larger body is determined by the expression:

$$V = \sqrt{V_X{}^2 + V_Y{}^2}. \quad (4'')$$

We can determine the angular momentum vector of the small body relative to the centre *O* of the larger body as a vector product:

$$\boxed{\overrightarrow{PM_O} = \vec{r} \times \left(m \bullet \vec{V}\right) \quad (5)}$$

This vector $\overrightarrow{PM_O}$ is perpendicular to the plane in which lie the radius vector $\vec{r}$ and the velocity vector $\vec{V}$ characterising the small body. Since the gravitational force $\vec{F}_G$ is a central force and the moment of a central force relative to the point O is equal to zero, the angular momentum vector (5) does not change over time. Therefore, the plane in which the trajectory of the small body lies does not change either, as it was mentioned above.

The vector (5) can be represented in the coordinate form as follows:

$$\overrightarrow{PM_O} = m \bullet \left(0,0, x \bullet V_Y - y \bullet V_X\right)$$

Therefore, the statement that $\overrightarrow{PM_O} = const$ for the small body mass m being constant is equivalent to the equation:

$$\boxed{x \bullet V_Y - y \bullet V_X = const. \quad (5')}$$

We can show that the equation of the small body motion (Newton's second law) as affected by the force (1):

$$\boxed{m \bullet \vec{a} = \overrightarrow{F_G} \quad (6)}$$

leads to the law of conservation of mechanical energy:

$$\boxed{\frac{m \bullet V^2}{2} - \frac{G \bullet m \bullet M}{r} = const. \quad (7)}$$

Further calculations require introducing new variables (a polar coordinate system) $(x, y) \longmapsto (r, \phi)$, where *r* is the length of the radius vector (2'). In this case the velocity (4") is determined by this equation:

$$\boxed{V = \sqrt{\left(\frac{dr}{dt}\right)^2 + r^2 \bullet \left(\frac{d\phi}{dt}\right)^2}, \qquad (4'')}$$

and expression (5') takes the form:

$$\boxed{r^2 \bullet \left(\frac{d\phi}{dt}\right) = const. \qquad (5'')}$$

We will then introduce the following designation:

$$\boxed{r^2 \bullet \left(\frac{d\phi}{dt}\right) = L_0. \qquad (8)}$$

Using the new variables, after cancelling out the small body mass *m*, the expression (7) will take the following form, taking into account (8):

$$\boxed{\frac{1}{2}\left(\left(\frac{dr}{dt}\right)^2 + r^2 \bullet \left(\frac{d\phi}{dt}\right)^2\right) - \frac{G \bullet M}{r} = const. \qquad (9)}$$

Since the distance along the desired motion trajectory depends on the angular variable $r = r(\phi)$, then after substituting $\rho = \frac{1}{r}$ the derivative $\frac{dr}{dt}$ is transformed as follows:

$$\boxed{\frac{dr}{dt} = \frac{dr}{d\phi} \bullet \frac{d\phi}{dt} = -\frac{1}{\rho^2}\frac{d\rho}{d\phi} \bullet \frac{d\phi}{dt}. \qquad (10)}$$

The expression (9), taking into account (8) and (10), leads to:

$$\boxed{\frac{1}{2} \bullet \left(\left(\frac{d\rho}{d\phi}\right)^2 \bullet {L_0}^2 + \rho^2 \bullet {L_0}^2\right) - G \bullet M \bullet \rho = const \qquad (9')}$$

After differentiating (9') with respect to the angular variable φ and cancelling $\frac{d\rho}{d\phi}$, we obtain a second-order ordinary differential equation:

$$\boxed{\frac{d^2\rho}{d\phi^2} \bullet {L_0}^2 + \rho \bullet {L_0}^2 = G \bullet M. \qquad (11)}$$

The solution to the equation (11) can be presented in general form:

$$\boxed{\rho = \frac{G \bullet M}{{L_0}^2} + C \bullet cos\left(\phi - \phi_0\right). \qquad (12)}$$

Taking into account the substitution $\rho = \frac{1}{r}$ and the introduction of the notation $p = \frac{{L_0}^2}{G \bullet M}$ and $\epsilon = \frac{C \bullet {L_0}^2}{G \bullet M}$, the equation for the small body motion trajectory follows from (12):

$$\boxed{r = \frac{p}{1 + \epsilon \bullet cos\left(\phi - \phi_0\right)} \qquad (13)}$$

The parameters included in (13) usually mean the following: *p* is the focal parameter, ε is the orbital eccentricity. The expression (13) is known to define a second-order curve, which can be considered as a conic section. The specific form taken by this curve is determined by the value of the parameter ε. In particular, the value ε = 0 corresponds to a circular trajectory, while for 0 < ε < 1 the trajectory is elliptical. At ε = 1 it is a parabola; for ε > 1 we have a hyperbola.

The values of the constants C and φ_0 in (12) can be determined by substituting expression (13) into (8) and (9) for specific values of the distance and velocity of the body. If we select $\varphi_0 = 0$ in (13), then the orbital curve will be symmetrical relative to the *Y* axis (Fig. 1). The distance from the small body to the centre of the larger body will be minimal in this case, when φ = 0. Let this distance be denoted as r_0. The velocity of the small body at this point will be designated as V_0.

As r_0 is the minimum distance, then the extremum condition $\frac{dr}{dt} = 0$ must be satisfied at φ = 0 and $r = r_0$; therefore, it follows from (4”) that:

$$V_0 = r_0 \bullet \left(\frac{d\phi}{dt}\right)_0$$

where $\left(\frac{d\phi}{dt}\right)_0$ is the angular velocity at φ = 0. Then the equation (8) assumes the form:

$$\boxed{V_0 \bullet r_0 = L_0. \qquad (14)}$$

This expression may be interpreted as follows: at the point closest to the centre of the larger body, the velocity vector $\vec{V_0}$ of the small body is perpendicular to the radius vector $\vec{r_0}$.

The equation (13) implies that for ε = 0 (that is, when the body moves in a circle), the velocity of the small body is $V_0 = \sqrt{\frac{G \bullet M}{r_0}}$. To satisfy the inequality 0 < ε < 1 (meaning that the orbit is an ellipse), the following double inequality must hold: $\sqrt{\frac{G \bullet M}{r_0}} < V_0 < \sqrt{2\frac{G \bullet M}{r_0}}$. For ε = 1 (when the orbit is a parabola), the velocity is $V_0 = \sqrt{2\frac{G \bullet M}{r_0}}$. The body will move along a hyperbola, i.e., ε > 1, if $V_0 > \sqrt{2\frac{G \bullet M}{r_0}}$.

2.2. Thought experiment

Consider the following thought experiment. Let the speed of the small body at the point closest to the larger body, i.e., at the distance r_0 from the centre of the larger body, be given as follows:

$$\boxed{V_0 = \beta \bullet \sqrt{\frac{G \bullet M}{r_0}}, \qquad (15)}$$

where β ≥ 1 is a constant coefficient. Then, according to the reasoning above, this coefficient determines the type of the small body trajectory. For example, we may note that the trajectory is a circle for β = 1.

Taking into account (15), the expression for the focal parameter in (13) takes the form $p = \beta^2 \bullet r_0$, and the eccentricity can be derived from (13), if we take into account that φ_0 was selected to be 0 and, consequently, the following should hold true: $r = r_0$ at φ = 0. Therefore, we arrive at the conclusion that $\varepsilon = \beta^2 - 1$, and the expression (13) takes the form:

$$\boxed{r = \frac{\beta^2 \bullet r_0}{1 + \left(\beta^2 - 1\right) \bullet cos(\phi)}. \qquad (16)}$$

Let *A* be the point closest to the centre of the larger body. Then the *Y* axis passes through point *A* and the centre of the larger body, that is, point *O* (Fig. 2). Suppose that at some point in time two identical small bodies simultaneously begin to move from point *A* with certain initial velocities; the mass of each body is equal to *m*. At the initial moment, let the velocity vectors of these bodies be perpendicular to the *Y* axis, pointing in opposite directions, and let the vector lengths be the same, as given by the expression (15). The motion trajectories for each body should be described by equations of the form (16). In what follows, the parameters of motion related to the small body that moves to the left will be denoted by the subscript *L*, while the parameters of the small body that moves to the right will be distinguished by the subscript *R*.

Accordingly, for the velocity vectors of both bodies at the point *A*, the following equation is true:

$$\vec{V}_{0L} = -\vec{V}_{0R}$$

These small bodies, having started their movement from point *A*, will move along trajectories that are symmetrical relative to the *Y* axis. They will travel the same distance over equal periods of time. As a result, these two bodies will simultaneously reach certain points B_R and B_L that are located on their trajectories symmetrically relative to the *Y* axis (Fig. 2). Taking these assumptions into account, we will now consider the motion of one of the bodies in detail. Let us take the right-hand one, for example.

As the masses of the bodies are very small in comparison with the mass of the larger body, the force of gravitational interaction between these two small bodies is negligible as compared to the forces of their gravitational interaction with the larger body.

Having reached the point B_R with coordinates $\left(r_R, \phi\right)$, for $\beta > 1$ the velocity vector of the right body $\vec{V}_R$ will no longer be perpendicular to the radius vector $\vec{r}_R$, and hence to the vector of the gravitational attraction force $\vec{F}_{GBR}$ (Fig. 3) (since in this case, the trajectory is not a circle). The expression for the gravitational force acting on the right-hand small body at the point B_R can be written in vector form in terms of the radius vector of the small body as follows:

$$\vec{F}_{GBR} = -G \bullet \frac{m \bullet M}{{r_R}^3} \vec{r}_R$$

This means that in order to find the projection of the body velocity vector $\overrightarrow{V_R}$ onto the direction of the force vector $\overrightarrow{F_{GBR}}$, it is sufficient to find the angle between the body velocity vector and the radius vector $\overrightarrow{r_R}$.

The angle θ between the radius vector $\overrightarrow{r_R}$ and the velocity vector $\overrightarrow{V_R}$ of the right-hand body at the point B_R can be derived using the dot product, the expression for which looks as follows in coordinate form:

$$cos(\theta) = \frac{x_R V_{RX} + y_R V_{RY}}{V_R r_R}. \qquad (17)$$

The coordinates of the right-hand body may be written in the form of the equations below stemming from (3) and (13):

$$\begin{cases} x_R = \dfrac{\beta^2 \bullet r_0 \bullet sin(\phi)}{1 + \left(\beta^2 - 1\right) \bullet cos(\phi)} \\ y_R = \dfrac{\beta^2 \bullet r_0 \bullet cos(\phi)}{1 + \left(\beta^2 - 1\right) \bullet cos(\phi)} \end{cases}, \qquad (18)$$

while (4) and (18) allow us to derive the velocity vector coordinates:

$$\begin{cases} V_{RX} = \beta^2 \bullet r_0 \bullet \frac{d\phi}{dt} \bullet \frac{cos(\phi) + \left(\beta^2 - 1\right) \bullet cos(2\phi)}{\left(1 + \left(\beta^2 - 1\right) \bullet cos(\phi)\right)^2} \\ V_{RY} = -\beta^2 \bullet r_0 \bullet \frac{d\phi}{dt} \bullet \frac{sin(\phi)}{\left(1 + \left(\beta^2 - 1\right) \bullet cos(\phi)\right)^2} \end{cases} . \quad (19)$$

Equations (2′) and (4′) enable us to find the values of respectively r_R and V_R in (17).

After substituting (18) and (19) into (17), we obtain the following expression:

$$cos(\theta) = \frac{sin(\phi) \bullet \left(\beta^2 - 1\right) \bullet cos(2\phi)}{\sqrt{1 + 2 \bullet \left(\beta^2 - 1\right) \bullet cos(\phi) \bullet cos(2\phi) + \left(\beta^2 - 1\right)^2 \bullet cos^2(2\phi)}} . \quad (20)$$

It should be noted that for β = 1, i.e., when the body moves in a circle (ε = 0), the equation (20) means that *cos*(θ) = 0 for any angle φ. Consequently, in this case, the velocity vector of the body is perpendicular to the vector of the force of gravitational attraction at each point of the trajectory, which is an obvious statement. The function (20) of the angle φ is plotted in Fig. 4 for different values of the parameter β > 1.

Consider a small section of the orbit in the vicinity of the point *A* (Fig. 3), i.e., close to the angle φ = 0, for the case when β > 1. It should be noted that it is possible to make estimates for the small value of the angle φ ≪ 1 as follows:

$$\begin{cases} cos(\phi) \approx 1 - \frac{\phi^2}{2} \\ sin(\phi) \approx \phi \end{cases} . \quad (21)$$

In this case, taking into account (21), an approximate expression is obtained from the equation (20):

$$cos(\theta) = \phi \bullet \frac{\left(\beta^2 - 1\right)}{\beta^2}. \quad (22)$$

Therefore, it follows from (22) that in the vicinity of the point *A*, i.e., when the angle φ is small, the inequality cos (θ) > 0 must be true for β > 1, therefore θ < 90°. This means that the angle between the velocity vector of the small body and the vector of the gravitational attraction force is greater than 90 degrees. Consequently, the component of the velocity vector of the small body that is parallel to the gravitational attraction force vector will point in the opposite direction to the gravitational attraction force vector in the vicinity of this point.

Suppose that at point B_R the right-hand small body bounces elastically from a certain wall, moving after impact in the opposite direction along the same trajectory. Having reached point *A*, the body bounces elastically from another wall and again moves along the same trajectory to point B_R, where it bounces back, etc. At point *A*, the velocity vector of the small body is perpendicular to the vector of the gravitational attraction force. Therefore, the vector of the force acting on the wall at point *A* while the small body bounces is also perpendicular to the vector of the gravitational attraction force.

However, as shown above, at point B_R this angle is no longer equal to 90 degrees. Therefore, the vector $\vec{F}_{BR}$ representing the force acting on the wall during the elastic bounce of the small body at the current point is no longer perpendicular to the vector of the gravitational attraction force $\vec{F}_{GBR}$ at point B_R (Fig. 5). Consequently, the force vector $\vec{F}_{BR}$ may be represented as a sum:

$$\vec{F}_{BR} = \vec{F}_{\perp BR} + \vec{F}_{\parallel BR}$$

in such a way that the vector $\vec{F}_{\perp BR}$ will be perpendicular to the vector of gravitational attraction $\vec{F}_{GBR}$, while the vector $\vec{F}_{\parallel BR}$ will be parallel but directed opposite to $\vec{F}_{GBR}$. This statement makes it possible to estimate the length of the projection $\vec{F}_{\parallel BR}$:

$$F_{\parallel BR} = F_{BR} \bullet \cos(\theta). \quad (23)$$

The change introduced to the vector of the small body momentum $\overrightarrow{\Delta p}$ by the body bouncing from the wall is equal to the impulse of the force (averaged over time) $\overrightarrow{F_{BR}}'$ acting on the small body upon impact from the direction of the wall :

$$\overrightarrow{\Delta p} = \overrightarrow{F_{BR}}' \bullet \Delta t. \qquad (24)$$

According to Newton's third law, our small body acts on the wall with a force of the same magnitude but in the opposite direction:

$$\overrightarrow{F_{BR}} = -\overrightarrow{F_{BR}}'. \qquad (24')$$

As projected onto the direction of motion during elastic impact, the change in the momentum of the small body during elastic impact is equal to:

$$\Delta p = -2mV_R \qquad (25)$$

The law of conservation of energy (7) allows us to determine the small body velocity {V}_{R} at the moment of impacting the wall at point B_R:

\frac{m{{V}_{0}}^{2}}{2}-G\frac{mM}{{r}_{0}}=\frac{m{{V}_{R}}^{2}}{2}-G\frac{mM}{{r}_{B}}. (26)

It follows from (16) that the following approximate expression is valid for the small angle φ:

$$ {r}_{B}\approx \frac{{r}_{0}}{1-\left(1-\frac{1}{{{\beta }}^{2}}\right)\bullet \frac{{{\phi }}^{2}}{2}}. \tag{27} $$

The equations (26) and (27) lead to:

$$ {V}_{R}\approx {V}_{0}\bullet \sqrt{1-\left(1-\frac{1}{{{\beta }}^{2}}\right)\bullet \frac{{{\phi }}^{2}}{{{\beta }}^{2}}} \tag{28} $$

or,

$$ {V}_{R}\approx {V}_{0}\bullet \left(1-\frac{1}{2}\bullet \left(1-\frac{1}{{{\beta }}^{2}}\right)\bullet \frac{{{\phi }}^{2}}{{{\beta }}^{2}}\right). \tag{28'} $$

The time interval Δt in (24) may be estimated as the time between two successive impacts of the right-hand small body against the wall at the point B_R. The duration estimate for this time interval for a small angle $\varphi \ll 1$ takes the form of a double inequality:

$$ \frac{2\bullet dl}{{V}_{0}}\le \varDelta t\le \frac{2\bullet dl}{{V}_{R}}, \tag{29} $$

where the small magnitude of the path length from point A to point B_R is determined by the approximate expression

$$dl\approx \sqrt{{\left(dr\right)}^{2}+{\left({r}_{0}\bullet {\phi }\right)}^{2}}. \qquad (30)$$

The change in the radius vector length $dr = r_B - r_0$ for a small angle $\varphi \ll 1$ can be derived from (27):

$$dr\approx {r}_{0}\bullet \left(1-\frac{1}{{{\beta }}^{2}}\right)\bullet \frac{{{\phi }}^{2}}{2}. \qquad (31)$$

Therefore, expressions (29) and (30) let us obtain:

$$dl\approx {r}_{0}\bullet {\phi }\bullet \left(1+{\left(1-\frac{1}{{{\beta }}^{2}}\right)}^{2}\frac{{{\phi }}^{2}}{8}\right). \qquad (32)$$

Projecting the expression (24) onto the direction of motion upon impact while taking into account (24’), (25) and (29), we obtain an estimate of the projection magnitude for the force from (24)

$$m{V}_{R}\frac{{V}_{R}}{dl}\le {F}_{BR}\le m{V}_{R}\frac{{V}_{0}}{dl}. \qquad (33)$$

Respectively, the inequality below follows from (33) for the average force projection value (23):

$$m{V}_{R}\frac{{V}_{R}}{dl}\text{c}\text{o}\text{s}\left({\theta }\right)\le {F}_{\parallel BR}\le m{V}_{R}\frac{{V}_{0}}{dl}\text{c}\text{o}\text{s}\left({\theta }\right). \qquad (34)$$

Taking into account (28) and (32), the limit of the expression (34) as $\varphi \to 0$ leads to the equation:

$${F}_{\parallel BR}=\frac{m{{V}_{0}}^{2}}{{r}_{0}}\frac{\left({{\beta }}^{2}-1\right)}{{{\beta }}^{2}}. \qquad (35)$$

Since the positions of both points B_R and B_L tend to the position of point *A* as φ → 0, we may say that both walls in the vicinity of point *A* will be subjected to the total average force generated by elastic collisions of small bodies, the magnitude of whose component parallel to the gravitational attraction force will be

$${F}_{\parallel }={F}_{\parallel BL}+{F}_{\parallel BR}$$

that is,

$${F}_{\parallel }=2\frac{m{{V}_{0}}^{2}}{{r}_{0}}\frac{\left({{\beta }}^{2}-1\right)}{{{\beta }}^{2}}. \qquad (36)$$

Since the value of expression (36) is positive for ${\beta }>1$, the vector of this force component points in the opposite direction to the vector of the gravitational attraction force at point *A*.

It should also be noted that for φ → 0 the force vectors $\overrightarrow{{F}_{\perp BR}}$ and $\overrightarrow{{F}_{\perp BL}}$ will be perpendicular to the vector of the gravitational attraction force at point *A*. Since these vectors satisfy the relation

$$\overrightarrow{{F}_{\perp BR}}=-\overrightarrow{{F}_{\perp BL}}$$

then we may say that the total force that will act on both walls during impacts in the vicinity of point *A* in the direction perpendicular to the vector of the gravitational attraction force is equal to zero:

$$\overrightarrow{{F}_{\perp }}=\overrightarrow{{F}_{\perp BR}}+\overrightarrow{{F}_{\perp BL}}=\overrightarrow{0}$$

Therefore, the vector of the total force acting on the walls during elastic impacts in the vicinity of point *A* turns out to be:

$$\overrightarrow{{F}_{BR}}+\overrightarrow{{F}_{BL}}=\overrightarrow{{F}_{\parallel }}. \qquad (36')$$

The direction of this vector will be opposite to the force of gravitational attraction. The magnitude of this force is determined by expression (36).

Now we may assume that these three walls – the left-hand one (at point B_L), the right-hand one (at point B_R) and the central one (at point A) – are connected to each other (Fig. 6). Small bodies move along sections of trajectories (16) between the walls, striking them elastically. Considering the walls and bodies as elements of a single system with a total mass of m_0, since this system is located in the inhomogeneous gravitational field of a massive spherical body M at a distance r_0 from its centre, we find that this system is subjected to a gravitational attraction force in the form (1):

$$F_{G0}=G\bullet \frac{m_{0}\bullet M}{r_{0}^{2}}, \qquad (37)$$

as well as to a repulsive force (36'), the vector of which lies in the direction opposite to the vector of the gravitational attraction force $\overrightarrow{F_{G0}}$ (37).

We should make the following comment. Suppose that the gravitational field is homogeneous: that is, force vectors have the same directions and lengths at any point of this field. Let our small bodies likewise commence moving from the starting point in directions perpendicular to the gravitational attraction force. In this case, further movement of bodies will lead to the fact that the angles between their velocity vectors and the vectors of the gravitational attraction force will be less than 90 degrees. Consequently, a repulsive force cannot be created in this case.

If there is no gravitational field at all, then by virtue of the law of conservation of momentum, a repulsive force cannot be created either, since the system will be closed and the forces of interaction between the bodies in the system will be internal, and internal forces cannot change the momentum of a closed system.

Therefore, the presence of an inhomogeneous gravitational field is a necessary condition for generating the repulsive force described herein.

3. Discussion

The results of the thought experiment described above indicate that it is possible in principle to generate a repulsive force without shedding momentum, with the force directed away from the gravitational centre. The magnitude of this force is determined by the equation (36).

If we substitute (15) into (36), then the equation (36) can be written in the form:

$${F}_{\parallel }=\left({{\beta }}^{2}-1\right)\bullet G\frac{2\bullet mM}{{{r}_{0}}^{2}}. \qquad (38)$$

However, the expression:

$${F}_{G}=G\frac{2\bullet mM}{{{r}_{0}}^{2}} \qquad (39)$$

determines the total force of gravitational attraction between both small bodies and the larger body. Therefore, the expression (38) defines a force that is directly proportional to the force of gravitational attraction (39) but pointing in the opposite direction. In this sense, we can say that the force (36) (or (38)) is quasi antigravitational.

We can also compare the magnitudes of the forces (36) and (37). The condition ${F}_{\parallel }\ge {F}_{G0}$ leads to the relation

$${\beta }\ge \sqrt{1+\frac{{m}_{0}}{2\bullet m}}. \qquad (40)$$

The inequality (40) determines the range of values for the parameter β where the magnitude of the repulsive force is greater than the force of gravitational attraction for the case when the system of mass m_0 is at a distance of r_0 from the centre of attraction.

4. Conclusion

This paper is the first to present a case of generating a repulsive force from the gravitational attraction of a massive body without shedding momentum.

An expression for the magnitude of this force is obtained.

It is shown that the magnitude of this force as a function of distance and body masses is identical to the expression for the force of gravitational attraction with an additional constant.

The paper also shows that the necessary condition for generating a repulsive force is the presence of an inhomogeneous gravitational field.

Declarations

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Declarations of interest: none. The author declares that there is no conflict of interest regarding the data and results presented herewith.

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Figures

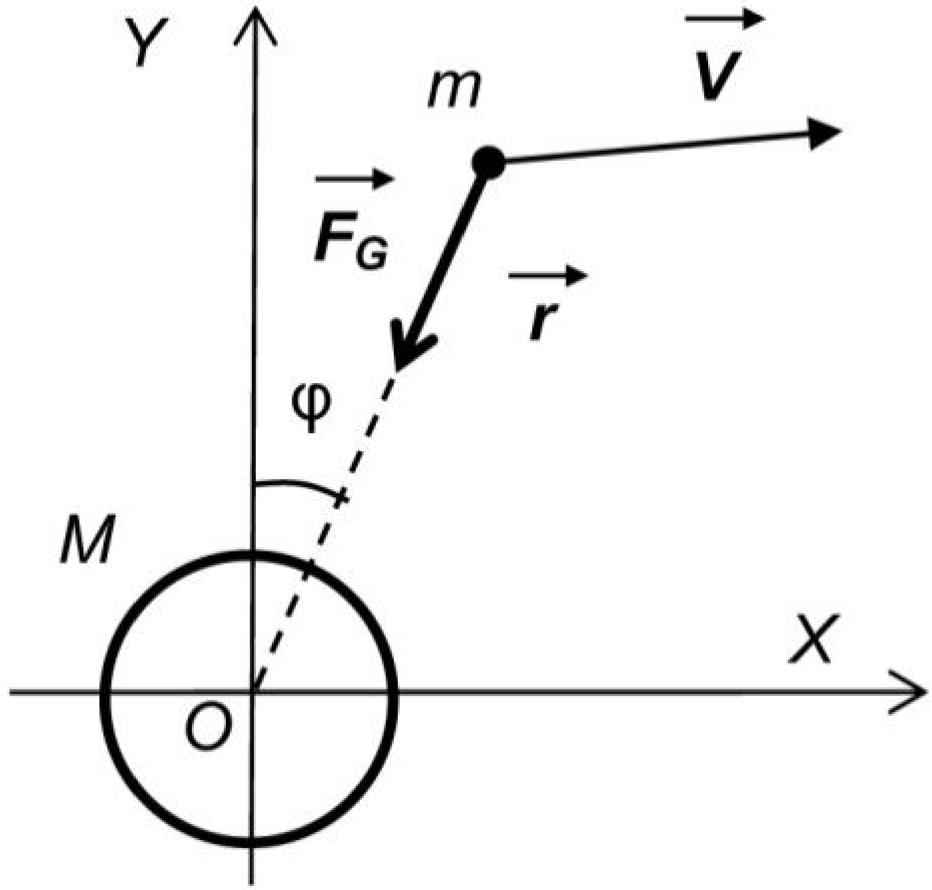

Figure 1

Describing the movement of the smaller body. The *Z* axis is not shown, but we may assume that it is directed towards the observer, perpendicular to the figure plane at point *O*.

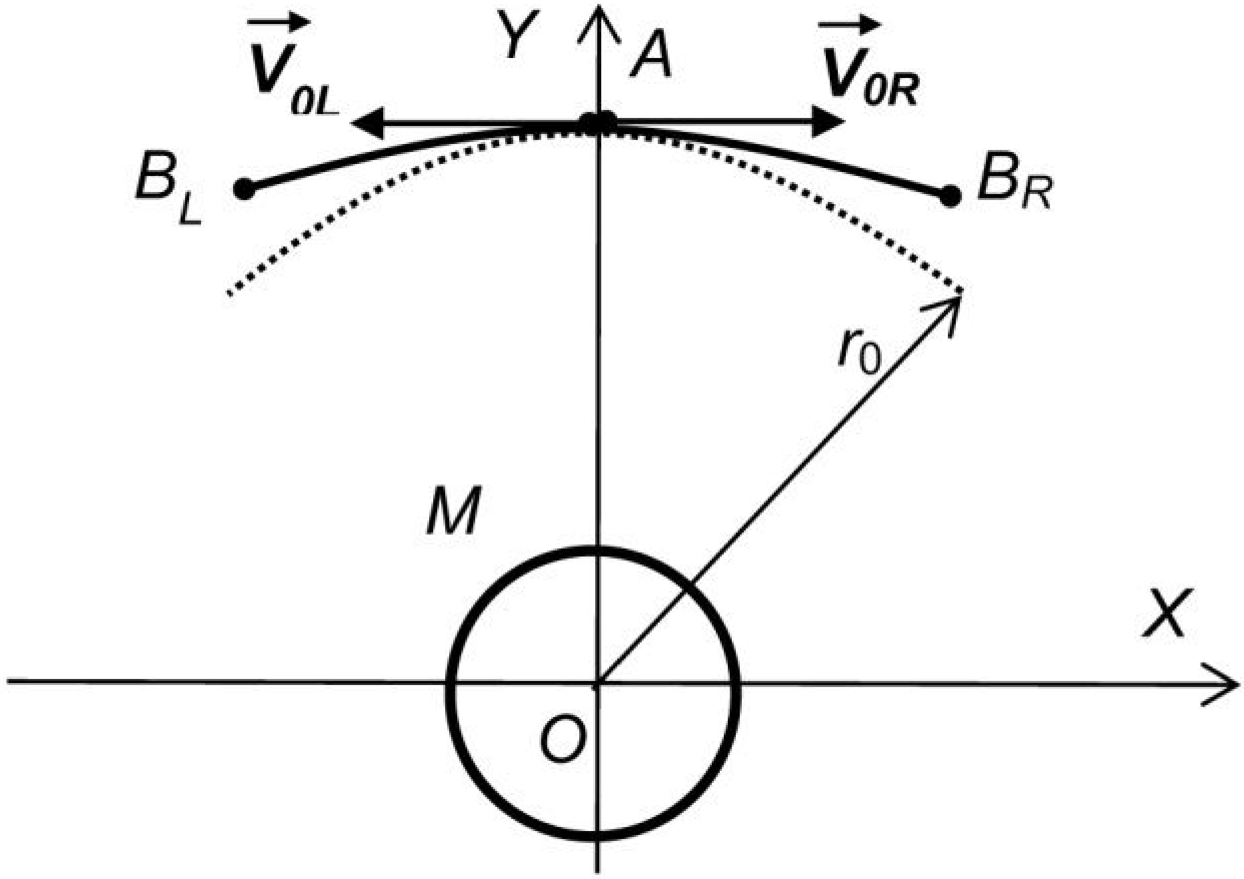

Figure 2

Motion trajectories of the left-hand (*L*) and right-hand (*R*) small bodies.

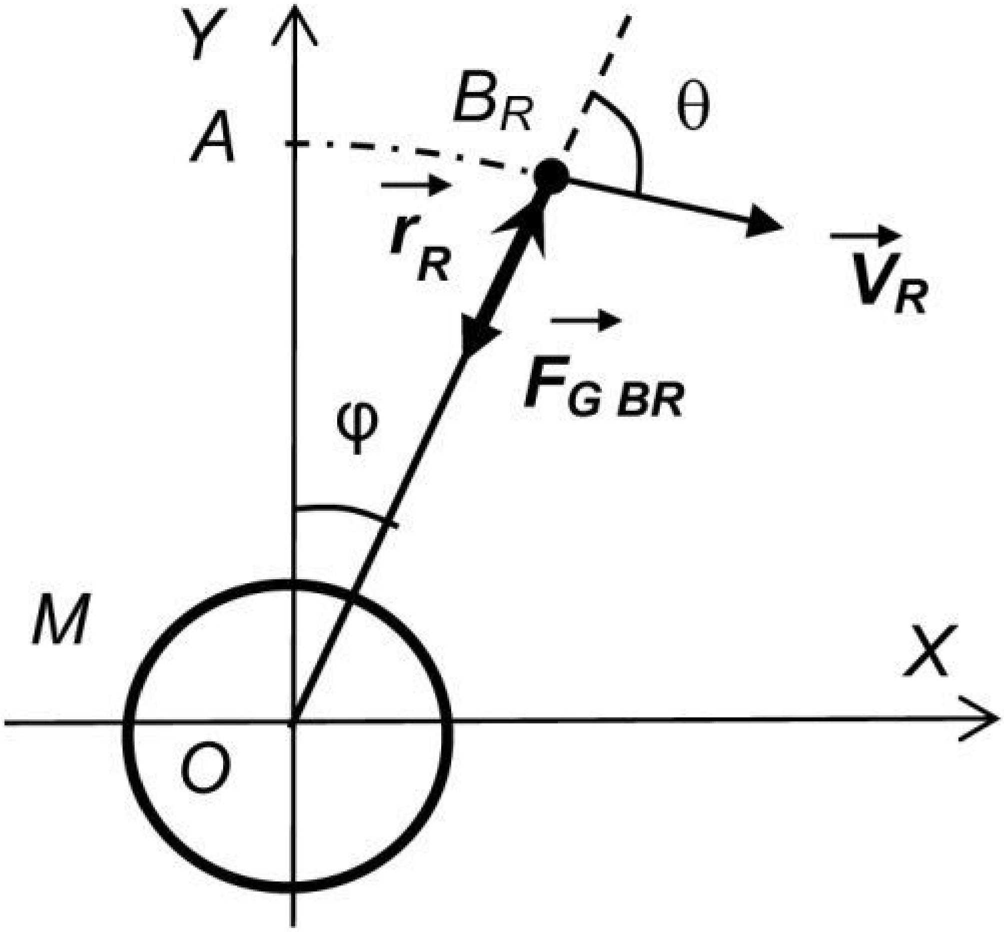

Fig. 3. Specifying the angle θ between the radius vector $\vec{r_R}$ of the right-hand body and its velocity vector $\vec{V_R}$ at the point B_R.

Figure 3

See image above for figure legend.

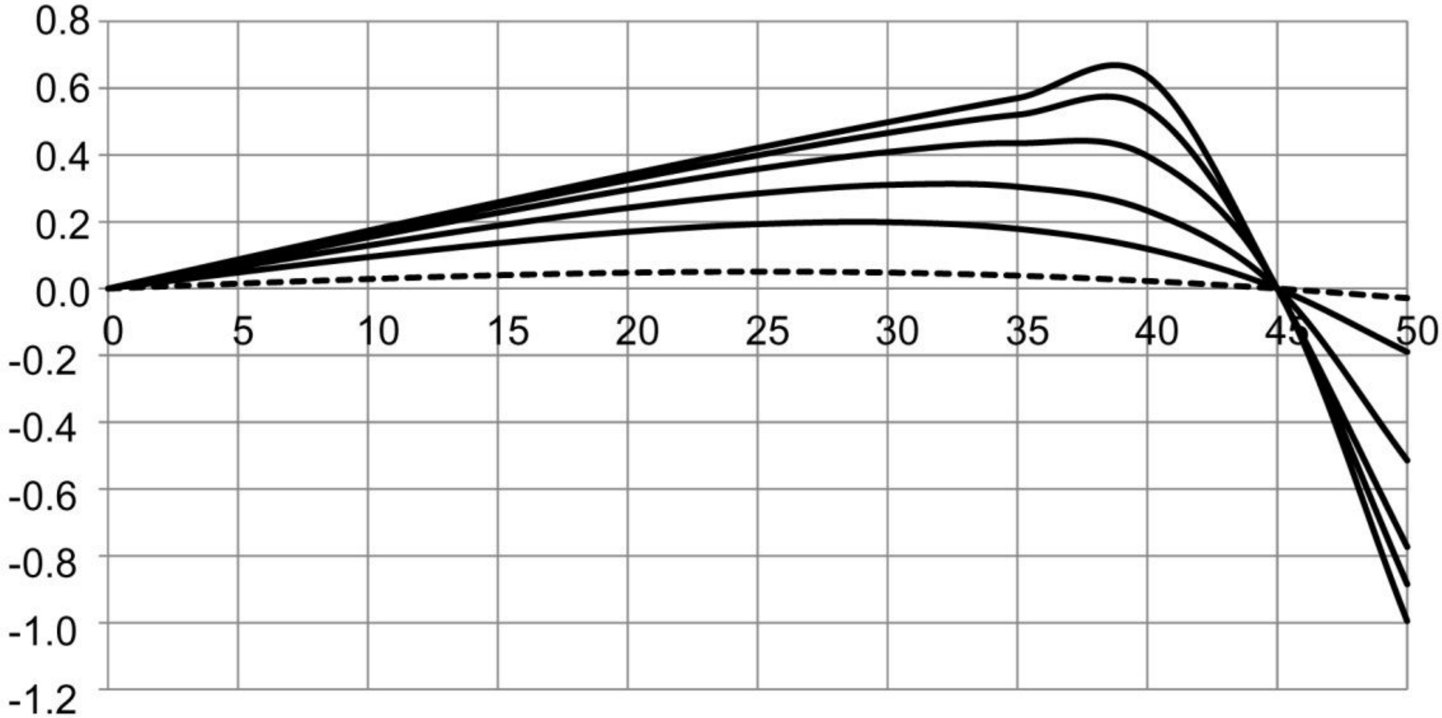

Figure 4

Plots of function (20) of the angle φ for different values of the parameter β. The function is plotted along the ordinate, the angle φ in degrees increases along the abscissa. The lower plot (dashed line) corresponds to the value β = 1.1; then, respectively upwards, the plots represent the values β = 1.5; β = 2.0; β = 3.0; β = 5.0; β = 20.0.

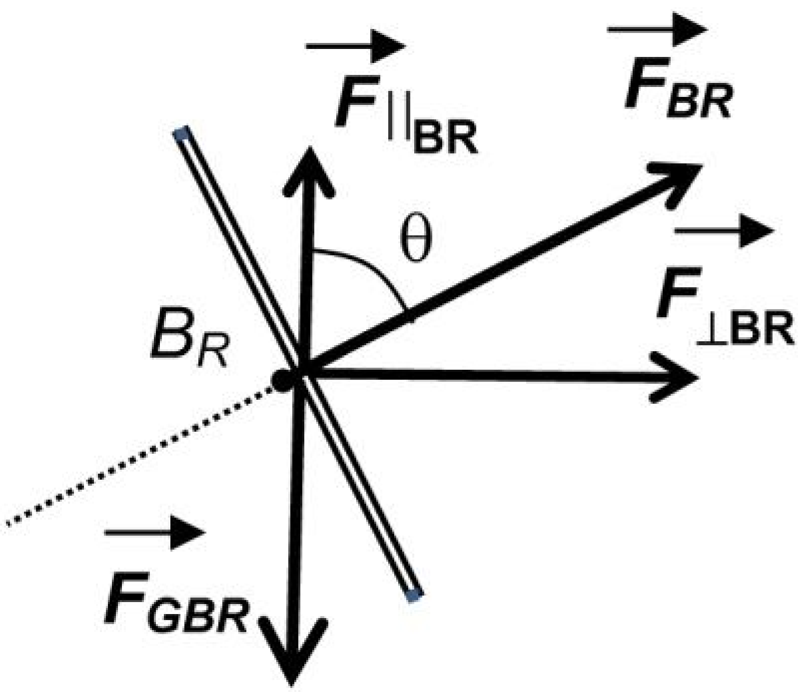

Figure 5

Impact of the right-hand body against the wall at point B_R. The wall is shown as a double stripe.

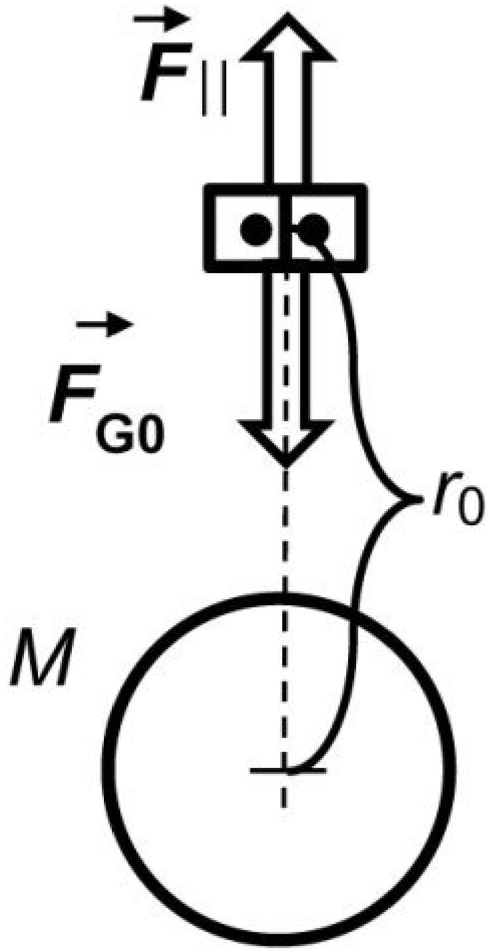

Figure 6

Forces acting on the system