DHHFL-MABAC Approach based on DHHFLDM-LCME for Multi-criteria Group Decision Making Involving Sewage Treatment Company Selection

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DHHFL-MABAC approach based on DHHFLDM-LCME for multi-criteria group decision making involving sewage treatment company selection

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Abstract

With the continuous acceleration of urban construction and development, ecological governance has become an important part of our green life. Therefore, how to select the appropriate governance company in the complex decision-making environment is very important. A double hierarchy hesitant fuzzy linguistic multi-attributive border approximation area method (DHHFL-MABAC) is proposed in this paper, which is based on distance measure and comprehensive weight. DHHFL-MABAC method not only considers the potential loss, but also has simple calculation and stable results. The double hierarchy hesitant fuzzy linguistic distance measure based on least common multiple (DHHFLDM-LCME) is proposed, which reduces the loss of original information and makes the result more accurate. Binary contrast method is proposed and combined with entropy weight to obtain comprehensive weight, which makes the determined weight more reasonable. Finally, this method is applied to the case selected by the sewage treatment company and proved its effectiveness.

Keywords: DHHFLDM-MABAC; double hierarchy hesitant fuzzy linguistic; DHHFLDM-LCME; binary contrast method; comprehensive weight.

1. Introduction

As the economy and society have progressed, the amount of decision-making information is increasing, and the uncertainty of decision-making environment and the complexity of problems are also increasing. The precise numerical and mathematical models cannot satisfy the requirement of decision-makers to describe the uncertainty and hesitation in actual decision-making. Base on this, in 1965, Zadeh first put forward the concept of fuzzy sets[1], which created a theoretical method for studying the problem of fuzziness or uncertainty. In 1975, Zadeh put forward the concept of linguistic variables[2]. Scholars conducted further research and proposed new forms of language

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expression. In 1967, Goguen proposed the L-fuzzy sets[3]. The interval valued fuzzy sets[4] was proposed by Tuksen in 1986,, Herrera put forward the concept of 2-tuple fuzzy sets[5] in 2000. However, the above expressions of fuzzy information only enrich and expand the value range of the membership function. They cannot broaden the dimensionality of information expression. In response to the above problems, scholars have made further research on it. The intuitionistic fuzzy sets[6] and interval intuitionistic fuzzy sets[7] were obtained by further extending the fuzzy sets, both of which take into account the information of the two dimensions of hesitation and membership, which makes the expression of fuzzy information more complete.

To better figure out the problem of decision-making in a more complex environment, Torra proposed the defination of hesitant fuzzy sets(HFS)[8] in 2010. It solves the problem that decision maker is hesitant between two or finite membership values. Based on this idea, some scholars have studied and extended the hesitant fuzzy sets[9, 10]. In a hesitant fuzzy linguistic environment, decision methods and models[11, 12, 13], which used to work out multi-attribute decision making(MADM), were proposed.

Soon, in order to increase the appropriacy of information expression in different decision-making situations, the double hierarchy linguistic term set(DHLTS) has been developed based on the linguistic term sets (LTS). Later, in view of the idea of HFS and DHLTS, Gou[14] put forward the concept of double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). When the decision-maker swings about different linguistic information, DHHFLTS can make the decision-maker express the evaluation more accurately. Gou[15] put forward the concept of distance and similarity measure between double hierarchy hesitant fuzzy linguistic elements(DHHFLEs) and DHHFLEs. DHHFLTS[16, 17, 18, 19, 20] was frequently applied to practical decision-making problems based on advantages of it. In MADM problems, the weight of attributes represents the relative importance of attributes. The right selection of decision-making procedures and the calculation of weight can assist decision-makers to make correct judgements and choices in a double hierarchy hesitant fuzzy linguistic (DHHFL) environment.

Therefore, determining the weight of attributes is important for ranking alternatives. Many scholars have made further research on weight calculation and decision-making methods. R. Krishankumar[21] proposed the VIKOR method based on the DHHFLTS, which ranked alternatives based on their proximity to the ideal solution. Xu and Zhang[22] put forward the method of DHHFL-BWM-DSET, which solved the MADM problem with unknown weights under the DHHFL environment. The COPRAS method[18] was proposed by R Krishankumar under the DHHFLTS, which distinguished and compares the priorities of objects from different perspectives. Liu[23] put forward the TODIM method of the DHHFLTS for multi-criteria group decision-making method. Wang[24] proposed ORESTE method based on the DHHFLTS.

According to the above literature analysis, we can find the following challenges:

(1) In DHHFLTS, the distance measure calculation of DHHFLEs of different lengths will have the problem of linguistic information loss.

(2) In the method of weight calculation, which cannot effectively express the importance of multiple attributes and decision maker's preference information.

(3) The existing decision-making methods are computationally complex and cannot sort alternatives efficiently and reasonably.

We are motivated by these challenges, and this paper has made some contributions:

(1) The double hierarchy hesitant fuzzy linguistic distance measure based on least common multiple(DHHFLDM-LCME) is proposed, which can reduce and overcome the loss of linguistic information.

(2) Binary contrast method is proposed, which can express the preference information of decision-makers well. On this basis, combined with entropy weight to obtain comprehensive weight.

(3) DHHFL-MABAC method based on DHHFLDM-LCME and comprehensive weight is proposed, the reasonability and effectiveness of this method are illustrated by the cases selected by the sewage treatment company.

The other sections of the paper are as below: In Section 2, the definitions of DHLTS and DHHFLTS are reviewed. In Section 3, the DHHFLDM-LCME is proposed. Section 4 proposes the DHHFL-MABAC method based on DHHFLDM-LCME and comprehensive weight. Section 5 introduces a case study on the selection of sewage treatment companies. Section 6 conducts a sensitivity analysis. In Section 7, a summary of this paper is given.

2. Preliminaries

In this part, we have made some preparations for the whole work.

2.1. Double hierarchy linguistic term set

During the decision-making process, to handle ambiguity and uncertainty, some scholars have proposed the HFLTS, which has made a breakthrough, and conducted in-depth research on decision-making and future directions [25, 26]. However, considering that the vocabulary we use to describe information in our daily life is becoming more and more abundant, Gou[14] put forward the concept of DHLTS and DHHFLTS, they are able to articulate more detailed linguistic information, which corresponds to people's perceptions. The following is the concept of DHLTS.

Definition 1. [14]Let $S = \{s_t | t = -\tau, \dots, -1, 0, 1, \dots, \tau\}$ be the first hierarchy LTS and $O = \{o_k | k = -\varsigma, \dots, -1, 0, -1, \dots, \varsigma\}$ be the second hierarchy LTS, which are completely independent of each other. The DHLTS S_O has the following mathematical expression::

$$S_O = \{s_{t < o_k} | t = -\tau, \dots, -1, 0, 1, \dots, \tau; k = -\varsigma, \dots, -1, 0, -1, \dots, \varsigma\} \quad (1)$$

When the first hierarchy linguistic term is s_t , o_k denotes the second hierarchy linguistic term. We refer $s_{t<o_k>}$ as the double hierarchy linguistic term (DHLT),

Remark 1. There are four cases according to different values of t [19]:

If $t < 0$, the first hierarchy LTS $S = \{s_t | t < 0\}$ has a negative connotation, and the second hierarchy LTS must be picked in decreasing order.

If $t \geq 0$, the first hierarchy LTS $S = \{s_t | t \geq 0\}$ has a non-negative connotation, and the second hierarchy LTS must be picked in ascending order.

if $t = -\tau$, then we only care about the first half of the second hierarchy of LTS, $O = \{o_k | k = 0, 1, \dots, \varsigma\}$.

if $t = \tau$, then we only care about the second hierarchy of LTS, $O = \{o_k | k = -\tau, \dots, -1, 0\}$.

2.2. Double hierarchy hesitant fuzzy linguistic term set

During the decision-making process, there are complex linguistic terms that cannot be clearly articulated in a single linguistic term, for example “*between a little high and very high*”. Therefore, Gou[14] defined the concept of DHHFLTS based on DHLTS. DHHFLTS provide decision makers a richer and more flexible environment to express complex linguistic information. The following are some fundamental DHHFLTS informations.

Definition 2. [14] Let X be a fixed set, S_O be a double hierarchy LTS, A double hierarchy hesitant fuzzy linguistic term set on X , H_{S_o} is expressed in terms of a membership function returning a subset of S_o when applied to X , and is expressed mathematically:

$$H_{s_o} = \{ \langle x_i, h_{s_o}(x_i) \rangle | x_i \in X \} \quad (2)$$

In which $h_{s_o}(x_i)$ is a set of values from S_o that represents the possible degree of membership of element $x_i \in X$ to the set H_{s_o} is:

$$h_{s_o}(x_i) = \{ s_{\phi_l < o_{\varphi_l}}(x_i) | s_{\phi_l < o_{\varphi_l}} \in S_o; l = 1, 2, 3 \dots, L; \phi_l = -\tau, \dots, -1, 0, 1, \dots, \tau; \varphi_l = -\varsigma, \dots, -1, 0, 1, \dots, \varsigma \} \quad (3)$$

with L being the number of DHLTs in $h_{s_o}(x_i)$ and $s_{\phi_l < o_{\varphi_l}}(l = 1, 2, 3, \dots, L)$ in each $h_{s_o}(x_i)$ being the continuous terms in S_o . $h_{s_o}(x_i)$ is called the double hierarchy hesitant fuzzy linguistic element(DHHFLE), which stands for the possible degree of the linguistic variable x_i to S_o . Next, in order to better understand DHHFLTS, we give an example.

Example 1. Suppose $S = \{s_{-2} = \text{null}, s_{-1} = \text{small}, s_0 = \text{medium}, s_1 = \text{big}, s_2 = \text{huge}\}$ and $O = \{o_{-2} = \text{slightly}, o_{-1} = \text{a little}, o_0 = \text{just right}, o_1 = \text{very}, o_2 = \text{entirely}\}$ are the first hierarchy and the second hierarchy LTSs, when we describe some complicated linguistic terms, for example, “just right medium” and “between a little small and entirely big”, we can use DHHFLEs $\{s_{0<o_0}>\}$ and $\{s_{0<o_0>}, s_{1<o_3}>\}$.

Definition 3. [15] Let S_O be a DHLTS and $h_{s_o}(x_i) = \{s_{\phi_l < o_{\varphi_l}}(x_i) | s_{\phi_l < o_{\varphi_l}} \in S_o; l = 1, 2, 3 \dots, L; l = 1, 2, \dots, \#h_{s_o}\}$ ($\#h_{s_o}$ represents the number of h_{s_o} in DHLTS) be a DHHFLE, after that we can define:

$$v(h_{s_o}(x_i)) = 1 - \frac{1}{\#h_{s_o}} \quad (4)$$

the hesitant degree of the DHHFLE $h_{s_o}(x_i)$. The degree of hesitation is a very important characteristic of DHHFLE, which reflects the degree of hesitation of the decision-maker. The quality of DHHFLE will become lower as the degree of hesitation increases.

In order to simplify the operation of DHHFLT, two equivalent conversion functions are developed to convert DHFLT to fuzzy numbers, as shown below:

Definition 4. [14] Let $\overline{S_o} = \{s_{t < o_k} | t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ be a virtual DHLTS, S_O be a DHLTS, $h_{s_o}(x_i)$ be a DHHFLE with L being the number of linguistic terms in h_{s_o} , and $h_\gamma = \{\gamma_l | \gamma_l \in [0, 1]; l = 1, 2, 3, \dots, L\}$ be a set of numerical scales. Then the numerical scale γ_l and the subscript (ϕ_l, φ_l) of the DHLT $s_{\phi_l < o_{\varphi_l}}$ that express the equivalent information to the numerical scale γ_l can be transformed to each other by two function f and f^{-1} :

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \rightarrow [0, 1], f(\phi_l, \varphi_l) = \frac{\varphi_l + \varsigma(\tau + \phi_l)}{2\tau\varsigma} = \gamma_l \quad (5)$$

$$\begin{aligned} f^{-1} : [0, 1] &\rightarrow [-\tau, \tau] \times [-\varsigma, \varsigma], \\ f^{-1}(\gamma_l) &= [2\tau\gamma_l - \tau] < o_{\varsigma(2\tau\gamma_l - \tau - [2\tau\gamma_l - \tau])} > = [2\tau\gamma_l - \tau] + 1 < o_{\varsigma((2\tau\gamma_l - \tau - [2\tau\gamma_l - \tau]) - 1)} > \end{aligned} \quad (6)$$

Let $\Phi \times \Psi$ as the set of all DHHFLEs on $\overline{S_o}$, Ω be the set of all numerical scales. Then the conversion function F and F^{-1} between h_{s_o} and h_γ based on f and f^{-1} are:

$$\begin{aligned} F : \Phi \times \Psi &\rightarrow \Omega, \\ F(h_{s_o}) &= F(\{s_{\phi_l < o_{\varphi_l}} | s_{\phi_l < o_{\varphi_l}} \in \overline{S_o}; l = 1, 2, \dots, L; \phi_l \in [-\tau, \tau]; \varphi_l \in [-\varsigma, \varsigma]\}) \\ &= \{\gamma_l | \gamma_l = f(\phi_l, \varphi_l); l = 1, 2, \dots, L\} = h_\gamma \end{aligned} \quad (7)$$

$$\begin{aligned} F^{-1} : \Omega &\rightarrow \Phi \times \Psi, \\ F^{-1}(h_\gamma) &= F^{-1}(\{\gamma_l | \gamma_l \in [0, 1]; l = 1, 2, \dots, L\}) = \{s_{\phi_l < o_{\varphi_l}} | \phi_l < o_{\varphi_l} > = f^{-1}(\gamma_l)\} = h_{s_o} \end{aligned} \quad (8)$$

Definition 5. [14] Let S_O be a DHLTS, h_{s_o} be a DHHFLE, L is the length of h_{s_o} . The following is the definition of the expected value of h_{s_o} .

$$E(h_{s_o}) = \frac{1}{L} \sum_{l=1}^L F(s_{\phi_l < o_{\varphi_l}}) \quad (9)$$

In different DHHFLT, the quantity of linguistic terms may be different. Therefore, it is necessary to standardize the DHHFLT, so that they contain the same quantity of linguistic terms.

To ensure the integrity of the DHHFLTS information before and after expansion, this paper gives the double hierarchy hesitant fuzzy linguistic least common multiple expansion method, which is as follows:

Definition 6. Let $h_{s_o} = \{s_{\phi_0 < o_{\varphi_0}} >, s_{\phi_1 < o_{\varphi_2}} >, \dots, s_{\phi_i < o_{\varphi_j}} >\}$ be a DHHFLE with L linguistic term on S_o , Namely, $h_{s_o} \in DHHFLTS(S_o)$. Then H_{s_o} is the set after expansion with h_{s_o} , which is defined as follows:

$$H_{s_o} = \underbrace{\{s_{\phi_0 < o_{\varphi_0}} >, s_{\phi_0 < o_{\varphi_0}} >, s_{\phi_0 < o_{\varphi_0}} >\}}_{t \text{ times}} \underbrace{\{s_{\phi_1 < o_{\varphi_2}} >, s_{\phi_1 < o_{\varphi_2}} >, s_{\phi_1 < o_{\varphi_2}} >\}}_{t \text{ times}} \dots \underbrace{\{s_{\phi_i < o_{\varphi_j}} >, s_{\phi_i < o_{\varphi_j}} >, s_{\phi_i < o_{\varphi_j}} >\}}_{t \text{ times}} \quad (10)$$

The t represents the multiplicity of each element s_k of h_{s_o} in H_{s_o} .

3. The double hierarchy hesitant fuzzy linguistic distance measure based on least common multiple

The existing literature defines a lot of distance measure methods, but most of these distance measure assume that the length of the DHHFLTSs is the same, or use the α criterion and β criterion to subjectively normalize the DHHFLEs of different lengths. However, these distance measures have certain shortcomings:

(1) In actual decision-making problems, few different decision makers give DHHFLTS of the same length.

(2) α and β criteria will destroy hesitation the completeness of the information before and after the expansion of the fuzzy linguistic term set.

Hence based on the idea of normalization method in literature[28], DHHFLDM-LCME is proposed in this paper.

Definition 7. Let $h_{s_o}^1$ and $h_{s_o}^2$ are two DHHFLEs of length L_1 and L_2 respectively. The following is a mathematical representation of DHHFLDM-LCME.

$$d_E(h_{s_o}^1, h_{s_o}^2) = \frac{1}{LCM} \sum_{k=1}^{LCM} |F(H_k^1) - F(H_k^2)| \quad (11)$$

In which LCM is the least common multiple of L_1 and L_2 , and F is a monotone function, H_k is the k -th element in the H_{s_o} set after h_{s_o} expanded.

Theorem Let $(h_{s_o}^1, h_{s_o}^2)$ are two DHHFLEs, then:

- (1) Boundary: $0 \leq d_E(h_{s_o}^1, h_{s_o}^2) \leq 1$;
- (2) Symmetry: $d_E(h_{s_o}^1, h_{s_o}^2) = d_E(h_{s_o}^2, h_{s_o}^1)$;
- (3) Reflexivity: $d_E(h_{s_o}^1, h_{s_o}^1) = 0$;
- (4) Inequality: $d_E(h_{s_o}^1, h_{s_o}^2) + d_E(h_{s_o}^2, h_{s_o}^3) \geq d_E(h_{s_o}^1, h_{s_o}^3)$.

Where (1), (2) and (3) are obvious, we will not prove it here. The proof of Theorem (4) will be given below:

Proof:

We will classify and discuss from two major aspects: when the lengths of the DHHFLEs are the same, we can get the following situations:

if $h_{s_o}^1 = h_{s_o}^2 = h_{s_o}^3$, according to Theorem (3), there are:

$$d_E(h_{s_o}^1, h_{s_o}^2) = d_E(h_{s_o}^2, h_{s_o}^3) = d_E(h_{s_o}^1, h_{s_o}^3) = 0, d_E(h_{s_o}^1, h_{s_o}^2) + d_E(h_{s_o}^2, h_{s_o}^3) \geq d_E(h_{s_o}^1, h_{s_o}^3)$$

Apparently established;

if $h_{s_o}^1 = h_{s_o}^2 \neq h_{s_o}^3$, according to Theorem (3), there are:

$$d_E(h_{s_o}^1, h_{s_o}^2) + d_E(h_{s_o}^2, h_{s_o}^3) = d_E(h_{s_o}^2, h_{s_o}^3), d_E(h_{s_o}^2, h_{s_o}^3) = d_E(h_{s_o}^1, h_{s_o}^3),$$

$$d_E(h_{s_o}^1, h_{s_o}^2) + d_E(h_{s_o}^2, h_{s_o}^3) = d_E(h_{s_o}^1, h_{s_o}^3).$$

In the same way, it can also be proved under the two conditions of $h_{s_o}^1 \neq h_{s_o}^2 = h_{s_o}^3$ and $h_{s_o}^1 = h_{s_o}^3 \neq h_{s_o}^2$.

When the lengths of the DHHFLEs are different, we set the lengths of $h_{s_o}^1$, $h_{s_o}^2$, and $h_{s_o}^3$ to be L_1 , L_2 , and L_3 respectively, then a , b and c are the least common multiples between them respectively, and the least common multiple of the three of them is n , according to Definition 7, there are:

$$d_E(h_{s_o}^1, h_{s_o}^2) = \frac{1}{a} \sum_{k=1}^a |F(H_k^1) - F(H_k^2)|$$

$$d_E(h_{s_o}^2, h_{s_o}^3) = \frac{1}{b} \sum_{k=1}^b |F(H_k^2) - F(H_k^3)|$$

$$d_E(h_{s_o}^1, h_{s_o}^3) = \frac{1}{c} \sum_{k=1}^c |F(H_k^1) - F(H_k^3)|$$

$d_E(h_{s_o}^1, h_{s_o}^2) = \frac{1}{a} \sum_{k=1}^a |F(H_k^1) - F(H_k^2)| = \frac{1}{ak_1} k_1 \sum_{l=1}^a |F(H_k^1) - F(H_k^2)|$ which k_1 , k_2 , k_3 are constants respectively, make $ak_1 = n$, $ak_2 = n$, $ak_3 = n$. Thoughts based on the principle of least common multiple, according to Definition 6, there are:

$$\frac{1}{ak_1} k_1 \sum_{l=1}^a |F(H_k^1) - F(H_k^2)| = \frac{1}{ak_1} \sum_{l=1}^{ak_1} |F(H_k^1) - F(H_k^2)| = \frac{1}{n} \sum_{l=1}^n |F(H_k^1) - F(H_k^2)|$$

In the same way, there are:

$$d_E(h_{s_o}^2, h_{s_o}^3) = \frac{1}{n} \sum_{k=1}^n |F(H_k^2) - F(H_k^3)|$$

$$d_E(h_{s_o}^1, h_{s_o}^3) = \frac{1}{n} \sum_{k=1}^n |F(H_k^1) - F(H_k^3)|$$

If we want to prove $d_E(h_{s_o}^1, h_{s_o}^2) + d_E(h_{s_o}^2, h_{s_o}^3) \geq d_E(h_{s_o}^1, h_{s_o}^3)$, we only need to prove:

$$\frac{1}{n} \sum_{k=1}^n |F(H_k^1) - F(H_k^2)| + \frac{1}{n} \sum_{k=1}^n |F(H_k^2) - F(H_k^3)| \geq \frac{1}{n} \sum_{k=1}^n |F(H_k^1) - F(H_k^3)|, \text{ and that,}$$

$$\frac{1}{n} \sum_{k=1}^n (|F(H_k^1) - F(H_k^2)| + |F(H_k^2) - F(H_k^3)| - |F(H_k^1) - F(H_k^3)|) \geq 0$$

we can get the following six situations:

if $F(H_k^1) > F(H_k^2) > F(H_k^3)$, there are:

$$\frac{1}{n} \sum_{k=1}^n (F(H_k^1) - F(H_k^2) + F(H_k^2) - F(H_k^3) - (F(H_k^1) - F(H_k^3)))$$

$$= \frac{1}{n} \sum_{k=1}^n (F(H_k^1) - F(H_k^3) - F(H_k^1) + F(H_k^3)) = 0$$

if $F(H_k^1) > F(H_k^3) > F(H_k^2)$, there are:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^n (F(H_k^1) - F(H_k^2) + F(H_k^3) - F(H_k^2) - (F(H_k^1) - F(H_k^3))) \\ &= \frac{1}{n} \sum_{k=1}^n (2F(H_k^3) - 2F(H_k^2)) > 0 \end{aligned}$$

if $F(H_k^2) > F(H_k^1) > F(H_k^3)$, there are:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^n (F(H_k^2) - F(H_k^1) + F(H_k^2) - F(H_k^3) - (F(H_k^1) - F(H_k^3))) \\ &= \frac{1}{n} \sum_{k=1}^n (2F(H_k^2) - 2F(H_k^1)) > 0 \end{aligned}$$

if $F(H_k^2) > F(H_k^3) > F(H_k^1)$, there are:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^n (F(H_k^2) - F(H_k^1) + F(H_k^2) - F(H_k^3) - (F(H_k^3) - F(H_k^1))) \\ &= \frac{1}{n} \sum_{k=1}^n (2F(H_k^2) - 2F(H_k^3)) > 0 \end{aligned}$$

if $F(H_k^3) > F(H_k^1) > F(H_k^2)$, there are:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^n (F(H_k^1) - F(H_k^2) + F(H_k^3) - F(H_k^2) - (F(H_k^3) - F(H_k^1))) \\ &= \frac{1}{n} \sum_{k=1}^n (2F(H_k^1) - 2F(H_k^2)) > 0 \end{aligned}$$

if $F(H_k^3) > F(H_k^2) > F(H_k^1)$, there are:

$$\begin{aligned} & \frac{1}{n} \sum_{k=1}^n (F(H_k^2) - F(H_k^1) + F(H_k^3) - F(H_k^2) - (F(H_k^3) - F(H_k^1))) \\ &= 0 \end{aligned}$$

In summary, the proof of Theorem.(4) is complete.

Example 2. Let $S_o = \{s_{-2} = \text{none}, s_{-1} = \text{small}, s_0 = \text{medium}, s_1 = \text{big}, s_2 = \text{huge}; o_{-2} = \text{slightly}, o_{-1} = \text{a little}, o_0 = \text{just right}, o_1 = \text{very}, o_2 = \text{entirely}\}$ is a DHHFLTS, $h_{s_o}^1 = \{s_{-1 < o_1 >}, s_{0 < o_1 >}, s_{1 < o_2 >}\}$ and $h_{s_o}^2 = \{s_{1 < o_1 >}, s_{2 < o_0 >}\}$ are two DHHFLEs, Further, based on the least common multiple expansion rules and standardized treatment are:

$$H_{s_o}^1 = \{s_{-1 < o_1 >}, s_{-1 < o_1 >}, s_{0 < o_1 >}, s_{0 < o_1 >}, s_{1 < o_2 >}, s_{1 < o_2 >}\},$$

$$H_{s_o}^2 = \{s_{1 < o_{-1} >}, s_{1 < o_{-1} >}, s_{1 < o_{-1} >}, s_{1 < o_1 >}, s_{1 < o_1 >}, s_{1 < o_1 >}\}.$$

Then we can use the above transformation equation(7) to get the new set:

$$F(H_{s_o}^1) = \{0.375, 0.375, 0.625, 0.625, 1, 1\}.$$

$$F(H_{s_o}^2) = \{0.625, 0.625, 0.625, 0.875, 0.875, 0.875\}.$$

And then according to the Eq.(11), we can figure out the distance between $h_{s_o}^1$ and $h_{s_o}^2$:

$$d_E(h_{s_o}^1, h_{s_o}^2) = \frac{1}{LCM} \sum_{k=1}^{LCM} |F(H_k^1) - F(H_k^2)| = 0.167.$$

4. The DHHFL-MABAC method based on DHHFLDM-LCME and comprehensive weight

Since DHHFLDM-LCME and comprehensive weight are prepared for DHHFL-MABAC method, so we first introduce the comprehensive weight based on binary contrast method and entropy weight, and then introduce the calculation steps of DHHFL-MABAC method in detail.

The comprehensive weighted method combines the entropy weight method[29][30] for calculating objective attribute weights with the binary contrast method for calculating subjective attribute

weight. It effectively overcomes the shortcomings of subjective attribute weight and objective attribute weight.

4.1. Binary contrast method

The described binary contrast method uses the empirical knowledge of decision makers to determine the subjective weight of attributes. Suppose the attribute set is $E = \{e_1, e_2, \dots, e_n\}$.

Step 1. Compare the superiority of adjacent attributes according to the constraints of people's life experience and environmental factors. If the first one has a higher degree of superiority than the second one, swap the two of them.

Step 2. Do the same work for each pair of adjacent attributes, from the first pair at the beginning to the last pair at the end. At this point, the last attribute should be the attribute with the highest degree of superiority.

Step 3. Repeat the above steps for all attributes except the last one.

Step 4. Repeat the above steps for fewer and fewer attributes until there is no pair of attributes to compare. We can get an ascending set of attributes sorted by superiority.

Step 5. According to the sorted set of attributes and weighting function, we can get the subjective weight of each attribute, $w_j^1 = \{w_1^1, w_2^1, \dots, w_n^1\}$.

$$w_i^1 = \frac{1}{2^{i-1}} \times \frac{2^{n-1}}{2^n - 1}, i = 1, 2, \dots, n. \quad (12)$$

Where n is the total quantity of attributes, $\sum_{i=1}^n w_i^1 = 1, 0 \leq w_i^1 \leq 1$.

4.2. Entropy weight based on DHHFLTS

The entropy weight method[30] calculates the weight of each attribute, and then evaluates each attribute based on its weight. In the DHHFL environment, the following steps are taken to calculate the weight through the entropy weight method.

Step 1. Obtain the decision information which take the form of DHHFLTS, and construct the evaluation matrix R .

$$R = \begin{matrix} & \begin{matrix} e_1 & e_2 & \cdots & e_n \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{matrix} & \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{1m} & r_{2m} & \cdots & r_{mn} \end{bmatrix} \end{matrix} \quad (13)$$

Step 2. Normalize evaluation matrix R into R^1 by Eq.(14).

$$r_{ij}^1 = \frac{r_{ij} - \min(r_j)}{\max(r_j) - \min(r_j)}, \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n. \quad (14)$$

$$R^1 = \begin{bmatrix} r_{11}^1 & r_{12}^1 & \cdots & r_{1n}^1 \\ r_{21}^1 & r_{22}^1 & \cdots & r_{2n}^1 \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^1 & r_{m2}^1 & \cdots & r_{mn}^1 \end{bmatrix} \quad (15)$$

In which r_{ij}^1 stands for the evaluation value of the j -th attribute under the i -th alternative after standardization in the decision matrix, the $\max(r_j)$ and $\min(r_j)$ are the maximum and minimum values of the j -th attribute, respectively.

Step 3. According to the following formula, we can get the information entropy of each attribute.

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^m P_{ij} \ln P_{ij}, \quad j = 1, 2, \dots, n; i = 1, 2, \dots, m. \quad (16)$$

$$P_{ij} = \frac{r_{ij}^1}{\sum_{i=1}^m r_{ij}^1} \quad (17)$$

Step 4. Each attribute's weight value is calculated using formula as follows.

$$w_j^2 = \frac{1 - E_j}{\sum_{j=1}^n (1 - E_j)} \quad (18)$$

4.3. The comprehensive weight

To compensate for the shortcomings of subjective and objective weights, the decision maker's preference for attributes and the importance of attributes themselves are considered, we combine binary contrast method and entropy weight to obtain comprehensive weights, so as to make the decision results true and reliable.

Suppose the subjective attribute weight obtained by the binary contrast method is $w_j^1 = \{w_1^1, w_2^1, \dots, w_n^1\}$, where $\sum_{i=1}^n w_i^1 = 1, 0 \leq w_i^1 \leq 1$, according to the entropy weight method, the objective attribute weight is $w_j^2 = \{w_1^2, w_2^2, \dots, w_n^2\}$, where $\sum_{i=1}^n w_i^2 = 1, 0 \leq w_i^2 \leq 1$. According to the calculated w_j^1 and w_j^2 , we can get the comprehensive weight of attributes:

$$w_j = \lambda w_j^1 + (1 - \lambda) w_j^2, j = 1, 2, \dots, n. \quad (19)$$

Where λ is a parameter, $\sum_{i=1}^j w_i = 1, 0 \leq w_i \leq 1$. When the decision maker has a preference for subjective weight and objective weight, λ changes with the decision maker's preference information.

The comprehensive weight is obtained by the linear combination of objective weight and subjective weight. According to additive effect, the value of comprehensive weight is positively correlated with the value of subjective and objective weight. Moreover, the comprehensive weights overcome the limitation of considering only the influence of subjective or objective factors.

4.4. The DHHFL-MABAC method based on DHHFLDM-LCME and comprehensive weight

MABAC method[32] is a MADM method. The operation method is simple and practical. The MABAC method's[31] basic idea is that to computer the border approximation area (BAA) of each attribute first. Then the distance between the BAA and each attribute is calculated. Finally, calculate the global distance of each alternative.

The classic MABAC method[33, 34] ranks the alternatives by calculating the global distance. When calculating the distance, there will be a problem of information loss, resulting in inaccurate sorting results. In complex decision-making problems, reasonable attribute weights are not considered. Based on this, this paper put forward a DHHFL-MABAC method based on DHHFLDM-LCME and comprehensive weight to solve the MADM problem.

It is assumed that the alternative set is $C = \{c_1, c_2, \dots, c_n\}$, the attribute set is $E = \{e_1, e_2, \dots, e_n\}$, suppose S_O and $h_{s_o}(x_i)$ are DHLTS and DHHFLE, respectively, use them to represent the j -th attribute value of the i -th alternative. The specific procedure of DHHFL-MABAC approach are as follows:

Step 1. Select important attribute indicators to construct a decision matrix R by the DHHFLTS, as shown in the following table.

$$R = \begin{matrix} & e_1 & e_2 & \cdots & e_n \\ \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{mn} \end{bmatrix} \end{matrix} \quad (20)$$

Next, we use Eq.(9) to get the numeric matrix $\bar{R}$.

$$\bar{R} = \begin{matrix} & e_1 & e_2 & \cdots & e_n \\ \begin{matrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{matrix} & \begin{bmatrix} a_{11}^1 & a_{12}^1 & \cdots & a_{1n}^1 \\ a_{21}^1 & a_{22}^1 & \cdots & a_{2n}^1 \\ \vdots & \vdots & \vdots & \vdots \\ a_{1m}^1 & a_{2m}^1 & \cdots & a_{mn}^1 \end{bmatrix} \end{matrix} \quad (21)$$

Step 2. Through the DHHFLDM-LCME and the Eq.(22), the initial matrix(R) are normalized.

$$h_{ij} = \frac{d_E(a_{ij}, a_i^-)}{d_E(a_i^+, a_i^-)}, i = 1, 2, \dots, m; j = 1, 2, \dots, n. \quad (22)$$

Where h_{ij} denotes the evaluation value of the i -th alternative under the j -th attribute after standardization in decision matrix(R), a_i^+ represents the DHHFLE corresponding to the maximum value of the i -th alternative under the j -th attribute in the numerical matrix $\bar{R}$, a_i^- represent the

DHHFLE correspond with the minimum value of the i -th alternative under the j -th attribute in the numerical matrix $\bar{R}$.

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \quad (23)$$

Step 3. The comprehensive weight w_j of attributes is obtained by binary contrast method and entropy weight. $j = 1, 2, \dots, n$.

$$w_j = \lambda w_j^1 + (1 - \lambda) \frac{1 - E_j}{\sum_{j=1}^n (1 - E_j)} \quad (24)$$

Construct the weighted standardized decision matrix ($\bar{H}$) using the following formula, w_j represents the weight of attribute j .

$$h_{ij}^* = w_j(1 + h_{ij}) \quad (25)$$

$$\bar{H} = \begin{bmatrix} h_{11}^* & h_{12}^* & \cdots & h_{1n}^* \\ h_{21}^* & h_{22}^* & \cdots & h_{2n}^* \\ \vdots & \vdots & \cdots & \vdots \\ h_{m1}^* & h_{m2}^* & \cdots & h_{mn}^* \end{bmatrix} \quad (26)$$

Step 4. Determine the BAA for each attribute by the Eq.(27), and then we can get the BAA matrix (G).

$$g_j = \left(\prod_{i=1}^m h_{ij}^* \right)^{\frac{1}{m}} \quad (27)$$

In which h_{ij}^* denotes the elements of weighted standardized decision matrix ($\bar{H}$), m stands for the total quantity of alternatives.

Step 5. Using the Eq.(28) to calculate the distance to the BAA . The distance(D) between the alternative and the BAA is determined as the difference between the element in the weighted matrix ($\bar{H}$) and the value of the BAA (G).

$$D = \bar{H} - G \quad (28)$$

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix} = \begin{bmatrix} h_{11}^* & h_{12}^* & \cdots & h_{1n}^* \\ h_{21}^* & h_{22}^* & \cdots & h_{2n}^* \\ \vdots & \vdots & \cdots & \vdots \\ h_{m1}^* & h_{m2}^* & \cdots & h_{mn}^* \end{bmatrix} - \begin{bmatrix} g_1 & g_2 & \cdots & g_n \\ g_1 & g_2 & \cdots & g_n \\ \vdots & \vdots & \cdots & \vdots \\ g_1 & g_2 & \cdots & g_n \end{bmatrix} \quad (29)$$

where h_{ij}^* denotes the weighted matrix of the elements $(\overline{H})$, g_n represents the border approximation area for attribute, i stand for the quantity of alternatives, the quantity of attributes is j . In particular, the alternative is closely related to the lower approximation area (LAA), the BAA and the upper approximation area (UAA).

According to the different values of d_{ij} , we can get the following situations: The alternatives belong to the $(LAA)G^-$ when $d_{ij} < 0$. The alternatives belong to the $(BAA)G$ when $d_{ij} = 0$. The alternative is part of the $UAA(G^+)$ when $d_{ij} > 0$.

Evidently, the best alternatives must be in the UAA and the worst alternatives must be in the LAA .

Step 6. According to the Eq.(30), the overall distance from each alternative to the BAA is calculated, and then sort the alternative according to the value of the distance.

$$D_i = \sum_{j=1}^n q_{ij}, \quad i = 1, 2, \dots, m. \quad (30)$$

Figure 1: The MADM method of DHHFL-MABAC.

5. A case study: sewage treatment company selection

5.1. Background description

Due to the continuous acceleration of urban construction and development, sewage treatment problem in urban environmental protection has become more and more serious, and the discharge of urban sewage has been increasing, which has caused great harm to water resources and the environment. Therefore, a certain government department aims to solve the problem of sewage treatment. Intends to choose the best one from the four sewage treatment companies for sewage treatment. The evaluated companies include c_1, c_2, c_3, c_4 . The evaluation attributes include four aspects: e_1 (Achieved water quality standards), e_2 (Invest funds), e_3 (Time required), e_4 (Sewage treatment equipment). The weight is completely unknown.

Let $S = \{s_{-3} = \text{terrible}, s_{-2} = \text{very bad}, s_{-1} = \text{bad}, s_0 = \text{medium}, s_1 = \text{good}, s_2 = \text{very good}, s_3 = \text{perfect}\}$ be an LTS for evaluating property performance of the attributes and $O =$

$\{o_{-3} = \text{far from}, o_{-2} = \text{little}, o_{-1} = \text{a little}, o_0 = \text{ordinary}, o_1 = \text{much}, o_2 = \text{very much}, o_3 = \text{entirely}\}$ be the second hierarchy linguistic term. In this case, the expert evaluated four sewage treatment companies based on four attributes. According to the language expression selected by the expert, the original assessment information can be converted into DHHFLEs.

5.2. Attribute weight calculation process

First, we need to figure out the weight of each attribute to prepare for the selection of candidate companies by the DHHFL-MABAC method. The following is the process of weight calculation.

Step 1. Calculate subjective weights based on the binary contrast method, we prioritize the attributes of sewage treatment companies by binary contrast method, the weight of each attribute is obtained. We can get the priority of the attributes: $e_1 \succ e_3 \succ e_2 \succ e_4$.

Table 1: The subjective weights matrix

	e_1	e_2	e_3	e_4
w_j^1	0.533	0.133	0.267	0.067

Step 2. Calculate objective weight based on the entropy weight.

First, we can get the normalized matrix from the evaluation matrix(R) in Table 6.

Table 2: The normalized matrix

	e_1	e_2	e_3	e_4
c_1	0.377	0.603	0.000	1.000
c_2	1.000	0.053	0.737	0.000
c_3	1.000	0.497	0.000	1.000
c_4	0.335	1.000	0.000	1.000

Next, we can get the information entropy of each attribute.

Table 3: The information entropy matrix

	e_1	e_2	e_3	e_4
E_j^2	0.186	0.257	0.000	0.792

Finally, we can calculate the each attribute's weight through information entropy.

Table 4: The objective weights matrix

	e_1	e_2	e_3	e_4
w_j^2	0.294	0.268	0.362	0.076

Step 3. Calculate the comprehensive weight of the attribute, in this case, $\lambda = 0.5$.

Table 5: The comprehensive weights matrix

	e_1	e_2	e_3	e_4
w_j	0.414	0.201	0.315	0.720

5.3. Process based on DHHFL-MABAC method

In order to prioritize sewage treatment companies and make reasonable selections, next, the flow of the method in this article is introduced.

Step 1. Construct the evaluation matrix R in the form of DHHFLEs.

Table 6: The evaluation matrix R

	e_1	e_2	e_3	e_4
c_1	$\{s_{-1<o_2}>\}$	$\{s_{0<o_{-1}}>\}$	$\{s_{-3<o_0}>, s_{-2<o_1}>, s_{-1<o_{-2}}>\}$	$\{s_{1<o_{-2}}>, s_{1<o_1}>\}$
c_2	$\{s_{1<o_0}>, s_{2<o_0}>, s_{3<o_0}>\}$	$\{s_{-1<o_0}>\}$	$\{s_{1<o_0}>, s_{2<o_{-2}}>\}$	$\{s_{-1<o_{-2}}>, s_{0<o_{-2}}>\}$
c_3	$\{s_{0<o_1}>\}$	$\{s_{-1<o_0}>, s_{0<o_2}>\}$	$\{s_{0<o_{-2}}>\}$	$\{s_{0<o_{-2}}>, s_{1<o_{-2}}>, s_{2<o_{-2}}>\}$
c_4	$\{s_{0<o_{-3}}>, s_{1<o_{-2}}>\}$	$\{s_{1<o_{-2}}>\}$	$\{s_{-1<o_0}>, s_{0<o_{-1}}>\}$	$\{s_{1<o_{-2}}>\}$

Next, we use Eq.(9) to get a numeric matrix $\bar{R}$.

Table 7: The numeric matrix $\bar{R}$

	e_1	e_2	e_3	e_4
c_1	0.444	0.444	0.222	0.639
c_2	0.833	0.333	0.694	0.305
c_3	0.556	0.472	0.389	0.556
c_4	0.445	0.556	0.389	0.556

Step 2. Standardize the elements of the initial matrix(R), and the normalized matrix (H) is obtained.

Table 8: The normalized matrix H

	e_1	e_2	e_3	e_4
c_1	0.000	0.498	0.000	1.000
c_2	1.000	0.000	1.000	0.000
c_3	0.416	0.623	0.546	0.250
c_4	0.208	1.000	0.546	0.250

Step 3. The weighted standardized decision matrix ($\bar{H}$) is constructed.

Table 9: The weighted standardized decision matrix H

	e_1	e_2	e_3	e_4
c_1	0.413	0.301	0.314	0.143
c_2	0.827	0.201	0.629	0.072
c_3	0.586	0.325	0.486	0.089
c_4	0.499	0.401	0.486	0.089

Step 4. Determining the BAA matrix (G).

Table 10: The BAA matrix

	e_1	e_2	e_3	e_4
g_j	0.562	0.297	0.464	0.095

Step 5. Calculate the distance to the $BAA(G)$.

Table 11: The distance to the border approximation area

	e_1	e_2	e_3	e_4
c_1	-0.149	0.004	-0.15	0.048
c_2	0.265	-0.096	0.165	-0.023
c_3	0.024	0.028	0.022	-0.006
c_4	-0.063	0.104	0.022	-0.006

Step 6. Rank the alternatives.

Table 12: Rank the alternatives

	D	$Rank$
c_1	-0.247	4
c_2	0.311	1
c_3	0.068	2
c_4	0.057	3

Sorting the alternatives through the DHHFL-MABAC method, we find that the best sewage treatment company is c_2 , and the worst sewage treatment company is c_1 .

6. Sensitivity analysis

The sensitivity analysis of the DHHFL-MABAC method has the following three parts. In the first parts, a series of standard weights are given to study the influence of weights on rankings,

which examines the solutions' stability in the face of weight standard changes. In the second part, sensitivity analysis on the impact of the change of λ parameter. The third parts is to compare with other methods and conduct consistency analysis.

6.1. Sensitivity analysis on the impact of the different weights

The result of the MADM method depends on a large extent on the weight value of the attribute. Sometimes, when the attribute's weight coefficient varies only little, the final choice will change. Therefore, the sensitivity of MADM results to these changes should usually be analyzed later. Sensitivity analysis is used to determine how changes in attribute weight affect the ranking of alternatives. Table 13 depicts the situations A through E, which correspond to the various weights assigned to each standard and the various priorities assigned to the standard..

Table 13: Situations with different weights of the attributes

Situations	Weights of the attributes			
	e_1	e_2	e_3	e_4
<i>A</i> — — — <i>equal weights of the attribute</i>	0.25	0.25	0.25	0.25
<i>B</i> — — — <i>priority achieved water quality standards</i>	0.55	0.15	0.10	0.20
<i>C</i> — — — <i>priority invest funds</i>	0.15	0.55	0.20	0.10
<i>D</i> — — — <i>priority time required</i>	0.10	0.15	0.50	0.15
<i>E</i> — — — <i>priority achieved water quality standards and invest funds</i>	0.35	0.45	0.15	0.05

According to the DHHFL-MABAC method, the candidate suppliers with different weight attributes are ranked, the Table 14 manifests the ranking results. Table 15 can more intuitively show the ranking of alternatives in different weight situations. We can see from the table that assigning different weight values to each attribute will lead to different ranking results. In other words, the DHHFL-MABAC method is more sensitive to these weights. We compare the optimal solutions in the two situations B and D with the results shown in Table 12, and we can see that their optimal solutions are the same. There is not much difference in the sequence of situations between situation B and situation D. The optimal solutions for situations A, C, and E are the same, and the order of the other alternatives is different. This suggests that changing attribute weights may aid in the confirmation of these rankings and consequently the selection of the best option.

Table 14: Rank the alternatives with different weights of attributes

Alternative	Ranking of the alternatives by situations				
	A	B	C	D	E
c_1	4	4	3	4	4
c_2	2	1	4	1	3
c_3	3	2	2	3	2
c_4	1	3	1	2	1

6.2. Sensitivity analysis on the impact of the change of λ parameter

Through the above analysis of different weight situations, we have obtained that the DHHFL-MABAC side is very sensitive to attribute weight values. The comprehensive weight of attributes is obtained by binary contrast method and entropy weight. Through the λ parameter, the proportion of subjective weight and objective weight is selected. When the selected subjective weight or objective weight parameter changes very little, the final choice will change. Therefore, we conduct a sensitivity analysis on the impact of the change of λ parameter. The following table shows the comprehensive weights under a variety of λ parameter values.

Table 15: The comprehensive weights under different values of the λ parameter

Different values of λ	The objectives weights				The subjective weights				The comprehensive weight			
	$e1$	$e2$	$e3$	$e4$	$e1$	$e2$	$e3$	$e4$	$e1$	$e2$	$e3$	$e4$
$\lambda = 0.0$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.294	0.268	0.362	0.076
$\lambda = 0.1$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.318	0.255	0.353	0.075
$\lambda = 0.2$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.342	0.241	0.343	0.074
$\lambda = 0.3$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.366	0.228	0.334	0.073
$\lambda = 0.4$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.390	0.214	0.324	0.072
$\lambda = 0.5$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.414	0.201	0.315	0.071
$\lambda = 0.6$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.437	0.187	0.305	0.071
$\lambda = 0.7$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.461	0.174	0.300	0.070
$\lambda = 0.8$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.485	0.160	0.286	0.069
$\lambda = 0.9$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.509	0.147	0.277	0.068
$\lambda = 1.0$	0.533	0.133	0.267	0.067	0.294	0.268	0.362	0.076	0.533	0.133	0.267	0.067

Figure 2: The total distance of each alternative under different values of the λ parameter.

From the above figure, we can see that with the continuous change of the value of λ , the comprehensive weight of attributes changes continuously. When the value of λ increases, the comprehensive weight of attribute e_1 increases and the comprehensive weight of attribute e_2 , e_3 and e_4 decreases. As the value of λ changes, the total distance of each alternative also changes. Among them, the total distance of c_1 and c_4 decreases, while the total distance of c_2 and c_3 increases. This shows that different λ values will change the comprehensive weight of attributes and affect the total distance of each alternative.

6.3. Compare with MULTIMOORA method and conduct consistency analysis

MULTIMOORA is a reasonable and effective multi-objective optimization method. Gou[35] extended the DHHFL-MULTIMOORA method based on the traditional MULTIMOORA method. The DHHFL-MULTIMOORA method takes the comprehensive consideration of the DHHFLRS, the DHHFLRP and the DHHFLFMF measures, and finally determines the most excellent plan. Next, we work out the problem of selection of sewage treatment companies by using the DHHFL-MULTIMOORA method, and compare it with the DHHFL-MABAC method to prove the improvement and advantages of the DHHFL-MABAC method.

Step 1. Convert the original evaluation information into a numeric matrix, shown in Table 16.

Table 17: The normalization results of all DHHFLEs

	e_1	e_2	e_3	e_4
c_1	0.195	0.246	0.131	0.311
c_2	0.366	0.184	0.450	0.148
c_3	0.244	0.261	0.230	0.270
c_4	0.195	0.308	0.230	0.270

Table 16: The numeric matrix $\bar{R}$

	e_1	e_2	e_3	e_4
c_1	0.444	0.444	0.222	0.639
c_2	0.833	0.333	0.694	0.305
c_3	0.556	0.472	0.389	0.556
c_4	0.445	0.556	0.389	0.556

Step 2. Use DHHFLRS to sort the alternatives.

$$h_{s_{oij}}^o = E(h_{s_{oij}}) / \sum_{i=1}^m E(h_{s_{oij}}) \quad (31)$$

The numerical matrix is normalized according to Eq.(31) and Eq.(9), Table 17 shows the outcomes. In addition, we can calculate the aggregate ratio Φ_i for each alternative by Eq.(32), shown in Table 18.

$$\Phi_i = \sum_{j=1}^{\theta} h_{s_{oij}}^o - \sum_{j=\theta+1}^m h_{s_{oij}}^o \quad (32)$$

In the case of this paper, consider that all attributes are indicators of profitability, where θ represents the number of benefit attribute and $m - \theta$ is the quantity of cost attribute. Thus Φ_i stands for the best performance value of i-th alternative.

Table 18: The final summarizing ratios and the rank of alternative

	c_1	c_2	c_3	c_4
Φ_i	0.883	1.148	1.005	1.003
rank	4	1	2	3

Step 3. The alternatives are ranked by using DHHFLRP. We calculate maximal objective reference points $M_j(i = 1, 2, 3, 4; j = 1, 2, 3, 4)$ by Eq.(33). Table 19 shows the results.

$$M_j = \begin{cases} \max_i \{h_{s_{oij}}^o\}, & \text{if } j \leq \theta \\ \min_i \{h_{s_{oij}}^o\}, & \text{if } j > \theta \end{cases} \quad (33)$$

Table 19: The maximal objective reference point M_j of each attribute

M_1	M_2	M_3	M_4
$\{s_{1<o_0>}, s_{2<o_0>}s_{3<o_0>}\}$	$\{s_{1<o_{-2}>}\}$	$\{s_{1<o_0>}, s_{2<o_{-2}>}\}$	$\{s_{1<o_{-2}>}, s_{1<o_1>}\}$

Next, according to the Eq.(34), the distance between each DHHFLE $h_{s_{o_{ij}}}^o$ and M_j is calculated. The calculation result was as shown in Table 20.

$$D(h_{s_{o_{ij}}}^o, M_j) = \sqrt{\frac{1}{L} \sum_{l=1}^L (\omega_1^l - \omega_2^l)^2} \quad (\omega_1^l \in F(h_{s_{o_{ij}}}^o), \omega_2^l \in F(M_j)) \quad (34)$$

Table 20: The distance between each DHHFLE $h_{s_{o_{ij}}}^o$ and M_j and the final ranking

	e_1	e_2	e_3	e_4	$\max_j \{D(h_{s_{o_{ij}}}^o, M_j)\}$	Ranking
c_1	0.412	0.111	0.553	0.000	0.553	3
c_2	0.000	0.222	0.000	0.333	0.333	2
c_3	0.309	0.162	0.307	0.118	0.309	1
c_4	0.561	0.000	0.307	0.118	0.561	4

Step 4. The alternatives are ranked by using DHHFLFMF. The calculation results are shown in Table 21.

$$U_i = \Im_i \otimes \aleph_i = \prod_{j=1}^{\theta} E(h_{s_{o_{ij}}}^o) \quad (35)$$

Table 21: The overall expected values of all alternatives and the rank of alternatives

	c_1	c_2	c_3	c_4
U_i	0.028	0.059	0.057	0.054
rank	4	1	2	3

Step 5. We finally obtained the ranking of alternatives by calculating and comparing the results of DHHFLRS, DHHFLRP and DHHFLFMF.

Table 22: The normalization results of all DHHFLEs

	$DHHFLRS$	$DHHFLRP$	$DHHFLFMF$	$DHHFL - MULTIMOORA$
c_1	4	3	4	4
c_2	1	2	1	1
c_3	2	1	2	2
c_4	3	4	3	3

Table 23: Comparison of alternatives sorting results

	c_1	c_2	c_3	c_4
$DHHFL - MABAC(\lambda=0.1)$	4	1	3	2
$DHHFL - MABAC(\lambda=0.3)$	4	1	3	2
$DHHFL - MABAC(\lambda=0.5)$	4	1	2	3
$DHHFL - MABAC(\lambda=0.7)$	4	1	2	3
$DHHFL - MABAC(\lambda=0.9)$	4	1	2	3
$DHHFL - MULTIMOORA$	4	1	2	3

By comparing with the DHHFL-MULTIMOORA method, the optimal alternative obtained by the two methods are consistent, which also illustrates the feasibility of the DHHFL-MABAC method. What's more, the DHHFL-MABAC method has more advantages in some aspects. The DHHFL-MABAC method can get a strong ranking, while the DHHFL-MULTIMOORA method cannot accurately rank the alternatives. If the DHHFL-MULTIMOORA method is adopted, the results of the three measures may be completely different, so that the comprehensive ranking cannot be obtained. The DHHFL-MABAC method sorts the alternatives by calculating the global distance of each alternative, and its calculation process is simple and efficient. Compared with the traditional method, the DHHFL-MABAC decision matrix contains more comprehensive and detailed evaluation information. The DHHFL-MABAC method considers potential losses in the calculation process, and also considers the preference information of decision-makers and the importance of attributes, making the calculation results more accurate and reasonable. When the parameter λ takes different values, the DHHFL-MABAC method sorts the alternatives differently. By setting the value of λ , a more reasonable order of the alternatives can be obtained.

7. Conclusion

This paper proposes the DHHFL-MABAC method based on distance measure and comprehensive weight. To this end, we first define a double hierarchy hesitant fuzzy linguistic distance measure based on the least common multiple to calculate the difference between DHHFLEs, and detailed research its properties. Next, we linearly combine the proposed binary contrast method with entropy weight to get the comprehensive weight, which not only considers the effectiveness of objective weights, but also considers the preference information of decision makers. This method can make the determined weight more effective and reasonable. Then, we apply the DHHFL-MABAC method to the MADM problem chosen by the sewage treatment company. Finally, a sensitivity analysis is performed on the DHHFL-MABAC method. The sensitivity analysis includes three aspects. The first is to sensitivity analysis on the impact of the change of λ parameter. The second is to detect the impact on the ranking of alternatives by giving different weight values to. The third is to compare the results with other MADM methods to analyze the consistency of the results. The research results of this paper prove that this method is an effective and useful tool for decision-making.

The research on determining attribute weights is very meaningful in the future application of DHHFL-MABAC method. In addition, we can also extend the DHHFL-MABAC method to work out the problem of multi-attribute group decision making.

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Declarations

1. Articles do not relying on clinical trials.
2. The authors declare that they have no conflict of interest.
3. All submitted manuscripts containing research which does not involves human participants and/or animal experimentation.

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