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Theoretical Solutions to The Problem of Seepage and Consolidation in Saturated Clay Based on The Spatial Axisymmetric Model

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Abstract: The series solutions to the problem of spatial axisymmetric consolidation are deduced under non-homogeneous boundary conditions. Firstly, differentiable step function is introduced to construct the homogeneous operation function. Secondly, the operation function is used to superimpose the non-homogeneous boundaries to obtain homogeneous boundaries, non-homogeneous fundamental equation and new initial condition. Finally, the method of variables separation is used to construct the eigenfunction, and due to the mathematical justification of complete orthogonality of the eigenfunction, the series expansions of the fundamental equation and initial condition are carried out to obtain solutions for the seepage and consolidation in saturated clay with a borehole boundary. The correctness of the theoretical solutions are verified by the strict mathematical and mechanics derivation and the law of space-time variation in seepage flow.

Key words: non-homogeneous boundary; differentiable step function; homogeneous operation; seepage and consolidation; theoretical solutions

1 Introduction

Surcharge preloading, vacuum preloading and pile compaction can produce excess pore water pressure in the saturated clay. The speed and degree of excessive-pore-water-pressure dissipation affect the construction progress, foundation treatment effects and the bearing capacity of pile foundation, and also reflect the average degree of consolidation and the law of variation in compression quantity with time of clay layer. The related seepage and consolidation research mainly includes the determination of the initial excess pore water pressure, the derivation of theoretical or numerical solutions to reflect the spatiotemporal variations of excess pore water pressure, and their applications in the engineering and environmental problems.

Piling mechanism and soil squeezing effects in saturated clay are described in the article [1]. In articles [2,3,4], the displacement, deformation and stability of soil under different drainage assumptions are researched. In the article [5], the K_0 consolidated drainage model is used to deduce the stress field of the pile squeezing in saturated clay, and the solution for the initial excess pore water pressure is obtained under the corresponding boundary conditions. In the article [6], the problem of pile compaction in saturated soft clay is studied, and logarithmic strain parameters are introduced to describe the characteristics of large deformation and softening. The stress field caused by piling compaction in soil is deduced, and the law of distribution of excess pore water pressure is described also. The excess pore water pressure caused by pile driving in saturated clay is estimated in the article [7], and a numerical model reflecting the consolidation of soil around the pile is established to calculate the average degree of consolidation. In articles [8,9], the additional displacement, strain, stress, excess pore water pressure caused by pile compression and its dissipation are studied, and the corresponding engineering problems are solved. However, the above-mentioned solutions to the consolidated seepage problem of soil are limited to the one-dimensional or homogeneous fundamental equation and boundary conditions basically.

For one-dimensional non-homogeneous boundary problems, the superposition constructor can be used to homogenize the boundary. However, for multi-dimensional problems, the homogeneous superposition operation in one boundary surface will superimpose non-zero values in the other boundary surface under normal conditions, making it difficult to realize the traditional homogeneous operation. To solve the conflict, we introduce the differentiable limit step function and construct the homogeneous operation function for multi-dimensional problems without values of interaction. The method of separation of variables is used to construct the

eigenfunction of the inhomogeneous generalized equation, and the solution for the seepage and consolidation with a borehole boundary is obtained with the validation of the eigenfunction's completeness and orthogonality.

2 Mathematical model

Consolidation model is established as shown in Fig. 1, including the top surface at $z=0$, bottom surface at $z=H$, drainage (or irrigation) well boundary at $r=r_w$ and the covering of the initial excess pore water pressure at $r=r_e$. The consolidation differential equation [10] of spacial axis-symmetric problem is shown as Eq. (1)

$$\frac{\partial u}{\partial t} = C_h \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + C_v \frac{\partial^2 u}{\partial z^2} \quad (1)$$

where u is excess pore water pressure, C_h , C_v are the horizontal and vertical consolidation coefficients and r, z are radial and vertical coordinates respectively. The initial condition is written as Eq. (2).

$$u|_{t=0} = f(r, z) \quad (2)$$

The boundary conditions are written as Eq. (3)

$$\left. \frac{\partial u}{\partial r} \right|_{r=r_w} = u_r(r_w, z, t), \quad u|_{r=r_e} = u(r_e, z, t) \quad (3a)$$

$$u|_{z=0} = u(r, 0, t), \quad \left. \frac{\partial u}{\partial z} \right|_{z=H} = u_z(r, H, t) \quad (3b)$$

where r_w is the radius of borehole, pumping or irrigation well, r_e is the influence radius of $f(r, z)$ and H is the thickness of saturated clay layer.

Fig. 1 Spatial axis-symmetric consolidation model

3 Boundary processing

For multidimensional problems, it is difficult to apply the method of superposition of additional function to obtain homogeneous boundary, because the superposition at boundary $r = r_w$ will affect the value at boundary $z = H$.

For Eq. (3b), let $u(r, z, t) = V(r, z, t) + W(r, z, t)$ and $W(r, z, t) = z \times u_z(r, H, t) + u(r, 0, t)$, that is, by superimposing function $W(r, z, t)$ we can obtain $V(r, z, t)$ shown in Eq. (6) that satisfies the homogeneous boundary conditions at $z = 0$ or H , but the values of $W(r, z, t)|_{r=r_e} = z \times u_z(r_w, H, t) + u(r_w, 0, t)$ and $W(r, z, t)|_{r=r_e} = z \times u_z(r_e, H, t) + u(r_e, 0, t)$ will also be superimposed at boundary $r = r_w$ and $r = r_e$ at the same time, making homogenization impossible to achieve.

On the contrary, the homogenization at the $r = r_w, r_e$ boundary will also generate additional superposition items at $z = 0, H$.

To solve the above-mentioned problem, the differentiable Eq. (4) and (5) are constructed. Fig. 2 can be obtained by calculating the situation of $r_w = 0.25\text{m}, r_e = 5.0\text{m}, z_0 = 0\text{m}, z_H = H = 30.9\text{m}$ based on Eq. (4) and (5), and they can converge to the step function as shown in Fig. 2 when $n \rightarrow \infty$.

$$S_{sfr} = \frac{\arctan[n(r - r_w)] - \arctan[n(r - r_e)]}{\pi} \quad (4)$$

$$S_{sfz} = \frac{\arctan[n(z - z_0)] - \arctan[n(z - z_H)]}{\pi} \quad (5)$$

(a) $n : S_{sfr}$

(b) $n : S_{sfz}$

Fig. 2 Differentiable limit step function

Combining with Eq. (4) and (5), making $W_1 = [u(r_e, z, t) + u_r(r_w, z, t) \cdot (r - r_e)] \cdot S_{sfz}$, $W_2 = [u(r, 0, t) + u_z(r, H, t) \cdot z] \cdot S_{sfr}$, $u(r, z, t) = V(r, z, t) + W_1 + W_2$, and substituting into Eq. (1), we can obtain Eq. (6)

$$\frac{\partial V}{\partial t} - C_h \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) - C_v \frac{\partial^2 V}{\partial z^2} = Q \quad (6)$$

where $Q = Q(r, z, t)$, Q is the inhomogeneous term obtained from substituting $V(r, z, t) + W_1 + W_2$ into Eq. (1), and $V(r, z, t)$ satisfies boundary conditions $V|_{r=r_e} = 0$, $(\partial V / \partial r)|_{r=r_w} = 0$, $V|_{z=0} = 0$, $(\partial V / \partial z)|_{z=H} = 0$ and initial condition $V|_{t=0} = f(r, z) - W_1(r, z, 0) - W_2(r, z, 0)$.

4 Solution derivation

The homogeneous equation corresponding to Eq. (6) is written as Eq. (7).

$$\frac{\partial V}{\partial t} - C_h \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) - C_v \frac{\partial^2 V}{\partial z^2} = 0 \quad (7)$$

Apply the separation of variables method, assuming $V(r, z, t) = R(r)Z(z)T(t)$, and make $T' = \frac{dT}{dt}$,

$$Z'' = \frac{d^2 Z}{dz^2}, \quad R' = \frac{dR}{dr}, \quad R'' = \frac{d^2 R}{dr^2}, \quad \text{and substitute it into Eq. (7) to get Eq. (8)}$$

$$RZT' - C_h DZT - C_v TRZ'' = 0 \quad (8)$$

where $D = R'' + \frac{1}{r}R'$.

By introducing parameters μ and α , Eq. (9) can be obtained by separating variables from Eq. (8).

$$Z'' + \mu Z = 0, \quad R'' + \frac{R'}{r} + \alpha^2 R = 0 \quad (9)$$

Eq. (9) satisfies boundary conditions $R'|_{r=r_w} = 0$, $R|_{r=r_e} = 0$, $Z|_{z=0} = 0$, $Z'|_{z=H} = 0$. Eq. (10) and (11) can be obtained

by solving Eq. (9) according to the boundary conditions.

$$Z_k(z) = C_k \sin \sqrt{\mu_k} z \quad (10)$$

In Eq. (10), $\mu_k = \pi^2 (2k+1)^2 / (4H^2)$, $(k=0,1,2,L)$.

$$R_i(r) = C_i Y_0(\alpha_i r) - C_i \frac{Y_0(\alpha_i r_e)}{J_0(\alpha_i r_e)} J_0(\alpha_i r) \quad (11)$$

The characteristic Eq. (12) can be obtained by combining the boundary condition $R'|_{r=r_w}=0$ and Eq. (11).

$$J_1(\alpha_i r_w) Y_0(\alpha_i r_e) - J_0(\alpha_i r_e) Y_1(\alpha_i r_w) = 0 \quad (12)$$

In Eq. (12), $\alpha_i (i=1,2,3,L)$ are characteristic values, J_0 and J_1 are class 1 Bessel functions, and Y_0 and Y_1 are class 2 Bessel functions.

The eigenfunction Eq. (13) of Eq. (6) can be obtained from Eq. (10) - (11).

$$B_{k,i}(r,z) = \left[\sin \sqrt{\mu_k} z \right] \cdot \left[Y_0(\alpha_i r) - \frac{Y_0(\alpha_i r_e)}{J_0(\alpha_i r_e)} J_0(\alpha_i r) \right] \quad (13)$$

$V(r,z,t)$ can be separated to obtain Eq. (14) using Eq. (13), and Eq. (15) can be obtained by substituting Eq. (14) into the inhomogeneous fundamental Eq. (6).

$$V(r,z,t) = \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} B_{k,i}(r,z) T_{k,i}(t) \quad (14)$$

$$\sum_{k=0}^{\infty} \sum_{i=1}^{\infty} \left[T'_{k,i}(t) + (\alpha_i^2 C_h + \mu_k C_v) T_{k,i}(t) \right] \cdot B_{k,i}(r,z) = Q(r,z,t) \quad (15)$$

It can be verified by both analytic and numerical integrations, when $k_1 \neq k_2$ or $i_1 \neq i_2$,

$$\int_0^H dz \int_{r_w}^{r_e} dr \left[r B_{k_1,i_1}(r,z) B_{k_2,i_2}(r,z) \right] = 0, \text{ and when } \alpha = 0, \quad R'' + \frac{R'}{r} + \alpha^2 R = 0 \text{ of Eq. (9) has only zero solution, that is,}$$

$B_{k,i}(r,z) (k=0,1,2L, i=1,2,3L)$ takes r as weight to form a complete orthogonal system. According to the complete orthogonality, Eq. (15) is integrated with the weight $r B_{k,i}$ to obtain Eq. (16).

$$\left[T'_{k,i} + (\alpha_i^2 C_h + \mu_k C_v) T_{k,i} \right] \int_0^H dz \int_{r_w}^{r_e} B_{k,i} r B_{k,i} dr = \int_0^H dz \int_{r_w}^{r_e} Q r B_{k,i} dr \Rightarrow T'_{k,i} + (\alpha_i^2 C_h + \mu_k C_v) T_{k,i} = \varphi(t) \quad (16)$$

Eq. (16) is ordinary differential equation of order 1 with t as the variable and the right end of the equal sign

$$\text{is } \varphi(t) = \int_0^H dz \int_{r_w}^{r_e} Q r B_{k,i} dr \Bigg/ \int_0^H dz \int_{r_w}^{r_e} B_{k,i} r B_{k,i} dr.$$

According to the complete orthogonality and $V(r,z,0) = \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} B_{k,i}(r,z) T_{k,i}(0)$, we can obtain the initial value

$T_{k,i}(0)$ of $T_{k,i}$ in Eq. (16) from Eq. (17).

$$\int_0^H dz \int_{r_w}^{r_e} T_{k,i}(0) \cdot B_{k,i} B_{k,i} r dr = \int_0^H dz \int_{r_w}^{r_e} V(r,z,0) r B_{k,i} dr \Rightarrow T_{k,i}(0) = C_{k,i} \quad (17)$$

In Eq. (17), $C_{k,i} = \frac{\int_0^H dz \int_{r_w}^{r_e} V(r, z, 0) r B_{k,i} dr}{\int_0^H dz \int_{r_w}^{r_e} B_{k,i} r B_{k,i} dr}$ takes a constant value.

With $T_{k,i}$ deduced from Eq. (16) - (17) and Eq. (13), theoretical solutions of Eq. (18) - (19) can be obtained for seepage and consolidation in saturated clay based on spatial axisymmetric model.

$$u(r, z, t) = \sum_{k=0}^{\infty} \sum_{i=1}^{\infty} B_{k,i}(r, z) T_{k,i}(t) + W_1 + W_2 \quad (18)$$

$$U(t) = 1 - \frac{\int_{r_w}^{r_e} \int_0^H u(r, z, t) 2\pi r dr dz}{\int_{r_w}^{r_e} \int_0^H f(r, z) 2\pi r dr dz} \quad (19)$$

5 Project case

5.1 Computing arguments

The bearing capacity of a test pile and its converted average degree of consolidation [8,11] are shown in Table 1.

Table 1 Bearing capacity and equivalent consolidation degree

Time /day	0	10	19	31	61	91
Bearing capacity $10^3/\text{kN}$	1.00	3.10	3.74	4.29	4.50	4.66
converted average degree / %	0.0	57.4	74.9	89.9	95.6	100

Pile cross section is $0.45\text{m} \times 0.45\text{m}$, pile length is 30.9 m and the equivalent radius is $r_w = \sqrt{d^2/\pi} = 0.2539\text{ m}$. Initial condition takes Eq. (20)

$$f_1(r, z) - W(r, z, 0) = \frac{1}{3 \ln a} \left[M_0 + \frac{c_a z}{r_w} + 2.7 \alpha_f c_u \right] \ln \frac{a}{\rho} - z \times u_2 - u_1 \quad (20)$$

where $M_0 = 2(2c_u - K_p \gamma' r_w \tan \phi) \ln \frac{R_p}{r_w}$, $\gamma' = \gamma - 9.8(\text{kN/m}^3)$, $K_p = \tan^2(45^\circ + \frac{\phi}{2})$,

$\alpha_f = \frac{\sqrt{2}}{2}(3A - 1)$, $\rho = \frac{r}{r_w}$, $a = \frac{r_e}{r_w}$, and $R_p = \sqrt{\frac{Er_0}{2(1+\mu)c_u}}$ is the radius of the plastic zone. ϕ ,

c_a and A are friction angle, cohesion and pore pressure parameters, μ is Poisson ratio, E is elastic modulus,

and c_u is undrained shear strength. $\gamma = 18.0\text{ kN/m}^3$, $A = 0.85$, $C_v = C_h = 0.27\text{ m}^2/\text{d}$, $E = 10.5\text{ MPa}$,

$\phi = 23^\circ$, $c_a = 11\text{ kPa}$, $c_u = 20\text{ kPa}$, $\mu = 0.25$ and $a = 20$ [12].

Using the above-mentioned geometric parameters and characteristic equation Eq. (12), eigenvalues can be obtained as shown in Fig. 3.

Fig. 3 Eigenvalues

5.2 Calculation and analysis

According to Eq. (18) - (19), excess pore pressure and average degree of consolidation are calculated for multiple boundary conditions. Using the calculation results, contour of excess pore water pressure and seepage velocity vector are drawn, as shown in Fig. 4-15. The curves of $U(t)$ are shown in Fig. 16 for different boundary conditions.

Based on the analysis of Fig. 4-15, the following conclusions can be drawn:

- (1) The distribution of water pressure lines are consistent at $t = 0$ day under different boundary conditions described in Fig. 4-15. The contour conforms to the distribution law described by the initial condition written as Eq. (19). The calculation shows that the theoretical series solutions in this paper can be convergent, and the correctness of the solutions are preliminarily verified.
- (2) From the seepage contour and velocity vector diagram in Fig. 4-15, it can be seen that the velocity vectors are always parallel to the boundary at $r = r_w$ and the contour line is perpendicular to the boundary when $u_r(r_w, z, t) = 0$, which also verifies the convergence and correctness of the solution.
- (3) The influence of boundary and initial conditions on the excessive-pore-water-pressure distribution and velocity vector varies with time. The $t = 0$ day distribution of contour and velocity vector is determined by the initial condition, and the distribution in the period of $t > 0$ day is determined by interaction of initial and boundary conditions, while in the later period, it is mainly determined by boundary conditions.
- (4) When the water pressure boundary takes a constant value, that is, $u(r, 0, t) = C_1$ or $u(r_e, z, t) = C_2$, where C_1, C_2 are constants, the distribution of contour in the area near the boundary plane approaches to and stabilizes at the boundary pressure value finally. The seepage velocity at the vertical boundary direction tends to 0 as the excess pore water pressure dissipates.
- (5) When the boundary hydraulic gradient takes a constant value, that is, $u_r(r_w, z, t) = C_3$ or $u_z(r, H, t) = C_4$, where C_3, C_4 are constants, the boundary will produce continuous drainage or water supply regardless of the variation of the excess pore water pressure value would be in the calculated area. If the boundary takes negative pore pressure gradient, the gradient surely will make the negative pore pressure at a certain time point in the calculated area and continue to decrease. If the boundary is positive pressure gradient, the gradient surely will make the positive pore pressure at a certain time point in the calculated area and continue to increase. In engineering, maintaining a constant hydraulic gradient requires continuously increasing power support, which is actually difficult to achieve.
- (6) The radial velocity component is larger in general while the radial path of the calculation example is

relatively short. For the model shown in Fig. 1 which describes the spatial axis-symmetric consolidation and the boundary area at $r=r_w$ which is much smaller than the area at $r=r_e$ or $z=0$ or $z=H$, the seepage will be congested at the flow from boundary $r=r_e$ to boundary $r=r_w$.

(7) The average degree of consolidation is shown in Fig. 14, corresponding to each boundary value. Boundary conditions affect the dissipation rate and final dissipation degree of excess pore water pressure. The boundary of positive hydraulic gradient or head pressure values would reduce the rate and degree of the consolidation, and would do the opposite when they take negative values.

(8) If the initial excess pore water pressure near the boundary is small, such as the pressure nears $r=r_e$ or $z=0$ in the calculation example in this paper, the higher boundary pressure value can make the average degree of consolidation at initial phase negative around $t=5$ day, as shown in the twelfth boundary condition in Fig. 16.

(9) The circled boundary condition in Fig. 16 represents the average degree of consolidation curve converted from field test bearing capacity results, and condition 1 represents the theoretical calculation results in this paper under the same boundary conditions as the circled. The curves of the two conditions above-mentioned are in good agreement, which further verifies the correctness of the formula derived from this article.

(10) The consolidation degree can be completed by about 80% in 45 days for most boundary conditions with regard to the engineering examples of this article. In order to facilitate the comparison, the contour and velocity vector diagram are drawn only from 0 to 45 days.

Fig. 4 Seepage contour and velocity vector

Fig. 5 Seepage contour and velocity vector

Fig. 6 Seepage contour and velocity vector

Fig. 7 Seepage contour and velocity vector

6 Conclusions

Aiming at the case of water pressure boundary at $z=0$ and $r=r_c$ and hydraulic gradient boundary at $r=r_w$ and $z=H$, the mathematical model of spatial axis-symmetric seepage is established, and the series solutions to the problem is deduced.

The homogeneous operation function is constructed according to the specific boundary, and the inhomogeneous boundary of spatial axis-symmetric problem is superimposed to obtain the new homogeneous boundary, generalized equation and initial condition.

Using the method of variable separation to construct the complete orthogonal characteristic function, the series expansion of the generalized equation and initial condition are carried out, and the consolidation series solution to the spatial axis-symmetric problem with a borehole boundary is obtained.

Based on the theoretical formula obtained from the above-mentioned derivation, contour of excess pore water pressure is drawn by combining the calculated values of the solution with different boundary functions to verify the applicability of the solution to the corresponding engineering problem.

Fig. 8 Seepage contour and velocity vector

Fig. 9 Seepage contour and velocity vector

Fig. 10 Seepage contour and velocity vector

Fig. 11 Seepage contour and velocity vector

Fig. 12 Seepage contour and velocity vector

Fig. 13 Seepage contour and velocity vector

Fig. 14 Seepage contour and velocity vector

Fig. 15 Seepage contour and velocity vector

Fig. 16 Consolidation degree variation with boundary conditions

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