Comparative Analysis of Spin, Paths, Temporal, and Spatial Modes

Carlos Martinez (✉ carlosc@hispatel.es)

Universidad Politecnica de Madrid <https://orcid.org/0000-0002-9304-9959>

Ewa Nieminsky

Hispatel - France

Research Article

Keywords: Poincare, polarization, wave plate, beam splitter, Bloch, Laguerre, temporal modes, OAM, optics, photonics, qubit, quantum

Posted Date: December 1st, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1099398/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

(rev.) Comparative Analysis of Spin, Paths, Temporal, and Spatial modes

Martinez, J. Carlos · Ewa Nieminsky

2021 Nov. 19th (24th with images)

Abstract Research on optical modes, such as orbital, temporal, or parity, bring much attention, for these new degrees of freedom allow larger quantum communication alphabets. Each lab usually adapts the Bloch or Poincare sphere to their experiment or light mode. This takes an extra effort and time and produces a plethora of spheres and notations. Yet, we miss a common framework or convention valid among diverse physical-modes. We aim to unite in one representation the best points from many different spheres. Such common-sphere could also help to compare distant experiments, for an intuitive understanding of quantum optical states. We built a common representation by mathematically aligning the Hilbert space and a three dimensional color space. We define a unique color for each one of the three Poincare axes and positive Pauli vectors. Beyond three primary colors and states, our equations associate each Hilbert state to a specific tonality, among the infinite combinations in Color space. These maths achieve a new ability to unequivocally represent any quantum state by its precise combination of colors. Thus, with these equations, quantum states ‘yellow’ or ‘magenta’ are not mere names, rather each one denotes an exact superposition in Hilbert space. To handle disparities between SO3 vs. SU2 space operations, we propose a darkness bit and a Hermite-inspired shape. A simulation of HG modes let us align distinct shapes to quantum optical states. Three examples of applications show our color sphere in practice. First, we apply the Hilbert-Color mapping in Polarization. Then, the same color-space is shown in Orbital Angular Momentum. We also represent location paths in this color-mapping. The simulations and practical comparisons let us refine the proposed color sphere convention. For higher-order and path-to-industry, any sphere section serves as color constellation diagram. One color-space sphere served as common ground to represent coexisting concepts among diverse physical areas. The introduced change diagrams are visual tools to communicate setups and operators. The examples showed a unique notation matches many physical processes. The resulting diagram of superposition of spatially separated optical paths is coherent with a Plate on Polarization or cylinder lens on OAM Hermite Gaussian modes. A unique change diagram describes the three examples. The meaning persist despite the physical implementation. Found also how this color space let us grasp visually some meaning. Thus, the amount of blue in a state representation indicated the degree of its phase shift. Overall, we presented math and visual tools to display and compare experiments. We showed examples in different physical modes, all linked to a unique color sphere.

Contents of this pre-print version

REVISION NOTES	2
1 Introduction	2
1.1 Problem: Multiple spheres and notations	2
1.2 Purpose: Consistent Representation.	3
1.3 Side benefits: Ability to compare paradigms	3
2 Theory, Mathematical definitions, and Simulations	3
2.1 Known representations of Qubits in R3 Sphere or Hilbert	3
2.2 Defining three primary Kets in H, R3, and Color spaces.	4
2.3 Mapping Hilbert and Color spaces for any state	4
2.4 Resulting Hilbert-Color States & Sphere	5
2.5 Further extensions, shapes for SO3 vs SU2	6
3 Examples comparing physical cases with one Color Sphere	6
3.1 A Quarter Wave Plane for Polarization	7
3.2 Lenses for Orbital Angular Momentum	7

3.3	Beam Splitter for Location Paths	8
3.4	Results: Color Kets Notation Table	8
3.5	Signal Constellation for Communications	9
4	Discussion	9
4.1	Representing Device Change	10
4.2	Consistency, despite physical area, and comparison.	10
4.3	Intuitive & Visual meaning.	10
4.4	Abstracting from or to different physical modes	10
4.5	Color convention proposed for representing any states	11
4.6	Consistency and Validation	11
5	Conclusions	11
	Declarations	12
5.1	Funding	12
5.2	Conflict of interest/Competing interests	12
5.3	Ethics approval	12
5.4	Consent to participate	12
5.5	Consent for publication	12
5.6	Availability of data and materials	13
5.7	Code availability	13
5.8	Authors' contributions	13

REVISION NOTES

Thanks for your notes this August. Kindly find the changes by your comments. (This adds images to 19Nov2021)
 We revisited the original December2019 submission and simplified: - removed several redundant and repeated comments. - improved short introductions (indented on pre-print only) to clarify the flow and abstract - removed past discussions-5.4-5 5.8 for future papers - added newer references, as suggested - this changed text, eqs, tables. touched margins (too much?). - we could simplify further if you advise so. Meanwhile, we rewrite images to vector (ongoing with pgfplot tikz)

Keywords Poincare, polarization, wave plate, beam splitter, Bloch, Laguerre, temporal modes, OAM, optics, photonics, qubit, quantum

1 Introduction

Research on optical modes, such as orbital, temporal, or parity, bring much attention, for these new degrees of freedom allow larger quantum communication alphabets.

New modes impacted quantum information for the opportunity coding qubits[1] in new physical modes. It also extends the use of polarization and interferometry devices from wave-plates or splitters[2]. Many labs succeeded to manipulate temporal[3] and spatial[4] optical modes. Its advantage comes from the many dimensions achievable by Hermite Gaussian (HG) and Laguerre (LG) modes[5]. Deterministic quantum gates also exploited the two-dimensional spatial-parity symmetry[6] of the transverse[7] single-photon field. Significant achievements raised with: temporal[8] modes and time-parity[9]; implementations coexisting[2] with traditional fibers; or orbital angular momentum (OAM)[10][5][11].

The importance of this research area is shown by the number of improvements and broad advances, ranging from new wavelengths [12] to atmospheric [13] propagation. Hence, new tools in this field can help many teams.

1.1 Problem: Multiple spheres and notations

Each lab usually adapts the Bloch or Poincare sphere to their experiment or light mode. This takes an extra effort and time and produces a plethora of spheres and notations. Yet, we miss a common framework or convention valid among diverse physical-modes.

All modes cited above use different new representations to study and communicate results. Different experiments used to tag their spheres according to spin, orbital, paths[14], temporal[3], or parity[15] modes. Visualizations include schematics of the light path or arrows for spin. The spheres brought the benefits of coordinates representation and geometrical analysis. Thus, experiments represent quantum states in the sphere, and show device operations as rotations.

Thus, For angular momentum, a team might denote $|N\rangle$ the state in the sphere north. In spin, the north is often named $|R\rangle$. For parity[6], a modified Poincare shows instead $|e\rangle$ from the even and odd modes. The produced bibliography has a plethora of different spheres, blueprints, and colors [16] to visualize states and their changes or rotations [17].

Communications combining two-eigenstates optical modes were shown in a LG-Bloch[10] sphere, based on the traditional Poincare sphere[18]. Other approaches extended the Bloch[19] sphere for OAM with further dimensions including hybrid[20] and higher-order[21] spheres.

1.2 Purpose: Consistent Representation.

We aim to unite in one representation the best points from many different spheres.

We seek logical state properties consistent across physical realizations: orbital, spin, temporal[22], spatial parity symmetry, paths. . . Thus, a common sphere could map to one degree of freedom at a time: including Parity or Time Energy LG modes[3].

Could a sphere offer enough value to serve many different experiments? Could we compare different cases in a unique common sphere? We see the mathematical tool and the color-shape sphere built in this paper shows so. Yet there is room where other researchers can improve this sphere.

1.3 Side benefits: Ability to compare paradigms

Such common-sphere could also help to compare distant experiments, for an intuitive understanding of quantum optical states.

We seek tools to compare quantum properties among different degrees of freedom. The ability to change perspectives and compare was a valuable research tool to find new insights. The bibliography of many researches shows so:

For instance, comparing classical polarization versus quantum behaviors [23] gave good results. Other Examples explored interactions between spin and orbital momentum [24] to bring new insights, or measured combined entanglement and orbitals modes [11].

Connecting fields let to build probabilistic CNot gates[14], linking a polarization encoded qubit into a spatially encoded one (optical path superposition). Physical Analysis of optical instruments [25] helped researchers explore the quantum nature. Key achievements included all-optical[26][27] OR-exclusive gate built with interferometry [28], or and comparing new wavelengths in fibers and beam splitters[29].

To compare experiments, we construct a color sphere representation framework next sec. 2. Later, a practical section sec. 3 will link each logical point to its physical state depending of the chosen degree of freedom.

2 Theory, Mathematical definitions, and Simulations

We built a common representation by mathematically aligning the Hilbert space and a three dimensional color space.

Readers can jump to 2.1 skipping this paragraph. This is a short side-note on the method used, derived from information modeling: We started by exploring many notations developed by teams to communicate designs; then named key states with abstract identifiers like $|0\rangle$ red. Then, we studied the logical states according to their form, e.g. Hilbert $(1\ 0)^t \in \mathbb{H}$. We relate to different modes or physical observable. Finally, it let us associate and compare several research areas.

2.1 Known representations of Qubits in R3 Sphere or Hilbert

Consider two orthogonal states $|0\rangle$ and $|1\rangle$ in a Hilbert space $\mathbb{H}$. These two form a basis spawning the infinite logical states in $\mathbb{H}$.

In order to represent each state in three dimensions, we use the three positive eigenvectors of Pauli[30] as a basis of $\mathbb{R}^3$ space, able to spawn all points.

Thus, we use the well-known representation from spherical coordinates to $z+x+y$. This common mapping between $\mathbb{H}$ and $\mathbb{R}^3$ makes $|0\rangle$ $|1\rangle$ align in the z axis, at the North & South poles of the sphere.

Consider the known spherical coordinates. This let us represent a quantum state $|\phi\rangle$ in spherical coordinates of colatitude $0 \leq \theta \leq \pi$ and longitude $0 \leq \phi < 2\pi$, and transform it into $\mathbb{R}^3$ coordinates eq. 3:

$$\forall \theta, \phi | 0 \leq \theta \leq \pi, 0 \leq \phi < 2\pi : |\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle = \quad (1)$$

$$= \cos(\theta/2)|0\rangle + (\cos\phi + i\sin\phi)\sin(\theta/2)|1\rangle = \quad (2)$$

$$= \sin\theta\cos\phi x + \sin\theta\sin\phi y + \cos\theta z \quad (3)$$

2.2 Defining three primary Kets in $\mathbb{H}$, $\mathbb{R}^3$, and Color spaces.

We define a unique color for each one of the three Poincare axes and positive Pauli vectors.

We start with red for $|0\rangle$ in line with papers depicting North pole in red, among other reviewed experiments and their representation. . .

We define three primary colors[31] and pictographs, mapping each to one of the positive states in Pauli.

Table 1: Proposed primary colors defined in relation to 3D axis, Quantum States and Spherical coordinates

Color-Ket	$\mathbb{R}^3$	Ket	Hilbert	Pauli	Colatitude	Longitude
r^+	$z+$	$ 0\rangle$	$(1\ 0)'$	S1	0	0
g^+	$x+$	$ +\rangle$	$(1\ 1)'$	S2	$\pi/2$	0
b^+	$y+$	$ i\rangle$	$(1\ i)'$	S3	$\pi/2$	$\pi/2$

The table tbl. 1 shows these definitions, as well as in equations eq. 6 :

$$r^+ \equiv \psi_z \equiv S1 \equiv |0\rangle \equiv Sph(0, 0) \quad (4)$$

$$g^+ \equiv \psi_x \equiv S2 \equiv |+\rangle \equiv |0\rangle + |1\rangle \equiv Sph(\pi/2, 0) \quad (5)$$

$$b^+ \equiv \psi_y \equiv S3 \equiv |i\rangle \equiv |0\rangle + i|1\rangle \equiv Sph(\pi/2, \pi/2) \quad (6)$$

Where three main color vectors red green blue correspond to three orthogonal axis $z \times y$ in $\mathbb{R}^3$. Thus, the name red r^+ denotes $|0\rangle$.

While this adds to previous two-color works [16], as in the photon-atom transfer sphere[32], our work goes further. We arrived here to a sphere of three primary colors, as used in temporal modes[3], and add more next.

2.3 Mapping Hilbert and Color spaces for any state

Beyond three primary colors and states, our equations associate each Hilbert state to a specific tonality, among the infinite combinations in Color space.

For a Bloch sphere, we could replace the $\mathbb{R}^3$ basis $\{x, y, z\}$ by three positive non-orthogonal kets $\{|+\rangle, |i\rangle, |0\rangle\}$. Then, from spherical eq. 3 on $\mathbb{R}^3$, we obtain a definition of state ψ as linear combination of three states in $\mathbb{H}$:

$$|\psi\rangle = \sin\theta\cos\phi|+\rangle + \sin\theta\sin\phi|i\rangle + \cos\theta|0\rangle \quad (7)$$

To build a Color sphere, we replace the x, y, z basis by three colors green, blue, red, respectively. This let us describe any state in color space[31], with three weights RGB and a color-basis $\{g, b, r\}$:

$$|\psi\rangle = \sin\theta\cos\phi|g\rangle + \sin\theta\sin\phi|b\rangle + \cos\theta|r\rangle = \quad (8)$$

$$= C_g|g\rangle + C_b|b\rangle + C_r|r\rangle \quad (9)$$

$$C_r = \cos\theta; C_g = \sin\theta\cos\phi; C_b = \sin\theta\sin\phi \quad (10)$$

The eq. 10 defines exact color coordinates (Cr,Cg,Cb) for quantum states and its combinations. This allows infinite tonalities varying each color contribution, in the positives amount from 0 to 1 (negative colors in sec:so2).

This extends the common Hilbert to R3 mapping one step forward to the color-space spawned by three primary colors. tbl. 2

2.4 Resulting Hilbert-Color States & Sphere

These maths achieve a new ability to unequivocally represent any quantum state by its precise combination of colors. Thus, with these equations, quantum states ‘yellow’ or ‘magenta’ are not mere names, rather each one denotes an exact superposition in Hilbert space.

The equations eq. 7 and eq. 10 can define as many Color-Kets qubits names as common colors we know. The table shows just a few color states and how they are defined with exact θ, ϕ coordinates or Hilbert state combination.

Table 2: Intermediate color qubit combinations in a sphere.

Ket	Color	Ket Combination	Colat(θ)	Long(ϕ)
y+	Yellow	$(+\rangle + 0\rangle) / \sqrt{2}$	$\pi/4$	0
c+	Cyan	$(+\rangle + i\rangle) / \sqrt{2}$	$\pi/2$	$\pi/4$
m+	Magenta	$(0\rangle + i\rangle) / \sqrt{2}$	$\pi/4$	$\pi/2$
o+	Orange	$\propto r\rangle + y\rangle$	$\pi/8$	0

This leads to the following color sphere, which completes the main achievement of this section.

Fig. 1 Color sphere with main states and orientations (PRINT-NOTE= we are rewriting this in TIKZ to make it vector)

This logical sphere maps logical states to color coordinates. This was simulated in Matlab and plotted in 3D but such sphere can be also coded in Latex Tikz.

The maths eq. 10 let anyone name a state “cyan” and that color defines a precise univocal state. This allows teams to cite this sphere and associating Color and Hilbert spaces to display qubits with unequivocally tones, and saving these demonstrations and examples.

2.5 Further extensions, shapes for SO3 vs SU2

To handle disparities between SO3 vs. SU2 space operations, we propose a darkness bit and a Hermite-inspired shape. A simulation of HG modes let us align distinct shapes to quantum optical states.

Leveraging color space has some drawbacks but potential benefits. This paper deals with two-dimensional modes, field-orthogonal eigenstates. As in the cited works on temporal[3] modes or orbital[11] angular momentum.

A color sphere seemed hindered by an apparent disparity between three primary colors versus six main states, and six Pauli vectors. We revisit this with few simulations to address mapping three colors with six states.

See simulations and figures at fig. 2 reminds us of the actual HG10 shape for better understanding. The three-dimensions figure shows the negative lobe or phase component.

Fig. 2 Numeric Sim HG10 showing the negative lobe. generated by tikz pgfplot

The numeric calculation fig. 2 highlights the negative HG lobe with a darker tone. Leveraging darkness lets us keep the color notation proposed to locate states in the Bloch sphere. These simulations lead to highlight x- with darker tones. The overall HG mode calculation follows the color proposed in table tbl. 3. The x- is the orthogonal state of x+, Likewise using darker for y- z-.

Table 3: Orthogonal opposites negative colors.

Ket	Hilbert	Axis	Colat(θ)	Long(ϕ)	pictograph
r+	$ 1\rangle$	z+	0	any	
r-	$ 0\rangle$	z-	π	any	

In order to form a basis, a set {red,green,blue} needs to combine negative colors. The solution taken for this initial setback was to use a darker-color pictogram for opposite or orthogonal state. Thus, if red+ is $|0\rangle$, the orthogonal $|1\rangle$ red - is darker red or upside down state, this let us treat it as red negative color. In table tbl. 3.

This dark side corresponds to the complex phase of HG modes. We worked with upside-down pictograph of its orthogonal. It eases a direct mapping from the proposed color-shape sphere into experiments with OAM, spatial-parity or energy-temporal modes. We adopt the convention of works[6][3] that signified the negative eigenstate using red color.

Hence, we linked each SO2 Quantum State with a univocal point in SU3 Color-Space using a dark-shape bit. Future papers can explore an alternative inner vs outer color representation.

The reader might intuit these equations bring a series of benefits. We present a few examples first in sec. 3 then discuss its implications.

3 Examples comparing physical cases with one Color Sphere

Three examples of applications show our color sphere in practice.

With this unique notation and diagram, let us see in practice to compare few setups: classical polarization, spin, orbital and spatial location paths.

We examine States and changes according to the observable or measurable. Let us denote Xi the variable to differentiate Paths, Spin, Transverse mode ... This is the contact between colors or Hilbert spaces and the physically observable they represent. (Physical Xi path, Ex polarization, parity $\hat{e}$, angular mode HG01...) (Physical Xi path, Ex polarization, parity $\hat{e}$, angular mode HG01...)

The device ability to sense, conditions the association we use from Hilbert to Xi.

3.1 A Quarter Wave Plane for Polarization

First, we apply the Hilbert-Color mapping in Polarization.

In classic polarization, the measurable Xi are two orthogonal electromagnetic field components $\hat{e}^x \otimes \hat{e}^y$. Thus, the red state $r+$ or $|0\rangle$ corresponds to a Jones vector of pure horizontal state $JH = (10)^t$, with no component on vertical $JV = (01)^t$. A horizontally polarized photon with logical form $1 \equiv |0\rangle$ is named red in the sphere fig. 1 and table tbl. 1.

In quantum description of photon spin, likewise we use two complex values $(c1, c2)^T \in C^2$. It is the photon state superposition projected into orthogonal eigenstates bases. This gives a straight-forward association from $|H\rangle |V\rangle$ polarization to logical eigenstates $|0\rangle |1\rangle$. Hence, in the sphere Horizontal links to $|0\rangle$ red. A circular polarization maps to pure blue in fig. 1.

Very briefly consider when a horizontally polarized photon hits a Quarter Wave Plane. We can see the QWP device applies a matrix operation on the 45 degrees incoming beam. Thus, if the incoming photon has logical form $(10)^t \equiv |0\rangle \equiv red+$, after the QWP the state is $(1i)^t \equiv |i\rangle \equiv blue+$. This is shown in eq. 13

$$M_{QWP} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad (11)$$

$$r^+ . M_{QWP} \rightarrow b^+ \quad (12)$$

$$b^+ = M_{QWP} . r^+ \quad (13)$$

Let us see other physical areas, next.

3.2 Lenses for Orbital Angular Momentum

Then, the same color-space is shown in Orbital Angular Momentum.

In angular momentum, the Xi is spawned by the basis eigenstates HG01 HG10 [4]. Using bi-dimensional Orbital momentum modes fits this sphere. See the decomposition[4] of Laguerre dough-nut modes by Hermite Gauss summarized here :

$$LG01 = HG01 + iHG10 \quad (14)$$

The basis[10] $HG10 \equiv (10)^t \equiv |0\rangle$ associates with $r+ \equiv |0\rangle$. Likewise for $HG01 \equiv r-$, as in tbl. 3. Hence, the mode decomposition in color notation:

$$|blue-\rangle = |red-\rangle + i|red+\rangle \quad (15)$$

In contact with two cylindrical lenses[4], the HG beam will suffer a Guoy phase shift. The lenses transform the incoming HG10 mode into the LG01 mode. This change from state $r+$ to $b+$ has the same change diagram, red to blue, as fig. 3 and this eq. 17:

$$M_{lenses}|0\rangle \rightarrow |i\rangle \quad (16)$$

$$M_{device}|red+\rangle \rightarrow |blue\rangle \quad (17)$$

Discussing HG implications. At this stage, this is just a visualization of change. Each state displayed with a color given by a sphere convention.

3.3 Beam Splitter for Location Paths

We also represent location paths in this color-mapping.

When dissecting a system by classical optical paths, X_i is the space coordinate denoting if lights takes a transmission X_t or reflection X_r path.

For photon states between two different optical paths. The physical transmission X_t and reflected X_r paths have its eigenstates in a Hilbert space. The logical state represented as $\text{red}+ = |0\rangle = (1\ 0)'$ means a photon in the transmission port.

A splitter divides the incoming light into two beams. Each deflected path associate to one of the eigenstates. Assuming a $\pi/2$ phase in the reflected path, the operator describing a dielectric 50/50 beam splitter[29] is well-known annihilator operators equations:

$$\hat{a}_2 = (\hat{a}_0 + i\hat{a}_1) / \sqrt{2}, \quad (18)$$

$$\hat{a}_3 = (i\hat{a}_0 + \hat{a}_1) / \sqrt{2} \quad (19)$$

$$M_{pbs} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \quad (20)$$

The operator eq. 20 coincides that of QWP, and produces the same logical change seen with the wave plate. The beam splitter (or fiber coupling) transforms the input state $|0\rangle$ red into a resulting $|i\rangle$ blue. This is shown in Change diagrams as eq. 17 and the following fig. 3:

Fig. 3 s (PRINT-NOTE= to redraw in latex TIKZ vector image)

Blue here means a phase-shift in the reflection path $|1\rangle = (0\ 1)'$. It represents simultaneously both optical paths, or fibers, with a phase shift.

Note how this fig. 3 matches the previos examples in OAM and Polarization too, so forth in other physical areas. Will see it in discussions. The HG modes like fig. 2 help us research temporal modes too, but that and HG01 simulations are left for future papers.

3.4 Results: Color Kets Notation Table

The simulations and practical comparisons let us refine the proposed color sphere convention.

See how these matches physical realizations, the main sphere states are three colors pictograms plus their inverses (darker lobes) as shown in the following table tbl. 4.

This table tbl. 4 shows color-names corresponding to the main states axis. Horizontally polarized $|0\rangle$ $|H\rangle$ in red; 45-polarization in green; circular in blue. Then, vertical optical $|V\rangle$ polarization associates with $|1\rangle$ red - which is darker.

Each color combination represents a well-defined logical state according to the equations. We can label it for polarization, as characterized by the Stokes parameters, or any oOther physical setups and modes shown in the examples sec. 3.

Table 4: Color-ket table for six Pauli and few mixed states

Ket	Hilbert	Axis	(θ)	(ϕ)	Compares With	pictographs
r+	$ 1\rangle$	z+	0	any	H HG10
r-	$ 0\rangle$	z-	π	-any	V HG01
g+	$ +\rangle$	x+	$\pi/2$	0	D+ TEM
g-	$ -\rangle$	x-	$\pi/2$	π	A- TEM
b+	$ i\rangle$	y+	$\pi/2$	$\pi/2$	R+ LG10
b-	$ i-\rangle$	y-	$\pi/2$	$-\pi/2$	L- LG0
y+	$ +\rangle + 0\rangle$	.	$\pi/4$	0	.	yellow
c+	$ +\rangle + i\rangle$	.	$\pi/2$	$\pi/4$	.	cyan
m+	$ 0\rangle + i\rangle$	.	$\pi/4$	$\pi/2$	.	magenta
o+	$ r\rangle + y\rangle$	.	$\pi/8$	0	.	orange

From the theoretical simulated data, highlighting phase in darker tonality, and a few experiments, we could produce these graphs sketch and pictographs.

Teams in different areas use similar vectors and manipulations. Some notations are repeated in tbl. 4 for clarification. This refined a colored Bloch or Poincare sphere to represent logical states, separated yet able to match many physical measurables.

3.5 Signal Constellation for Communications

For higher-order and path-to-industry, any sphere section serves as color constellation diagram.

These sphere projections constellations [33][34] are common in telecoms. Our color math brings value to speeding up representation as the quantum communication technology moves into broader usage.

Fig. 4 Color constellation diagram, a sphere projection of fundamental states

In spatial paths, Blue means reflection port shift. In polarization, the Blue is circular polarization. With first-order Gaussian, the blue shows Laguerre Gaussian modes.

Showing N-dimensional constellations and relating their rotations is left for future work. We observe what could be a limitation for many other dimensions and more degrees of freedom. Yet this is a start and the two dimensions fit well the concrete [11] cases.

4 Discussion

One color-space sphere served as common ground to represent coexisting concepts among diverse physical areas.

An advantage of this Hilbert-Color map is when comparing different experiments and modes of light.

One unique state and change diagram worked in many cases sec. 3: superposition of spin, orbital, or of both reflected transmission paths. In classical, we can think of superposition of E_x, E_y fields.

The logical state blue describes that superposition.

4.1 Representing Device Change

The introduced change diagrams are visual tools to communicate setups and operators.

Change diagrams as eq. 17 or fig. 3: correspond to transformations, device operations, rotations[25] in the sphere. This is proposed to speed up prototypes. The cases in the results section represent the superposition and state change in such diagrams.

4.2 Consistency, despite physical area, and comparison.

The examples showed a unique notation matches many physical processes. The resulting diagram of superposition of spatially separated optical paths is coherent with a Plate on Polarization or cylinder lens on OAM Hermite Gaussian modes. A unique change diagram describes the three examples. The meaning persist despite the physical implementation.

Thus, either polarization, optical path, or orbitals associated to the same color sphere. We achieved the goal to synthesize a set of logical states useful among different experiments.

A generic quantum logical state associates to different physical measurable perspectives. That common point let us implement a state or operation in different physical degrees of freedom.

We show different physical cases with a very similar description and change diagrams. Only the meaning associated to the process varies from one environment to another.

These are proposed tools to compare quantum superposition degrees, regardless of physical implementation. One value of the presented state and change diagrams is to explore fundamentally similar quantum states.

This leaves the color-sphere and change-diagrams as tools to explore the nature of information, despite their implementations.

4.3 Intuitive & Visual meaning.

Found also how this color space let us grasp visually some meaning. Thus, the amount of blue in a state representation indicated the degree of its phase shift.

Any state with some phase shift (imaginary component) displays an easily detected blue tone in its visual representation. In this sense, the color sphere facilitates visual comprehension and communication of designs. For instance, full blue in our example means light state being in transmit and reflect paths simultaneously, with a phase shift in between.

Likewise, the state we call green serves to show the superposition of our basis eigenstates. The superposition with green means photon being in transmit and reflect paths. In other words, the representation let us grasp the simultaneity of being in both paths. Certain simultaneous states will be named $|+\rangle$ green, others might just have a light tone of green.

Thus, colors are proposed to understand some combinations intuitively. The intuitive visualization let us study the intrinsic properties of quantum states.

4.4 Abstracting from or to different physical modes

This approach allows to translate a design or model into alternative physical degrees of freedom. Orbital vs. Polarization vs. Parity vs. Temporal vs. Path, . . . Realization is possible by first abstracting the intrinsic of quantum states and operators. Subsequently, it let us design alternative realizations using other degrees of freedom.

Its potential is inter-paradigm comparison: this approach helps translate experiments. Given an operation or experiment commonly implemented with polarization, we can plan to implement it with orbitals, parity or temporal modes. In that manner, we expect this could facilitate the research and production of complex systems.

Assume a polarization setup gives us certain intuition on a superposition (with phase). Having a common diagram let us use that intuition in a different paradigm. Thus, the results show same diagrams used in optical paths in an interferometer.

4.5 Color convention proposed for representing any states

In summary, we have equations eq. 3 to handle the logical states with common notations and representation. The table tbl. 4 provides a visual map to research quantum systems from a common perspective. Each state corresponds to unique coordinates in color space, as well as in a Poicare Bloch sphere. Consequently, each color represents a quantum state combination.

We saw ket $|0\rangle$ $(1\ 0)$ for instance as generic physically independent perspective of quantum states. The precise component of three colors RGB let us automate diagrams using the native computing support of colors.

Previous works showed few colors [16], this paper adds the option to combine colors precisely for each state. Our proposed notation for logical states follows colors and orientations to represent all states. Hence, in a three dimensional space basis. The eq. 10 allows a direct mapping from state components into computer color. This serves to represent states into a computer application. For instance, a yellow state is the exact combination of the $|0\rangle$ red with $|+\rangle$ green. Likewise. the quantum state called magenta has a precise definition $|0\rangle+|i\rangle$ as table tbl. 2 lists most common states.

4.6 Consistency and Validation

We compared three different cases in the results section. The structured analysis of states and changes serve also compare if the cases are coherent consistent among them. Separating physical description from logical ones let us work with the quantum superposition. A disparity between different experiments should alert us of a miscalculation or a cause to investigate.

A traditional approach might get a physical observable ($|R\rangle$ or $|N\rangle$) and analyze its time phase component. Instead, this direct mapping to RGB allows detecting the phase in the color representation of a state. If the comparison and calculus are in discrepancy, we need to search for a cause of the inconsistency. For instance, if we expected a phase shift but do not see a blue tone in the diagram, then we would suspect a mistake. In these cases, the comparative approach serves as a validation tool.

5 Conclusions

Overall, we presented math and visual tools to display and compare experiments. We showed examples in different physical modes, all linked to a unique color sphere. Hence, when handling new modes, researchers could save time and effort by simply using these color sphere equations.

The resulting sphere and color-states maps not only well-known Pauli states, $|0\rangle$ $|1\rangle$ $|+\rangle$ $|-\rangle$ $|1+i\rangle$ $|1-i\rangle$ but also any complex superposition. The mapping from Hilbert to Color Sphere allows any combinations, as each intermediate color defines its quantum state exactly. Hence a qubit named magenta unequivocally represents the state as in tbl. 2.

These equations save time or hassle or tuning or re-designing a new Bloch-like sphere per case. This Hilbert to color-space mapping gives unambiguous definition of where each states sits in the sphere.

We linked plate operations on polarization with very different devices and fields. Thus, Cylinder lenses acting on orbital momentum matched the same color representation. Furthermore, the same diagrams fit optical path manipulation by splitters or fiber couplers. A unique diagram described all cases. A polarization change with a wave plate; vs. orbital momenta, vs. spatial paths. The abstraction and color convention of qubits applies to various degrees of freedom. We present the equations for color coding, and summarize the most common representations in a table. It associates with possible encoding: spin, path, temporal or orbital modes.

This showed how comparing one logical expression against different physical implementation helps validate consistency. The examples in this paper showed how the color sphere and change diagrams served to investigate intrinsic quantum behavior. Meaning of blue tone was discussed for its intuitive value, it worked despite what physical mode we used.

Researchers use sections of the sphere, and polar or equator walks intuitively. We propose similar section diagrams or constellations to work with quantum information and communications. These are adapted from the classical tools to represent coding symbols.

A value added by this approach is keeping the consistency check. The comparative method can save us time, or reassure us when our calculations are correct. By keeping track of how similar quantum states behave in other experiments.

This sphere can help present future results, by reducing the overhead needed to build new spheres or demonstrating a setup. We also discussed the color tones as intuitive help to design, build, and analyze quantum behaviors.

Furthermore, the sphere subject of this paper, led to build constellations and diagrams. Shown briefly to simplify this communication, more uses would be expanded in future publications.

As discussed, validating against common optical modes, we demonstrate how the proposed sphere is a suitable to represent many optical implementations, by using color and shape combinations.

This research continues with more details, and future papers can expand on the devices operations using this color-space examples. In other line of research, we aim to explore ways to visualise SU3 vs SO2 operations.

At this stage, we presented this work on the color space mapping, proposed sphere, HG, and few examples. We hope our colleagues can save time and effort by simply referring to these equations and color ket. We would re-build these equations and sphere openly if the community suggested different colors or shape conventions.

Supplementary information. This article is helped by few simulations, the data will be published in github or alternative repo before end 2021.

Acknowledgements The author wishes to acknowledge useful discussions with scientists P.Benitez, A. de Guzman, . This research was partially funded by the Hispatel-Ing' Grant 2018jun.

Declarations

5.1 Funding

Partially funded by the Hispatel-Ing' Grant 2018jun

5.2 Conflict of interest/Competing interests

Not applicable

5.3 Ethics approval

Not applicable

5.4 Consent to participate

We actively encourage research teams and institutions to participate, since we maintain several cooperation projects both within EU and internationally.

5.5 Consent for publication

Yes, currently pending review.

5.6 Availability of data and materials

Complementary data will be published in github or alternative repo before end 2021.

5.7 Code availability

Complementary code will be published in github or alternative repo before end 2021.

5.8 Authors' contributions

All authors contributed significantly to this work.

- [1] M. Motazedifard A., "Measurement of entropy and quantum coherence properties of two type-i entangled photonic qubits". *Opt Quant Electron* 53, 378 (2021). <https://doi.org/10.1007/s11082-021-03067-8>.
- [2] R. V. et al., "Measurement-device-independent quantum key distribution coexisting with classical communication," *Quantum Sci. Tec.*, v4, n4 (2019).
- [3] C. S. B Brecht DV Reddy, "Photon temporal modes: A complete framework for quantum information science," *Physical Review X* 5, 041017 (2015).
- [4] G. W. A Mair A Vaziri, "Entanglement of the orbital angular momentum states of photons". *Nature* 412, 313-316 (2001).
- [5] T. Xia C., "Vortex mode excitation in a multimode fiber by the laguerre-gaussian laser beams." *Opt Quant Electron* 51, 266 (2019). <https://doi.org/10.1007/s11082-019-1982-x>.
- [6] AF. A. KH. Kagalwala1 G DiGiuseppe, "Single-photon three-qubit quantum logic using spatial light modulators," *Nature Communications* 8(1) 2017.
- [7] G. Pruszyńska-Karbownik E., "Optical polarization shift in beams emitted by quantum cascade lasers". *Opt Quant Electron* 51, 325 (2019). <https://doi.org/10.1007/s11082-019-2043-1>.
- [8] M. A. V Ansari G Harder, "Temporal-mode measurement tomography of a quantum pulse gate," *Physical Review A*, 2017 – APS DOI: 10.1103/PhysRevA.96.063817.
- [9] F.-N. Tabeu S. B., "Imaginary resistor based parity-time symmetry electronics dimers." *Opt Quant Electron* 51, 335 (2019). <https://doi.org/10.1007/s11082-019-2048-9>, 2019.
- [10] P. S. Galvez EJ; Crawford, "Geometric phase associated with mode transformations of optical beams bearing orbital angular momentum," *PHYSICAL REVIEW LETTERS Vol90 Iss20 203901 may2003*.
- [11] M. L. Fickler R; Krenn, "Real-time imaging of quantum entanglement". In *Scientific Reports* 3, 1914 EP DOI: 10.1038/srep01914.
- [12] et al BS Ryvkin EA Avrutin, "Asymmetric-waveguide, short cavity designs with a bulk active layer for high pulsed power eye-safe spectral range laser diodes," *Semicond. Sci. Technol.* 35 085008.
- [13] S. Zhang S., "Entanglement performance of light through the composite free space channel". *Opt Quant Electron* 53, 81 (2021). <https://doi.org/10.1007/s11082-020-02706-w>.
- [14] G. J. ;. W. O'Brien J. L.; Pryde, "Demonstration of an all-optical quantum controlled-NOT gate". *Nature*. 426 (6964): 264–267. *arXiv:quant-ph/0403062*. *Bibcode:2003Natur.426..264O*. *doi:10.1038/nature02054*. *PMID 14628045*.
- [15] G. D. AF. Abouraddy TM. Yarnall, "Encoding arbitrary four-qubit states in the spatial parity of a photon pair," *Physical Review A* 85(6) June 2012 DOI: 10.1103/PhysRevA.85.062317.
- [16] N. D. Mermin, "Shedding (red and green) light on bell inequalities," *Physics Today*, April 1985, 38-47.
- [17] J. W. W Kiefer R Löw, "Na-faraday rotation filtering: The optimal point," *Scientific Reports volume 4, Article number: 6552 (2014)*.
- [18] I. M. JL Martínez J Alberro, "Analysis of optical polarization modulation systems through the pancharatnam connection," *Optics Communications* 285 (2012) 393–401 Elsevier.
- [19] Z. Zheng X., "Quantum key distribution with two-way authentication". *Opt Quant Electron* 53, 304 (2021). <https://doi.org/10.1007/s11082-021-02845-8>.

- [20] Y. L. Yi Xunong; Liu, "Hybrid-order poincare sphere," *PHYSICAL REVIEW A Volume: 91 Issue: 2 Article Number: 023801 Published: FEB 2 2015*.
- [21] D. A. N. Giovanni Milione H. I. Sztul and R. R. Alfano, "Higher-order poincaré sphere, stokes parameters, and the angular momentum of light," *Phys. Rev. Lett.* 107, 053601 – Published 25 July 2011.
- [22] M. A. V Ansari JM Donohue, "Tomography and purification of the temporal-mode structure of quantum light," *PHYSICAL REVIEW LETTERS* 120, 213601 (2018).
- [23] et al B. N. Simon S. Simon, "Nonquantum entanglement resolves a basic issue in polarization optics," *Phys. Rev. Lett.* 104, 023901 – Published 13 January 2010.
- [24] Y. C. Z Shao Jiangbo Zhu, "Spin-orbit interaction of light induced by transverse spin angular momentum engineering," *Nature Communications volume 9, Article number: 926 (2018)* <https://www.nature.com/articles/s41467-018-03237-5>.
- [25] U. Leonhardt, "Quantum physics of simple optical instruments," *Rept.Prog.Phys.* 66 (2003) 1207-1250 doi:10.1088/0034-4885/66/7/203 arXiv:quant-ph/0305007.
- [26] A. S. Sendy Phang Peter Bienstman, "A chaotic microresonator structure for an optical implementation of an artificial neural network," *Proceedings Volume 11283, Integrated Optics: Devices, Materials, and Technologies XXIV; 1128313 (2020)* <https://doi.org/10.1117/12.2548716>.
- [27] R. Kumar A., "Implementation of optical gray code converter and even parity checker using the electro-optic effect in the mach-zehnder interferometer." *Opt Quant Electron* 47, 2117–2140 (2015). <https://doi.org/10.1007/s11082-014-0087-9>, 2015.
- [28] H. D. H. Sun Q. Wang and N. K. Dutta, "XOR performance of a quantum dot semiconductor optical amplifier based mach-zehnder interferometer," *Optics Express Vol. 13, Issue 6, pp. 1892-1899 (2005)* <https://doi.org/10.1364/OPEX.13.001892>.
- [29] B. Badraoui N., "An experimental millimeter wave radio over fiber link with double polarization multiplexing". *Opt Quant Electron* 53, 533 (2021). <https://doi.org/10.1007/s11082-021-03178-2>.
- [30] T. Duarte F. J., "On the probability amplitude of quantum entanglement and the pauli matrices". *Opt Quant Electron* 52, 106 (2020). <https://doi.org/10.1007/s11082-020-2205-1>.
- [31] S. et al., "A three-dimensional color space from the 13th century," *J. Opt. Soc. Am. A / Vol. 29, No. 2 (2012)*.
- [32] S. R. O Bechler A Borne, "A passive photon–atom qubit swap operation," *Nature Physics*, 14(10) · 2018.
- [33] K. Kaur G., "WRELM based CO-OFDM system with improved performance, spectral efficiency and computational complexity". *Opt Quant Electron* 53, 373 (2021). <https://doi.org/10.1007/s11082-021-02996-8>, 2021.
- [34] M. Mandal P., "A bidirectional hybrid WDM-OFDM network for multiservice communication employing self-injection locked qdash laser source based on elimination of rayleigh backscattering noise technique," *Opt Quant Electron* 53, 263 (2021). <https://doi.org/10.1007/s11082-021-02948-2>.