

DHMM-Based Asynchronous Finite-Time Sliding Mode Control for Markovian Jumping Lur'e Systems

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DHMM-based asynchronous finite-time sliding mode control for Markovian jumping Lur'e systems

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Abstract—This paper is concerned with the asynchronous problem for a class of Markovian jumping Lur's system (MJLSs) via sliding mode control (SMC) in continuous-time domain. Specifically, the discrete hidden Markovian model (DHMM) is employed to describe the nonsynchronization between the controller modes and the MJLSs modes. In particular, considering the nonlinearity of MJLSs, a novel Lur'e-integral-type sliding surface is constructed. In order to ensure the finite-time stability of sliding mode dynamics and the accessibility of the specified sliding surface, the asynchronous Lur'e-type SMC law of the detector mode is presented. Finally, an example of DC motor is provided to demonstrate the effectiveness of the proposed technique.

Index Terms—Markovian jumping Lur'e systems, asynchronous sliding mode control, discrete hidden Markov model, finite-time stabilization.

I. INTRODUCTION

NONLINEAR systems have received extensive attention and a series of gratifying results are available in the past decades, see, e.g., [2], [3], [4], [5] and many references therein. As an important kind of nonlinear systems, Lur'e system is composed of a linear system and an unknown static nonlinear system with certain constraints through negative feedback [6]. For more topics of Lur'e systems one can refer to [7], [8]. Additionally, many nonlinear dynamic systems in engineering and economic fields, such as manufacturing process, network communication system, power circuit system and economic system, are susceptible to random variations and abrupt changes caused by external environment, internal structure or human intervention in the operation process. Thus, Markovian jumping systems (MJSs) are developed to model these more general nonlinear systems and notable results have been produced [9], [10], [11]. Recently, the stability analyses and control problems of MJSs have received great interests among researchers [12], [13], [14]. However, there are few researches on the Markovian jumping Lur'e systems (MJLSs), which makes this paper have certain research significance.

It should be noted that in the ideal MJLSs, each state of the Markov chain corresponds to an observable physical quantity. But the practical situation is complex, so all the

real states observation is notoriously difficult to conduct. In this case, the hidden Markov model (HMM) is introduced to describe such phenomenon. As a statistical analysis model, HMM has become a hot topic over the last few years, and its application including but not limited to speech processing [15], target tracking [16], digital communication [17], biomedical engineering [18] and finance [19]. It's worth mentioning that the aforementioned HMM includes two random processes, i.e., the random process caused by the Markov chain, and the other process brought from the observed variables. According to the observed values, the HMM can be divided into two types. When the observed value is discrete, it is called the discrete HMM (DHMM). Otherwise, when the observed value is continuous and can be described as a function, such HMM is called continuous HMM (CHMM). However, to the best of our knowledge, most of the existing research are mainly focused on interpreting DHMM in discrete time domain, see [20], [21], [22], and rarely studied in continuous time domain. Therefore, this paper addresses a longstanding open issue on DHMM in continuous time domain.

Based on DHMM, the asynchronous control scheme [23], [24] offers an effective alternative for dynamic system fully utilized the potential information in the presence of time delays and data loss. In [25], when the switching between the candidate controller and the system mode is asynchronous, the stability of a class of linear systems with mean residence time is studied by using asynchronous switching control. In [26], an asynchronous elastic controller is designed to ensure the stability of the nonlinear switched system subject to time delay and uncertainty for the asynchronization problem. In [27], considering the asynchronous problem between the controller and the system modes, the stability of uncertain time-delay switched nonlinear systems is studied by using asynchronous switching control. However, for nonlinear systems with external disturbance, the control objective is not only to guarantee system stability but also to accelerate the convergence and improve the robustness. Sliding mode control (SMC) has been widely applied in practical engineering in terms of its fast response speed and strong robustness to uncertainties and disturbances. In [28], the H_∞ control problem in commodity pricing has been solved by SMC method. Reference [29] develops an adaptive SMC scheme integrating fuzzy control with SMC control for interval type-2 fuzzy systems in the presence of uncertainty parameters, and the issue of SMC for fuzzy singularly perturbed systems with application to electric circuit is investigated in [30]. Furthermore, the extended application to missile system, communication network system and robot control system, has enabled finite-time boundedness to become

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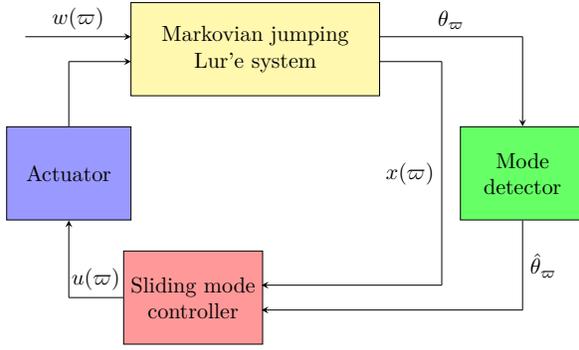


Fig. 1. Structure graph of asynchronous SMC for MJLSs.

considerably embraced [31], [32], [33].

Inspired by the above facts, we investigate the finite-time stabilization problem for a class of MJLSs via SMC scheme under the framework of DHMM which is illustrated in Fig. 1. The main contributions of the proposed scheme can be concluded as:

- (1) Asynchronous SMC is used for the first time to solve the finite-time stabilization problem of the MJLSs in continuous time domain under the framework of DHMM.
- (2) An asynchronous Lur'e-type SMC law is designed to drive state trajectories onto the specified sliding surface during the prescribed finite-time interval.
- (3) Nonlinear terms are added to the integral-type sliding surface in order to reduce the conservatism.
- (4) Compared with [34], we avoid the coupling problem caused by mode in the derivation process by selecting the appropriate Lyapunov function in the proof of reachability.

The rest of this paper is organized as follows. The MJLSs with stochastic perturbation is described in Section II followed by the main results in Section III. The effectiveness of the proposed method is illustrated in Section IV. Finally, Section V draws a remarking conclusion.

II. PROBLEM FORMULATION AND PRELIMINARIES

Given a probability space $(\Omega, \mathcal{F}, \mathbf{Prob})$, where Ω is the sample space, \mathcal{F} represents the algebra of events and \mathbf{Prob} denotes the probability measure defined on \mathcal{F} . $\{\theta_\varpi = i, \varpi \geq 0\}$ indicates a Markov chain taking values in a finite set $\mathcal{N} = \{1, 2, \dots, N\}$ with a transition probability matrix $\Pi \triangleq [\pi_{ij}]$, $i, j \in \mathcal{N}$. The transition probability from mode i at time ϖ to mode j at time $\varpi + \Delta\varpi$ can be described by:

$$\mathbf{Prob}\{\theta_{\varpi+\Delta\varpi} = j | \theta_\varpi = i\} = \begin{cases} \pi_{ij}\Delta\varpi + o(\Delta\varpi), & i \neq j \\ 1 + \pi_{ii}\Delta\varpi + o(\Delta\varpi), & i = j \end{cases} \quad (1)$$

in which $\pi_{ij} \in [0, 1], \forall i, j \in \mathcal{N}$ and $\sum_{j=1}^N \pi_{ij} = 1$. The MJLSs with stochastic perturbation can be written as:

$$\begin{cases} \dot{x}(\varpi) = A(\theta_\varpi)x(\varpi) + A_d(\theta_\varpi)x(\varpi - \tau) + B(\theta_\varpi)u(\varpi) \\ \quad + D(\theta_\varpi)w(\varpi) + F(\theta_\varpi)\varrho(\varpi) \\ \varrho(\varpi) = M_1x(\varpi) \\ x(\varpi) = h(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0] \end{cases} \quad (2)$$

where $x(\varpi) \in \mathfrak{X}^n$ is the state vector, $u(\varpi) \in \mathfrak{X}^m$ is the control input, $w(\varpi) \in \mathfrak{X}^p$ is the input disturbance and $\varphi(\cdot)$ denotes the nonlinear vector.

Some assumptions and the corresponding definitions as well as lemmas are given to facilitate the stability analysis of MJLSs (2).

Assumption 1: The nonlinear function $\varphi(\varrho(\varpi)) : \mathfrak{X}^p \rightarrow \mathfrak{X}^p$ is an additive vector depending on the vector $\varrho(\varpi)$. Thus, $\varphi(\cdot)$ satisfies:

$$\begin{cases} SC(\varphi(\cdot), \eta, L) = \varphi(\eta)^\top L[\varphi(\eta) - \Omega\eta] \leq 0 \\ \varphi(0) = 0, \forall \eta \in \mathfrak{X}^p \end{cases} \quad (3)$$

where $L \in \mathfrak{X}^{p \times p} = L^\top$, Ω is known as a priori.

Assumption 2: Define a finite-time interval \mathcal{T} , the unknown input $w(\varpi)$ satisfies:

$$\int_0^T w^\top(s)w(s)ds \leq \delta, \delta \geq 0. \quad (4)$$

Definition 1:[5] Given a time interval $[0, \mathcal{T}]$, positive scalars ξ, ς , with $0 < \xi < \varsigma$ and a mode-dependent weighting matrix $R_i > 0$, MJLSs (2) is finite-time bounded (FTB) if

$$\mathbf{E}\{x^\top(0)R_ix(0)\} \leq \xi \Rightarrow \mathbf{E}\{x^\top(\varpi)R_ix(\varpi)\} \leq \varsigma, \forall \varpi \in [0, \mathcal{T}]. \quad (5)$$

Definition 2: [35] Given a finite-time interval $[0, \mathcal{T}]$, positive scalars ξ, ς , with $0 < \xi < \varsigma$ and a mode-dependent weighting matrix $R_i > 0$, MJLSs (2) is finite-time stabilizable (FTS) with $w(\varpi) = 0$ if condition (5) is fulfilled.

Lemma 1: (Partitioning Strategy)[36] For MJLSs (2) with the specified parameters set $(\iota_1, \iota_2, [0, \mathcal{T}], R_i, \delta)$, the closed-loop system is FTB with respect to $(\iota_1, \iota_2, [0, \mathcal{T}], R_i, \delta)$, if and only if there exists an auxiliary scalar c^* satisfying $\iota_1 < \iota^* < \iota_2$ such that it is FTB with respect to $(\iota_1, \iota^*, [0, \mathcal{T}_1], R_i, \delta)$ during the reaching phase and FTB with respect to $(\iota^*, \iota_2, [\mathcal{T}_1, \mathcal{T}], R_i, \delta)$ during the sliding motion phase.

III. MAIN RESULTS

A. Asynchronous Sliding Surface and SMC Law Design

Due to packet loss, transmission delay or random disturbance involved in practice along with the inaccurate model information of the system, there is a need of the asynchronization between the controller modes and the system modes. In this section, we introduce the DHMM $(\theta_\varpi, \hat{\theta}_\varpi)$ to describe the asynchronous phenomenon described above. More precisely, a stochastic variable $\hat{\theta}_\varpi$ can be utilized to estimate θ_ϖ with a known Π -dependent conditional probability matrix $\Phi \triangleq [\delta_{il}]$, in which the probability δ_{il} is defined as:

$$\mathbf{Prob}\{\hat{\theta}_\varpi = l | \theta_\varpi = i\} = \delta_{il}, \quad (6)$$

where $0 \leq \delta_{il} \leq 1$ and $l \in \mathcal{M}$, $\sum_{l=1}^M \delta_{il} = 1$ for all $i \in \mathcal{N}$.

Remark 1: It should be mentioned that $\hat{\theta}_\varpi$ is introduced as a detector to represent the controller mode, and take values in a finite set $\mathcal{M} = \{1, 2, \dots, M\}$. Notice that $\hat{\theta}_\varpi$ depends on θ_ϖ according to the given conditional probability.

By reserving the index i for the modes of the Markov chain θ_ϖ and l for the detector $\hat{\theta}_\varpi$, we design an Lur'e-integral-type sliding surface function as:

$$s(\varpi) = G_i x(\varpi) - \int_0^\varpi G_i B_i E_l \varphi(M_1 x(s)) ds - \int_0^\varpi G_i B_i K_l x(s) ds - \int_0^\varpi G_i F_i \varphi(M_1 x(s)) ds \quad (7)$$

in which $G_i \in \mathbb{R}^{n \times n}$ is a real matrix such that $G_i B_i$ is nonsingular.

Then, the asynchronous Lur'e-type SMC law can be achieved by:

$$u(\varpi) = K_l x(\varpi) + E_l \varphi(M_1 x(\varpi)) - \beta_l(\varpi) \text{sign}(s(\varpi)) \quad (8)$$

where K_l is the controller gain and E_l is the nonlinear feedback gain.

B. FTS Analysis of Sliding Mode Dynamics

By applying the partitioning strategy, the FTS problem of MJLSs (2) can be addressed in two phases via asynchronous SMC, i.e., the reaching phase within $[0, \mathcal{T}_1]$ and the sliding motion phase within $[\mathcal{T}_1, \mathcal{T}]$.

When the system states move on $[0, \mathcal{T}_1]$, the system trajectories will not reach the Lur'e-integral-type sliding surface (7). Substituting the above asynchronous Lur'e-type SMC law (8) into MJLSs (2), the closed-loop MJLSs can be rewritten as:

$$\begin{cases} \dot{x}(\varpi) = A_{il}x(\varpi) + A_{di}x(\varpi - \tau) + E_{il}\varphi(M_1x(\varpi)) \\ \quad + D_i w(\varpi) - B_i \bar{\beta}(\varpi) \\ x(\varpi) = h(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0] \end{cases} \quad (9)$$

where $A_{il} = A_i + B_i K_l$, $E_{il} = B_i E_l + F_i$, $\bar{\beta}(\varpi) = \beta_l(\varpi) \text{sign}(s(\varpi))$.

Theorem 1: For a given finite-time interval $[0, \mathcal{T}_1]$, by implementing the asynchronous Lur'e-type SMC law (8), MJLSs (2) is FTB, i.e., the closed-loop MJLSs (9) is FTB with respect to (c_1, c^*, R_i, δ) , if there exists a scalar $\mu > 0$, positive-definite symmetric matrices P_i, Q_i and positive-definite matrices Q, O_1, O_2, R , such that the following inequalities hold for all $i \in \mathcal{N}$ and $l \in \mathcal{M}$:

$$\begin{bmatrix} \Theta_3 & \Theta_4 & \Theta_5 & P_i B_i & -P_i D_i & \sum_{l=1}^M \delta_{il} \tau A_{il}^\top \\ * & Q_i - O_2 & 0 & 0 & 0 & \tau A_{di}^\top \\ * & * & 2\Delta & 0 & 0 & \sum_{l=1}^M \delta_{il} \tau E_{il}^\top \\ * & * & * & \mu I & 0 & -\tau B_i^\top \\ * & * & * & * & \mu I & \tau D_i^\top \\ * & * & * & * & * & \tau R^{-1} \end{bmatrix} > 0 \quad (10)$$

$$\frac{c_1(\lambda_1 + \tau\lambda) + 4\mu\gamma^2 \mathcal{T} + 4\mu\alpha_3^2 \delta^2 \mathcal{T}}{\lambda_2 e^{-\mu\mathcal{T}}} < c^* \quad (11)$$

$$4\mu\alpha_1^2 \leq O_1 e^{-\mu\mathcal{T}} \quad (12)$$

$$4\mu\alpha_2^2 \leq O_2 e^{-\mu\mathcal{T}} \quad (13)$$

$$\lambda_2 R_i < P_i < \lambda_1 R_i \quad (14)$$

$$0 < Q_i < \lambda R_i \quad (15)$$

$$\sum_{j=1}^N \pi_{ij} Q_j < Q \quad (16)$$

Proof: Consider the following Lyapunov-Krasovskii functional for MJLSs (2):

$$V_1(x(\varpi)) = V_{11}(x(\varpi)) + V_{12}(x(\varpi)) + V_{13}(x(\varpi)) \quad (17)$$

in which

$$\begin{aligned} V_{11}(x(\varpi)) &= x^\top(\varpi) P_i x(\varpi), \\ V_{12}(x(\varpi)) &= \int_{\varpi-\tau}^\varpi x^\top(s) Q_i x(s) ds + \int_{-\tau}^0 \int_{\varpi+\rho}^\varpi x^\top(s) Q x(s) ds d\rho \\ &\quad + \int_{-\tau}^0 \int_{\varpi+\rho}^\varpi \dot{x}^\top(s) R \dot{x}(s) ds d\rho, \\ V_{13}(x(\varpi)) &= \int_0^\varpi x^\top(s) O_1 x(s) ds + \int_0^\varpi x^\top(s - \tau) O_2 x(s - \tau) ds. \end{aligned}$$

Then, along the state trajectories of MJLSs (2), one has the infinitesimal operator of $V_1(x(\varpi))$ which is indicated as $\mathfrak{V}V_1(x(\varpi))$.

According to Newton-Leibniz Formula, we have

$$2x^\top(\varpi) W_i [x(\varpi) - x(\varpi - \tau) - \int_{\varpi-\tau}^\varpi \dot{x}(s) ds] = 0. \quad (18)$$

Under the condition of (16), we can obtain the following inequality by substituting (18) into (17)

$$\begin{aligned} \mathfrak{V}V_1(x(\varpi)) &\leq \sum_{l=1}^M \delta_{il} 2x^\top(\varpi) P_i \dot{x}(\varpi) + x^\top(\varpi) \left(\sum_{j=1}^N \pi_{ij} P_j \right) x(\varpi) \\ &\quad + x^\top(\varpi) Q_i x(\varpi) - x^\top(\varpi - \tau) Q_i x(\varpi - \tau) + \tau x^\top(\varpi) Q x(\varpi) \\ &\quad + \tau \dot{x}^\top(\varpi) R \dot{x}(\varpi) - \int_{\varpi-\tau}^\varpi \dot{x}^\top(s) R \dot{x}(s) ds + x^\top(\varpi) O_1 x(\varpi) \\ &\quad + x^\top(\varpi - \tau) O_2 x(\varpi - \tau) - \int_{\varpi-\tau}^\varpi x^\top(s) W_i R^{-1} W_i^\top x(s) ds \\ &\quad + 2x^\top(\varpi) W_i x(\varpi) - 2x^\top(\varpi) W_i x(\varpi - \tau) \\ &\quad - 2x^\top(\varpi) W_i \int_{\varpi-\tau}^\varpi \dot{x}(s) ds + \tau x^\top(\varpi) W_i R^{-1} W_i^\top x(\varpi). \end{aligned} \quad (19)$$

Under Assumption 1, for any given Ω , there exists Δ such that

$$\begin{aligned} SC[\varphi(\cdot), M_1 x(\varpi), \Delta] &= \varphi^\top(M_1 x(\varpi)) \Delta [\varphi(M_1 x(\varpi)) - \Omega M_1 x(\varpi)] \\ &\leq 0. \end{aligned} \quad (20)$$

Recalling to (19) and (20), we know that

$$\begin{aligned}
\mathfrak{J}V_1(x(\varpi)) &\leq \sum_{l=1}^M \delta_{il} 2x^\top(\varpi) P_i \dot{x}(\varpi) \\
&+ x^\top(\varpi) \left(\sum_{j=1}^N \pi_{ij} P_j \right) x(\varpi) + x^\top(\varpi) Q_i x(\varpi) \\
&- x^\top(\varpi - \tau) Q_i x(\varpi - \tau) + \tau x^\top(\varpi) W_i R^{-1} W_i^\top x(\varpi) \\
&+ \tau x^\top(\varpi) Q_i x(\varpi) + \tau \dot{x}^\top(\varpi) R \dot{x}(\varpi) + x^\top(\varpi) O_{1i} x(\varpi) \\
&+ x^\top(\varpi - \tau) O_{2i} x(\varpi - \tau) - 2SC[\varphi(\cdot), M_1 x(\varpi), \Delta] \\
&+ 2x^\top(\varpi) W_i x(\varpi) - 2x^\top(\varpi) W_i x(\varpi - \tau) \\
&- \int_{\varpi-\tau}^{\varpi} [W_i^\top x(\varpi) + R \dot{x}(s)]^\top R^{-1} [W_i^\top x(\varpi) + R \dot{x}(s)] ds \\
&= \chi^\top(\varpi) Z_1 \chi(\varpi) + \tau \dot{x}^\top(\varpi) R \dot{x}(\varpi) \\
&- \int_{\varpi-\tau}^{\varpi} [W_i^\top x(\varpi) + R \dot{x}(s)]^\top R^{-1} [W_i^\top x(\varpi) + R \dot{x}(s)] ds \quad (21)
\end{aligned}$$

where

$$Z_1 = \begin{bmatrix} \Theta_1 & P_i A_{di} - W_i & \Theta_2 & -P_i B_i & P_i D_i \\ * & O_2 - Q_i & 0 & 0 & 0 \\ * & * & -2\Delta & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\chi(\varpi) \triangleq [x^\top(\varpi) \ x^\top(\varpi - \tau) \ \varphi^\top(M_1 x(\varpi)) \ \bar{\beta}^\top(\varpi) \ w^\top(\varpi)]^\top,$$

$$\begin{aligned}
\Theta_1 &= \sum_{l=1}^M \delta_{il} (A_{il}^\top P_i + P_i A_{il}) + \sum_{j=1}^N \pi_{ij} P_j + Q_i + \tau Q \\
&+ W_i + W_i^\top + \tau W_i R^{-1} W_i^\top + O_{1i}, \\
\Theta_2 &= \sum_{l=1}^M \delta_{il} P_i E_{il} + (\Delta \Omega M_1)^\top.
\end{aligned}$$

Next, we define an auxiliary variable Ξ as:

$$\begin{aligned}
\Xi &= \mu V_1(x(\varpi)) + \mu \bar{\beta}^\top(\varpi) \bar{\beta}(\varpi) + \mu w^\top(\varpi) w(\varpi) \\
&- \mathfrak{J}V_1(x(\varpi)). \quad (22)
\end{aligned}$$

Further defining the auxiliary variable Ξ as follows:

$$\begin{aligned}
\Phi &= \Xi - \mu V_{12}(x(\varpi)) - \mu V_{13}(x(\varpi)) \\
&- \int_{\varpi-\tau}^{\varpi} [W_i^\top x(\varpi) + R \dot{x}(s)]^\top R^{-1} [W_i^\top x(\varpi) + R \dot{x}(s)] ds \\
&\geq \mu V_{11}(x(\varpi)) + \mu \bar{\beta}^\top(\varpi) \bar{\beta}(\varpi) - \chi^\top(\varpi) Z_1 \chi(\varpi) \\
&- \tau \dot{x}^\top(\varpi) R \dot{x}(\varpi) \\
&= \chi^\top(\varpi) Z_2 \chi(\varpi) \quad (23)
\end{aligned}$$

where

$$Z_2 = \begin{bmatrix} \Theta_3 & \Theta_4 & \Theta_5 & P_i B_i & -P_i D_i & \sum_{l=1}^M \delta_{il} \tau A_{il}^\top \\ * & Q_i - O_2 & 0 & 0 & 0 & \tau A_{di}^\top \\ * & * & 2\Delta & 0 & 0 & \sum_{l=1}^M \delta_{il} \tau E_{il}^\top \\ * & * & * & \mu I & 0 & -\tau B_i^\top \\ * & * & * & * & \mu I & \tau D_i^\top \\ * & * & * & * & * & \tau R^{-1} \end{bmatrix},$$

$$\Theta_3 = \mu P_i - \Theta_1, \quad \Theta_4 = W_i - P_i A_{di},$$

$$\Theta_5 = - \sum_{l=1}^M \delta_{il} P_i E_{il} - (\Delta \Omega M_1)^\top.$$

From (10), one can infer $\Phi > 0$ and the following inequality holds:

$$\begin{aligned}
\Xi &> \mu V_{12}(x(\varpi)) + \mu V_{13}(x(\varpi)) \\
&+ \int_{\varpi-\tau}^{\varpi} [W_i^\top x(\varpi) + R \dot{x}(s)]^\top R^{-1} [W_i^\top x(\varpi) + R \dot{x}(s)] ds \\
&> 0. \quad (24)
\end{aligned}$$

From (22) and (24), it is easily checked that

$$\mathfrak{J}V_1(x(\varpi)) < \mu V_1(x(\varpi)) + \mu \bar{\beta}^\top(\varpi) \bar{\beta}(\varpi) + \mu w^\top(\varpi) w(\varpi). \quad (25)$$

Next, we perform a series of equivalent transformations on (25). By multiplying the left and right sides of the (25) with $e^{-\mu\varpi}$, and then integrate the result from 0 to ϖ with $\varpi \in [0 \ \mathcal{T}_1]$ on both sides, we obtain

$$\begin{aligned}
\mathbf{E}\{e^{-\mu\varpi} V_1(x(\varpi))\} &< V_1(0) + \mu \int_0^{\varpi} e^{-\mu s} \bar{\beta}^\top(s) \bar{\beta}(s) ds \\
&+ \mu \int_0^{\varpi} e^{-\mu s} w^\top(s) w(s) ds. \quad (26)
\end{aligned}$$

Define $\tilde{P}_i = R_i^{-\frac{1}{2}} P_i R_i^{-\frac{1}{2}}$, $\tilde{Q}_i = R_i^{-\frac{1}{2}} Q_i R_i^{-\frac{1}{2}}$, we have

$$\begin{aligned}
\mathbf{E}\{V_1(x(\varpi))\} &< e^{\mu T} [\iota_1(\lambda_1 + \tau\lambda) + 4\mu\alpha_1^2 \int_0^{\varpi} x^\top(s)x(s) ds \\
&+ 4\mu\gamma^2 \mathcal{T} + 4\mu\alpha_2^2 \int_0^{\varpi} x^\top(s-\tau)x(s-\tau) ds + \mu\delta \\
&+ 4\mu\alpha_3^2 \delta^2 \mathcal{T}] \quad (27)
\end{aligned}$$

where $\bar{\beta}^\top(\varpi) \bar{\beta}(\varpi) \leq 4\gamma^2 + 4\alpha_1^2 x^\top(\varpi)x(\varpi) + 4\alpha_2^2 x^\top(\varpi - \tau)x(\varpi - \tau) + 4\alpha_3^2 w^\top(\varpi)w(\varpi)$.

More specifically, it follows:

$$\begin{aligned}
\mathbf{E}\{V_1(x(\varpi))\} &\geq x^\top(\varpi) P_i x(\varpi) + \int_0^{\varpi} x^\top(s) O_{1i} x(s) ds \\
&+ \int_0^{\varpi} x^\top(s-\tau) O_{2i} x(s-\tau) ds. \quad (28)
\end{aligned}$$

Based on (13) and (14), it yields:

$$x^\top(\varpi) R_i x(\varpi) < \frac{\iota_1(\lambda_1 + \tau\lambda) + 4\mu\gamma^2 \mathcal{T} + 4\mu\alpha_3^2 \delta^2 \mathcal{T}}{\lambda_2 e^{-\mu\mathcal{T}}}. \quad (29)$$

Hence, it is easily to find that $x^\top(\varpi) R_i x(\varpi) < \iota^*$ for all $\varpi \in [0 \ \mathcal{T}_1]$ under condition (11). Thus, we complete the proof of FTB for MJLSs (2) within $[0 \ \mathcal{T}_1]$.

Next, we give the FTS proof for MJLSs (2) during the finite-time interval $[\mathcal{T}_1 \ \mathcal{T}]$, i.e., the sliding motion phase. According to $\dot{s}(\varpi) = 0$, the equivalent control law can be written as:

$$\begin{aligned}
u_{eq} &= K_l x(\varpi) + E_l \varphi(M_1 x(\varpi)) - (G_i B_i)^{-1} G_i A_i x(\varpi) \\
&- (G_i B_i)^{-1} G_i A_{di} x(\varpi - \tau) - (G_i B_i)^{-1} G_i D_i w(\varpi). \quad (30)
\end{aligned}$$

By substituting the equivalent control law (30) into the MJLSs (2), the closed-loop MJLSs become

$$\begin{cases} \dot{x}(\varpi) &= \bar{A}_i x(\varpi) + \Gamma A_{di} x(\varpi - \tau) + E_{il} \varphi_l(M_1 x(\varpi)) \\ &+ \Gamma D_i w(\varpi) \\ x(\varpi) &= h(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0] \end{cases} \quad (31)$$

$$\begin{bmatrix} L_1(X_i, Y_i) & \Gamma A_{di} X_i & \sum_{l=1}^M \delta_{il} E_{il} + X_i (\Delta \Omega M_1)^\top \\ * & -\mathcal{P}_{3i} & 0 \\ * & * & -2\Delta \\ * & * & * \\ * & * & * \\ & & \Gamma D_i & M(X_i) \\ & & 0 & 0 \\ & & 0 & 0 \\ & & -\mu I & 0 \\ & & * & N(X_i) \end{bmatrix} < 0 \quad (45)$$

$$\begin{bmatrix} -\iota^* e^{-\mu \mathcal{T}} + 4\mu\gamma^2 \mathcal{T} + 4\mu\alpha_3^2 \delta \mathcal{T} & \sqrt{\iota_1} \\ * & -\sigma_1 - \tau\sigma_2 \end{bmatrix} < 0 \quad (46)$$

$$\begin{bmatrix} -\iota_2 e^{-\mu T} + \mu\delta & \iota^* \\ * & -\sigma_1 \iota^* \end{bmatrix} < 0 \quad (47)$$

$$\begin{bmatrix} -Z_1 e^{-\mu \mathcal{T}} & 2\alpha_1 Z_1 \\ * & -\mu^{-1} I \end{bmatrix} < 0 \quad (48)$$

$$\begin{bmatrix} -Z_2 e^{-\mu \mathcal{T}} & 2\alpha_2 Z_2 \\ * & -\mu^{-1} I \end{bmatrix} < 0 \quad (49)$$

$$\sigma_1 R_i^{-1} < X_i < R_i^{-1} \quad (50)$$

$$\sum_{j=1}^N \pi_{ij} \mathcal{Q}_{ij} < \mathcal{F}_{2i} \quad (51)$$

$$\iota_1 < \iota^* < \iota_2 \quad (52)$$

where

$$L(X_i, Y_i) = \sum_{l=1}^M \delta_{li} (X_i A_l^\top + Y_l^\top B_l^\top + A_l X_i + B_l Y_i) + \mathcal{F}_{1i} + \tau \mathcal{F}_{2i} + \mathcal{F}_{3i} + \mathcal{F}_{3i}^\top + (\pi_{ii} - \mu) X_i,$$

$$L_1(X_i, Y_i) = \sum_{l=1}^M \delta_{li} (X_i A_l^\top \Gamma^\top + Y_l^\top B_l^\top + \Gamma A_l X_i + B_l Y_i) + \mathcal{P}_{3i} + (\pi_{ii} - \mu) X_i,$$

$$M(X_i) = [\sqrt{\pi_{i1}} X_i, \dots, \sqrt{\pi_{i(i-1)}} X_i, \sqrt{\pi_{i(i+1)}} X_i, \dots, \sqrt{\pi_{iN}} X_i],$$

$$N(X_i) = -\text{diag}\{X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_N\},$$

$$\Lambda_2 = A_{di} X_i - \mathcal{F}_3, \quad \Lambda_3 = \sum_{l=1}^M \delta_{il} B_l E_l + F_i + X_i (\Delta \Omega M_1)^\top,$$

$$\Lambda_4 = \sum_{l=1}^M \delta_{li} \tau (X_i A_l^\top + Y_l^\top B_l^\top),$$

$$\Lambda_5 = -\tau (X_i + X_i^\top - R).$$

Thus, the asynchronous controller gains can be obtained by $K_l = Y_l X_i^{-1}$ and $E_l = B_l^{-1} (E_{il} - F_i)$.

Proof: Generally, the matrix inequalities in Theorem 2 and Theorem 3 are difficult to solve directly. Based on certain equivalent transformation and linear matrix inequalities (LMIs) techniques, we give a sufficient condition.

Denote $X_i = P_i^{-1}$, $Y_i = K_i X_i$, $\mathcal{F}_{1i} = X_i \mathcal{Q}_i X_i$, $\mathcal{F}_{2i} = X_i \mathcal{Q}_i X_i$, $\mathcal{F}_{3i} = X_i W_i X_i$, $\mathcal{Q}_{ij} = X_i \mathcal{Q}_j X_i$, $Z_1 = O_1^{-1}$, $Z_2 = O_2^{-1}$, $\mathcal{P}_{3i} = X_i O_3 X_i$.

Then, the congruence transformation of (10) can be performed by a block-diagonal matrix $\text{diag}\{P_i^{-1}, P_i^{-1}, I, I, I, I, P_i^{-1}\}$. Based on Schur's complement properties, we can get (44). Similarly, we can obtain (45) by taking (32) through a congruence transformation with a block-diagonal matrix $\text{diag}\{P_i^{-1}, P_i^{-1}, I, I, I\}$.

Subsequently, in view of Schur's complement, it is easily to achieve LMIs (46)-(49) under conditions (11)-(13) and (33). This completes the proof.

Remark 3: It is easily checked that $0 \leq (X_i - \mathcal{Q}_i) \mathcal{Q}_i^{-1} (X_i - \mathcal{Q}_i)^\top = X_i \mathcal{Q}_i^{-1} X_i^\top - X_i - X_i^\top + \mathcal{Q}_i$, which implies $-X_i \mathcal{Q}_i^{-1} X_i^\top \leq -X_i - X_i^\top + \mathcal{Q}_i$. Following this method, one immediately obtains $-\tau P_i^{-1} R^{-1} P_i^{-1} \leq -\tau (X_i + X_i^\top - R)$.

C. Analysis of Reachability

The control objective in this paper is to design a asynchronous Lur'e-type SMC law (8) that ensures system trajectories reach the predetermined sliding surface within a finite time interval.

Theorem 4: Consider MJLSS (2), the reachability of the specified sliding surface (7) can be guaranteed in a finite time \mathcal{T}_1 with $0 < \mathcal{T}_1 < \mathcal{T}$ by the designed asynchronous Lur'e-type SMC law (8). The robust term $\beta_l(\varpi)$ is chosen as

$$\beta_l(\varpi) = \gamma_i + \alpha_{1i} \|x(\varpi)\| + \alpha_{2i} \|x(\varpi - \tau)\| + \alpha_{3i} \delta, \quad (53)$$

in which $\alpha_{1i} = \|(G_i B_i)^{-1} G_i A_i\|$, $\alpha_{3i} = \|(G_i B_i)^{-1} G_i D_i\|$, $\alpha_{2i} = \|(G_i B_i)^{-1} G_i A_{di}\|$, and the adjust scalar γ_i is determined by:

$$\gamma_i \geq \frac{\|G_i x(0)\|}{\mathcal{T} \{\sqrt{\lambda_{\min}(G_i B_i B_i^\top G_i^\top)}\}}. \quad (54)$$

Proof: Consider the following Lyapunov-Krasovskii functional as:

$$V_3(x(\varpi)) = \frac{1}{2} s^\top(\varpi) s(\varpi). \quad (55)$$

Then, it follows from (7) that

$$\begin{aligned} \dot{V}_3(x(\varpi)) &= s^\top(\varpi) \dot{s}(\varpi) \\ &= s^\top(\varpi) G_i [A_i x(\varpi) + A_{di} x(\varpi - \tau) + B_i u(\varpi) \\ &\quad + F_i \varphi(M_1 x(\varpi)) + D_i w(\varpi) - B_i K_l x(\varpi) \\ &\quad - B_i E_l \varphi(M_1 x(\varpi)) - F_i \varphi(M_1 x(\varpi))] \\ &= [G_i B_i s^\top(\varpi)] [(G_i B_i)^{-1} G_i A_i x(\varpi) + u(\varpi) \\ &\quad + (G_i B_i)^{-1} G_i A_{di} x(\varpi - \tau) + (G_i B_i)^{-1} G_i D_i w(\varpi) \\ &\quad - K_l x(\varpi) - E_l \varphi(M_1 x(\varpi))] \\ &= [G_i B_i s^\top(\varpi)] [(G_i B_i)^{-1} G_i A_i x(\varpi) + K_l x(\varpi) \\ &\quad + (G_i B_i)^{-1} G_i A_{di} x(\varpi - \tau) + E_l \varphi(M_1 x(\varpi)) \\ &\quad - \beta_l(\varpi) \text{sign}(s(\varpi))] \\ &\quad + (G_i B_i)^{-1} G_i D_i w(\varpi) - K_l x(\varpi) - E_l \varphi(M_1 x(\varpi))] \\ &\leq \|B_i^\top G_i^\top s^\top(\varpi)\| \left[\|(G_i B_i)^{-1} G_i A_i\| \|x(\varpi)\| \right. \\ &\quad \left. + \|(G_i B_i)^{-1} G_i A_{di}\| \|x(\varpi - \tau)\| - \beta_l(\varpi) \text{sign}(s(\varpi)) \right] \end{aligned}$$

$$\begin{aligned}
& + \|(G_i B_i)^{-1} G_i D_i\| \|w(\varpi)\| \\
& \leq -\gamma_i \|B_i^T G^T s(\varpi)\| \\
& \leq -\hat{\gamma}_i V_3^{\frac{1}{2}}(x(\varpi)), \tag{56}
\end{aligned}$$

where $\hat{\gamma}_i = \sqrt{2}\gamma_i \{\sqrt{\lambda_{\min}(G_i B_i B_i^T G_i^T)}\} > 0$. As shown in (56), there exists a positive scalar \mathcal{T}_1 :

$$\mathcal{T}_1 \leq \frac{2V_3^{\frac{1}{2}}(0)}{\hat{\gamma}_i}. \tag{57}$$

such that $V_3(x(\varpi)) = 0$ when $\varpi \geq \mathcal{T}_1$, and we have $s(\varpi) = 0$. By (55), it follows that

$$V_3(0) \leq \frac{\|G_i x(0)\|^2}{2}. \tag{58}$$

Substituting (58) into (57), it yields:

$$\mathcal{T}_1 \leq \frac{\|G_i x(0)\|}{\gamma_i \{\sqrt{\lambda_{\min}(G_i B_i B_i^T G_i^T)}\}}. \tag{59}$$

Under condition of (54), the following inequality can be derived by (59)

$$\mathcal{T}_1 \leq \mathcal{T}. \tag{60}$$

Thus, we can also find a time \mathcal{T}_1 within $[0, \mathcal{T}]$ to guarantee the accessibility of the sliding mode surface. This completes the proof.

IV. ILLUSTRATIVE EXAMPLE

In this section, a DC motor circuit [37] is used to evaluate the effectiveness of the proposed methodology, which is shown in Fig. 2. The dynamics is described by:

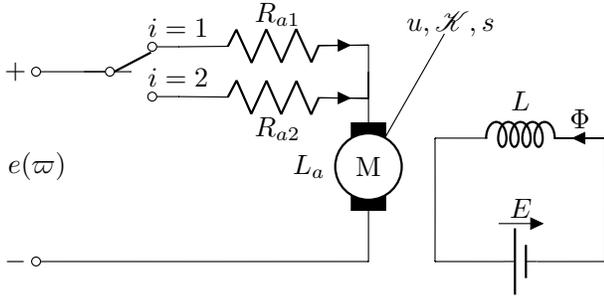


Fig. 2. The dynamic equivalent circuit of armature-controlled DC motor

$$\begin{cases} R_{ai} i_a + L_a \frac{di_a}{dt} + \mathcal{E}_e u = e, & i = 1, 2. \\ \mathcal{K} \frac{du}{dt} + su - \mathcal{C}_M i_a = 0, \end{cases} \tag{61}$$

The physical meaning of each symbol is listed in Table I.

Let $x_1 = u$, $x_2 = i_a$, $y = u$. Then the dynamic nonlinear model can be described in the form of

$$\dot{x}(\varpi) = A_i x(\varpi) + B_i u(\varpi) \tag{62}$$

where

$$A_i = \begin{bmatrix} -\frac{s}{L_a} & \frac{\mathcal{C}_M}{L_a} \\ -\frac{\mathcal{C}_e}{L_a} & -\frac{R_{ai}}{L_a} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix}, i = 1, 2.$$

TABLE I
LIST OF SYMBOLS

R_{ai}	brake resistance
i_a	brake current
L_a	armature inductance
u	shaft angular velocity
\mathcal{K}	motor inertia
s	viscous friction coefficient
\mathcal{C}_e	back electromotive-force coefficient
\mathcal{C}_M	electromagnetic torque coefficient

TABLE II
PARAMETERS OF THE DC MOTOR

$L_a(\text{H})$	0.5
$R_{a1}/R_{a2}(\Omega)$	2/1
$\mathcal{K}(kg \cdot m^2)$	0.003
$s(\text{N} \cdot \text{m} \cdot \text{s}/\text{rad})$	0.015
$\mathcal{C}_e(\text{V}/\text{K} \cdot \text{RPM})$	67.2
$\mathcal{C}_M(\text{N} \cdot \text{m}/\text{A})$	1.066

The parameters are given in Table II.

Considering the time-delays, disturbances and nonlinearities involved, we have the following system

$$\begin{cases} \dot{x}(\varpi) = A_i x(\varpi) + A_{di} x(\varpi - \tau) + B_i u(\varpi) \\ \quad + D_i w(\varpi) + F_i \varphi(\varrho(\varpi)) \\ \varrho(\varpi) = M_1 x(\varpi) \\ x(\varpi) = h(\varpi), \varpi \in [\varpi_0 - \tau, \varpi_0] \end{cases} \tag{63}$$

Then, we assume that system (63) and the designed controller contain two modes respectively, i.e., $N = 2$ and $M = 2$. In addition, we select the initial conditions as $x_i(0) = [0.4; 0.1]$ and $\mathcal{T} = 0.1$. The other relevant parameters are given in Table III.

TABLE III
SYSTEM PARAMETERS

τ	μ	ι_1	ι_2	σ_1	σ_2	δ
0.2	0.02	0.59	6.26	0.27	1.246	0.136

Besides, the random nonlinearity function $\varphi(\cdot)$ is taken as $\varphi(M_1 x(\varpi)) = 0.5\Omega M_1 x(\varpi)(0.1 + 0.2\cos(M_1 x(\varpi)))$ with $\Omega = 2.6$. Then, by solving LMIs (44) – (52) in Theorem 3, and the circularly optimizing parameter ι^* is chosen as $\iota_{min}^* = 1.9683$, we have:

$$\begin{aligned}
K_1 &= [-183.2166 \quad -243.1673], \\
K_2 &= [-182.4272 \quad -263.5429], \\
E_1 &= [0.1233 \quad 0.0444], \\
E_2 &= [0.1233 \quad 0.0444].
\end{aligned}$$

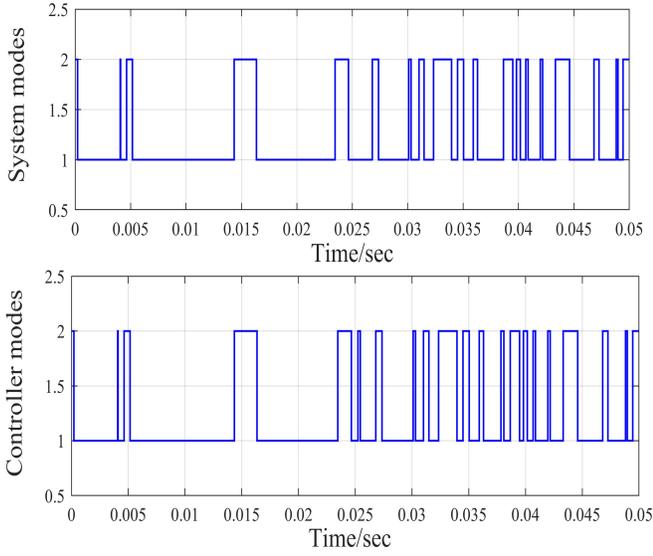
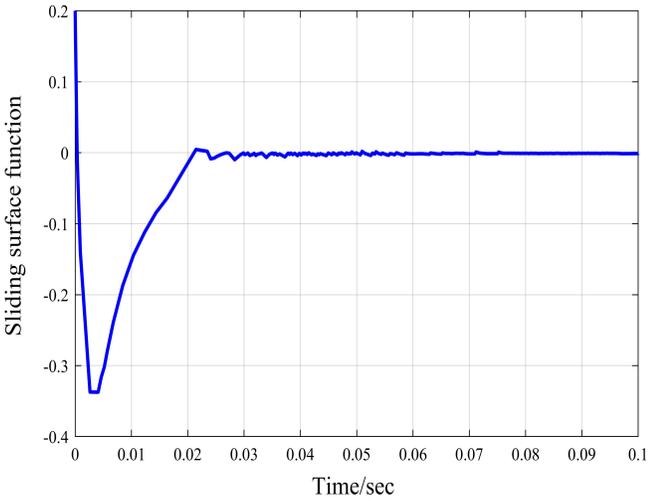


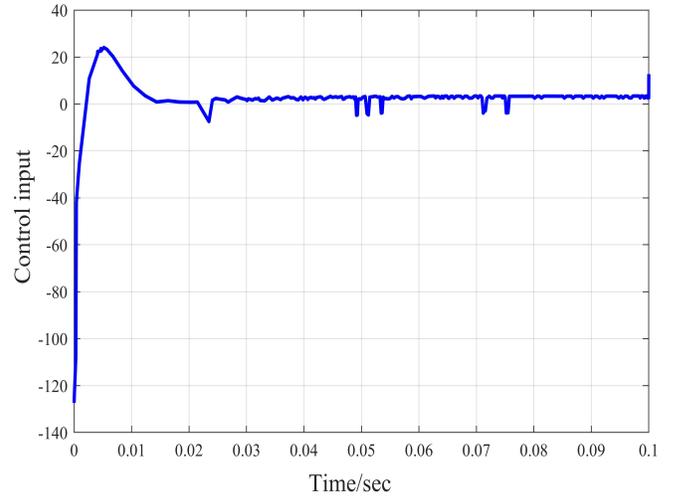
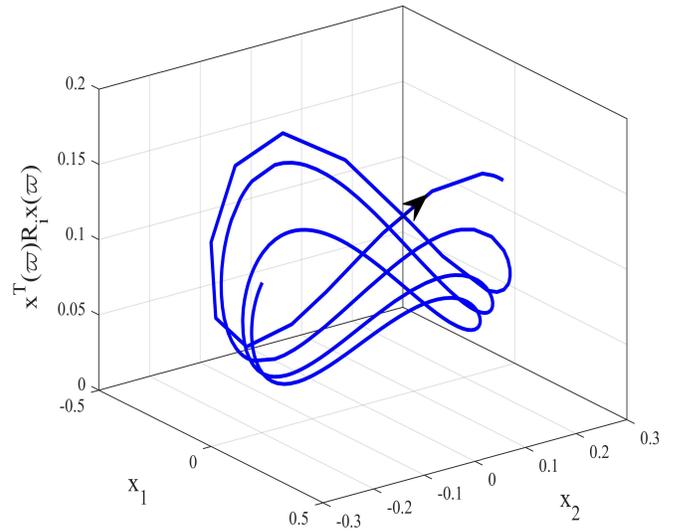
Fig. 3. Switching signals of the system and the controller


 Fig. 4. The sliding surface function $s(\varpi)$

Thus, the sliding surface in (7) is:

$$s(\varpi) = \begin{cases} = [0 \ 2]x(\varpi) \\ - \int_0^t [-1.10680.5774]\varphi(M_1x(s))ds \\ - \int_0^{\varpi} [-732.8664 - 972.6690]x(s)ds \quad i = 1, l = 1 \\ = [0 \ 2]x(\varpi) \\ - \int_0^{\varpi} [-1.10680.5774]\varphi(M_1x(s))ds \\ - \int_0^{\varpi} [-729.7 - 1054.2]x(s)ds \quad i = 1, l = 2 \\ = [0 \ 2]x(\varpi) \\ - \int_0^{\varpi} [1.0932 - 0.6226]\varphi(M_1x(s))ds \\ - \int_0^{\varpi} [-732.8664 - 972.6690]x(s)ds \quad i = 2, l = 1 \\ = [0 \ 2]x(\varpi) \\ - \int_0^{\varpi} [1.0932 - 0.6226]\varphi(M_1x(s))ds \\ - \int_0^{\varpi} [-729.7 - 1054.2]x(s)ds \quad i = 2, l = 2 \end{cases}$$

Select the adjustable parameters $\gamma_1 = 0.0018$ and $\gamma_2 = 0.0015$. Then the asynchronous Lur'e-type SMC law in (8) can be


 Fig. 5. SMC input $u(\varpi)$

 Fig. 6. Evolutions of $x^T(\varpi)R_l x(\varpi)$ in open-loop case

calculated as:

$$u(\varpi) = \begin{cases} = [-183.2166 \ -243.1673]x(\varpi) + [0.1233 \\ 0.0444]\varphi(M_1x(\varpi)) - \beta_1(\varpi)\text{sign}(s(\varpi)) \\ \quad i = 1, l = 1 \\ = [-182.4272 \ -263.5429]x(\varpi) + [0.1233 \\ 0.0444]\varphi(M_1x(\varpi)) - \beta_2(\varpi)\text{sign}(s(\varpi)) \\ \quad i = 1, l = 2 \\ = [-183.2166 \ -243.1673]x(\varpi) + [0.1233 \\ 0.0444]\varphi(M_1x(\varpi)) - \beta_1(\varpi)\text{sign}(s(\varpi)) \\ \quad i = 2, l = 1 \\ = [-182.4272 \ -263.5429]x(\varpi) + [0.1233 \\ 0.0444]\varphi(M_1x(\varpi)) - \beta_2(\varpi)\text{sign}(s(\varpi)) \\ \quad i = 2, l = 2 \end{cases}$$

The simulation results are shown in Figs. 3-7, where Fig. 3 depicts a possible sequence of the system/controller modes.

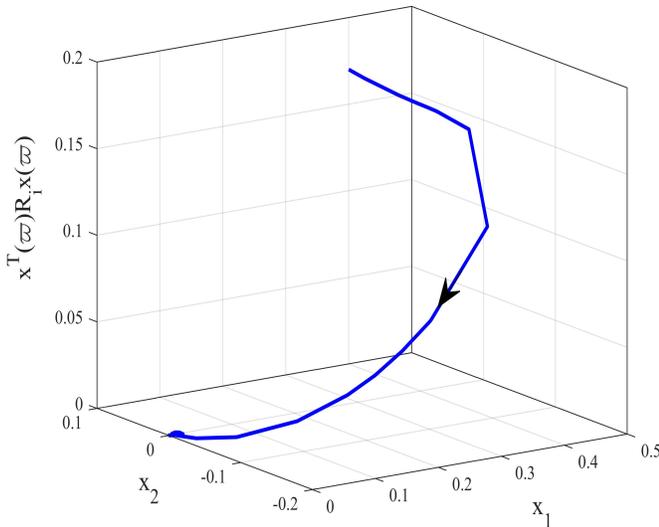


Fig. 7. Evolutions of $x^T(\varpi)R_i x(\varpi)$ in closed-loop case

The responses of the sliding variable $s(\varpi)$ and the asynchronous SMC input $u(\varpi)$ are depicted in Fig. 4 and Fig. 5, respectively. From Figs. 6-7, we can see that MJLSs (2) is FTS during the finite-time interval with the designed asynchronous controller.

V. CONCLUSION

This paper investigates the asynchronous SMC problem of a class of MJLSs. Firstly, the asynchronous problem of system modes and controller modes is modeled by HMM. Then, asynchronous sliding mode controller with Lur'e nonlinear information is designed to guarantee the reachability of sliding surface. Finally, according to the partitioning strategy, sufficient conditions are given to ensure the finite-time stabilization of the MJLSs. Future work will be directed at integrating the CHMM theory with SMC and fuzzy control.

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Data availability statement The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of interest The authors declare that they have no conflicts of interest.

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