Experimental Verification of Analysis Model of Seismic Isolation With High-Static-Low-Dynamic Stiffness

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Experimental verification of analysis model of seismic isolation with high-static-low-dynamic stiffness

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Abstract This paper verifies the model of high-static-low-dynamic stiffness (HSLDS) for seismic isolation based on an experiment. Seismic isolation is widely used in several countries. Moreover, the number of seismically isolated buildings has rapidly increased in these few decades. Seismic isolation extends a natural period of a building and decreases the absolute acceleration to reduce a seismic force. However, as there is a trade-off between displacement and absolute acceleration, it might result in the maximum displacement beyond an allowable range. HSLDS is nonlinear, and its restoring force can be approximated cube of a displacement.

Thus, HSLDS applies a large restoring force for significant displacement, and the force is small for small perturbation around an equilibrium position. To improve the control performance of seismic isolation for displacement, we apply HSLDS for seismic isolation. This paper conducts an experiment and

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compares the results with a time-history analysis to verify a numerical model of HSLDS.

Keywords High-static-low-dynamic stiffness (HSLDS) · Quasi-zero stiffness (QZS) · Duffing oscillator · Base isolation · Seismic isolation · Passive structural control · Nonlinear isolation

1 Introduction.

Passive isolation has been applied to several fields [1]. In civil engineering, seismic isolation has been used to protect buildings from large earthquakes. To date, several buildings employ seismic isolation. In particular, in Japan, the number of seismically isolated buildings has rapidly increased after the Great Hanshin earthquake (Kobe earthquake) [2].

Seismic isolation introduces a soft layer into a building to extend a natural period to suppress absolute acceleration. However, as there is a trade-off between displacement and absolute acceleration, the displacement of a seismically isolated layer might exceed an allowable range. To overcome this disadvantage, nonlinear devices are used and invented such as lead damper, steel damper, and high-damping rubber bearing[3]. In recent times, a high-static-low-dynamic stiffness (HSLDS) or quasi-zero stiffness (QZS), which is a Duffing equation-based spring, has been applied to isolation systems. The restoring force of HSLDS is derived from the cube of displacement. Thus, the effect is small caused by small perturbations, and it becomes large caused by significant displacement. The combination of seismic isolation and HSLDS suppresses absolute acceleration for small displacement, and it suppresses the displacement for large earthquakes. To improve the control performance of isolation systems, HSLDS has been applied for vibration isolation, e.g., [4]-[12].

The dynamic behavior of a linear system with HSLDS (HSLDS model) is complicated because the jump phenomenon, which causes bifurcation, occurs. Several studies have estimated the performance of the HSLDS model.

Tamas et al. considered a forced and free response of the HSLDS model and compared the dynamic behavior of it and a linear system [4]. While a linear single-degree-of-freedom (SDOF) system possesses one resonance frequency, an HSLDS model with weak damping exhibits a secondary resonance frequency, which is sub-harmonic resonances. Carella et al. defined a simple equation that estimates the peak gain of transmissibility of an external force to displacement [5]. The peak transmissibility of a linear system depends on a damping ratio. In contrast, the transmissibility of an HSLDS model depends on not only the damping ratio but also the coefficient of HSLDS and maximum steady-state response.

In recent times, HSLDS has been applied to vibration isolation. HSLDS can be achieved using magnets, electrical devices, or multiple springs [5–7]. The advantages of using electronic or magnetronic devices are a long stroke, long life span. Zhang et al. presented a torsion-translational vibration isolator based on convex ball-roller mechanisms that isolate torsion and translational

Fig. 1 HSLDS-based isolation models: (a) original model and (b) equivalent model.

vibration [8]. Wang et al. presented a dual quasi-zero stiffness-based vibration isolator that uses additional springs to improve the control performance of the HSLDS model [9].

HSLDS is used for seismic isolation to protect a building. Watanabe et al. applied HSLDS for seismic isolation with inertia. They showed that HSLDS is effective to reduce displacement, and inertia reduces the absolute acceleration [10]. Zhou et al. used HSLDS to suppress a vertical vibration of a building and showed that HSLDS is effective to reduce transmissibility [11]. Liu et al. analyzed the control performance of an isolation system with HSLDS for near-fault ground motion and demonstrated that HSLDS exhibits good control with regard to the displacement of earthquake waves [12].

Previous studies demonstrated that the combination of HSLDS and seismic isolation yields good control performance. However, most studies are based on analytical and theoretical ways. Unlike these studies, this paper conducts experiments to verify a numerical model of HSLDS. The experimental results are compared with time history analysis (THA) and theoretical results. This paper is organized as follows: Section 2 derives the maximum displacement of an HSLDS model for a sinusoidal wave and defines an equation that estimates the maximum absolute acceleration of an HSLDS model. Section 3 describes the experimental setup and specimen. Section 4 compares the experimental results and THA. Section 5 concludes the paper.

2 Dynamics and maximum response of isolation with HSLDS.

First, a steady-state response and a resonance frequency of HSLDS model are derived. Second, a deformation response factor, R_d , i.e., the ratio of a resonant

response and static response is derived. Then, an equation that estimates absolute acceleration that is based on [5] is presented, and its validity is verified.

2.1 Steady state response of HSLDS model.

HSLDS is achieved using orthogonal two springs (Fig. 1), and the equation of motion of an HSLDS model is as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + F_n(x(t)) = -m\ddot{x}_g(t), \quad (1)$$

where m is a mass, c is a damping coefficient, k is stiffness, $x(t)$ is a displacement, $F_n(x(t))$ is a restoring force of HSLDS, and $\ddot{x}_g(t)$ is ground acceleration. The restoring force of HSLDS, $F_n(x(t))$, can be approximated by third Taylor polynomial of the function [5, 15]:

$$F_n(x(t)) = \Lambda_{N1}x(t) + \Lambda_{N3}x^3(t), \quad (2)$$

where Λ_{N1} and Λ_{N3} are the coefficients of HSLDS, and they are derived from a free length of a spring, d_0 :

$$\begin{cases} \Lambda_{N1} = 2k_{ort} \left(1 - \frac{d_0}{d}\right), \\ \Lambda_{N3} = k_{ort} \frac{d_0}{d^3}. \end{cases} \quad (3)$$

In this study, we used unstretched springs, i.e. $d_0 = d$. Thus, the restoring force of HSLDS, $F_n(x(t))$ can be approximated using the following equation:

$$F_n(x(t)) = \Lambda_{N3}x^3(t). \quad (4)$$

Eq. (4) showed that the restoring force of HSLDS is yielded based on the cube of displacement. In other words, stiffness depends on displacement and is given by the square of displacement. Thus, the stiffness of HSLDS is small for small perturbation and large for big displacement. This point is one of the big differences to linear stiffness. The derivations of Eqs. (2) and (3) are explained in Appendix A.

Rewriting Eq. (1) yields the following equation:

$$\ddot{x}(t) + 2\xi\omega_s\dot{x}(t) + \omega_s^2x(t) + \lambda x^3(t) = -\ddot{x}_g(t), \quad (5)$$

where

$$\begin{cases} \xi = \frac{c}{2m\omega_s}, \quad \omega_s = \sqrt{\frac{k}{m}}, \\ \omega_s^2 = \frac{k}{m}, \quad \lambda = \frac{\Lambda_{N3}}{m}. \end{cases} \quad (6)$$

To consider the steady-state response of the system, we assume that the ground motion $\ddot{x}_g(t) = -a_e \cos \Omega t$. Moreover, we assume that the response of system Eq. (1) can be approximated by following equations:

$$\begin{cases} x(t) = r \cos(\Omega t + \phi), \\ \dot{x}(t) = -r\Omega \sin(\Omega t + \phi), \\ \ddot{x}(t) = -r\Omega^2 \cos(\Omega t + \phi). \end{cases} \quad (7)$$

Substituting Eq. (7) into Eq. (5) yields that

$$\begin{aligned} \left(\omega_s^2 r + \frac{3\lambda}{4} r^3 - r\Omega^2 \right) \cos(\Omega t + \phi) + \frac{\lambda}{4} r^3 \cos 3(\Omega t + \phi) \\ - 2\xi\omega_s r \Omega \sin(\Omega t + \phi) = a_e \cos \Omega t. \end{aligned} \quad (8)$$

Note that the following trigonometric identity is used to calculate Eq. (8):

$$\cos 3(\Omega t + \phi) = 4 \cos^3(\Omega t + \phi) - 3 \cos(\Omega t + \phi). \quad (9)$$

In the above equation, $a_e \cos \Omega t$ can be rewritten as

$$a_e \cos(\Omega t + \phi - \phi) = a_e \cos \phi \cos(\Omega t + \phi) + a_e \sin \phi \sin(\Omega t + \phi). \quad (10)$$

Substituting Eq. (10) into Eq. (8) and considering only the excitation frequency, Ω , the following equation has been derived:

$$\begin{cases} \omega_s^2 r + \frac{3\lambda}{4} r^3 - r\Omega^2 = a_e \cos \phi, \\ -2\xi\omega_s r \Omega = a_e \sin \phi. \end{cases} \quad (11)$$

Regrouping Eq. (11) using $\sin^2 \phi + \cos^2 \phi = 1$ gives

$$\begin{aligned} a_e^2 &= (2\xi\omega_s \Omega)^2 r^2 + \left(\omega_s^2 + \frac{3\lambda}{4} r^2 - \Omega^2 \right)^2 r^2, \\ &= \left[(2\xi\omega_s \Omega)^2 + \left(\omega_s^2 + \frac{3\lambda}{4} r^2 - \Omega^2 \right)^2 \right] r^2. \end{aligned} \quad (12)$$

Thus, the amplitude of a steady-state response r is

$$r = \frac{a_e}{D(\Omega)}, \quad (13)$$

where

$$D(\Omega) = \sqrt{(2\xi\omega_s \Omega)^2 + \left(\omega_s^2 + \frac{3\lambda}{4} r^2 - \Omega^2 \right)^2}. \quad (14)$$

2.2 Estimation of maximum response of HSLDS.

Suppose that the damping ratio is much smaller than 1, $\xi \ll 1$; then Eq. (13) considers peak value if Ω satisfies $\omega_s^2 + \frac{3}{4}\lambda r_p^2 - \Omega^2 = 0$. In this case, Ω is defined as a resonance frequency, Ω_p :

$$\Omega_p = \sqrt{\omega_s^2 + \frac{3}{4}\lambda r_p^2}. \quad (15)$$

The resonance frequency of an undamped linear system, $\Omega_{p,lin}$, is given as follows:

$$\Omega_{p,lin} \approx \omega_s. \quad (16)$$

ω_s is the same as Eq. (15) with $\lambda = 0$. Since a stiffness of the HSLDS model is a function of a displacement, Eq. (15) reflects the characteristic of it. Substituting Eq. (15) into Eq. (13) yields the peak amplitude r_p :

$$r_p = \frac{a_e}{D(\Omega_p)} = \frac{a_e}{2\xi\omega_s\Omega_p}. \quad (17)$$

Let us consider a deformation response factor, R_d , that is a ratio of r_p to static displacement of a linear system, x_{st} :

$$R_d = \frac{r_p}{x_{st}}, \quad x_{st} = \frac{ma_e}{k}. \quad (18)$$

Substituting Eqs. (15) and (17) into Eq. (18) yields

$$R_d = \frac{1}{2\xi\sqrt{1 + \frac{3}{4\omega_s^2}\lambda r_p^2}}. \quad (19)$$

Note that the deformation response factor of a linear system $R_{d,lin}$ is as follows [14]:

$$R_{d,lin} = \frac{1}{2\xi}. \quad (20)$$

This value is identical to R_d with $\lambda = 0$. Equation Eq. (20) shows that the deformation response factor of a linear system only depends on the damping ratio, ξ . In contrast, the factor of an HSLDS model depends not only on the damping ratio but also the displacement because the stiffness of HSLDS depends on the square of displacement. Substituting Eq. (15) into Eq. (17) gives the following equation:

$$\frac{3}{4}\lambda r_p^4 + \omega_s^2 r_p^2 - \frac{a_e^2}{4\xi^2\omega_s^2} = 0. \quad (21)$$

r_p^2 is defined using the quadratic formula for Eq. (21):

$$r_p^2 = \frac{-\omega_s^2 \pm \sqrt{\omega_s^4 + \frac{3}{4\xi^2\omega_s^2}\lambda a_e^2}}{\frac{3}{2}\lambda}. \quad (22)$$

Therefore, r_p is given by

$$r_p = \pm \sqrt{\frac{2}{3\lambda} \left(\sqrt{\omega_s^4 + \frac{3\lambda}{4\xi^2\omega_s^2} a_e^2} - \omega_s^2 \right)}. \quad (23)$$

This equation is identical to [5].

The stiffness of HSLDS increases for large displacement. We assume that the absolute acceleration of an HSLDS model takes maximum value when the displacement is maximum, x_{max} . The equation that estimates the maximum absolute acceleration of an HSLDS model:

$$|\ddot{x}(t) + \ddot{x}_g|_{\max} \approx \frac{1}{m} (kx_{max} + \Lambda_{N3}x_{max}^3). \quad (24)$$

Since the velocity, which is the derivative of displacement, is 0 if the displacement takes a maximum value; the maximum absolute acceleration can be estimated based on the displacement. Note that the maximum absolute acceleration of the HSLDS model for sinusoidal wave is given by substituting $x_{max} = r_p$ in Eq. (24).

Next, we check Eq. (24) through numerical analysis. The parameters of the model are as follows:

Mass of the model (m): 4038 kg.

Natural period of the model (T_u): 1.0 s, 2.0 s, 3.0 s, and 4.0 s.

The damping ratio for the natural period (ξ): 0.01, 0.04, 0.07, 0.1, 0.15, and 0.20.

The stiffness, k , of the HSLDS model is

$$k = \frac{4\pi^2}{T_u^2} m. \quad (25)$$

The HSLDS coefficient, Λ_{N3} , is designed in a way so that nonlinear restoring force, $F_n(x) = \Lambda_{N3}x^3(t)$, is the same as the linear restoring force at the selected displacement, x_{sel} . Thus, Λ_{N3} is given by

$$\Lambda_{N3} = k_{ort} \frac{1}{x_{sel}^2}. \quad (26)$$

In this subsection, x_{sel} is selected to be 0.3 m.

We performed THA considering 44 earthquakes for numerical models. The waves are recommended to evaluate vibration-suppression performance by the Federal Emergency Management Agency (FEMA) P695 [16]. The detail of the waves are listed in Tabs. 1 and 2.

Figure 2 compares the absolute acceleration obtained using THA and Eq. (24). In Fig. 2, the maximum displacement yielded by THA is used in Eq. (24).

Fig. 2 Comparison between Eq. (24) and results of THA for numerical models: (a) $T_u = 1.0$ s, (b) $T_u = 2.0$ s, (c) $T_u = 3.0$ s, and (d) $T_u = 4.0$ s.

Table 1 Earthquakes for evaluation [16].

No.	M	Year	Record No.	Low freq. [Hz]	Name	PGA [g]	PGV [cm/s]
1-1	6.7	1994	953	0.25	NORTHR/MUL009 (Component1)	0.42	59
1-2	6.7	1994	953	0.25	NORTHR/MUL279 (Component2)	0.52	63
2-1	6.7	1994	960	0.13	NORTHR/LOS000 (Component1)	0.35	22
2-2	6.7	1994	960	0.13	NORTHR/LOS270 (Component2)	0.48	45
3-1	7.1	1999	1602	0.06	DUZCE/BOL000 (Component1)	0.73	56
3-2	7.1	1999	1602	0.06	DUZCE/BOL090 (Component2)	0.82	62
4-1	7.1	1999	1787	0.04	HECTOR/HEC000 (Component1)	0.27	29
4-2	7.1	1999	1787	0.04	HECTOR/HEC090 (Component2)	0.34	42
5-1	6.5	1979	169	0.06	IMPVALL/H-DLT262 (Component1)	0.24	26
5-2	6.5	1979	169	0.06	IMPVALL/H-DLT352 (Component2)	0.35	33
6-1	6.5	1979	174	0.25	IMPVALL/H-E11140 (Component1)	0.36	34
6-2	6.5	1979	174	0.25	IMPVALL/H-E11230 (Component2)	0.38	42
7-1	6.9	1995	1111	0.13	KOBE/NIS000 (Component1)	0.51	37
7-2	6.9	1995	1111	0.13	KOBE/NIS090 (Component2)	0.50	37
8-1	6.9	1995	1116	0.13	KOBE/SHI000 (Component1)	0.24	38
8-2	6.9	1995	1116	0.13	KOBE/SHI090 (Component2)	0.21	28
9-1	7.5	1999	1158	0.24	KOCAELI/DZC180 (Component1)	0.31	59
9-2	7.5	1999	1158	0.24	KOCAELI/DZC270 (Component2)	0.36	46
10-1	7.5	1999	1148	0.09	KOCAELI/ARC000 (Component1)	0.22	18
10-2	7.5	1999	1148	0.09	KOCAELI/ARC090 (Component2)	0.15	40
11-1	7.3	1992	900	0.07	LANDERS/YER270 (Component1)	0.24	51
11-2	7.3	1992	900	0.07	LANDERS/YER360 (Component2)	0.15	30

Table 2 Earthquakes for evaluation [16].

No.	M	Year	Record No.	Low freq. [Hz]	Name	PGA [g]	PGV [cm/s]
12-1	7.3	1992	848	0.13	LANDERS/CLW-LN (Component1)	0.42	42
12-2	7.3	1992	848	0.13	LANDERS/CLW-TR (Component2)	0.28	28
13-1	6.9	1989	752	0.13	LOMAP/CAP000 (Component1)	0.53	35
13-2	6.9	1989	752	0.13	LOMAP/CAP090 (Component2)	0.44	29
14-1	6.9	1989	767	0.13	LOMAP/G03000 (Component1)	0.56	36
14-2	6.9	1989	767	0.13	LOMAP/G03090 (Component2)	0.37	45
15-1	7.4	1990	1633	0.13	MANJIL/ABBAR-L (Component1)	0.52	42
15-2	7.4	1990	1633	0.13	MANJIL/ABBAR-T (Component2)	0.50	41
16-1	6.5	1987	721	0.13	SUPERST/B-ICC000 (Component1)	0.36	46
16-2	6.5	1987	721	0.13	SUPERST/B-ICC090 (Component2)	0.26	41
17-1	6.5	1987	725	0.25	SUPERST/B-POE270 (Component1)	0.45	36
17-2	6.5	1987	725	0.25	SUPERST/B-POE360 (Component2)	0.36	33
18-1	7	1992	829	0.07	CAPEMEND/RIO270 (Component1)	0.39	44
18-2	7	1992	829	0.07	CAPEMEND/RIO360 (Component2)	0.55	42
19-1	7.6	1999	1244	0.05	CHICHI/CHY101-E (Component1)	0.35	71
19-2	7.6	1999	1244	0.05	CHICHI/CHY101-N (Component2)	0.44	115
20-1	7.6	1999	1485	0.05	CHICHI/TCU045-E (Component1)	0.50	13
20-2	7.6	1999	1485	0.05	CHICHI/TCU045-N (Component2)	0.36	21
21-1	6.6	1971	68	0.25	SFERN/PEL090 (Component1)	0.21	19
21-2	6.6	1971	68	0.25	SFERN/PEL180 (Component2)	0.17	15
22-1	6.6	1976	125	0.13	FRIULI/A-TMZ000 (Component1)	0.35	22
22-2	6.6	1976	125	0.13	FRIULI/A-TMZ270 (Component2)	0.32	31

Fig. 3 Experimental system of HSLDS based isolation.

The results of THA (Fig. 2) shows that Eq. (24) accurately estimates the maximum absolute acceleration of the HSLDS model. The results demonstrate that it is reasonable to suppose that the absolute acceleration can be maximum value when the displacement is maximum. That is, the maximum absolute acceleration can be estimated considering the maximum displacement.

3 Experimental setup.

This section illustrates the experimental setup. The specimen was developed at the Institute of Technology of Shimizu Corporation.

The specimen comprises a mass, vertical four linear springs, and HSLDS, which is achieved by two wires and disc springs (Figs. 3 and 4 (a)-(d)). The mass is 4038 kg, the stiffness of the model is 13597 N/m, and the damping ratio, which is caused by friction, is estimated to be 0.04. x_{sel} is selected to be 0.3 m in Eq. (26), and Λ_{N3} is 158530 N/m³. Λ_{N3} can be adjusted by the stiffness of disc springs (Fig. 4(d)).

The mass is guided by two linear-motion guides. The mass oscillates along the horizontal direction. In this experiment, the following responses and forces are measured: the displacement and absolute acceleration of the specimen, the restoring force of HSLDS and linear springs, and the acceleration of the shaking table.

(a)

(b)

(c)

(d)

Fig. 4 Specimen: (a) specimen overview, (b) linear springs, (c) HSLDS (wires), and (d) disc springs.

Fig. 5 Sweep Up (a) accelerogram and (b) dominant frequency vs. time.

Fig. 6 Sweep Down (a) accelerogram and (b) dominant frequency vs. time.

Fig. 7 Hachinohe wave (a) accelerogram and (b) pseud velocity spectrum.

4 Comparison between time-history analysis (THA) and experimental results.

This section compares THA and experimental analysis.

Fig. 8 Kobe wave (a) accelerogram and (b) pseud velocity spectrum.

Fig. 9 Random wave (a) accelerogram and (b) pseud velocity spectrum.

This paper uses two sweep waves (chirp signals), two recorded waves, and one artificial wave:

- 1) Sweep Up (Fig. 5): The amplitude is 0.3 m/s², and the initial and target frequencies are 0.1 and 1.2.
- 2) Sweep Down (Fig. 6): The amplitude is 0.3 m/s², and the initial and target frequencies are 1.2 and 0.1.
- 3) Hachinohe wave (Fig. 7): Tokachi earthquake, 1968.
- 4) Kobe wave (Fig. 8): Great Hanshin earthquake, 1995.
- 5) Random wave (Fig. 9): An artificial wave and the pseud velocity spectrum, $_pS_V$ is approximately 0.8 m/s for a structure whose natural period is longer than 0.64 s and the damping ratio is 5%.

In this study, the sweep wave is described as

$$y(t) = A_p \sin \left(2\pi f_s \frac{T(\gamma^{\frac{t}{T}} - 1)}{\ln(\gamma)} \right), \quad \gamma = \frac{f_e}{f_s}. \quad (27)$$

where A_p is the amplitude of the wave, f_s is the initial frequency, f_e is the target frequency, and T is the length of the time. The frequency of the signal $f_{sw}(t)$ is given by

$$f_{sw}(t) = f_s \gamma^{\frac{t}{T}}. \quad (28)$$

Table 3 Comparison among maximum responses of experiment, THA, and estimation.

	Exp.	THA	Est. (Eqs. (23) and (24))	Exp./Est.	THA/Est
x_{max}	0.44	0.45	0.45	98.3%	99.7%
$ \ddot{x} + \ddot{x}_g _{max}$	4.83	5.79	5.03	96.0%	115.1%

As Eq. (28) showed that the frequency of the signal is a function of time, and it is varied continuously between f_s and f_e .

This paper uses a scaled Hachinohe wave to asses the effectiveness of HSLDS, and the scaling factor is 1.47. Figure 7 shows the scaled wave. The pseudo velocity response spectra, ${}_pS_V$, for these waves show that the dominant period of the Hachinohe wave is about 2.5 s and that of the Kobe wave is about 1.0 s. On the other hand, the spectrum of the Random wave is about 0.8 m/s for a building with a damping ratio of 5%, and the natural period is longer than 0.64 s. This section shows the results of the two sweep waves. The results of three earthquake waves are shown in Appendix B.

Figures 10 and 11 show the results of Sweep Up, and Figs. 12 and 13 show those of Sweep Down. Figures 12(c) and 13(c) show that the envelope curves of the maximum displacement and f_{sw} of the waves, and F_{Res} is sum of the restoring forces of HSLDS and linear spring, i.e.,

$$F_{Res} = \Lambda_{N3}x^3(t) + kx(t). \quad (29)$$

An HSLDS model, which is one of the Duffing's oscillator, depends on the "direction" of an excitation frequency. This implies that if an excitation frequency varies from low level to a high level, the response amplitude increases until r_p , which is known as resonant branch. However, if an excitation frequency decreases, the maximum amplitude is lower than r_p (Fig. 14). Figure 14 shows the envelope curves of maximum displacement and f_{sw} for Sweep Up and Sweep Down. Equation (23) estimates the maximum displacement for resonant branch, and the estimated value is close to the result for Sweep Up. These figures also show that Eq. (24) accurately estimates the maximum absolute acceleration of the system for Sweep Up. Note that in this section, r_p , which is obtained from Eq. (23), is used in (24).

Figures 10-13 compare the experimental and THA results. The results of THA closely agree with the experimental results. Furthermore, this figure compares the estimation equations, which are Eqs. (23) and (24) with the parameters $\omega_s = 1.84$ rad/s, $\lambda = 39.2$, and $\zeta = 0.04$. Moreover, the estimated maximum responses are close to the experimental and THA results. The experimental, THA, and the estimation results are listed Tab.3.

Since the restoring force with regard to HSLDS is approximated by the cube of the displacement, the restoring force sharply increases as the displacement increases to suppress the maximum displacement (Figs. 11 (b) and 13 (b)). On the other hand, for the small perturbation, the absolute acceleration of HSLDS model reduces because the restoring force of HSLDS decreases. This is one of the major differences between HSLDS and large linear stiffness.

Fig. 10 Responses for Sweep Up: (a) displacement, (b) absolute acceleration, and (c) envelope curves of x_{max} and $f_{sw}(t)$.

5 Conclusion.

This paper dealt with seismic isolation with high-static-low-dynamic stiffness (HSLDS). Seismic isolation uses a low stiffness component to decrease the influence of high-frequency waves. However, it might result in significant displacement and causes beyond the allowable range. In contrast, since stiffness

Fig. 11 Comparison of results of experiment and THA for Sweep Up: (a) absolute acceleration vs. displacement, and (b) Restoring force vs. displacement.

of HSLDS depends on the square of displacement, the stiffness is large for significant displacement and it is small for small perturbations. Thus, the combination of HSLDS and seismic isolation is effective to protect a building. To verify dynamic characteristics and a numerical model of HSLDS in an actual device, this paper conducted experiments and demonstrated the validity of the proposed method. This paper clarified the following points:

- The comparison between the time history analysis (THA) and the experimental results presented that THA accurately estimated the experimental results.
- For Sweep waves, the maximum responses of an HSLDS model depend on the direction of input frequency, and this characteristic is also confirmed by the experiment.
- The comparison between the experimental and THA results shows that the restoring force of HSLDS can be estimated by the cube of the displacement of an HSLDS model.

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Fig. 12 Responses for Sweep Down: (a) displacement, (b) absolute acceleration, and (c) envelope curves of x_{max} and $f_{sw}(t)$.

Data availability statement

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Fig. 13 Comparison between the experimental and THA results for Sweep Down: (a) absolute acceleration vs. displacement, and (b) restoring force vs. displacement.

Fig. 14 Envelope curves for Sweep Up and Sweep Down (experimental results).

Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix A Polynomial approximation of $F_n(x(t))$

This section shows the HSLDS can be modeled using Duffing's equation with regard to stiffness. First, we explain the derivation of $F_n(x(t))$. Then, we explain the consideration of polynomial approximation.

Appendix A.1 Derivation of $F_n(x(t))$.

As shown in Fig. 1, the restoring force of HSLDS, $F_n(t)$, (Eq. (1)) is given by

$$F_n(x(t)) = 2k_{ort}(1 - \frac{d_0}{\sqrt{x^2(t) + d^2}})x(t). \quad (\text{A.1:})$$

Carella et al showed that the restoring force of an HSLDS can be described by Taylor polynomial of approximation [15], and usually a third Taylor polynomial of it is used (e.g. [4,5,15]). The third Taylor polynomial of restoring force, $F_n(x)$, is

$$F_n(x(t)) = F_n(0) + \frac{F_n^{(1)}(0)}{1!}x(t) + \frac{F_n^{(2)}(0)}{2!}x^2(t) + \frac{F_n^{(3)}(0)}{3!}x^3(t), \quad (\text{A.2:})$$

where $F_n^{(N)}(x(t))$ is the n th derivative of $F_n(x(t))$ based on $x(t)$. Since the first to third derivative of $F_n(x)$ are

$$F_n^{(1)}(x(t)) = 2k_{ort}d_0(x^2(t) + d_0^2)^{-\frac{3}{2}}x^2(t) + 2k_{ort} - 2k_{ort}d_0(x^2(t) + d^2)^{-\frac{1}{2}}, \quad (\text{A.3:})$$

$$F_n^{(2)}(x(t)) = -6k_{ort}d_0(x^2(t) + d^2)^{-\frac{5}{2}}x^3(t) + 6k_{ort}d_0(x^2(t) + d^2)^{-\frac{3}{2}}x(t), \quad (\text{A.4:})$$

$$F_n^{(3)}(x(t)) = 30k_{ort}d_0(x^2(t) + d^2)^{-\frac{7}{2}}x^4(t) - 36k_{ort}d_0(x^2(t) + d^2)^{-\frac{5}{2}}x^2(t) + 6k_{ort}d_0(x^2(t) + d^2)^{-\frac{3}{2}}. \quad (\text{A.5:})$$

$F_n^{(1)}(0)$, $F_n^{(2)}(0)$, $F_n^{(3)}(0)$, which are the coefficients of $x(t)$, $x^2(t)$, and $x^3(t)$, respectively are defined as

$$F_n^{(1)}(0) = 2k_{ort}\left(1 - \frac{d_0}{d}\right), \quad F_n^{(2)}(0) = 0, \quad F_n^{(3)}(0) = 6k_{ort}\frac{d_0}{d^3}. \quad (\text{A.6:})$$

In particular, if $d_0 = d$, which means the springs are unstretched at the initial position, these are given by

$$F_n^{(1)}(0) = 0, \quad F_n^{(2)}(0) = 0, \quad F_n^{(3)}(0) = 6k_{ort}\frac{1}{d^2}. \quad (\text{A.7:})$$

Appendix A.2 Influence of high-order terms.

Next, we consider the polynomial approximation for HSLDS.

Regrouping Eq. (A.1:) yields the following equation:

$$F_n(x(t)) = 2k_{ort} \left(1 - \frac{d_0}{\sqrt{x^2(t) + d^2}} \right) x(t), \quad (\text{A.8:})$$

$$= 2k_{ort} \left(d - d_0 + d_0 - \frac{d_0}{\sqrt{(x(t)/d)^2 + 1}} \right) \left(\frac{x(t)}{d} \right), \quad (\text{A.9:})$$

$$= 2k_{ort}(d - d_0) \left(\frac{x(t)}{d} \right) + 2k_{ort}d_0 \left(1 - \frac{1}{\sqrt{(x(t)/d)^2 + 1}} \right) \left(\frac{x(t)}{d} \right) \quad (\text{A.10:})$$

If $|x(t)/d| < 1$, then $\frac{1}{\sqrt{(x(t)/d)^2 + 1}}$ can be approximated by using the binomial series:

$$\frac{1}{\sqrt{(x(t)/d)^2 + 1}} = \sum_{j=0}^{\infty} -\frac{1}{2} C_j \left(\frac{x(t)}{d} \right)^{2j}, \quad (\text{A.11:})$$

$$= 1 - \frac{1}{2} \left(\frac{x(t)}{d} \right)^2 + \frac{3}{8} \left(\frac{x(t)}{d} \right)^4 - \frac{5}{16} \left(\frac{x(t)}{d} \right)^6 + \dots \quad (\text{A.12:})$$

In the aforementioned equation, ${}_m C_n$ indicates a combination of m and n :

$${}_m C_n = \frac{m(m-1)(m-2)\dots(m-n+1)}{n!}. \quad (\text{A.13:})$$

Substituting (A.12:) into (A.10:) yields

$$\begin{aligned} F_n(x(t)) &= 2k_{ort}(d - d_0) \left(\frac{x(t)}{d} \right) + 2k_{ort}d_0 \left\{ \frac{1}{2} \left(\frac{x(t)}{d} \right)^2 - \frac{3}{8} \left(\frac{x(t)}{d} \right)^4 + \dots \right\} \left(\frac{x(t)}{d} \right) \\ &= 2k_{ort}(d - d_0) \left(\frac{x(t)}{d} \right) + k_{ort}d_0 \left(\frac{x(t)}{d} \right)^3 - \frac{3}{4}k_{ort}d_0 \left(\frac{x(t)}{d} \right)^5 + \dots \end{aligned} \quad (\text{A.14:})$$

In particular, if $d = d_0$, $F_n(x(t))$ can be approximated as

$$F_n(x(t)) = k_{ort}d_0 \left(\frac{x(t)}{d_0} \right)^3 - \frac{3}{4}k_{ort}d_0 \left(\frac{x(t)}{d_0} \right)^5 + \dots \quad (\text{A.15:})$$

Note that $\frac{F_n^{(N)}(0)}{N!}$, which is the coefficient of $x^N(t)$ in the Taylor polynomial series, is the same as the coefficient of $\left(\frac{x(t)}{d_0} \right)^N$ multiplied by d_0^{-N} .

This is worth mentioning that if $|x(t)/d_0| > 1$, the value of approximation will not converge. In other words, the restoring force of HSLDS can be approximated if the displacement, $x(t)$, is smaller than d_0 . Figures 11 and 13 demonstrated that the third-order polynomial equation appropriately approximates the restoring force of HSLDS in this paper.

Appendix B Comparison of results for THA and experiment for earthquake waves.

The THA and experimental results for three earthquake waves are compared. The results show that the analytical model is in close agreement with the experimental results for response and the restoring force. Thus, the behavior of an HSLDS model can be estimated using, Eq. (1).

Fig. 15 Responses for Hachinohe wave: (a) displacement and (b) absolute acceleration.

Fig. 16 Comparison between THA and experimental results for Hachinohe wave: (a) absolute acceleration vs. displacement, and (b) restoring force vs. displacement.

Fig. 17 Responses for Kobe wave: (a) displacement and (b) absolute acceleration.

Fig. 18 Comparison between THA and experimental results for Kobe wave: (a) absolute acceleration vs. displacement, and (b) restoring force vs. displacement.

Fig. 19 Responses for Random wave: (a) displacement and (b) absolute acceleration.

Fig. 20 Comparison between THA and experimental results for Random wave: (a) absolute acceleration vs. displacement, and (b) restoring force vs. displacement.