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Junghyun Bae (✉ bae43@purdue.edu)

Purdue University West Lafayette

Stylianos Chatzidakis

Purdue University West Lafayette

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Fieldable muon spectrometer using multi-layer pressurized gas Cherenkov radiators and its applications

Junghyun Bae^{1,*}, Stylianos Chatzidakis¹

¹School of Nuclear Engineering, Purdue University, West Lafayette, IN 47906

Abstract

Cosmic ray muons have been considered as a non-conventional radiation probe in various applications. To utilize cosmic ray muons in engineering applications, two important quantities, trajectory and momentum, must be known. The muon trajectories are easily reconstructed using two-fold detector arrays with a high spatial resolution. However, precise measurement of muon momentum is difficult to be achieved without deploying large and expensive spectrometers such as solenoid magnets. Here, we propose a new method to estimate muon momentum using multi-layer pressurized gas Cherenkov radiators. This is accurate, portable, compact ($< 1\text{m}^3$), and easily coupled with existing muon detectors without the need of bulky magnetic or time-of-flight spectrometers. The results show that not only our new muon spectrometer can measure muon momentum with a resolution of $\pm 0.5\text{ GeV}/c$ in a momentum range of 0.1 to 10.0 GeV/c , but also we can reconstruct cosmic muon spectrum with high accuracy ($\sim 90\%$).

*Corresponding Author: Junghyun Bae
Email: bae43@purdue.edu

Introduction

Cosmic ray muons present a large part of background radiation¹ and they have recently been explored as a non-conventional radiation probe for monitoring or imaging the contents of dense and large objects, otherwise not feasible with conventional radiographic techniques². It is noteworthy how the recent developments on cosmic ray muon applications have been successfully acknowledged in various engineering fields, including nuclear reactor and spent nuclear fuel cask imaging³⁻⁶, homeland security⁷⁻¹⁰, geotomography¹¹⁻¹⁵, and even archeology¹⁶. Despite the potential and success, the wide application of cosmic ray muons is limited by the naturally low muon intensity, approximately $10^4 \text{ m}^{-2} \text{ min}^{-1}$ at sea level. Since it is not practical to deploy neither a large hadron collider nor an accelerator in the field to provide muon beams, it is important to measure muon momentum to maximize the utilizability of each cosmic ray muon^{17,18}. However, it is still challenging to measure muon momentum in the field without resorting to large solenoid or toroidal magnets, Cherenkov ring imagers, or time-of-flight detectors¹⁹⁻²¹. Although recent efforts at CNL and INFN aim to infer momentum knowledge from multiple Coulomb scattering measurements^{22,23}, at present no portable spectrometer exists that can measure muon momentum in the field.

Here, we develop a new concept to measure muon momentum using multi-layer pressurized gas Cherenkov radiators. Because muons are charged particle, they can induce Cherenkov radiation in transparent media. Unlike solid or liquid Cherenkov radiators, the refractive index of gas radiators can be varied by changing gas pressure and temperature. We can then find the optimal muon threshold momentum levels by carefully selecting the gas pressure for each radiator. As a result, radiators will emit only when the threshold momentum is less than the actual muon momentum even though a muon passes through all radiators (Fig. 1). By measuring the Cherenkov signals in each radiator, we can then estimate the actual muon momentum (Fig. 2). The benefit of such an approach is compact, lightweight, and portable spectrometer that can be deployed in the field to improve existing or explore new engineering applications: cosmic ray muon tomography, geological studies, and cosmic radiation measurement in the International Space Station.

In this paper, we present the design concept, performance, and feasibility of our proposed muon spectrometer. This includes (i) theoretical background and (ii) principle of operation of the Cherenkov spectrometer and (iii) Geant4 modeling and simulations. The simulation results demonstrate that our spectrometer can measure muon momentum with high accuracy ($\sim 90\%$) for a wide muon momentum range ($0.1 - 10.0 \text{ GeV/c}$) and a resolution of $\pm 0.5 \text{ GeV/c}$ which are sufficient for engineering applications. At the end, we demonstrate the benefits of muon momentum measurement in two real-world applications, (i) identification and (ii) imaging of special nuclear materials (SNMs) using muon multiple Coulomb scattering. The results show that SNM identification and imaging capability can be significantly improved when we integrate muon momentum knowledge.

Fig. 1 Characteristics of photon emission. Cherenkov radiation, scintillation, and transition radiation can be emitted in radiators. Radiator A (top) emits Cherenkov photons since $p_\mu > p_{th}$ whereas radiator B (bottom) does not because $p_\mu < p_{th}$. However, both radiators emit scintillation and transition radiation regardless of actual muon momentum. Due to the forward-biased directional photon emission of Cherenkov radiation, scintillation photon signals can be efficiently discriminated.

Fig. 2 Signal flow diagram. The simplified schematic of signal flow to estimate muon momentum. In this example, only first four radiators emit Cherenkov radiation because the actual muon momentum is 3.1 GeV/c . Regardless of muon momentum, there always exists noise due to the scintillation, transition radiation, electronic noise. Using a linear discriminator it correctly identify the actual muon momentum with a resolution of $\pm 0.5 \text{ GeV/c}^{24}$.

RESULTS

Various mono-energetic muons are simulated to evaluate the functionality of the muon spectrometer. The measured photon signals are classified into two categories, Cherenkov photons (signals) and others (noise). In the first simulation, optical photon emissions by the scintillation and transition radiation are excluded to focus on the Cherenkov photon emission by muons. Two mono-energetic muons, $p_\mu = 3.25$ and 10.0 GeV/c , vertically enter our muon spectrometer and passes through all radiators (Fig. 3). When $p_\mu = 3.25 \text{ GeV/c}$, only the first four radiators emit conical shape Cherenkov radiation because their threshold momenta are less than the simulated muon momentum. On the other hand, all radiators emit Cherenkov radiation because threshold momentum levels for all radiators are lower than 10.0 GeV/c . In the next simulation, scintillation and transition radiation photon emissions are included in addition to the Cherenkov radiation, however, all the other parameters remain unchanged. More optical photons are found in radiators due to the presence of the scintillation and transition radiation. It is noted that scintillation photon signals can be efficiently differentiated from Cherenkov photon signals based on, a) characteristic photon emission direction (θ_c vs 4π) and b) light flash duration (Fig. 4). There was no transition radiation observed in Fig. 3 because it is a rare event for a few GeV muons and for the small number of boundary systems.

It is noteworthy that some additional Cherenkov photon emissions ($N_{ch} \neq 0$ when $p_\mu < p_{th}$) are found in Fig. 3 and they are also recorded (Fig. 5). It is because of either Compton scattering or Cherenkov photon emissions by secondary particles (mainly electrons) which can be produced by muon decays and muon to electron conversions²⁵.

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \quad (1)$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \quad (2)$$

$$\mu N_{Al} \rightarrow e N_{Al} \quad (3)$$

Equation (1) and (2) represent radiative decays of $\mu^\pm$ with a mean lifetime of $\tau_\mu \cong 2.2 \mu\text{sec}$. Radiative muon decays are the primary source of secondary charged particles. Equation (3) represents the muon to electron conversion. A muon can be captured by the $1s$ orbital of aluminum and mono-energetic electron emission follows²⁶. Although the muon capture cross-section is insignificant, it is considered because the photon absorbers are made of aluminum foils.

The results of total number of optical photons and Cherenkov photons as a function of muon energy are shown in Fig. 5. The rapid increase in the total number of photon emissions is observed at each muon threshold momentum level (vertical dashed line) for each radiator due to the Cherenkov threshold momentum for muons. Then, it gradually increases as muon energy increases. It demonstrates that our proposed spectrometer is feasible because muon energies are clearly identified by analyzing optical photon signals from radiators.

Fig. 3 Geant4 simulation results: (i) Cherenkov radiation only (left column), (ii) Cherenkov radiation, scintillation, and transition radiation (right column) when muon momenta are 3.25 (top row) and 10.0 GeV/c (bottom row). Green and red represent optical photons and negative muons, respectively.

Fig. 4 Cherenkov and scintillation light flash times. Cherenkov light flashing time has a fast pulse ($< 1 \times 10^{-9}$ sec) because it is the result of instant physical disorder caused by the incident muon whereas scintillation light flash time has a slow distribution with a tail due to the decay constant ($\gg 1 \times 10^{-9}$ sec) because it is a fluorescence process of scintillation.

Fig. 5 Expected number of optical photon emission. The rapid increments are observed at threshold momentum levels (vertical dashed lines) because of the Cherenkov radiation emission above the threshold momentum levels. Then, it gradually increases as muon energy increases. The italic numbers (1 to 6) on the right represent the radiator IDs.

Performance Evaluation

We evaluate the performance of our muon spectrometer by reconstructing a cosmic ray muon spectrum. The experimental results and mathematical expressions for cosmic ray muon spectrum at sea level are found in references^{27–29}. We used the “Muon_generator_v3”³⁰ which is developed to simulate cosmic ray muon spectra based on validate semi-analytical models^{31,32}. To quantify the muon spectrometer measurement accuracy, we introduced a classification rate (CR) which is a probability that the system identifies actual correct muon momentum. A reconstructed cosmic muon spectra (< 10 GeV/c) using 10^4 muon samples with (i) perfect and (ii) practical (noise is included) muon spectrometers are shown in Fig. 6. The computed CRs for the practical muon spectrometer are also presented.

The next simulation results are the computed CRs using 10^4 mono-energy muons as a function of muon momenta with various discriminator levels, 0, 1, and 2 (Fig. 7). Simple linear discriminators were used in the final stage of signal process to cut off predictable noise signals from total signals. When $p_{th} < 1.0$ GeV/c, overall CR is less than 60% because of high scintillation photon intensity in the glass radiator. Hence, a CR is improved as the discrimination level increases. When $1.0 < p_{th} < 4.0$ GeV/c, all gas radiators show high CRs because Cherenkov and scintillation signals are clearly differentiated using a linear discriminator. However, when $4.0 < p_{th} < 6.0$ GeV/c, CRs begin to decrease since Cherenkov photon yields are too low due to the rarefied CO₂ ($\sim$ a half atmosphere pressure) gas for high muon threshold momentum. When $p_{th} > 6.0$ GeV/c, on the other hand, CR rebounds because Cherenkov photon yields gradually increase as muon momentum increases. However, it is not efficient to use any linear discriminator in high p_{th} (> 5.0 GeV/c). In addition, repeated sudden drops in CR are also noteworthy phenomena in Fig. 7. This is because p_{μ} near p_{th} boundaries, 0.1, 1.0, ... 5.0 GeV/c, tends to have a high false classification rate.

Fig. 6 Reconstructed cosmic ray muon spectra.

Reconstructed cosmic ray muon spectra are reconstructed using 10^4 muon samples with (i) perfect and (ii) practical muon spectrometers. Differential muon intensity increases and classification rate decreases at > 5.0 GeV/c because all muons that have a momentum greater than 5.0 GeV/c are categorized in the largest p_{th} level. The computed classification rates for measurement classes are also indicated.

Fig. 7 Classification rate as a function of muon momentum.

Using 10^4 mono-energy muons, the classification rates are plotted as a function of muon momentum with various discriminator levels, 0, 1, and 2, from 0.1 to 10.0 GeV/c. Repeated sudden drops of CR are observed because muon momentum near p_{th} boundaries tends to have a high false classification rate.

Applications

Separation and Identification of SNMs

One potential application of our proposed muon spectrometer is to monitor hidden special nuclear materials. Cosmic ray muons have been considered as an unconventional radiation probe for monitoring of SNMs due to their high-penetrative nature and charge. Muons are not easily absorbed by SNMs but undergo multiple Coulomb scattering (MCS) and they are deflected with characteristic angles depending on the Z number of materials and momentum. The estimated muon scattering distribution using the MCS approximation is known to follow Gaussian distribution with a zero mean and it is given by ³³:

$$f(\theta|0, \sigma_\theta^2) = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left(-\frac{1}{2} \frac{\theta^2}{\sigma_\theta^2}\right) \quad (4)$$

$$\sigma_\theta = \frac{13.6 \text{ MeV}}{\beta_\mu p_\mu c} \sqrt{\frac{X}{X_0} \left[1 + 0.088 \log\left(\frac{X}{X_0}\right)\right]} \quad (5)$$

where θ is the muon scattering angle, σ_θ is the standard deviation of scattering angle, X is the thickness of the scattering medium, and X_0 is the radiation length.

To investigate the effect of including muon momentum in SNM monitoring, high enriched uranium (HEU), low enriched uranium (LEU), plutonium (Pu), and lead (Pb) shielding were studied using Monte Carlo simulations. We used 10^3 muons (approximately translate to a few minutes of scanning time) to analyze their scattering angle variance distributions in order to identify SNMs and separate non-SNM from them. The results for SNM separation and identification with and without muon momentum knowledge are shown in Fig. 8. By measuring muon momentum, lead is clearly separated from SNMs and

identification of each type of SNM is also possible even when they are surrounded by 30 cm of lead shielding. This last scenario is almost inseparable without muon momentum knowledge.

Muon Scattering Tomography

Our proposed muon spectrometer can be integrated with a muon scattering tomography system which reconstructs images of hidden high-density materials using the point-of-closest approach (PoCA) algorithm³⁴. As shown in (5), the expected muon scattering angle distribution is determined by two variables, material property and muon momentum. In other words, materials can be identified by measuring muon scattering angle distribution and momentum. Using a modified PoCA algorithm which coupled with the muon momentum term, we successfully improved the image resolution as shown in Fig. 9. The experimental configuration includes cylindrical tungsten (radius = 5.5 cm, height = 5.7 cm) at the center and two stainless beams on the sides. This example was firstly introduced by K. Borozdin et al³⁵. Without muon momentum measurement, many incorrect voxels are found between tungsten and steel beams. However, the resolution of reconstructed image is significantly improved by incorporating a proposed muon spectrometer.

Fig. 8 Muon scattering angle variance distributions. Scattering angle variance distribution for HEU, LEU, Pu, and Pb using 10^3 muon samples (it can translate to ~min of scanning time) without (left) and with (right) muon momentum knowledge. By measuring muon momentum, we can separate lead SNMs and identify each type of SNM even when they are surrounded by thick lead shielding (30 cm)¹⁸.

Fig. 9 Reconstructed images using a modified PoCA. The image of hidden materials (a cylindrical tungsten and two steel beams) is reconstructed based on a modified PoCA algorithm using 10^4 muon samples. Resolution of reconstructed image is significantly improved by incorporating a proposed

muon spectrometer (right)

DISCUSSION AND CONCLUSION

In this paper, we presented a new concept to measure muon momentum using multi-layer pressurized gas Cherenkov radiators. This spectrometer relies on pressurized gas Cherenkov radiators, thus eliminating the need for bulky and expensive magnetic spectrometers. Six sequential threshold momentum levels were set using a solid radiator (SiO_2) and pressurized CO_2 gases, $p_{th} = 0.1, 1.0, 2.0, 3.0, 4.0$, and 5.0 GeV/c. Each radiator can emit Cherenkov radiation when a muon passes through it if the actual muon momentum exceeds the threshold muon momentum of this radiator. On the other hand, radiators will emit scintillation photons and transition radiation even though the actual muon momentum is lower than the threshold momentum.

We performed detailed Geant4 simulations to assess the performance and demonstrate the functionality of the proposed muon spectrometer by investigating the Cherenkov, scintillation, and transition radiation photon emission in radiators. Using the proposed muon spectrometer, we were able to measure muon momentum with a resolution of ± 0.5 GeV/c within the momentum ranges of 0.1 to 10.0 GeV/c. This is sufficient for most engineering applications. Moreover, a cosmic ray muon spectrum was successfully reconstructed with a high accuracy ($\sim 90\%$).

To further demonstrate the potential of measuring muon momentum in special nuclear material monitoring and muon scattering tomography applications, we compared the results of both applications with and without muon momentum measurement. In the SNM monitoring application, we showed that we are not only able to separate lead from SNMs but also identify each type of SNMs (HEU, LEU, and Pu). In the muon scattering tomography, we developed a modified PoCA algorithm in order to integrate muon momentum into the original PoCA algorithm. When we reconstructed using the modified PoCA algorithm, imaging resolution was significantly improved.

METHODS

Cherenkov muon spectrometer

A threshold muon momentum is required to emit Cherenkov radiation in a given radiator and it is derived as follows ³⁶:

$$p_{th}c = \frac{m_\mu c^2}{\sqrt{n^2 - 1}} \quad (6)$$

where p_{th} is the threshold muon momentum, n is the refractive index of the medium, and $m_\mu c^2$ is the muon rest mass energy. The threshold momentum for muons is determined by the refractive index of the radiator. In gas radiators, the refractive index depends on both pressure and temperature of gas radiator and it is approximated by the Lorentz-Lorenz equation:

$$n \approx \sqrt{1 + \frac{3A_m p}{RT}} \quad (7)$$

where A_m is the molar refractivity, p is the gas pressure, T is the absolute temperature, and R is the universal gas constant. Therefore, the threshold muon momentum is given by:

$$p_{th} = m_{\mu}c \sqrt{\frac{R}{3A_m p} T} \quad (8)$$

Equation (8) shows that muon threshold momentum can be varied by changing the pressure or temperature without replacing radiator materials. For gas selection, we considered, C_4F_{10} , C_3F_8 , R-12, and CO_2 . C_4F_{10} and CO_2 are used in the Thomas Jefferson National Laboratory as gas Cherenkov radiators³⁷. R-12 gas refrigerant was used for the gas radiator in the past, however, the usage is limited due to its high Ozone Depletion Potential (ODP). Hence, a refrigerant, R-1234yf ($C_3H_2F_4$), becomes a promising alternative gas radiator to replace R-12 (CCl_2F_2)³⁸. C_3F_8 is the alternative gas radiator for C_4F_{10} due to the better stability in the high pressure. Gas pressure cannot exceed 3 atm using the C_4F_{10} at room temperature.

Optical Photon Emission

Cherenkov radiation

Cherenkov radiation photon is the emission of electromagnetic radiation when polarized molecules in the dielectric medium, while a charged particle travels, return to the normal states. The continuous Cherenkov photon wavelength spectrum extends from the near ultraviolet (UV) to the visible (VIS) region. Assuming that muon phase velocity, β , is a constant, the analytical expression of the Cherenkov photon intensity within the wavelength interval between λ_1 and λ_2 is given by³⁹:

$$\frac{dN_{ch}}{dx} = 2\pi\alpha \int_{\lambda_1}^{\lambda_2} \left(1 - \frac{1}{n^2(\lambda)\beta^2}\right) \frac{d\lambda}{\lambda^2} \quad (9)$$

where dN_{ch} is the expected number of Cherenkov photons in a unit path length dx , α is the fine structure constant and $n(\lambda)$ is the refractive index of the radiator gas. Cherenkov photons are emitted almost instantly because it is the result of instant physical disorder caused by the incident muon in contrast to the fluorescence process of scintillation. The Cherenkov light flash duration, Δt_{ch} , is given by^{40,41}:

$$\Delta t_{ch} = \frac{r}{\beta_{\mu}c} [\tan \theta_c(\lambda_2) - \tan \theta_c(\lambda_1)] \quad (10)$$

where $\theta_c(\lambda_1)$ and $\theta_c(\lambda_2)$ are the Cherenkov angles for λ_1 and λ_2 , and r is the distance to the observer. The estimated Cherenkov light flash duration for a high-energy muon is less than 1×10^{-9} sec.

Scintillation

Scintillation photon emission is associated with the molecular process of energy absorption, excitation, and relaxation. The emitted scintillation photon wavelength spectrum varies depending on scintillating materials, however, it generally extends within 200 to 700 nm. The scintillation photon intensity depends on the muon energy deposition in the medium. The expression for the expected number of photons was derived by Birks⁴²:

$$\frac{dN_{sc}}{dx} = S \frac{dE/dx}{1 + k_B(dE/dx)} \quad (11)$$

where dN_{sc} is the expected number of scintillation photons in a unit path length dx , S is the scintillation efficiency, k_B is the Birks' coefficient, and dE/dx is the muon mass stopping power that can be found using the Bethe equation ⁴³. The duration of scintillation light flash (decay of fast fluorescence component) is given by:

$$I/I_0 = f(t)e^{-t/\tau} \quad (12)$$

where I/I_0 is the normalized scintillation photon intensity, $f(t)$ is the characteristic Gaussian function, and τ is the decay constant. The estimated scintillation light durations are approximately the order of several nanoseconds for organic and microseconds for inorganic scintillators ⁴⁴.

The characteristics of Cherenkov and scintillation light durations for pressurized CO₂ gas were estimated using Geant4 simulations and the expected detector (timer) responses are shown in Fig. 5.

Transition radiation

Transition radiation is the electromagnetic radiation emission when a muon passes through a boundary between two inhomogeneous media. The emitted transition radiation photon wavelength includes optical and x-ray regions ⁴⁵. The transition radiation photon intensity is logarithmically proportional to the Lorentz factor, γ , of muons and it is given by ⁴⁶:

$$N_{tr}|_{\text{per boundary}} = \frac{\alpha}{\pi} \left[(\ln \gamma - 1)^2 + \frac{\pi^2}{12} \right] \quad (13)$$

where α is the fine structure constant ($e^2/\hbar c = 1/137$). The transition radiation photon intensity is insignificant ($N_{tr} < 10^{-3}$) for non-ultrarelativistic muons and small number of inhomogeneous media boundaries ⁴⁷.

Prototype Design

The underlying principle of the proposed muon spectrometer is illustrated in Fig. 1. It shows characteristics of photon emissions by Cherenkov, scintillation, and transition radiation. All surfaces of gas radiator tanks are surrounded by strong photon absorbers so that emitted photons are isolated within radiators. We chose CO₂ gas because it can be pressurized up to approximately 5.7 MPa without condensation although the refractive index of CO₂ gas is relatively lower than other gas radiators. Five sequential threshold momentum levels are obtained using pressurized CO₂ gas radiators, $p_{th} = 1.0, 2.0, \dots, 5.0$ GeV/c, however, we inevitably had to use a solid radiator (SiO₂) for the lowest threshold momentum level, $p_{th} = 0.1$ GeV/c, because it is impossible to reach that threshold momentum level with pressurized CO₂ at room temperature.

Geant4 Simulations

To demonstrate the functionality of the proposed muon spectrometer, a high-fidelity stochastic muon transport simulation using Geant4 was performed. The overall length of the muon spectrometer is 51.7 cm and the surface area is $20 \times 20 \text{ cm}^2$. It consists of one glass radiator (1 cm), five CO₂ gas radiators (10 cm), and thin photon absorber foils (10^{-1} cm). For glass radiator, a standard SiO₂ with refractive index of 1.45 is chosen, and for gas radiators, sequentially pressurized CO₂ gases are used to provide desirable refractive indices and muon threshold momenta. Some CO₂ gas radiators are technically depressurized because Cherenkov threshold momentum at atmospheric pressure is approximately 3.5 GeV/c. Therefore, some are depressurized to achieve $p_{th} = 4$ and 5 GeV/c. Any optical photon events are terminated once they arrive either at the photon absorbers or outside of “world” volume during the simulations. The overview of field muon spectrometer configuration is shown in Fig. 10. All major physics are included in the Geant4 reference physics list, QGSP_BERT⁴⁸.

Fig. 10 Overview of the fieldable muon spectrometer. The schematic of our proposed muon spectrometer. It shows dimensions and materials of components. In addition, threshold momentum and CO₂ gas pressure of each radiator are also presented. Note: The figure is not proportional to the actual size.

Signal Analysis

Optical sensors and photon absorbers are installed on one side of each radiator. Photon absorbers prevent photons from escaping and help efficiently discriminate scintillation photon signals given that scintillation photons are emitted in all directions whereas Cherenkov photons are emitted with a forward-biased direction. Fig. 2 shows an example of signal flows when a single muon ($p_\mu = 3.1 \text{ GeV/c}$) passes through all radiators. First four radiators emit Cherenkov radiation (“triggered”) because their threshold momentum levels are lower than the incident muon momentum, 3.1 GeV/c. In this example, the system records ‘1’ for triggered radiators and ‘0’ for intact radiators. Due to the presence of scintillation and transition radiation, noise signals always exist. Therefore, a use of discriminator is suggested to efficiently remove noise from signals. After the signal process, a final signal successfully identifies the correct muon momentum with a resolution of $\pm 0.5 \text{ GeV/c}$, even though the muon momentum is very close to the threshold momentum level.

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Author contributions

J. B. carried out the analytical derivations, numerical computations, and paper writing. S. C. conceived the proposed scheme and analyzed the results of numerical simulations. All authors reviewed the paper.

Competing interests

The authors declare no competing interests.

Additional Information

Muon generator_v3 is available online: <https://www.mathworks.com/matlabcentral/fileexchange/51203-muon-generator>.

Correspondence and requests for materials should be addressed to J. Bae.