Shared Manufacturing in A Differentiated Duopoly with Capacity Constraints

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Shared Manufacturing in A Differentiated Duopoly with Capacity Constraints

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Abstract:
This study sets up a differentiated duopoly model considering capacity constraints and shared manufacturing, investigates the equilibrium results, examines the effects of product differentiation and capacity constraints in three scenarios, and compares the equilibrium outcomes in three cases under Cournot and Stackelberg competition. We find that capacity constraints affect the relationships among product differentiation, equilibrium results, and the market share of enterprises. Shared manufacturing impacts the degree of excess capacity, profits, consumer surplus, and social welfare; however, it may sometimes play a negative role in alleviating excess capacity. Moreover, Cournot competition is a better choice for enterprises with capacity constraints compared to Stackelberg competition.
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JEL Classification: L11; L13; M11

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Abstract:

This study sets up a differentiated duopoly model considering capacity constraints and shared manufacturing, investigates the equilibrium results, examines the effects of product differentiation and capacity constraints in three scenarios, and compares the equilibrium outcomes in three cases under Cournot and Stackelberg competition. We find that capacity constraints affect the relationships among product differentiation, equilibrium results, and the market share of enterprises. Shared manufacturing impacts the degree of excess capacity, profits, consumer surplus, and social welfare; however, it may sometimes play a negative role in alleviating excess capacity. Moreover, Cournot competition is a better choice for enterprises with capacity constraints compared to Stackelberg competition.

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1 Introduction

Capacity sharing, which is commonplace in many fields, such as the logistics industry and clothing manufacturing, plays an important role in alleviating supply-demand imbalances and improving capacity allocation in space and time (Melo, Macedo, and Baptista, 2019). There are various forms of capacity sharing, among which one of the most popular forms is shared manufacturing. Enterprises with insufficient capacity can trade capacity with shared manufacturing enterprises, resulting in a win-win situation (Geissinger et al., 2019; Dai and Nu, 2020; Schwarz & Tan, 2021). In China, new production modes of "capacity sharing" and "factory sharing," which rely on modern information technology, are emerging all over the country. A series of favorable policies issued by the government aims to promote the rapid development of capacity sharing and enable more shared manufacturing to be developed based on big data (He, Zhang, and Gu, 2019; Yan et al., 2019; Yang, Shan, and Jin, 2017). By sharing manufacturing, an effective allocation of production capacity can be achieved. When faced with insufficient capacity, shared manufacturing enterprises can alleviate capacity constraints by selling capacity. For example, China's "Tao Factory" as well as various foundry factories use their capacity to meet the needs of other enterprises, thereby optimizing the entire market's capacity. In addition, shared manufacturing enterprises can provide a wide range of capacity services for various segments with similar production technologies. However, what are the conditions under which both sides engage in capacity sharing? What impacts will capacity sharing have on the market? Will capacity sharing alleviate or exacerbate excess capacity? Will the competition modes of enterprises affect capacity sharing? To answer these questions, based on the precondition of capacity constraints, this paper discusses the strategic interaction between enterprises in two duopoly competition modes, namely Cournot and Stackelberg, investigates the relationship between enterprises with capacity constraints and shared manufacturing, and examines the conditions and impacts of capacity sharing, especially on capacity and production matching, which provides new ideas for solving the problem of excess capacity.

Shared manufacturing is essentially a type of capacity sharing. Capacity sharing reflects competition and cooperation among stakeholders. The strategic behaviors of all players would exert great influence on equilibrium results and the capacity sharing effect. Based on specific objectives, such as reducing costs or improving efficiency, various studies have analyzed the formation, effect, and optimal strategies of capacity sharing using mathematical models and methods (Wang, Tang, and Huo, 2018). These include game theory, the Monte Carlo method, the ant colony algorithm, and the queuing model. These studies include two main aspects. The first focuses on the practical application of enterprise production and operations management and uses various algorithms and technical tools to help enterprises make capacity decisions. Chevalier-Roignant et al. (2021) explored the problem of capacity decision-making under flexible output conditions and found that when uncertainty in the economic environment increased, investment in capacity increased as well. Moghaddam and Nof (2016) developed a mechanism for real-time resource allocation, order management, and process monitoring for demand and capacity sharing among enterprises. Legros (2019) investigated a decision support tool for a bike-sharing system based on the Markov decision process method. Some scholars focus on game theory to reveal the formation mechanism of strategy to investigate its effects on participants. These studies are generally based on certain markets and enterprise constraints, such as the type of market competition (Cournot or Stackelberg), product differentiation (homogeneous or heterogeneous), production technology (increasing or constant returns to scale), number of enterprises (two or more), budget constraints (whether there are financial constraints), and negotiation strategies (negotiation function design). This indicates that a capacity sharing strategy is affected by many factors, which provide theoretical support for the model setting and analysis of the results. Xie and Han (2020) built game-theoretical models and used chaos theory to analyze the effect of capacity sharing and capacity investment on manufacturers with sufficient environmentally friendly or limited capacity. Roels and Tang (2017) studied the sharing of manufacturing capabilities and distribution capabilities in strategic alliances and found that the contract mechanism could reduce disputes over capacity allocation and increase profits for both parties. Fang and Wang (2020) investigated horizontal capacity sharing among competitors with asymmetric information. Qin et al. (2020) considered capacity sharing between competing companies and compared the equilibrium results of shared and non-shared capacity. They found that capacity sharing increased the profits of companies with insufficient capacity; however, it was not always better for companies with overcapacity to share capacity. Zhang et al. (2020) explored the implementation of sharing in the GSHP district system to determine whether it could enhance the reduction of carbon emissions and showed that individual and central district systems were applicable.

Several researchers have adopted the duopoly model, which has strong applicability for analyzing capacity sharing strategies. Therefore, a duopoly model was adopted for our research.

Capacity sharing has multiple and complex impacts. Extensive literature has shown that capacity sharing influences a series of market competition factors, such as market price, competition intensity, profit, and utility function, thereby affecting equilibrium results and the performance of the whole market. Some scholars believe that shared manufacturing is positive, as it can help alleviate capacity shortages and improve the resource allocation efficiency of the whole economy (Aloui and Lebsi, 2016; Tae, Luo, Lin, 2020; Nunez, Bai, and Du, 2021). However, others note that the positive effect of shared manufacturing should be based on certain conditions, which may not be beneficial or even be negative (Chen, Wang, and Liu, 2020; Guo and Wu, 2018; Levi et al., 2020). This study aims to comprehensively analyze the influence of capacity constraints and capacity sharing on various equilibrium results under different duopoly competition modes and examine their effects by incorporating matching situations of capacity and production into a cost function to further improve the practical significance of the results.

Compared with the existing literature, the main contributions of this study are as follows. First, the existing research mainly focuses on capacity sharing under one type of duopoly and lacks comparisons between different types of duopoly competition. In contrast, our study compares various market equilibrium results and the effects of capacity sharing considering three scenarios under two competition modes. Chen, Xie, and Liu (2020) constructed a capacity sharing model in a supply chain and revealed the impacts of government policies and different duopoly competition types on equilibrium results, but shared manufacturing enterprises and charges were not considered and are covered by their study. Second, there is plenty of literature exploring capacity sharing. Liu et al. (2018) clarified that sharing can be profitable and beneficial for both operators and consumers, meaning that effectiveness and efficiency can be a temporary boost. In our study, product differentiation is introduced to investigate its impact on the equilibrium results. This is a beneficial extension of existing research. Although researchers have examined product differentiation deeply in different settings (Chen, Wang, and Chu, 2020), capacity decisions were not incorporated. In our study, decisions regarding capacity and output are separated into two stages, consistent with Bárcena-Ruiz and Garzon (2010), Fanti and Meccheri (2017), and Chen et al. (2019). Third, many studies have been conducted on excess capacity based on the monopoly model (Dagdeviren, 2016; Murphy, 2017; Chen, Liu, and Qin, 2019). However, shared manufacturing was seldom involved. This study not only introduces a shared manufacturing enterprise to investigate its impact on excess capacity but also explores the impact of product differentiation and capacity constraints on excess capacity. This is valuable for the enrichment of excess capacity research.

The remainder of this paper is organized as follows. Section 2 constructs a duopoly model that considers product differentiation and excess capacity. Section 3 examines the equilibrium results in the cases of two enterprises with sufficient capacity (Model CSS and Model SSS), an enterprise with capacity constraints without a shared manufacturing enterprise (Model CSI and Model SSI), and an enterprise with capacity constraints and a shared manufacturing enterprise (Model CSP and Model SSP) under Cournot and Stackelberg competition. In addition, it analyzes the influence of product and capacity constraints on excess capacity and the other results in equilibrium and presents a comparative analysis of profits, consumer surplus, social welfare, and excess capacity under three conditions of two competition modes to further reveal the effects of capacity sharing. Section 4 presents the conclusions of the research.

2 The Model

This study establishes a duopoly model consisting of Enterprise 1 and Enterprise 2. The duopoly competition between the two enterprises is either Cournot or Stackelberg competition.

The inverse demand functions of the enterprises are denoted as $p_i = a - q_i - rq_j$, i $\neq$ j, i, j = 1 or 2, where r is the degree of product homogeneity and $r \in (0,1]$. The higher the value of r, the lower the degree of product differentiation. Moreover, capacity is incorporated into the cost functions of the enterprises. It is assumed that the cost of the enterprise includes fixed marginal costs and excess costs caused by the mismatch of capacity. If the capacity is greater or less than the necessary production, the precipitation cost or overload operation of machines will result in excess costs. The balance between capacity and output will help reduce these costs. To reflect the total cost brought to enterprises by the imbalance of capacity and output, the cost functions of the two enterprises are denoted as $c_i = (x_i - q_i)^2 + bq_i$, referring to the method widely adopted by scholars such as Vives (1986) and Tomaru, Nakamura, and Saito (2011). The profit functions of the enterprises are denoted by $\pi_i = p_i q_i - c_i$, where $a > b > 0$. Social welfare is expressed as $SW = \pi_1 + \pi_2 + CS$, where $CS = \frac{{q_1}^2 + {q_2}^2 + 2rq_1q_2}{2}$.

Enterprises are first faced with capacity decisions and then output decisions to pursue profit maximization. We assume that the enterprise has sufficient or insufficient capacity. When there is insufficient capacity, the enterprise decides whether to purchase capacity from a shared manufacturing enterprise. There are three scenarios: 1) both enterprises have sufficient capacity (each can achieve optimal capacity; denoted as superscript CSS and SSS); 2) one enterprise has sufficient capacity, while the other does not, and there is no shared manufacturing enterprise (denoted as superscript CSI for the enterprise with sufficient capacity and superscript SSI for the enterprise with insufficient capacity); and 3) one enterprise has sufficient capacity, while the other does not, and there is a shared manufacturing enterprise (denoted as superscript CSP for the enterprise with sufficient capacity and superscript SSP for the enterprise with insufficient capacity).

We assume Enterprise 1 always has sufficient capacity, and we denote the capacity of Enterprise 2 as k, which is lower than its optimal capacity without capacity constraints if the capacity of Enterprise 2 is insufficient, that is, $k > 0$. If Enterprise 2 purchases capacity from a shared manufacturing enterprise, Enterprise 2's profit is $\pi_2 = p_2 q_2 - c_2 - p_T X$, where the inverse demand function of the shared manufacturing enterprise is $p_T = e - Q - X > 0$; X represents the quantity of capacity purchased for Enterprise 2, and the capacity purchased for other industries is denoted as Q, which is a positive constant, and the shared manufacturing enterprise cost function is $c_T = \frac{(Q+X)^2}{2}$.

When a shared manufacturing enterprise pursues profit maximization, $MR = MC$, we obtain $\mathrm{X} = \frac{1}{3}e - Q$. However, because the price is affected by the purchase of capacity by Enterprise 2, the shared manufacturing enterprise is willing to sell capacity when $0 < \mathrm{X} \leq \frac{1}{3}e - Q$. At this time, the profit of the shared manufacturing enterprise increases as the purchased capacity of Enterprise 2 increases.

According to the above assumptions, the two enterprises play a two-stage sequential game. In the first stage, the two enterprises determine their capacity to maximize profit. Moreover, if an enterprise faces insufficient capacity, it should decide whether to purchase capacity from a shared manufacturing enterprise. In the second stage, two enterprises compete in a Cournot or Stackelberg competition for profit maximization.

3 Model analysis

3.1 Cournot Competition

3.1.1 Model CSS

The equilibrium results of the two competition modes are analyzed by backward induction.

First, the scenario in which both enterprises have sufficient capacity is analyzed. In the second stage, $\frac{\partial \pi_i}{\partial q_i} = 0$ must be satisfied. Then, the equilibrium outputs can be obtained as

$$q_i^{CSS} = \frac{(4-r)(a-b)-2rx_j+8x_i}{16-r^2} \tag{1}$$

Next, Equation (1) is substituted into the profit function of the enterprise. To maximize profit, $\frac{\partial \pi_i}{\partial x_i} = 0$ should be satisfied. Then, the equilibrium capacities can be derived as

$$x_i^{CSS} = \frac{16(a-b)}{32+16r-4r^2-r^3} \tag{2}$$

Lemma 1 *When both enterprises have sufficient capacity, the equilibrium results are as follows.*

$q_i^{CSS} = \frac{(16-r^2)(a-b)}{32+16r-4r^2-r^3}$, $x_i^{CSS} - q_i^{CSS} = \frac{r^2(a-b)}{32+16r-4r^2-r^3} > 0$, $\pi_i^{CSS} = \frac{2(a-b)^2(r^4-32r^2+128)}{(32+16r-4r^2-r^3)^2}$

$CS^{CSS} = \frac{(r^5+r^4-32r^3-32r^2+256r+256)(a-b)^2}{r^6+8r^5-16r^4-192r^3+1024r+1024}$, $SW^{CSS} = \frac{(r^5+5r^4-32r^3-160r^2+256r+768)(a-b)^2}{r^6+8r^5-16r^4-192r^3+1024r+1024}$.

It can be seen that in the case of sufficient capacity, both enterprises have excess capacity, which indicates that excess capacity can occur under the incentive of profit without government intervention.

Although excess capacity leads to certain waste of resources, it will also increase output and thus improve profits, making up for the loss of excess capacity. This can also account for the existence of excess capacity as a common phenomenon in the market economy. Next, we analyze the impacts of product differentiation on the equilibrium results in Proposition 1.

Proposition 1 *The effect of* r*:*

$$\frac{\partial {x_i}^{CSS}}{\partial r} < 0,\ \frac{\partial {q_i}^{CSS}}{\partial r} < 0, \frac{\partial {\pi_i}^{C1}}{\partial r} < 0,\ \frac{\partial({x_i}^{CSS} - {q_i}^{CSS})}{\partial r} > 0,\ \frac{\partial SW^{CSS}}{\partial r} < 0.$$

If $0 < r \leq 0.99$, then $\frac{\partial CS^{CSS}}{\partial r} \leq 0$; if $0.99 < r < 1$, then $\frac{\partial CS^{CSS}}{\partial r} > 0$.

In Model CSS, the capacities, outputs, and profits of Enterprises 1 and 2 as well as social welfare are negatively correlated with product homogeneity, but the degree of excess capacity is positively correlated with it. At first, the relationship between consumer surplus and r is negative and then turns positive when product homogeneity is higher than a certain level. With an increase in r, enterprises will reduce their capacities appropriately. Meanwhile, an increase in r reduces outputs, which further reduces profits and social welfare. Through market competition, enterprises are encouraged to expand product differentiation, and excess capacity can be alleviated. Therefore, the improvement of product differentiation is conducive to increasing the capacities and outputs of enterprises, further increasing their profits, and promoting capacity management. However, from the perspective of consumers, when productive homogeneity is high, product differentiation efforts are not beneficial to them.

3.1.2 Model CSI

Then, we changed the assumption of sufficient capacity to insufficient capacity for Enterprise 2. By maximizing the profits of the two enterprises, the equilibrium outputs can be obtained as

$$ {q_i}^{CSI} = \frac{(4-r)(a-b) - 2rx_j + 8x_i}{16 - r^2} \tag{3}$$

Because the capacity of Enterprise 2 is insufficient, which means $k < \frac{16(a-b)}{32+16r-4r^2-r^3}$, the capacity of Enterprise 2 can be derived as

$$x_2 = \begin{cases} k, & 0 \leq x_1 \leq \frac{16[(a-b)(4-r)-2rk]}{r^4-32r^2+128} \\ \frac{16[(a-b)(4-r)-2rx_1}{r^4-32r^2+128}, & x_1 > \frac{16[(a-b)(4-r)-2rk]}{r^4-32r^2+128} \end{cases} \tag{4}$$

To maximize profits, the equilibrium capacities can be obtained as

$$ {x_1}^{CSI} = \frac{16[(a-b)(4-r)-2rk]}{r^4-32r^2+128},\ {x_2}^{CSI} = k \tag{5}$$

where $0 < k < \frac{16(a-b)}{32+16r-4r^2-r^3}$.

Lemma 2 *The equilibrium results of Model CSI can be derived as follows.*

$$q_1{}^{CSI} = \frac{(r^3-4r^2-16r+64)(a-b)+2kr(r^2-16)}{r^4-32r^2+128}, \quad q_2{}^{CSI} = \frac{(r^3-4r^2-16r+32)(a-b)+8k(8-r^2)}{r^4-32r^2+128};$$

$$x_1{}^{CSI} - q_1{}^{CSI} = \frac{r^2[(a-b)(4-r)-2kr]}{r^4-32r^2+128} > 0;$$

$x_2{}^{CSI} - q_2{}^{CSI} = \frac{k(r^4-24r^2+64)-(r^3-4r^2-16r+32)(a-b)}{r^4-32r^2+128}$, if $0 < k \leq \frac{(r^3-4r^2-16r+32)(a-b)}{r^4-24r^2+64}$, then $x_2{}^{CSI} - q_2{}^{CSI} \leq 0$; if $\frac{(r^3-4r^2-16r+32)(a-b)}{r^4-24r^2+64} < k < \frac{16(a-b)}{32+16r-4r^2-r^3}$, then $x_2{}^{CSI} - q_2{}^{CSI} > 0$.

$$\pi_1{}^{CSI} = \frac{2[(a-b)(r-4)+2k]^2}{r^4-32r^2+128};$$

$$\pi_2{}^{CSI} = \frac{\begin{array}{c}2\left[(r^6-8r^5-16r^4+192r^3-1024r+1024)(a-b)^2-32k(r^5-4r^4-24r^3+64r^2+128r-256)(a-b)\right]- \\ k^2(r^8-64r^6+1152r^4-6144r^2+8192)\end{array}}{(r^4-32r^2+128)^2};$$

$$CS^{CSI} = \frac{\left\{\begin{array}{c}(r^7-7r^6-24r^5+208r^4+96r^3-1664r^2+512r+2560)(a-b)^2+ \\ 2(r^7-7r^6-24r^5+176r^4+96r^3-1024r^2+512r+1024)(a-b)k \\ -2k^2(7r^6-176r^4+1024r^2-1024)\end{array}\right\}}{(r^4-32r^2+128)^2};$$

$$SW^{CSI} = \frac{\left\{\begin{array}{c}(r^7-3r^6-56r^5+144r^4+992r^3-2432r^2-3584r+8704)(a-b)^2+ \\ 2(r^7-3r^6-56r^5+112r^4+992r^3-1536r^2-3584r+5120)(a-b)k \\ -k^2(r^8-58r^6+1056r^4-5120r^2+6144)\end{array}\right\}}{(r^4-32r^2+128)^2}.$$

It can be seen that when the capacity of Enterprise 2 is lower than a certain level ($\frac{(r^3-4r^2-16r+32)(a-b)}{r^4-24r^2+64}$), there is insufficient capacity; otherwise, there is excess capacity. Therefore, the degree of excess capacity is closely related to the capacity of Enterprise 2. This is different from Model CSS because capacity constraints are added. For Enterprise 1, consistent with the first situation, it has excess capacity.

Proposition 2

(1) The effects of r*:*

$\frac{\partial x_1{}^{CSI}}{\partial r} < 0$, $\frac{\partial x_2{}^{CSI}}{\partial r} = 0$, $\frac{\partial q_1{}^{CSI}}{\partial r} < 0$, $\frac{\partial q_2{}^{CSI}}{\partial r} < 0$, $\frac{\partial \pi_1{}^{CSI}}{\partial r} < 0$, $\frac{\partial \pi_2{}^{CSI}}{\partial r} < 0$, $\frac{\partial (x_1{}^{CSI}-q_1{}^{CSI})}{\partial r} > 0$, $\frac{\partial (x_2{}^{CSI}-q_2{}^{CSI})}{\partial r} > 0$, $\frac{\partial CS^{CSI}}{\partial r} > 0$, $\frac{\partial SW^{CSI}}{\partial r} < 0$

(2) The effects of k*:*

$\frac{\partial x_1{}^{CSI}}{\partial k} < 0$, $\frac{\partial x_2{}^{CSI}}{\partial k} > 0$, $\frac{\partial q_1{}^{CSI}}{\partial k} < 0$, $\frac{\partial q_2{}^{CSI}}{\partial k} > 0$, $\frac{\partial \pi_1{}^{CSI}}{\partial k} < 0$, $\frac{\partial \pi_2{}^{CSI}}{\partial k} > 0$, $\frac{\partial (x_1{}^{CSI}-q_1{}^{CSI})}{\partial k} < 0$, $\frac{\partial (x_2{}^{CSI}-q_2{}^{CSI})}{\partial k} > 0$, $\frac{\partial CS^{CSI}}{\partial k} > 0$, $\frac{\partial SW^{CSI}}{\partial k} > 0$.

As shown in Proposition 2, when the capacity of Enterprise 2 is constrained, compared with Model CSS, the relationship between consumer surplus and r changes from a negative to a positive correlation, and the other relationships remain the same. The results indicate that when Enterprise 2 is restrained by capacity, capacity constraints have a greater impact than product differentiation on consumer surplus. When Enterprise 2 faces an increase in r, capacity does not change accordingly. However, the output is reduced appropriately, which increases consumer surplus. Therefore, when there are capacity constraints, it is more beneficial for consumers to moderately homogenize products. Nevertheless, production homogenization intensifies the degree of excess capacity and leads to a reduction in social welfare. Therefore, increasing the degree of product differentiation to increase profits can reduce consumer surplus and increase social welfare.

The capacity, output, profit, and excess capacity of Enterprise 1 are negatively correlated with the capacity constraints of Enterprise 2, while others are positively correlated with it. The lower the capacity constraints of Enterprise 2, the higher the capacity, output, and profit of Enterprise 1 and the higher the excess capacity. Enterprise 2's capacity, output, and profit as well as the degree of excess capacity will be reduced; however, Enterprise 1 is reluctant to alleviate excess capacity at the cost of profit reduction. A contradiction between capacity constraints and excess capacity arises. Moreover, consumer surplus and social welfare increase as capacity constraints are released because when Enterprise 2 is limited in capacity, Enterprise 1 will occupy the market share of Enterprise 2 and obtain higher profits but simultaneously reduce consumer surplus and social welfare. Therefore, the capacity constraints of enterprises are beneficial to competitors but harmful to enterprises, consumers, and society, and conflicts among these stakeholders must be resolved. Thus, the next step is to introduce a shared manufacturing enterprise to explore whether capacity sharing can solve this contradiction.

3.1.3 Model CSP

We consider a capacity sharing manufacturing enterprise and define $A = a - b, B = e - Q$. The equilibrium outputs of the two companies can be derived as:

$$q_1^{CSP} = \frac{(4-r)A-2rx_2+8x_1}{16-r^2},\ q_2^{CSP} = \frac{(4-r)A-2rx_1+8x_3}{16-r^2} \tag{6}$$

Then, substituting Equation (6) into the profit functions of the enterprises, $\frac{\partial \pi_i}{\partial x_i} = 0$ must be satisfied, and the equilibrium capacities can be derived as

$$x_1^{CSP} = \frac{32A-(16-r^2)r(B+2k)}{8(8-r^2)},\quad x_2^{CSP} = \frac{(2048-640r^2+48r^4-r^6)(B+2k)-32(32-16r-4r^2+r^3)A}{256(8-r^2)} \tag{7}$$

Lemma 3 *The equilibrium results when Enterprise 2 has the insufficient capacity with a shared manufacturing enterprise are as follows.*

$q_1^{CSP} = \frac{32(16-r^2)A-(256-32r^2+r^4)(B+2k)r}{128(8-r^2)},\ q_2^{CSP} = \frac{(16-r^2)(B+2k)}{32};$

$x_1^{CSP} - q_1^{CSP} = \frac{32Ar^2-(16-r^2)(B+2k)r^3}{128(8-r^2)} > 0;$

$x_2^{CSP} - q_2^{CSP} = \frac{(1024-448r^2+40r^4-r^6)(B+2k)-32(32-16r-4r^2+r^3)}{256(8-r^2)};$

If $0 < k \leq -\frac{(1024-448r^2+40r^4-r^6)B-32(32-16r-4r^2+r^3)A}{2(1024-448r^2+40r^4-r^6)}$, then $x_2{}^{CSP} - q_2{}^{CSP} \leq 0$;

If $k > -\frac{(1024-448r^2+40r^4-r^6)B-32(32-16r-4r^2+r^3)A}{2(1024-448r^2+40r^4-r^6)}$, then $x_2{}^{CSP} - q_2{}^{CSP} > 0$;

$$\pi_1{}^{CSP} = \frac{\begin{array}{c}1024(128-32r^2+r^4)A^2-64(2048-640r^2+48r^4-r^6)rA(B+2k)+\\(32768-12288r^2+1408r^4-64r^6+r^8)(B+2k)^2r^2\end{array}}{8192(8-r^2)^2};$$

$$\pi_2{}^{CSP} = \frac{64(32-16r-4r^2+r^3)A(B+2k)-(2048-768r^2+56r^4-r^6)B^2-4(1024-640r^2+56r^4-r^6)(B+k)k}{512(8-r^2)};$$

$$CS^{CSP} = \frac{\begin{array}{c}1024(256-32r^2+r^4)A^2+64(4096-1280r^2+112r^4-3r^6)rA(B+2k)+\\(262144-294912r^2+78848r^4-8448r^6+400r^8-7r^{10})(B+2k)^2\end{array}}{32768(8-r^2)^2};$$

$$SW^{CSP} = \frac{\left\{\begin{array}{c}1024(768-160r^2+5r^4)A^2+64(16384-12288r-4096r^2+2816r^3+256r^4-144r^5+r^7)A(B+2k)\\-(786432-360448r^2+48128r^4-1280r^6-80r^8+3r^{10})B^2-\\4(262144-229376r^2+39936r^4-1280r^6-80r^8+3r^{10})(B+k)k\end{array}\right\}}{32768(8-r^2)^2}.$$

It can be seen that Enterprise 1 still has excess capacity. For Enterprise 2, similar to Model CSI, the degree of excess capacity depends on the level of limited capacity. The difference is the cut-off value, which is $k = \frac{(1024-448r^2+40r^4-r^6)B+32(32-16r-4r^2+r^3)A}{(1024-448r^2+40r^4-r^6)}$, which is lower than that in Model CSI.

To ensure that shared manufacturing companies are willing to sell capacity, $0 < X \leq \frac{1}{3}e - Q$. For Enterprise 2, when faced with capacity constraints, the profit after purchasing capacity from the shared manufacturing enterprise is greater than before the purchase. According to this, we obtain the ranges of making a shared deal as $k \in (k_1{}^{CSP}, k_2{}^{CSP})$, when $0 < r < r^{CSP}$, and $k \in (k_1{}^{\mathrm{CSP}}, k_3{}^{\mathrm{CSP}})$ when $r^{CSP} \leq r < 1$. The value of purchase capacity is $X^{CSP} = \frac{(2048-640r^2+48r^4-r^6)B+2(1024-512r^2+48r^4-r^6)k-32(32+16r-4r^2-r^3)A}{2048-256r^2}$.

Proposition 3

(1) The effects of r*:*

$\frac{\partial x_1{}^{CSP}}{\partial r} > 0$, $\frac{\partial x_2{}^{CSP}}{\partial r} > 0$, $\frac{\partial q_1{}^{CSP}}{\partial r} < 0$, $\frac{\partial q_2{}^{CSP}}{\partial r} < 0$, $\frac{\partial \pi_1{}^{CSP}}{\partial r} < 0$, $\frac{\partial \pi_2{}^{CSP}}{\partial r} < 0$, $\frac{\partial (x_1{}^{CSP}-q_1{}^{CSP})}{\partial r} > 0$, $\frac{\partial (x_2{}^{CSP}-q_2{}^{CSP})}{\partial r} > 0$, $\frac{\partial CS^{CSP}}{\partial r} > 0$, $\frac{\partial SW^{CSP}}{\partial r} < 0$.

(2) The effects of k*:*

$\frac{\partial x_1{}^{CSP}}{\partial k} < 0$, $\frac{\partial x_2{}^{CSP}}{\partial k} > 0$, $\frac{\partial q_1{}^{CSP}}{\partial k} < 0$, $\frac{\partial q_2{}^{CSP}}{\partial k} > 0$, $\frac{\partial \pi_1{}^{CSP}}{\partial k} < 0$, $\frac{\partial \pi_2{}^{CSP}}{\partial k} > 0$, $\frac{\partial (x_1{}^{CSP}-q_1{}^{CSP})}{\partial k} < 0$, $\frac{\partial (x_2{}^{CSP}-q_2{}^{CSP})}{\partial k} > 0$, $\frac{\partial CS^{CSP}}{\partial k} > 0$, $\frac{\partial SW^{CSP}}{\partial k} > 0$.

Proposition 3 implies that the introduction of the shared manufacturing enterprise makes the capacities of the two enterprises positively related to r, and the other cases are consistent with Model CSS. Compared with Model CSI, in addition to capacity, the excess capacity of Enterprise 2 and r become positively correlated as well. After the introduction of the shared manufacturing enterprise, an increase in r causes a decrease in the output of Enterprise 2, but the capacities increase more significantly under the role of the shared manufacturing enterprise, leading to the aggravation of excess capacity. This indicates that capacity sharing aggravates the degree of excess capacity when production homogeneity increases. Therefore, the decision concerning whether to implement capacity sharing must still fully consider the degree of product differentiation between enterprises.

Regarding capacity constraints, the situation of Enterprise 1 is the same as that of Model CSI. The capacity and excess capacity of Enterprise 2 are positively related to constrained capacity, and other correlations are also consistent with Model CSI. The lower the capacity constraints of Enterprise 2, the lower the capacity and excess capacity of Enterprise 2, and the lower the output and profit. For Enterprise 1, capacity sharing and product differentiation play a synergistic role in jointly easing excess capacity. For Enterprise 2, the effect of shared manufacturing enterprise differs. Therefore, Enterprise 2 should make trade-offs according to its situation to stabilize capacity at an appropriate level.

3.2 Stackelberg Competition

3.2.1 Model SSS

First, we assume that both enterprises have sufficient capacity, and Enterprise 2 is the follower. The equilibrium outputs are as follows.

$$q_1{}^{SSS} = \frac{(a-b+2x_2)r-4(a-b)-8x_1}{2(r^2-8)},\quad q_2{}^{SSS} = \frac{(a-b+2x_2)(r^2-16)+4(a-b+2x_1)r}{8(r^2-8)} \tag{8}$$

Next, capacity satisfies $\frac{\partial \pi_i}{\partial x_i} = 0$, $x_1{}^{SSS}$, and $x_2{}^{SSS}$ can be obtained as

$$x_1{}^{SSS} = \frac{4(r^3-3r^2-8r+16)(a-b)}{7r^4-72r^2+128},\ x_2{}^{SSS} = \frac{(r^4+4r^3-24r^2-64r+128)(a-b)}{2(7r^4-72r^2+128)} \tag{9}$$

Lemma 4 *When both enterprises have sufficient capacity, the equilibrium results are as follows.*

$$q_1{}^{SSS} = \frac{4(r^3-3r^2-8r+16)(a-b)}{(r^2-8)(7r^2-16)},\quad q_2{}^{SSS} = \frac{(r^2+4r-8)(a-b)}{7r^2-16},\quad x_1{}^{SSS} - q_1{}^{SSS} = 0,\quad x_2{}^{SSS} - q_2{}^{SSS} = \frac{(r^2-4r-8)(a-b)r^2}{2(r^2-8)(16-7r^2)} > 0,\quad \pi_1{}^{SSS} = \frac{4[(r^8-6r^7-11r^6+104r^5-4r^4-576r^3+384r^2+1024r-1024)(a-b)^2]}{(r^2-8)(16-7r^2)(7r^4-72r^2+128)};$$

$$\pi_2{}^{SSS} = \frac{[(7r^8+56r^7-96r^6-1216r^5+704r^4+8192r^3-6144r^2-16384r+16384)(a-b)^2]}{4(r^2-8)^2(7r^2-16)^2},$$

$$CS^{SSS} = \frac{(9r^8+16r^7-288r^6-288r^5+2832r^4+1280r^3-9728r^2+8192)(a-b)^2}{4(r^2-8)^2(7r^2-16)^2};$$

$$SW^{SSS} = \frac{(9r^8+184r^7-496r^6-3456r^5+6432r^4+19968r^3-31744r^2-32768r+49152)(a-b)^2}{4(r^2-8)^2(7r^2-16)^2}.$$

Under Stackelberg competition, Enterprise 1 does not have excess capacity, while Enterprise 2 does, which means that Enterprise 1 may have the advantage of incumbents in reducing its excess capacity.

This result can guide enterprises to make decisions on their action sequences and the degree of excess capacity. Next, we analyze the impacts of product differentiation on the equilibrium results, and Proposition 4 can be proposed.

Proposition 4 *The effects of* r*:*

$\frac{\partial x_2{}^{SSS}}{\partial r} < 0,\ \frac{\partial q_2{}^{SSS}}{\partial r} < 0,\ \frac{\partial \pi_1{}^{SSS}}{\partial r} < 0,\ \frac{\partial \pi_2{}^{SSS}}{\partial r} < 0, \frac{\partial (x_1{}^{SSS} - q_1{}^{SSS})}{\partial r} = 0, \frac{\partial (x_2{}^{SSS} - q_2{}^{SSS})}{\partial r} > 0,\ \frac{\partial SW^{SSS}}{\partial r} < 0.$

If $0 < r \le 0.99$, then $\frac{\partial x_1{}^{SSS}}{\partial r} \le 0,\ \frac{\partial q_1{}^{SSS}}{\partial r} \le 0$; if $0.99 < r < 1$, then $\frac{\partial x_1{}^{SSS}}{\partial r} > 0,\ \frac{\partial q_1{}^{SSS}}{\partial r} > 0$.

If $0 < r \le 0.408$, then $\frac{\partial CS^{SSS}}{\partial r} \le 0$; if $0.408 < r < 1$, then $\frac{\partial CS^{SSS}}{\partial r} > 0$.

In Model SSS, capacity is sufficient for Enterprise 2, and the effect of r is similar to that of Model CSS; when r is at a low level, the capacity and output of Enterprise 1 decrease with it. Then, the degree of excess capacity becomes irrelevant to r, and consumer surplus is positively related to r after a negative correlation. The critical value is 0.408. In other words, when r increases, the capacity, output, and excess capacity of Enterprise 2 and the profits of both enterprises decrease. At this point, the two enterprises still have the motivation to improve product differentiation.

3.2.2 Model SSI

By changing the assumption of sufficient capacity to insufficient capacity for Enterprise 2, we obtain the following equilibrium outputs:

$$q_1{}^{SSI} = \frac{(a-b+2x_2)r - 4(a-b) - 8x_1}{2(r^2-8)},\ q_2{}^{SSI} = \frac{(a-b+2x_2)(r^2-16) + 4(a-b+2x_1)r}{8(r^2-8)} \tag{10}$$

Next, we consider the first stage of the game. The capacity of Enterprise 2 is insufficient, that is, $k < \frac{(r^4+4r^3-24r^2-64r+128)(a-b)}{2(7r^4-72r^2+128)}$, and the capacity of Enterprise 2 can be expressed as follows:

$$x_2 = \begin{cases} k,\ 0 \le x_1 \le \frac{(a-b+2k)r - 4(a-b)}{2(r^2-4)} \\ \frac{(a-b)(r^4+4r^3-32r^2-64r-256) + 8x_1 r(r^2-16)}{2(7r^4-96r^2+256)}, x_1 > \frac{(a-b+2k)r - 4(a-b)}{2(r^2-4)} \end{cases} \tag{11}$$

$$x_1{}^{SSI} = \frac{(a-b+2k)r - 4(a-b)}{2(r^2-4)},\ x_2{}^{SSI} = k \tag{12}$$

Lemma 5 *The equilibrium outcomes are as follows.*

$q_1{}^{SSI} = \frac{(r-4)(a-b)+2kr}{2(r^2-4)},\ q_2{}^{SSI} = \frac{(r^2+4r-8)(a-b) + 2k(r^2-16)}{8(r^2-4)}$;

$x_1{}^{SSI} - q_1{}^{SSI} = 0,\ x_2{}^{SSI} - q_2{}^{SSI} = \frac{(r^2+4r-8)(a-b) - 2k(3r^2-8)}{8(4-r^2)}$;

If $0 < k \leq \frac{(a-b)(r^2+r-8)}{2(3r^2-8)}$, then $x_2{}^{SSI} - q_2{}^{SSI} \leq 0$; if $\frac{(a-b)(r^2+r-8)}{2(3r^2-8)} < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$, then $x_2{}^{SSI} - q_2{}^{SSI} > 0$.

$$\pi_1{}^{SSI} = \frac{(a-b)^2(r^2-8r+16)+4kr[(a-b)(r-4)+kr]}{16(4-r^2)};$$

$$\pi_2{}^{SSI} = \frac{(a-b)^2(r^4+8r^3-64r+64)+4k[(r^4+4r^3-16r^2-32r+64)(a-b)]-4k^2(7r^4-48r^2+64)}{32(r^2-4)^2};$$

$$CS^{SSI} = \frac{(a-b)^2(9r^4+8r^3-176r^2+64r+320)+4k[(9r^4+4r^3-64r^2+32r+64)(a-b)]+4k^2(9r^4-64r^2+64)}{128(r^2-4)^2};$$

$$SW^{SSI} = \frac{\begin{array}{c}(a-b)^2(5r^4+104r^3-272r^2-448r+1088)+4k[(5r^4+52r^3-96r^2-224r+320)(a-b)] \\ -4k^2(27r^4-160r^2+192)\end{array}}{128(r^2-4)^2}.$$

When Enterprise 2's capacity is below a certain level of ($\frac{(a-b)(r^2+r-8)}{2(3r^2-8)}$), the capacity is insufficient, and the capacity and output are matched at this level. It can be seen that the capacity ceiling of Enterprise 2 is closely related to the degree of excess capacity. For Enterprise 1, consistent with the first case, the capacity is equal to the output.

Proposition 5

(1) The effects of r*:*

$\frac{\partial x_2{}^{SSI}}{\partial r} = 0$, $\frac{\partial q_2{}^{SSI}}{\partial r} < 0$, $\frac{\partial \pi_1{}^{SSI}}{\partial r} < 0$, $\frac{\partial \pi_2{}^{SSI}}{\partial r} < 0$, $\frac{\partial (x_1{}^{SSI} - q_1{}^{SSI})}{\partial r} = 0$, $\frac{\partial (x_2{}^{SSI} - q_2{}^{SSI})}{\partial r} > 0$, $\frac{\partial CS^{SSI}}{\partial r} > 0$, $\frac{\partial SW^{SSI}}{\partial r} < 0$;

If $0 < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$ and $0 < r \leq 4 - 2\sqrt{3}$, then $\frac{\partial x_1{}^{SSI}}{\partial r} \leq 0, \frac{\partial q_1{}^{SSI}}{\partial r} \leq 0$;

If $0 < k \leq \frac{(a-b)(4+8r-r^2)}{2(r^2+4)}$ and $4 - 2\sqrt{3} < r < 1$, then $\frac{\partial x_1{}^{SSI}}{\partial r} \geq 0, \frac{\partial q_1{}^{SSI}}{\partial r} \geq 0$;

If $\frac{(a-b)(4+8r-r^2)}{2(r^2+4)} < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$ and $4 - 2\sqrt{3} < r < 1$, then $\frac{\partial x_1{}^{SSI}}{\partial r} < 0$, $\frac{\partial q_1{}^{SSI}}{\partial r} < 0$.

(2) The effects of k:

$\frac{\partial x_1{}^{SSI}}{\partial k} < 0$, $\frac{\partial x_2{}^{SSI}}{\partial k} > 0$, $\frac{\partial q_1{}^{SSI}}{\partial k} < 0, \frac{\partial q_2{}^{SSI}}{\partial k} > 0$, $\frac{\partial \pi_1{}^{SSI}}{\partial k} < 0$, $\frac{\partial \pi_2{}^{SSI}}{\partial k} > 0$, $\frac{\partial (x_1{}^{SSI} - q_1{}^{SSI})}{\partial k} = 0$, $\frac{\partial (x_2{}^{SSI} - q_2{}^{SSI})}{\partial k} > 0$, $\frac{\partial CS^{SSI}}{\partial k} > 0$, $\frac{\partial SW^{SSI}}{\partial k} > 0$.

In Model SSI, the relationship between the profit and r of the two enterprises is similar to that in Model SSS, but consumer surplus becomes positively correlated with r, and the relationship between the capacity and output of Enterprise 1 is more complex. When r is at a low level, the capacity and output of Enterprise 1 are negatively correlated with product homogeneity. When r is at a very high level, the relationship between them may be influenced by k. When Enterprise 2 capacity is insufficient, the capacity and output of Enterprise 1 and r are positively related. When the capacity shortage of Enterprise 2 is not serious, the capacity and output of Enterprise 1 are negatively correlated with r. In other words, only when r is high and the shortage of capacity of Enterprise 2 is more serious will Enterprise 1 still increase its capacity and output with an increase in r; otherwise, Enterprise 1 will reduce its capacity and output. These results have significant implications for enterprises when making capacity and production decisions depending on r.

Except for the relationship between the degree of excess capacity and capacity constraints, the other relationships are the same as in Model CSI. The excess capacity of Enterprise 1 is not affected by the constrained capacity of Enterprise 2. The excess capacity of Enterprise 2, consumer surplus, and social welfare are positively correlated with the constrained output of Enterprise 2. The greater the capacity constraint of Enterprise 2, the lower its excess capacity, but its output and profit will decrease, which will lead to a decline in consumer surplus and social welfare. Furthermore, the capacity constraints of Enterprise 2 will increase its degree of excess capacity, but its profit will decline; therefore, its willingness to increase capacity is still strong.

3.2.3 Model SSP

For simplicity, we define $A = a - b$ and $B = e - Q$.

First, in considering the second stage, the equilibrium outputs are derived as

$$q_1{}^{SSP} = \frac{(A+2x_2)r-4A-8x_1}{2(r^2-8)}, \quad q_2{}^{SSP} = \frac{(A+2x_2)(r^2-16)+4(A+2x_1)r}{8(r^2-8)} \tag{13}$$

Next, the first stage of the game is analyzed, and $\frac{\partial \pi_i}{\partial x_i} = 0$ is satisfied. The equilibrium outputs are obtained as

$$x_1{}^{SSP} = -\frac{4[(r^2-16)A-(B+2k)(r^3-8r)]}{r^4-24r^2+128}, \; x_2{}^{SSP} = -\frac{(r^4+4r^3-24r^2-64r+128)A-8(B+2k)(r^4-12r^2+32)}{2(r^4-24r^2+128)} \tag{14}$$

Lemma 6 *When the two enterprises have sufficient capacity, the equilibrium results are as follows.*

$q_1{}^{SSP} = -\frac{4[(r^2-16)A-(B+2k)(r^3-8r)]}{r^4-24r^2+128}, \; q_2{}^{SSP} = \frac{(B+2k)(r^2-8)}{r^2-16}$;

$x_1{}^{SSP} - q_1{}^{SSP} = 0, \; x_2{}^{SSP} - q_2{}^{SSP} = \frac{2(3r^4-32r^2+64)(B+2k)-(r^4-4r^3-24r^2-64r+128)A}{2(8-r^2)(16-r^2)}$;

If $0 < k \le \frac{(r^4-4r^3-24r^2-64r+128)A-2(3r^4-32r^2+64)B}{2(8-r^2)(16-r^2)}$, then $x_2{}^{SSP} - q_2{}^{SSP} \le 0$;

If $k > \frac{(r^4-4r^3-24r^2-64r+128)A-2(3r^4-32r^2+64)B}{2(8-r^2)(16-r^2)}$, then $x_2{}^{SSP} - q_2{}^{SSP} > 0$;

$$\pi_1{}^{SSP} = \frac{4\begin{bmatrix}(r^6-36r^4+384r^2-1024)A^2-(B+2k)A(2r^7-56r^5+448r^3-1024r)+ \\ (r^8-20r^6+104r^5-128r^4-256r^2)(B+2k)^2\end{bmatrix}}{(r^2-8)(16-r^2)(r^4-24r^2+128)};$$

$$\pi_2{}^{SSP} = \frac{\begin{matrix}(r^6+4r^5-40r^4-128r^3+512r^2+1024r-2048)A(B+2k)- \\ 4(r^6-32r^4+256r^2-512)B^2-2(7r^6-216r^4+1536r^2 2048)(B+k)k\end{matrix}}{2(r^2-8)(r^2-16)^2};$$

$$CS^{SSP} = \frac{\begin{matrix}16(r^4-32r^2+256)A^2-8(r^7-28r^5+224r^3-512r)A(B+2k)+ \\ (9r^8-208r^6+1664r^4-5120r^2+4096)(B+2k)^2\end{matrix}}{2(r^2-8)^2(r^2-16)^2};$$

$$SW^{SSP} = \frac{\begin{matrix}8(r^6-38r^4+448r^2-1536)A^2- \\ (r^8+12r^7-48r^6-384r^5+832r^4+3840r^3-6144r^2-12288r+16384)A(B+2k) \\ +(3r^8-112r^6+1408r^4-7168r^2+12288)B^2+2(5r^8-176r^6+1984r^4-8192r^2+8192)(B+2k)k\end{matrix}}{-2(r^2-8)^2(r^2-16)^2}.$$

It can be seen that Enterprise 1 still lacks the excess capacity for Enterprise 1. For Enterprise 2, similar to Model SSI, the degree of excess capacity depends on the level of limited capacity. The difference is that the cut-off point is $k = \frac{(r^4-4r^3-24r^2-64r+128)A-2(3r^4-32r^2+64)B}{2(8-r^2)(16-r^2)}$, which is lower than that in Model SSI.

To ensure that both the shared manufacturing enterprise and the enterprises that purchase capacity are profitable, they need to satisfy $0 < \mathrm{X} \le \frac{1}{3}e - Q$ and $\pi_2{}^{\mathrm{SSP}} > \pi_2{}^{SSP}$, and the range of k is obtained. Similar to Model CSP, if $0 < r < r^{SSP}$, the value range of k is $(k_1{}^{SSP}, k_2{}^{SSP})$, and if $r^{\mathrm{SSP}} \le r < 1$, $k \in (k_1{}^{SSP}, k_3{}^{SSP})$.

The purchasing capacity is $X = \frac{8(r^4-12r^2+32)B+2(7r^4-72r^2+128)k-(r^4+4r^3-24r^2-64r+128)A}{2(r^4-24r^2+128)}$.

Proposition 6

(1) The effects of r*:*

$\frac{\partial x_1{}^{SSP}}{\partial r} < 0, \frac{\partial x_2{}^{SSP}}{\partial r} > 0, \frac{\partial q_1{}^{SSP}}{\partial r} < 0, \frac{\partial q_2{}^{SSP}}{\partial r} < 0$ $\frac{\partial(x_1{}^{SSP}-q_1{}^{SSP})}{\partial r} = 0, \frac{\partial(x_2{}^{SSP}-q_2{}^{SSP})}{\partial r} > 0$， $, \frac{\partial \pi_1{}^{SSP}}{\partial r} < 0, \frac{\partial \pi_2{}^{SSP}}{\partial r} < 0, \frac{\partial CS^{SSP}}{\partial r} > 0, \frac{\partial SW^{SSP}}{\partial r} < 0.$

(2) The effects of k:

$\frac{\partial x_1{}^{SSP}}{\partial k} < 0$， $\frac{\partial x_2{}^{SSP}}{\partial k} > 0$， $\frac{\partial q_1{}^{SSP}}{\partial k} < 0$， $\frac{\partial q_2{}^{SSP}}{\partial k} > 0$， $\frac{\partial \pi_1{}^{SSP}}{\partial k} < 0$， $\frac{\partial \pi_2{}^{SSP}}{\partial k} > 0$， $\frac{\partial(x_1{}^{SSP}-q_1{}^{SSP})}{\partial k} = 0$， $\frac{\partial(x_2{}^{SSP}-q_2{}^{SSP})}{\partial k} > 0$, $\frac{\partial CS^{SSP}}{\partial k} < 0, \frac{\partial SW^{SSP}}{\partial k} > 0.$

With the emergence of the shared manufacturing enterprise, the capacity, output, and social welfare of Enterprise 1 are negatively correlated with r, and other correlations are consistent with Model SSI. With a decrease in r, the capacity, output, and profit of Enterprise 1 will increase, while the capacity and consumer surplus of Enterprise 2 will decrease, but the output, profit, and social welfare will increase. Therefore, after the emergence of the shared manufacturing enterprise, the improvement of product differentiation is beneficial to both enterprises and society. This indicates that after the introduction of shared manufacturing enterprises, product differentiation is still an effective means for enterprises to improve consumer surplus, increase their profits and social welfare.

The capacity constraints of Enterprise 2 are positively correlated with capacity and negatively correlated with consumer surplus, while the other correlations are the same as in Model SSI. If the capacity constraints decrease, the capacity, output, profit, and excess capacity of Enterprise 2 decrease accordingly, but the capacity, output, and profit of Enterprise 1 increase. Its capacity and output can also be coordinated. In addition, consumer surplus increases, but social welfare decreases because Enterprise 2's profit is affected by the constraints, which is more than the positive effects of Enterprise 1 and consumers. Therefore, a too strict capacity constraint is not conducive to the promotion of social welfare.

3.3 Comparison

Comparing the three cases under Cournot competition, including excess capacity, profit, consumer surplus, and social welfare, Corollary 1 can be proposed.

Corollary 1 *Comparison results under Cournot model*

$({x_1}^{CSI} - {q_1}^{CSI}) > ({x_1}^{CSS} - {q_1}^{CSS}) > ({x_1}^{CSP} - {q_1}^{CSP})$;

$({x_2}^{CSS} - {q_2}^{CSS}) > ({x_2}^{CSP} - {q_2}^{CSP}) > ({x_2}^{CSI} - {q_2}^{CSI})$;

${\pi_1}^{CSI} > {\pi_1}^{CSP} > {\pi_1}^{CSS}$, ${\pi_2}^{CSS} > {\pi_2}^{CSP} > {\pi_2}^{CSI}$;

$CS^{CSS} > CS^{CSP} > CS^{CSI}$, $SW^{CSS} > SW^{CSP} > SW^{CSI}$.

From Corollary 1, it can be seen that the level of excess capacity of Enterprise 1 is the lowest when Enterprise 2 has the insufficient capacity with the shared manufacturing enterprise under Cournot competition because shared manufacturing helps improve equipment efficiency and stable production. However, Enterprise 1's profit is the highest when Enterprise 2 has insufficient capacity in the absence of a shared manufacturing enterprise. The degree of excess capacity and profits of Enterprise 2, as well as the consumer surplus and social welfare, are the highest when both enterprises have sufficient capacity when Enterprise 2 has the insufficient capacity with shared manufacturing enterprise is the second high, and when Enterprise 2 has the insufficient capacity with the shared manufacturing enterprise is the lowest. It can be seen that during Cournot competition, Enterprise 1 is in an advantageous position when Enterprise 2 has insufficient capacity in the absence of a shared manufacturing enterprise. The emergence of the shared manufacturing enterprise weakens this advantage and improves the situation of Enterprise 2, consumer surplus, and social welfare. Therefore, the introduction of shared manufacturing improves the degree of excess capacity and profit of enterprises with capacity constraints (Enterprise 2) and reduces the degree of excess capacity of Enterprise 1. Moreover, consumers and society benefit from capacity sharing.

Under Stackelberg competition, including the degree of excess capacity, profits, consumer surplus, and social welfare, a comparative analysis of the three models is obtained as follows.

Corollary 2 *Comparison results under Stackelberg competition*

$(x_1{}^{SSS} - q_1{}^{SSS}) = (x_1{}^{SSI} - q_1{}^{SSI}) = (x_1{}^{SSP} - q_1{}^{SSP}) = 0;$

$(x_2{}^{SSS} - q_2{}^{SSS}) > (x_2{}^{SSP} - q_2{}^{SSP}) > (x_2{}^{SSI} - q_2{}^{SSI});$

$\pi_1{}^{SSI} > \pi_1{}^{SSP} > \pi_1{}^{SSS},\ \pi_2{}^{SSS} > \pi_2{}^{SSP} > \pi_2{}^{SSI};$

$CS^{SSI} > CS^{SSS} > CS^{SSP},\ SW^{SSI} > SW^{SSS} > SW^{SSP}.$

Unlike Corollary 1, under Stackelberg competition, the excess capacity of Enterprise 1 does not exist in all three cases. Consumer surplus and social welfare are higher when Enterprise 2 has insufficient capacity in the absence of a shared manufacturing enterprise than when both enterprises have sufficient capacity and when Enterprise 2 has the insufficient capacity with a shared manufacturing enterprise. Therefore, the fact that both enterprises have sufficient capacity is better for Enterprise 2, consumer surplus, and social welfare, explaining the widespread excess capacity phenomenon.

By comparing the excess capacity, profit, consumer surplus, and social welfare under the two competition modes, we obtain Corollary 3.

Corollary 3 *Comparison results between two competition modes*

$(x_1{}^{CSS} - q_1{}^{CSS}) > (x_1{}^{SSS} - q_1{}^{SSS}),\ (x_2{}^{CSS} - q_2{}^{CSS}) < (x_2{}^{SSS} - q_2{}^{SSS});$

$\pi_1{}^{CSS} < \pi_1{}^{SSS},\ \pi_2{}^{CSS} > \pi_2{}^{SSS},\ CS^{CSS} < CS^{SSS},\ SW^{CSS} < SW^{SSS};$

$(x_1{}^{CSI} - q_1{}^{CSI}) > (x_1{}^{SSI} - q_1{}^{SSI}),\ (x_2{}^{CSI} - q_2{}^{CSI}) < (x_2{}^{SSI} - q_2{}^{SSI});$

$\pi_1{}^{CSI} < \pi_1{}^{SSI},\ \pi_2{}^{CSI} > \pi_2{}^{SSI},\ CS^{CSI} < CS^{SSI}, SW^{CSI} < SW^{SSI};$

$(x_1{}^{SSP} - q_1{}^{SSP}) > (x_1{}^{CSP} - q_1{}^{CSP}),\ (x_2{}^{SSP} - q_2{}^{SSP}) < (x_2{}^{CSP} - q_2{}^{CSP});$

$\pi_1{}^{CSP} > \pi_1{}^{SSP},\ \pi_2{}^{CSP} > \pi_2{}^{SSP},\ CS^{CSP} < CS^{SSP},\ SW^{CSP} < SW^{SSP}.$

Corollary 3 shows that in the three cases, the excess capacity of Enterprise 1 and the profit of Enterprise 2 are higher under Cournot than Stackelberg competition, while the excess capacity, consumer surplus, and social welfare of Enterprise 2 are higher under Stackelberg competition. Enterprise 1's profit is more complex than the above. It is higher under the Stackelberg competition in Model SSS and Model SSI but changes after the emergence of the shared manufacturing enterprise. Hence, shared manufacturing enterprises will not only affect the degree of excess capacity, corporate profits, consumer surplus, and social welfare but also have an impact on the enterprises' competition mode selection.

4 Conclusion

This study examines the market equilibrium results in the three scenarios under Cournot and Stackelberg competitions and investigates the effects of product differentiation and capacity sharing. The main conclusions are summarized as follows.

First, capacity constraints affect the relationship between product differentiation and equilibrium results, thereby having an impact on the market share of enterprises. Under Cournot competition, consumer surplus changes from positive and then negative to an always positive correlation with r. When r increases, the output of Enterprise 2 is reduced owing to its capacity constraints, thereby increasing consumer surplus. Under Stackelberg competition, only when r is at a high level and Enterprise 2's capacity is seriously insufficient will Enterprise 1 still increase its capacity and output with an increase in r to occupy Enterprise 2's market share and increase profit. Hence, when the capacity constraint is lower than a certain threshold, it promotes the governance of excess capacity and the improvement of consumer surplus. However, when the capacity constraint is more serious, it threatens the survival and development of enterprises with insufficient capacity.

Second, the introduction of a shared manufacturing enterprise affects the degree of excess capacity, corporate profits, consumer surplus, and social welfare. Under the restriction that platform and Enterprise 2 are both profitable, shared manufacturing enterprises can exist and have an impact on equilibrium results. Under Cournot competition, after introducing a shared manufacturing enterprise, the degree of excess capacity, consumer surplus, and social welfare are higher than that of Enterprise 2, which has insufficient capacity without a shared manufacturing enterprise. However, under Stackelberg competition, the results are reversed. According to the results, the government should conduct an analysis before designing and issuing policies to support the development of the shared manufacturing enterprise.

Third, the competition mode affects the equilibrium results, and the difference in the equilibrium results further influences the decision in the competition mode. Under the two cases of competition with sufficient capacity and capacity constraints and in the absence of a shared manufacturing enterprise, the profit of Enterprise 1 under Stackelberg competition is higher than under Cournot competition. However, after the emergence of a shared manufacturing enterprise, the situation may change. Under Cournot competition, enterprises with capacity constraints can obtain a favorable competitive situation and higher profit, regardless of whether a shared manufacturing enterprise is introduced or not. The results provide a reference for enterprises to select a competition mode according to the choice of competitors from the perspective of strategy, considering capacity constraints and shared manufacturing enterprises.

APPENDIX A

Proof of Proposition 1

The effects of r:

$\frac{\partial {x_i}^{CSS}}{\partial r}=\frac{16(a-b)(3r^2+8r-16)}{(32+16r-4r^2-r^3)^2}<0$; $\frac{\partial {q_i}^{CSS}}{\partial r}=\frac{(64r+32r^2-256-r^4)(a-b)}{(32+16r-4r^2-r^3)^2}<0$; $\frac{\partial ({x_i}^{CSS}-{q_i}^{CSS})}{\partial r}=\frac{r(r^3+16r+64)(a-b)}{(32+16r-4r^2-r^3)^2}>0$; $\frac{\partial {\pi_i}^{CSS}}{\partial r}=\frac{4[(r^6-48r^4-64r^3+384r^2-2048)(a-b)^2]}{(32+16r-4r^2-r^3)^3}<0$.

$\frac{\partial CS^{CSS}}{\partial r}=\frac{(r^6-2r^5-48r^4-64r^3+640r^2+1536r-2048)r(a-b)^2}{(32+16r-4r^2-r^3)^3}$, if $0<r\le 0.99$, then $\frac{\partial CS^{CSS}}{\partial r}<0$; if $0.99<r<1$, then $\frac{\partial CS^{CSS}}{\partial r}>0$.

$\frac{\partial SW^{CSS}}{\partial r}=\frac{(r^7+6r^6-48r^5-448r^4+128r^3+4608r^2-2048r-16384)(a-b)^2}{(32+16r-4r^2-r^3)^3}<0$.

APPENDIX B

Proof of Proposition 2

(1) The effects of r:

$\frac{\partial {x_1}^{CSI}}{\partial r}=\frac{16[(3r^4-16r^3-32r^2+256r-128)(a-b)+2(3r^4-32r^2-128)k]}{(r^4-32r^2+128)^2}<0$, $\frac{\partial {x_2}^{CSI}}{\partial r}=0$;

$\frac{\partial {q_1}^{CSI}}{\partial r}=-\frac{(r^6-8r^5-16r^4+256r^3+128r^2-3072r+2048)(a-b)+2(r^6-16r^4+128r^2+2048)k}{(r^4-32r^2+128)^2}<0$;

$\frac{\partial {q_2}^{CSI}}{\partial r}=-\frac{(r^6-8r^5-16r^4+128r^3+128r^2-1024r+2048)(a-b)-16(r^5-16r^3+128r)k}{(r^4-32r^2+128)^2}<0$;

$\frac{\partial {\pi_1}^{CSI}}{\partial r}=-\frac{4[(r^5-12r^4+32r^3+128r^2-640r+512)(a-b)^2+4(r^5-6r^4+64r^2-128r+256)(a-b)k+4(r^5-128r)k^2]}{(r^4-32r^2+128)^2}<0$;

$$\frac{\partial {\pi_2}^{CSI}}{\partial r}=\frac{4\begin{bmatrix}(r^9-12r^8+352r^6-384r^5-4096r^4+8192r^3+12288r^2-65536r+65536)(a-b)^2\\-8(3r^8-16r^7-88r^6+384r^5+1024r^4-4096r^3-3072r^2+16384r-16384)(a-b)k+\\128(r^7-24r^5+256r^3-1024r)k^2\end{bmatrix}}{-(r^4-32r^2+128)^3}<0;$$

$\frac{\partial({x_1}^{CSI}-{q_1}^{CSI})}{\partial r}=\frac{r[(r^5-8r^4+32r^3-384r+1024)(a-b)+2rk(r^4+32r^2-384)]}{(r^4-32r^2+128)^2}>0$;

$\frac{\partial({x_2}^{CSI}-{q_2}^{CSI})}{\partial r}=\frac{(r^6-8r^5-16r^4+128r^3+128r^2-1024r+2048)(a-b)-16(r^5-16r^3+128r)k}{(r^4-32r^2+128)^2}>0$;

$$\frac{\partial CS^{CSI}}{\partial r}=\frac{\begin{matrix}(r^{10}-14r^9+24r^8+384r^7-1184r^6-4608r^5+15872r^4+20480r^3-86016r^2+98304r-65536)(a-b)^2\\+2(r^{10}-14r^9+24r^8+256r^7-1184r^6-768r^5+15872r^4-16384r^3-86016r^2+131072r-65536)(a-b)k\\-4(7r^9-128r^7+384r^5+8192r^3-65536r)k^2\end{matrix}}{-(r^4-32r^2+128)^3}>0;$$

$$\frac{\partial SW^{CSI}}{\partial r}=\frac{\begin{matrix}(r^{10}-6r^9-72r^8+384r^7-2272r^6-12288r^5-20992r^4+151552r^3-36864r^2-491520r+458752)(a-b)^2\\+2(r^{10}-6r^9-72r^8+25r^7-2272r^6-6912r^5-20992r^4+81920r^3-36864r^2-262144r+458752)(a-b)k\\-4(3r^9-128r^7+3456r^5-40960r^3+131072r)k^2\end{matrix}}{-(r^4-32r^2+128)^3}<0.$$

(2) The effects of k:

$\frac{\partial x_1{}^{CSI}}{\partial k}=\frac{-32r}{r^4-32r^2+128}<0$, $\frac{\partial x_2{}^{C2}}{\partial k}=1>0$; $\frac{\partial q_1{}^{CSI}}{\partial k}=\frac{2r(r^2-16)}{r^4-32r^2+128}<0$, $\frac{\partial q_2{}^{C2}}{\partial k}=\frac{-8(r^2-8)}{r^4-32r^2+128}>0$;

$\frac{\partial \pi_1{}^{CSI}}{\partial k}=\frac{8r[(a-b)(r-4)+2kr]}{r^4-32r^2+128}<0$, $\frac{\partial \pi_2{}^{C2}}{\partial k}=\frac{-2\left[\begin{array}{c}16(r^5-4r^4-24r^3+64r^2+128r-256)(a-b)+\\(r^8-64r^6+1152r^4-6144r^2+8192)k\end{array}\right]}{(r^4-32r^2+128)^2}>0$;

$\frac{\partial (x_1{}^{CSI}-q_1{}^{CSI})}{\partial k}=-\frac{2r^3}{r^4-32r^2+128}<0$, $\frac{\partial (x_2{}^{C2}-q_2{}^{C2})}{\partial k}=\frac{r^4-24r^2+64}{r^4-32r^2+128}>0$;

$\frac{\partial CS^{CSI}}{\partial k}=\frac{2[(r^7-7r^6-24r^5+176r^4+96r^3-1024r^2+512r+1024)(a-b)-2(7r^6-176r^4+1024r^2-1024)k]}{(r^4-32r^2+128)^2}>0$;

$\frac{\partial SW^{CSI}}{\partial k}=\frac{2\left[\begin{array}{c}(r^7-3r^6-56r^5+112r^4+992r^3-1536r^2-3584r+5120)(a-b)\\-(r^8-58r^6+1056r^4-5120r^2+6144)k\end{array}\right]}{(r^4-32r^2+128)^2}>0.$

Appendix C

Proof of Proposition 3

$\pi_2{}^{CSP}-\pi_2{}^{CSI}=$

$$\frac{\begin{array}{c}\{1024(r^8-8r^7-24r^6+256r^5+128r^4-2560r^3+1024r^2+8192r-8192)A^2+\\64(r^{11}-4r^{10}-80r^9+288r^8+2304r^7-7168r^6-28673r^5+73728r^4+147456r^3-327680r^2-262144r+524288)AB\\+128(r^{11}-4r^{10}-80r^9+288r^8+2176r^7-6656r^6-24576r^5+61440r^4+106496r^3-229376r^2-131072r+262144)Ak\\+(r^{14}-120r^{12}+5632r^{10}-131072r^8+1589248r^6-9830400r^4+29360128r^2-33554432)(B+2k)^2\}\end{array}}{512(128-32r^2+r^4)^2(8-r^2)}$$

From $\pi_2{}^{CSP}-\pi_2{}^{CSI}=0$, we can obtain:

$k_1{}^{CSP}=\frac{(-32r^3+128r^2+512r-1024)A+(-r^6+48r^4-640r^2+2048)B}{2(r^6-48r^4+512r^2-1024)}$;

$k_2{}^{CSP}=\frac{(-32r^5+128r^4+768r^3-2048r^2-4096r+8192)A+(-r^8+72r^6-1536r^4+9216r^2-16384)B}{2(r^8-72r^6+1408r^4-7168r^2+8192)}$.

From $0<X^{CSP}<\frac{1}{3}e-Q$, we derive $k_1{}^{CSP}<k<k_2{}^{CSP}$, where $k_3{}^{CSP}=-\frac{96(r^3-4r^2-16r+32)A+(3r^6-144r^4+1664r^2-4096)B+512(r^2-8)Q}{6(r^6-48r^4+512r^2-1024)}$. Thus, $k_1{}^{CSP}<k<k_3{}^{CSP}$.

(1) The effects of r:

$\frac{\partial x_1{}^{CSP}}{\partial r}=\frac{64Ar-(B+2k)(r^4-8r^2+128)}{8(r^2-8)^2}>0$;

$\frac{\partial x_2{}^{CSP}}{\partial r}=\frac{(1024-64r^2+8r^4)A+(B+2k)(r^7-36r^5+384r^3-1536r)}{64(r^2-8)^2}>0$;

$\frac{\partial q_1{}^{CSP}}{\partial r}=\frac{512Ar+(B+2k)(3r^6-72r^4+512r^2-2048)}{128(r^2-8)^2}<0$, $\frac{\partial q_2{}^{CSP}}{\partial r}=-\frac{(B+2k)r}{16}<0$;

$\frac{\partial \pi_1{}^{CSP}}{\partial r}=\frac{\begin{array}{c}163843A^2r^3+32(B+2k)(3r^8-104r^6+1280r^4-9216r^2+16384)\\+(B+2k)^2(3r^{11}-168r^9+3456r^7-33792r^5+163840r^3-262144r)\end{array}}{4096(r^2-8)^3}<0$;

$$\frac{\partial \pi_2{}^{CSP}}{\partial r}=-\frac{16A(B+2k)\left(r^4-128r^2+2048\right)+(B+2k)^2(r^7-40r^5+448r^3-2048)}{128(r^2-8)^2}<0;$$

$$\frac{\partial\left(x_1{}^{CSP}-q_1{}^{CSP}\right)}{\partial r}=\frac{r\left[512A-(B+2k)(3r^5-56r^3+384r)\right]}{128(r^2-8)^2}>0;$$

$$\frac{\partial\left(x_2{}^{CSP}-q_2{}^{CSP}\right)}{\partial r}=\frac{\left(1024-64r^2+8r^4\right)A-(B+2k)(r^7-32r^5+320r^3-1280r)}{64(r^2-8)^2}>0;$$

$$\frac{\partial CS^{CSP}}{\partial r}=\frac{\begin{array}{c}16384A^2r\left(r^2-16\right)-32\left(9r^8-280r^6+3200r^4-18432r^2+32768\right)A(B+2k)\\-(21r^{10}-1080r^8+21248r^6-202752r^4+96656r^2-1835008)(B+2k)^2r\end{array}}{16384(r^2-8)^3}>0;$$

$$\frac{\partial SW^{CSP}}{\partial r}=\frac{\begin{array}{c}16384\left(5r^2-16\right)A^2r+32\left(3r^8-200r^6+2944r^4-30720r^2+98304\right)A(B+2k)\\-(9r^{10}-280r^8+1280r^6+30720r^4-409600r^2+1310720)(B+2k)^2r\end{array}}{16384(r^2-8)^3}<0.$$

(2) The effects of k:

$$\frac{\partial x_1{}^{CSP}}{\partial k}=-\frac{r\left(r^2-16\right)}{4(r^2-8)}<0,\ \frac{\partial x_3{}^{CSP}}{\partial k}=\frac{r^6-48r^4+640r^2-2048}{128(r^2-8)}>0;$$

$$\frac{\partial q_1{}^{CSP}}{\partial k}=\frac{r(r^4-32r^2+256)}{64(r^2-8)}<0,\quad \frac{\partial q_2{}^{CSP}}{\partial k}=-\frac{1}{16}r^2+1>0;$$

$$\frac{\partial \pi_1{}^{CSP}}{\partial k}=\frac{r\left[32\left(r^6-48r^4+640r^2-2048\right)A+(B+2k)\left(r^9-64r^7+1408r^5-12288r^3+32768r\right)\right]}{2048(r^2-8)^2}<0;$$

$$\frac{\partial \pi_2{}^{CSP}}{\partial k}=\frac{32A\left(r^3-4r^2-16r+32\right)+(B+2k)(r^6-56r^4+640r^2-1024)}{128(8-r^2)}>0;$$

$$\frac{\partial(x_1{}^{CSP}-q_1{}^{CSP})}{\partial k}=-\frac{r^3\left(r^2-16\right)}{64(r^2-8)}<0,\ \frac{\partial(x_2{}^{CSP}-q_2{}^{CSP})}{\partial k}=\frac{r^6-40r^4+448r^2-1024}{128(r^2-8)}>0;$$

$$\frac{\partial CS^{CSP}}{\partial k}=\frac{32Ar\left(4096-1280r^2+112r^4-3r^6\right)-(B+2k)\left(7r^{10}-400r^8+8448r^6-78848r^4+294912r^2-262144\right)}{8192(r^2-8)^2}>0;$$

$$\frac{\partial SW^{CSP}}{\partial k}=\frac{\begin{array}{c}32A\left(r^7-144r^5+256r^4+2816r^3-4096r^2-12288r+16384\right)\\-(B+2k)\left(3r^{10}-80r^8-1280r^6+39936r^4-229376r^2+262144\right)\end{array}}{8192(r^2-8)^2}>0.$$

Appendix D

Proof of Proposition 4

The effects of r:

$\frac{\partial x_1{}^{SSS}}{\partial r}=-\frac{4\left[\left(7r^6-42r^5-96r^4+448r^3+192r^2-1536r+1024\right)(a-b)\right]}{(7r^4-72r^2+128)^2}$, if $0<r\le 0.99$, then $\frac{\partial x_1{}^{SSS}}{\partial r}\le 0$; if $0.99<r<1$, then $\frac{\partial x_1{}^{SSS}}{\partial r}>0$;

$$\frac{\partial x_2{}^{SSS}}{\partial r}=-\frac{2\left[\left(7r^6-48r^5-264r^4+768r^3+768r^2-3072r+2048\right)(a-b)\right]}{(7r^4-72r^2+128)^2}<0;$$

$\frac{\partial q_1{}^{SSS}}{\partial r}=-\frac{4[(7r^6-42r^5-96r^4+448r^3+192r^2-1536r+1024)(a-b)]}{(r^2-8)^2(7r^2-16)^2}$, if $0<r\le 0.99$, then $\frac{\partial q_1{}^{S1}}{\partial r}\le 0$; if $0.99<r<1$, then $\frac{\partial q_1{}^{S1}}{\partial r}>0$. $\frac{\partial q_2{}^{SSS}}{\partial r}=-\frac{4\left[(7r^2-20r+16)(a-b)\right]}{(7r^2-16)^2}<0$;

$$\frac{\partial \pi_1{}^{SSS}}{\partial r} = \frac{8[(21r^{10}-67r^9-444r^8+1360r^7+3648r^6-12288r^5-12544r^4+55296r^3-98304r+65536)(a-b)^2]}{(r^2-8)(16-7r^2)(7r^4-72r^2+128)^2} < 0;$$

$$\frac{\partial \pi_2{}^{SSS}}{\partial r} = \frac{-2\left[\begin{pmatrix}49r^{10}+84r^9-1680r^8-160r^7+18624r^6-23040r^5-76800r^4+\\180224r^3+49152r^2-39316r+262144\end{pmatrix}(a-b)^2\right]}{(r^2-8)^3(7r^2-16)^3} < 0;$$

$$\frac{\partial (x_1{}^{SSS}-q_1{}^{SSS})}{\partial r} = 0,\ \frac{\partial (x_2{}^{SSS}-q_2{}^{SSS})}{\partial r} = \frac{2r[(7r^5+8r^4+72r^3-128r^2-384r+512)(a-b)]}{(r^2-8)^2(7r^2-16)^2} > 0;$$

$\frac{\partial CS^{SSS}}{\partial r} = \frac{-8[(7r^{10}-90r^9-162r^8+1788r^7+608r^6-11712r^5+5760r^4+25600r^3-30720r^2+8192r)(a-b)^2]}{(r^2-8)^3(7r^2-16)^3}$, if $0 < r \leq 0.408$, then $\frac{\partial CS^{S1}}{\partial r} \leq 0$;if $0.408 < r < 1$, then $\frac{\partial CS^{S1}}{\partial r} > 0$;

$$\frac{\partial SW^{SSS}}{\partial r} = \frac{-2\left[\begin{pmatrix}161r^{10}-544r^9-4104r^8+12432r^7+35648r^6-119040r^5-103936r^4+503808r^3\\-73728r^2-753664r+524288\end{pmatrix}(a-b)^2\right]}{(r^2-8)^3(7r^2-16)^3} < 0.$$

Appendix E

Proof of Proposition 5

(1) The effects of r:

$\frac{\partial x_1{}^{SSI}}{\partial r} = -\frac{(r^2-8r+4)(a-b)+2k(r^2+4)}{2(r^2-4)^2}$, if $0 < k < \frac{(a-b)(4+8r-r^2)}{2(r^2+4)}$ and $0 < r < 4-2\sqrt{3}$, then $\frac{\partial x_1{}^{SSI}}{\partial r} < 0$; if $0 < k < \frac{(a-b)(4+8r-r^2)}{2(r^2+4)}$ and $4-2\sqrt{3} < r < 1$, then $\frac{\partial x_1{}^{SSI}}{\partial r} > 0$; if $\frac{(a-b)(4+8r-r^2)}{2(r^2+4)} < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$ and $4-2\sqrt{3} < r < 1$, then $\frac{\partial x_1{}^{SSI}}{\partial r} < 0$.

$\frac{\partial x_2{}^{SSI}}{\partial r}$=0, $\frac{\partial q_1{}^{SSI}}{\partial r} = -\frac{(r^2-8r+4)(a-b)+2k(r^2+4)}{2(r^2-4)^2}$, if $0 < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$ and $0 < r < 4-2\sqrt{3}$, then $\frac{\partial q_1{}^{SSI}}{\partial r} < 0$;if $0 < k < \frac{(a-b)(4+8r-r^2)}{2(r^2+4)}$ and $4-2\sqrt{3} < r < 1$, then $\frac{\partial q_1{}^{SSI}}{\partial r} > 0$; if $\frac{(a-b)(4+8r-r^2)}{2(r^2+4)} < k < \frac{(a-b)(r^4+4r^3-24r^2-64r+128)}{2(7r^4-72r^2+128)}$ and $4-2\sqrt{3} < r < 1$, then $\frac{\partial q_1{}^{SSI}}{\partial r} < 0$.

$$\frac{\partial q_2{}^{SSI}}{\partial r} = -\frac{(r^2-2r+4)(a-b)-4kr}{2(r^2-4)^2} < 0;$$

$$\frac{\partial \pi_1{}^{SSI}}{\partial r} = -\frac{[(a-b)^2+2k(a-b)]r^2-[5(a-b)^2+4kr(a-b+k)]+4(a-b)^2+8k(a-b)}{2(r^2-4)^2} < 0;$$

$$\frac{\partial \pi_2{}^{SSI}}{\partial r} = -\frac{(a-b)^2(r^4+2r^3-12r^2+32r-32)+2k[(r^4-4r^3-12r^2+32r-32)(a-b)]+64k^2r}{4(r^2-4)^3} < 0;$$

$$\frac{\partial (x_1{}^{SSI}-q_1{}^{SSI})}{\partial r} = 0,\ \frac{\partial (x_2{}^{SSI}-q_2{}^{SSI})}{\partial r} = \frac{(r^2-2r+4)(a-b)-4kr}{2(r^2-4)^2} > 0;$$

$$\frac{\partial CS^{SSI}}{\partial r} = -\frac{(a-b)^2(r^4-26r^3+36r^2-16r+32)+2k[(r^4+4r^3+36r^2-64r+32)(a-b)]+8k^2r(r^2-16)}{16(r^2-4)^3} > 0;$$

$$\frac{\partial SW^{SSI}}{\partial r} = -\frac{\begin{array}{c}(a-b)^2(13r^4-58r^3-12r^2+272r-224)+\\2k[(13r^4-28r^3-12r^2+128r-224)(a-b)]-8k^2r(7r^2-32)\end{array}}{16(r^2-4)^3} < 0.$$

(2) The effects of k:

$\frac{\partial x_1{}^{SSI}}{\partial k} = \frac{r}{r^2-4} < 0$, $\frac{\partial x_2{}^{SSI}}{\partial k} = 1 > 0$; $\frac{\partial q_1{}^{SSI}}{\partial k} = \frac{r}{r^2-4} < 0$, $\frac{\partial q_2{}^{SSI}}{\partial k} = \frac{r^2-8}{4(r^2-4)} > 0$; $\frac{\partial \pi_1{}^{SSI}}{\partial k} = \frac{r[(a-b)(r-4)+2kr]}{4(4-r^2)} < 0, \frac{\partial \pi_2{}^{SSI}}{\partial k} = -\frac{(a-b)(r^4+4r^3-16r^2-32r+64)-2k(7r^4-48r^2+64)}{8(r^2-4)^2} > 0$; $\frac{\partial (x_1{}^{SSI}-q_1{}^{SSI})}{\partial k} = 0$, $\frac{\partial (x_2{}^{SSI}-q_2{}^{SSI})}{\partial k} = \frac{8-3r^2}{4(4-r^2)} > 0$; $\frac{\partial CS^{SSI}}{\partial k} = \frac{(a-b)(9r^4+4r^3-64r^2+32r+64)+2k(9r^4-64r^2+64)}{32(r^2-4)^2} > 0$, $\frac{\partial SW^{SSI}}{\partial k} = \frac{(a-b)(5r^4+52r^3-96r^2-224r+320)-2k(27r^4-160r^2+192)}{32(r^2-4)^2} > 0$.

Appendix F

Proof of Proposition 6

$$\pi_2{}^{SSP} - \pi_2{}^{SSI} = -\frac{\begin{array}{c}(r^{10}+8r^9-40r^8-384r^7+576r^6+6656r^5-4608r^4-49152r^3+32768r^2+131072r+131072)A^2- \\ 16(r^{10}+4r^9-48r^8-160r^7+848r^6+2112r^5-6784r^4-10240r^3+24576r^2+16384r-32768)AB- \\ (28r^{10}+112r^9-1312r^8-4352r^7+22272r^6+54272r^5-165888r^4-229376r^3+524288r^2+262144r-524288)Ak+ \\ 64(r^{10}-r^8-64r^6+1152r^4-6144r^2+8192)Bk+4(49r^{10}-1848r^8+21440r^6-100864r^4+196608r^2-131072)k^2\end{array}}{32(r^2-8)^2(r^2-16)^2(r^2-4)^2}$$

From $\pi_2{}^{SSP} - \pi_2{}^{SSI} = 0$, we can obtain:

$k_1{}^{SSP} = \frac{(r^4+4r^3-24r^2-64r+128)A-8(r^4-12r^2+32)B}{2(7r^4-72r^2+128)}$;

$k_2{}^{SSP} = \frac{(r^6+4r^5-32r^4-96r^3+320r^2+512r-1024)A-8(r^6-28r^4+160r^2-256)B}{2(7r^6-192r^4+960r^2-1024)}$;

If $\pi_2{}^{SSP} - \pi_2{}^{SSI} > 0$, then $k_1{}^{SSP} < k < k_2{}^{SSP}$.

From $0 < \mathrm{X} < \frac{1}{3}e - Q$, we can derive $k < x_2{}^{SSP}$ and $k < x_2{}^{SSS}$, thus $k_1{}^{SSP} < k < k_3{}^{SSP}$.

(1) The effects of r:

$\frac{\partial x_1{}^{SSP}}{\partial r} = \frac{4[2(r^5-32r^3+256r)A-(8r^6-768r^2+4096)(B+2k)]}{(r^4-24r^2+128)^2} < 0$;

$\frac{\partial x_2{}^{SSP}}{\partial r} = \frac{2[(r^6-24r^4+2048)A-48(r^5-16r^3+64)(B+2k)]}{(r^4-24r^2+128)^2} > 0$;

$\frac{\partial q_1{}^{SSP}}{\partial r} = \frac{4[2(r^5-32r^3+256r)A-(r^6-192r^2+1024)(B+2k)]}{(r^2-8)^2(r^2-16)^2} < 0$, $\frac{\partial q_2{}^{SSP}}{\partial r} = -\frac{16r(B+2k)}{(r^2-16)^2} < 0$;

$$\frac{\partial \pi_1{}^{SSP}}{\partial r} = \frac{\begin{array}{c}8\{(r^9-48r^7+788r^5-4096r^3)A^2-(r^{10}-12r^8-448r^6-8960r^4+49152r^2-65536)A(B+2k)+ \\ 4(7r^9-184r^7+1728r^5-6656r^3+8768r)(B+2k)^2\}\end{array}}{(r^2-8)^3(r^2-16)^3} < 0;$$

$$\frac{\partial \pi_2{}^{SSP}}{\partial r} = \frac{\begin{array}{c}-2(r^8-40r^6+384r^4+2048r^2-32768)AB-4(r^8-40r^6+384r^4+2048r^2-32768)kA+ \\ 32(r^7-48r^5+576r^3-2048r)B^2+128(r^7-48r^5+576r^3-2048r)k(B+k)\end{array}}{(r^2-8)^2(r^2-16)^3} < 0;$$

$\frac{\partial (x_1{}^{SSP}-q_1{}^{SSP})}{\partial r} = 0$, $\frac{\partial (x_2{}^{SSP}-q_2{}^{SSP})}{\partial r} = \frac{2(r^6-24r^4+1024)A-80(r^5-16r^3+64r)(B+2k)}{(r^2-8)^2(r^2-16)^2} < 0$;

$$\frac{\partial CS^{SSP}}{\partial r} = -\frac{4\left\{\begin{array}{c}8(r^7-48r^5+768r^3-4096r)A^2-(r^{10}-12r^8-448r^6-8960r^4+49152r^2-65536)A(B+2k)+ \\ 56(r^9-28r^7+288r^5-1280r^3+2048r)(B+2k)^2\end{array}\right\}}{(r^2-8)^3(r^2-16)^3} > 0;$$

$$\frac{\partial SW^{SSP}}{\partial r}=\frac{2\left\{\begin{matrix}4\left(r^9-52r^7+960r^5-7168r^3+16384r\right)A^2\\-3\left(r^{10}-24r^8-64r^6+5632r^4-49152r^2+131072\right)A(B+2k)\\+16\left(r^9-44r^7+672r^5-4352r^3+10240r\right)(B+2k)^2\end{matrix}\right\}}{(r^2-8)^3(r^2-16)^3}<0.$$

(2) The effects of k:

$$\frac{\partial {x_1}^{SSP}}{\partial k}=\frac{8r}{r^2-16}<0, \frac{\partial {x_3}^{SSP}}{\partial k}=\frac{8(r^2-4)}{r^2-16}>0;\ \frac{\partial {q_1}^{SSP}}{\partial k}=\frac{8r}{r^2-16}<0\ , \frac{\partial {q_2}^{SSP}}{\partial k}=\frac{2r^2-16}{r^2-16}>0;$$

$$\frac{\partial {\pi_1}^{SSP}}{\partial k}=\frac{16\left\{r\left(r^4-20r^2+64\right)A-r^2(r^4-12r^2+32)(B+2k)\right\}}{(r^2-8)(r^2-16)^2}<0;$$

$$\frac{\partial {\pi_2}^{SSP}}{\partial k}=\frac{\left\{\left(r^6+4r^5-40r^4-128r^3+512r^2+1024r-2048\right)A-\left(7r^6-216r^4+1536r^2-2048\right)(B+2k)\right\}}{(r^2-8)(r^2-16)^2}>0;$$

$$\frac{\partial({x_1}^{SSP}-{q_1}^{SSP})}{\partial k}=0,\ \frac{\partial({x_3}^{SSP}-{q_2}^{SSP})}{\partial k}=\frac{2(3r^2-8)}{r^2-16}>0;$$

$$\frac{\partial CS^{SSP}}{\partial k}=-\frac{2\left\{4r\left(r^4-20r^2+64\right)A-\left(9r^6-136r^4+576r^2-512\right)(B+2k)\right\}}{(r^2-8)(r^2-16)^2}<0;$$

$$\frac{\partial SW^{SSP}}{\partial k}=\frac{\left\{\left(r^4+12r^3-32r^2-192r+256\right)A-(5r^4-96r^2+128)(B+2k)\right\}}{(r^2-16)^2}>0.$$

Appendix G

Proof of Corollary 1, Corollary 2 and Corollary 3

(1) Comparison of different cases under the Cournot competition:

$$({x_1}^{CSP}-{q_1}^{CSP})-({x_1}^{CSS}-{q_1}^{CSS})=\frac{r^3\left[32\left(16-r^2\right)A-\left(r^5+r^4-32r^3-96r^2+256r+512\right)(B+2k)\right]}{128(r^2-8)(r^3+4r^2-16r-32)}>0;$$

$$({x_1}^{CSP}-{q_1}^{CSP})-({x_1}^{CSI}-{q_1}^{CSI})=-\frac{r^3\left[32\left(r^3-4r^2-16r+32\right)A+\left(r^6-48r^4+640r^2-2048\right)(B+2k)\right]}{128(r^2-8)(r^4-32r^2+128)}<0;$$

$$({x_1}^{CSI}-{q_1}^{CSI})-({x_1}^{CSS}-{q_1}^{CSS})=-\frac{2r^3\left[16A+\left(r^3+4r^2-16r-32\right)k\right]}{(r^3+4r^2-16r-32)(r^4-32r^2+128)}>0;$$

$$({x_2}^{CSP}-{q_2}^{CSP})-({x_2}^{CSS}-{q_2}^{CSS})=\frac{\begin{matrix}\{32\left(r^6-40r^4+448r^2-1024\right)A+\\\left(\begin{matrix}r^9+4r^8-56r^7-192r^6+1088r^5+3072r^4-8192r^3\\-18432r^2+16384r+32768\end{matrix}\right)(B+2k)\}\end{matrix}}{256(r^2-8)(r^3+4r^2-16r-32)}<0;$$

$$({x_2}^{CSP}-{q_2}^{CSP})-({x_2}^{CSI}-{q_2}^{CSI})=\frac{\begin{matrix}\{32\left(r^7-4r^6-40r^5+128r^4+448r^3-1024r^2-1024r+2048\right)A+\\\left(r^{10}-72r^8+1856r^6-20480r^4+90112r^2-131072\right)(B+2k)\}\end{matrix}}{256(r^2-8)(r^4-32r^2+128)}>0;$$

$$({x_2}^{CSI}-{q_2}^{CSI})-({x_2}^{CSI}-{q_2}^{CSI})=$$

$$\frac{\{16\left(r^4-24r^2+64\right)A-\left(r^7+4r^6-40r^5-128r^4+448r^3+1024r^2-1024r-2048\right)k\}}{(r^3+4r^2-16r-32)(r^4-32r^2+128)}<0;$$

$${\pi_1}^{CSP}-{\pi_1}^{CSP}=$$

$$\frac{r\left\{\begin{matrix}1024A^2\left(r^9+8r^8-64r^7-448r^6+1408r^5+8192r^4-12288r^3-57344r^2+32768r+131072\right)\\+64A(B+2k)\left(\begin{matrix}r^{12}+8r^{11}-64r^{10}-576r^9+1408r^8+15360r^7-11264r^6-188416r^5-16384r^4\\+1048576r^3+655360r^2-2097152r-2097152\end{matrix}\right)\\+\left(\begin{matrix}r^{15}+8r^{14}-80r^{13}-704r^{12}+2432r^{11}+24576r^{10}-33792r^9-434176r^8+163840r^7+\\4063232r^6+917504r^5-18874368r^4-12582912r^3+33554432r^2+33554432r\end{matrix}\right)(B+2k)^2\end{matrix}\right\}}{8192(r^2-8)^2(r^3+4r^2-16r-32)^2}>0;$$

$$\pi_1{}^{CSP}-\pi_1{}^{CSI}=\frac{r\left\{\begin{matrix}1024A^2\left(r^7-80r^5+128r^4+1280r^3-2048r^2-5120r+8192\right)+\\64AB\left(r^{10}-80r^8+2304r^6-28672r^4+147456r^2-262144\right)+\\128Ak\left(r^{10}-80r^8+2304r^6-512r^5-26624r^4+8192r^3+114688r^2-32768r-131072\right)\\+\left(r^{13}-96r^{11}+3584r^9-65536r^7+606208r^5-2621440r^3+4194304r\right)(B+2k)^2\end{matrix}\right\}}{8192(r^2-8)^2(r^4-32r^2+128)}<0;$$

$$\pi_1{}^{CSI}-\pi_1{}^{CSS}=\frac{8r\left\{\begin{matrix}16A^2\left(r^4-32r^2+16r+128\right)+Ak\left(r^7+4r^6-48r^5-128r^4+768r^3+1024r^2-3072r-4096\right)+\\\left(r^7+8r^6-16r^5-192r^4+1024r^2+1024r\right)k^2\end{matrix}\right\}}{(r^3+4r^2-16r-32)^2(r^4-32r^2+128)}>0;$$

$$\pi_2{}^{CSP}-\pi_2{}^{CSS}=\frac{\begin{matrix}1024A^2\left(r^6-40r^4+384r^2-1024\right)+64A(B+2k)\begin{pmatrix}r^9+4r^8-64r^7-224r^6+1280r^5+\\3584r^4-9216r^3-20480r^2+16384r+32768\end{pmatrix}\\+B^2\begin{pmatrix}r^{12}+8r^{11}-72r^{10}-640r^9+1664r^8+17920r^7-13312r^6-221184r^5\\-24576r^4+1179648r^3+786432r^2-2097152r-2097152\end{pmatrix}+\\4k(B+k)\begin{pmatrix}r^{12}+8r^{11}-72r^{10}-640r^9+1536r^8+16896r^7-10240r^6-188416r^5-\\40960r^4+851968r^3+655360r^2-1048576r-1048576\end{pmatrix}\end{matrix}}{-512(r^3+4r^2-16r-32)^2(r^2-8)}<0\,;$$

$$\pi_2{}^{CSP}-\pi_2{}^{CSI}=\frac{\begin{matrix}1024\left(r^8-8r^7-24r^6+256r^5+128r^4-2560r^3+1024r^2+8192r-8192\right)A^2+\\64\begin{pmatrix}r^{11}-4r^{10}-80r^9+288r^8+2304r^7-7168r^6-28673r^5+73728r^4+147456r^3\\-327680r^2-262144r+524288\end{pmatrix}A(B+2k)\\+\begin{pmatrix}r^{14}-120r^{12}+5632r^{10}+131072r^8+1589248r^6-9830400r^4+\\29360128r^2-33554432\end{pmatrix}(B+2k)^2\end{matrix}}{512(128-32r^2+r^4)^2(8-r^2)}>0;$$

$$\pi_2{}^{CSI}-\pi_2{}^{CSS}=-\frac{\begin{matrix}256A^2\left(r^8-48r^6+640r^4-4096r^2+8192\right)+\\32Ak\begin{pmatrix}r^{11}+4r^{10}-72r^9-256r^8+1792r^7+5376r^6-19456r^5-49152r^4\\+90112r^3+196608r^2-131072r-262144\end{pmatrix}\\+\begin{pmatrix}r^{14}+8r^{13}-80r^{12}-704r^{11}+2176r^{10}+22528r^9-23552r^8-335872r^7+\\40960r^6+2424832r^5+1048576r^4-6291456r^2+8388608\end{pmatrix}k^2\end{matrix}}{(r^3+4r^2-16r-32)^2(r^4-32r^2+128)^2}<0;$$

$$CS^{CSP}-CS^{CSS}=\frac{\begin{matrix}1024A^2\begin{pmatrix}r^{10}-24r^9-80r^8+1088r^7+2304r^6-17408r^5-29696r^4+\\114688r^3+163840r^2-262144r-262144\end{pmatrix}-\\64A(B+2k)\begin{pmatrix}3r^{13}+24r^{12}-160r^{11}-1472r^{10}+3072r^9+34816r^8-21504r^7\\-393216r^6-49152r^5+2097152r^4+1310720r^3-4294304r^2-4294304r\end{pmatrix}\\-\begin{pmatrix}7r^{16}+56r^{15}-512r^{14}-4544r^{13}+14848r^{12}+151552r^{11}-206848r^{10}-2662400r^9\\+1146880r^8+26148864r^7+3670016r^6-139460608r^5-\\76546048r^4+352321536r^3+301989888r^2-268435456r-268435456\end{pmatrix}(B+2k)^2\end{matrix}}{32768(r^3+4r^2-16r-32)^2(r^2-8)^2}<$$

$0;$

$$CS^{CSP}-CS^{CSI}=$$

$$\frac{\begin{matrix}1024A^2\begin{pmatrix}r^{12}-32r^{11}+128r^{10}+1280r^9-6656r^8-17408r^7+\\108544r^6+81920r^5-753664r^4+65536r^3+2097152r^2-1048576r-1048576\end{pmatrix}\\-64AB\left(3r^{15}-304r^{13}+12288r^{11}-253952r^9+2867200r^7-17563648r^5+54525952r^3-67108864r\right)-\\128Ak\begin{pmatrix}3r^{15}-304r^{13}+12288r^{11}-3584r^{10}-274432r^9+147456r^8+3145728r^7-2195456r^6-18874368r^5\\+1468006r^4+534777376r^3-41943040r^2-5033164r+33554432\end{pmatrix}\\-\begin{pmatrix}7r^{18}-848r^{16}+43008r^{14}-1188864r^{12}+19546112r^{10}-195821568r^8+1178599424r^6-\\4043309056r^4+6979321856r^2-4294967296\end{pmatrix}(B+2k)\\-\begin{pmatrix}7r^{18}-848r^{16}+43008r^{14}-1188864r^{12}+19431424r^{10}-191102976r^8+1108344832r^6-\\3573547008r^4+5637144576r^2-3221225472\end{pmatrix}k^2\end{matrix}}{32768(r^3+4r^2-16r-32)^2(r^2-8)^2}>0;$$

$$CS^{CSI}-CS^{CSS}=$$

$$\frac{\begin{matrix}16A^2\left(r^{10}-3r^9-68r^8+160r^7+1520r^6-2688r^5-13568r^4+16384r^3+45056r^2-32768r-49152\right)\\-Ak\begin{pmatrix}r^{13}+r^{12}-96r^{11}-96r^{10}+3232r^9+2560r^8-49152r^7-28672r^6+352256r^5+\\163840r^4-1146880r^3-524288r^2+1572864r+1048576\end{pmatrix}\\-\begin{pmatrix}7r^{12}+56r^{11}-288r^{10}-2752r^9+3840r^8+49152r^7-10240r^6-\\385024r^5-163840r^4+1245184r^3+1048576r^2-1048576r-1048576\end{pmatrix}k^2\end{matrix}}{(r^3+4r^2-16r-32)^2(r^4-32r^2+128)^2}<0;$$

$$SW^{CSP}-SW^{CSS}=$$

$$\frac{\begin{array}{c}1024A^2\left(\begin{array}{c}5r^{10}+8r^9+400r^8-704r^7+11008r^6+15360r^5-123904r^4-\\114688r^3+57056r^2+262144r-786432\end{array}\right)+\\64A(B+2k)\left(\begin{array}{c}r^{13}+8r^{12}-160r^{11}-1088r^{10}+7168r^9+43008r^8-138240r^7-704512r^6+\\1228800r^5+5242880r^4-4456448r^3-16777216r^2+4194304r+16777216\end{array}\right)-\\\left(\begin{array}{c}3r^{16}+24r^{15}-128r^{14}-1216r^{13}+8192r^{11}+71680r^{10}+548864r^9-1212416r^8-13434880r^7+5242880r^6\\+124780544r^5+36700160r^4-520093696r^3-369098752r^2+805306368r+805306368\end{array}\right)(B+2k)^2\end{array}}{32768(r^4-32r^2+128)^2(r^2-8)^2}<0;$$

$$SW^{CSP}-SW^{CSI}=$$

$$\frac{\begin{array}{c}1024A^2\left(\begin{array}{c}5r^{12}-32r^{11}-384r^{10}+2304r^9+11264r^8-62464r^7-137216r^6+737280r^5+557056r^4\\-3866624r^3+524288r^2+7340032r-5242880\end{array}\right)+\\64AB\left(\begin{array}{c}r^{15}-208r^{13}+256r^{12}+13312r^{11}-20480r^{10}-385024r^9+606208r^8+5586944r^7-8388608r^6\\-41156608r^5+58720256r^4+146800640r^3-201326592r^2+201326592r+268435456\end{array}\right)+\\128Ak\left(\begin{array}{c}r^{15}-208r^{13}+256r^{12}+12800r^{11}-18944r^{10}-348160r^9+524288r^8+4587520r^7-6586368r^6\\-29360128r^5+39845888r^4+84934656r^3-109051904r^2-83886080r+100663296\end{array}\right)\\-\left(\begin{array}{c}3r^{18}-272r^{16}+7680r^{14}+3072r^{12}-434528r^{10}+94633984r^8-926941184r^6+\\4747952128r^4-12348030976r^2+12884901888\end{array}\right)B^2\\-4Bk\left(\begin{array}{c}3r^{18}-272r^{16}+7680r^{14}-5120r^{12}-3719168r^{10}+75235328r^8-658505728r^6+\\2868903936r^4-5905580032r^2+4294967296\end{array}\right)\\-4k^2\left(\begin{array}{c}3r^{18}-272r^{16}+7680r^{14}-13312r^{12}-3112960r^{10}+58458112r^8-447741952r^6+\\1593835520r^4-2415919104r^2+1073741824\end{array}\right)\end{array}}{32768(r^3+4r^2-16r-32)^2(r^2-8)^2}>0;$$

$$SW^{CSI}-SW^{CSS}=\frac{\begin{array}{c}32A^2\left(\begin{array}{c}r^{10}+r^9-76r^8-96r^7+1968r^6+2432r^5-20736r^4-\\16384r^3+86016r^2+32768r-114688\end{array}\right)+\\2Ak\left(\begin{array}{c}r^{13}+5r^{12}-96r^{11}-480r^{10}+3360r^9+16384r^8-55296r^7-249856r^6\\+450560r^5+1736704r^4-1540096r^3-5242880r^2+1572864r+5242880\end{array}\right)\\-\left(\begin{array}{c}r^{14}+8r^{13}-74r^{11}-656r^{10}+1984r^9+20608r^8-303104r^7+28672r^6+\\2113536r^5+983040r^4-6422528r^3-5242880r^2+6291456r+6291456\end{array}\right)k^2\end{array}}{(r^3+4r^2-16r-32)^2(r^4-32r^2+128)^2}<0.$$

(2) Comparison of different cases under the Stackelberg competition:

$$({x_2}^{SSP}-{q_2}^{SSP})-({x_2}^{SSS}-{q_2}^{SSS})=\frac{\begin{array}{c}(3r^6+12r^5-80r^4-224r^3+576r^2+512r-1024)A-\\(21r^6-272r^4+960r^2-1024)(B+2k)\end{array}}{-(r^2-8)(r^2-16)(7r^2-16)}<0;$$

$$({x_2}^{SSP}-{q_2}^{SSP})-({x_2}^{SSI}-{q_2}^{SSI})=\frac{\begin{array}{c}(3r^6+12r^5-80r^4-224r^3+576r^2+512r-1024)A-\\8(3r^6-44r^4+192r^2-256)B-2(21r^6-272r^4+960r^2-1024)k\end{array}}{-8(r^2-8)(r^2-16)(r^2-4)}>0;$$

$$({x_2}^{SSI}-{q_2}^{SSI})-({x_2}^{SSS}-{q_2}^{SSS})=\frac{\begin{array}{c}(3r^6+12r^5-80r^4-224r^3+576r^2+512r-1024)A-\\2(21r^6-272r^4+960r^2-1024)k\end{array}}{-8(r^2-8)(r^2-16)(r^2-4)}<0;$$

$${\pi_1}^{SSP}-{\pi_1}^{SSS}=\frac{4\left[\begin{array}{c}A^2\left(\begin{array}{c}r^{11}-6r^{10}-92r^9+296r^8+2592r^7-5440r^6-29440r^5+46080r^4+131072r^3\\-180224r^2-196608r+262144\end{array}\right)\\+2A(B+2k)(49r^{10}-1596r^8+17504r^6-82432r^4+172032r^2-131072)\\-(49r^{11}-1024r^9+11008r^7-46336r^5+90112r^3-65536)(B+2k)^2\end{array}\right]}{(7r^4-72r^2+128)(7r^2-16)(r^2-16)(r^4-24r^2+128)}>0;$$

$${\pi_1}^{SSP}-{\pi_1}^{SSI}=\frac{r\left\{\begin{array}{c}A^2(r^9-8r^8-96r^7+384r^6+2624r^5-6656r^4-26624r^3+49152r^2+81920r-131072)\\+128AB(r^8-32r^6+336r^4-1408r^2+2048)+\\4Ak(r^9+60r^8-48r^7-1856r^6+832r^5+18176r^4-6144r^3-655362r^2+16384r+65536)\\-64(r^9-24r^7+208r^5-768r^3+1024r)(B+2k)^2\end{array}\right\}}{16(r^2-4)(r^2-8)(r^2-16)(r^4-24r^2+128)}<0;$$

$${\pi_1}^{SSI}-{\pi_1}^{SSS}=\frac{r\left\{\begin{array}{c}A^2\left(\begin{array}{c}15r^9+8r^8-736r^7+128r^6+11712r^5-7680r^4-67584r^3\\+65536r^2+114688r-131072\end{array}\right)-\\4Ak\left(\begin{array}{c}49r^9-196r^8-1008r^7+4032r^6+6976r^5-27904r^4-18432r^3\\+73728r^2+16384r-65536\end{array}\right)\\-4(49r^9-1008r^7+6976r^5-18432r^3+16384r)k^2\end{array}\right\}}{16(r^2-4)(r^2-8)(7r^2-16)(r^4-72r^2+128)}>0;$$

$$\pi_2{}^{SSP}-\pi_2{}^{SSS}=\frac{\begin{gathered}A^2\begin{pmatrix}7r^{12}+56r^{11}-320r^{10}-3008r^9+5568r^8+61440r^7-53248r^6-589824r^5\\+393216r^4+2621440r^3-2097152r^2-4194304r+4194304\end{pmatrix}-\\2A(B+2k)\begin{pmatrix}49r^{12}+196r^{11}-2576r^{10}-8736r^9+51776r^8+137216r^7-499712r^6-\\901120r^5+2392064r^4+2359296r^3-5242880r^2-2097152r+4194304\end{pmatrix}\\+8\left(49r^{12}-2184r^{10}+34304r^8+34304r^6-250368r^4+905216r^2+1048576\right)(B+2k)^2\end{gathered}}{-4(r^2-8)^2(r^2-16)^2(7r^2-16)^2}<0;$$

$$\pi_2{}^{SSP}-\pi_2{}^{SSI}=\frac{\begin{gathered}\begin{pmatrix}r^{10}+8r^9-40r^8-384r^7+576r^6+6656r^5-4608r^4-49152r^3+\\32768r^2+131072r+131072\end{pmatrix}A^2-\\16\begin{pmatrix}r^{10}+4r^9-48r^8-160r^7+848r^6+2112r^5-6784r^4-10240r^3+\\24576r^2+16384r-32768\end{pmatrix}AB-\\\begin{pmatrix}28r^{10}+112r^9-1312r^8-4352r^7+22272r^6+54272r^5-165888r^4-\\229376r^3+524288r^2+262144r-524288\end{pmatrix}Ak\\+64\left(r^{10}-r^8-64r^6+1152r^4-6144r^2+8192\right)Bk+\\4\left(49r^{10}-1848r^8+21440r^6-100864r^4+196608r^2-131072\right)k^2\end{gathered}}{-32(r^2-8)(r^2-16)^2(r^2-4)^2}>0;$$

$$\pi_2{}^{SSI}-\pi_2{}^{SSS}=\frac{\begin{gathered}\begin{pmatrix}7r^{12}+56r^{11}-208r^{10}-2112r^9+2560r^8+30208r^7-23552r^6-217088r^5+\\151552r^4+786432r^3-655360r^2-1048576r+1048576\end{pmatrix}A^2\\-4\begin{pmatrix}49r^{12}+196r^{11}-1792r^{10}-5600r^9+26240r^8+60160r^7-194560r^6-\\296960r^5+757760r^4+655360r^3-1441792r^2-524288r+1048576\end{pmatrix}Ak\\+4\left(49r^{10}-1848r^8+21440r^6-100864r^4+196608r^2-131072\right)k^2\end{gathered}}{-32(r^2-8)(7r^2-16)^2(r^2-4)^2}<0;$$

$$CS^{SSP}-CS^{SSS}=\frac{\begin{gathered}\begin{pmatrix}9r^{12}+16r^{11}-576r^{10}-800r^9+13568r^8+14592r^7-145408r^6-114688r^5+\\724992r^4+327680r^3-1703936r^2+1048576\end{pmatrix}A^2\\-8\left(49r^{11}-1596r^9+17504r^7-82432r^5+172032r^3-131072r\right)A(B+2k)\\+\begin{pmatrix}441r^{12}-12208r^{10}+130432r^8-676864r^6+1773568r^4-\\2228224r^2+1048576\end{pmatrix}(B+2k)^2\end{gathered}}{-2(r^2-8)^2(7r^2-16)^2(r^2-16)^2}<0;$$

$$CS^{SSP}-CS^{SSI}=\frac{\begin{gathered}\begin{pmatrix}9r^{12}+8r^{11}-608r^{10}-320r^9+15744r^8+3584r^7-196608r^6+4096r^5+1224704r^4\\-262144r^3-3538944r^2+1048576r+3145728\end{pmatrix}A^2-\\256\left(r^{11}-36r^9+464r^7-2752r^5+7680r^3-8192r\right)AB+\\4\begin{pmatrix}9r^{12}+132r^{11}-496r^{10}-4768r^9+10624r^8+61184r^7-111616r^6-350208r^5\\+593920r^4+851968r^3-1441792r^2-5524288r+1048576\end{pmatrix}Ak\\-32\left(9r^{12}-280r^{10}+3472r^8-21760r^6+71680r^4-114688r^2+65536\right)B(B+4k)\\-4\left(279r^{12}-8464r^{10}+100480r^8-584704r^6+1699840r^4-2228224r^2+1048576\right)k^2\end{gathered}}{-64(r^2-8)^2(r^2-4)^2(r^2-16)^2}<0;$$

$$CS^{SSI}-CS^{SSS}=\frac{\begin{gathered}\begin{pmatrix}153r^{12}-120r^{11}-6176r^{10}+8384r^9+86912r^8-131584r^7-532480r^6+\\774144r^5+1421312r^4-1703936r^3-1703936r^2+1048576r+1048576\end{pmatrix}A^2\\-8\left(49r^{11}-1596r^9+17504r^7-82432r^5+172032r^3-131072r\right)A(B+2k)\\+\begin{pmatrix}441r^{12}-12208r^{10}+130432r^8-676864r^6+1773568r^4-\\2228224r^2+1048576\end{pmatrix}(B+2k)^2\end{gathered}}{64(r^2-8)^2(7r^2-16)^2(r^2-4)^2}>0;$$

$$SW^{SSP}-SW^{SSS}=-\frac{\begin{gathered}\begin{pmatrix}9r^{12}+184r^{11}-9344r^9-8768r^8+177664r^7+126976r^6-1556480r^5-253952r^4\\+6160384r^3-2359296r^2-8388608r+6291456\end{pmatrix}A^2-\\2\begin{pmatrix}49r^{12}+588r^{11}-2576r^{10}-21504r^9+51776r^8+277248r^7-499712r^6-\\1560576r^5+2392064r^4+3735552r^3-5242880r^2-3145728r+4194304\end{pmatrix}A(B+2k)\\-2\left(147r^{12}-6160r^{10}+94848r^8-695296r^6+2568192r^4-4587520r^2+3145728\right)B^2\\-4(245r^{12}-9744r^{10}+137920r^8-890880r^6+2744320r^4-\\3932160r^2+2097152)(B+k)k\end{gathered}}{4(r^2-8)^2(7r^2-16)^2(r^2-16)^2}<0;$$

$$SW^{SSP}-SW^{SSI}=\frac{\begin{gathered}\begin{pmatrix}7r^{12}+56r^{11}-304r^{10}-2880r^9+5504r^8+55808r^7-66560r^6-503808r^5+\\610304r^4+2097152r^3-3014656r^2-3145728r+5242880\end{pmatrix}A^2-\\32\begin{pmatrix}r^{12}+12r^{11}-56r^{10}-480r^9+1232r^8+7104r^7-13568r^6-49152r^5+78848r^4\\+159744r^3-229376r^2-196608r+262144\end{pmatrix}AB-\\4\begin{pmatrix}9r^{12}+164r^{11}-480r^{10}-6240r^9+9856r^8+85760r^7-98304r^6-534528r^5+\\495616r^4+1507328r^3-1179648r^2-1572864r+1048576\end{pmatrix}\\+32\left(3r^{12}-136r^{10}+2352r^8-20224r^6+92160r^4-212992r^2+196608\right)B^2+\\64\left(5r^{12}-216r^{10}+3472r^8-26880r^6+105472r^4-196608r^2+131072\right)Bk+\\4\left(71r^{12}-2976r^{10}+45696r^8-331776r^6+1191936r^4-196608r^2+1048576\right)k^2\end{gathered}}{-64(r^2-8)^2(r^2-16)^2(r^2-4)^2}<0;$$

$$SW^{SSI}-SW^{SSS}=\frac{\begin{array}{c}\left(\begin{array}{c}199r^{12}-200r^{11}-8944r^{10}+12992r^{9}+140416r^{8}-224768r^{7}-943104r^{6}+1593344r^{5}\\+2658304r^{4}-4849664r^{3}-2228224r^{2}+5242880r-1048576\end{array}\right)A^2\\+4\left(\begin{array}{c}343r^{12}+1372r^{11}-10976r^{10}-32928r^{9}+138880r^{8}+292096r^{7}-880640r^{6}-\\1185792r^{5}+2928640r^{4}+2228224r^{3}-4849664r^{2}-1572864r+3145728\end{array}\right)Ak\\-4\left(441r^{12}-11424r^{10}+114304r^{8}-565248r^{6}+1478656r^{4}-196608r^{2}+1048576\right)k^2\end{array}}{64(r^2-8)^2(7r^2-16)^2(r^2-4)^2}>0.$$

$$({x_1}^{CSS}-{q_1}^{CSS})-({x_1}^{SSS}-{q_1}^{SSS})=-\frac{r^2A}{r^3+4r^2-16r-32}>0;$$

$$({x_2}^{CSS}-{q_2}^{CSS})-({x_2}^{SSS}-{q_2}^{SSS})=\frac{r^4A(r^3-6r^2-8r+16)}{2(r^2-8)(7r^2-16)(r^3+4r^2-16r-32)}<0;$$

$${\pi_1}^{CSS}-{\pi_1}^{SSS}=\frac{2r^4A^2\left(\begin{array}{c}2r^{10}+4r^9-101r^8-160r^7+1736r^6+1728r^5\\-12992r^4-6656r^3+43008r^2+8192r-49152\end{array}\right)}{(r^2-8)(r^3+4r^2-16r-32)^2(7r^2-16)(7r^4-72r^2+128)}<0;$$

$${\pi_2}^{CSS}-{\pi_2}^{SSS}=-\frac{r^3\left\{A^2\left(\begin{array}{c}7r^{11}+112r^{10}-152r^9-4224r^8+2368r^7+58880r^6\\-17920r^5-372736r^4+32768r^3+1048576r^2-1048576\end{array}\right)\right\}}{4(r^2-8)^2(7r^2-16)^2(r^3+4r^2-16r-32)^2}>0;$$

$$CS^{CSS}-CS^{SSS}=-\frac{r^2\left\{A^2\left(\begin{array}{c}9r^{12}-10r^{11}-402r^{10}+576r^9+7216r^8-10496r^7-67456r^6\\+78848r^5+327680r^4-245760r^3-704512r^2+262144r+524288\end{array}\right)\right\}}{2(r^2-8)^2(7r^2-16)^2(r^3+4r^2-16r-32)^2}<0;$$

$$SW^{CSS}-SW^{SSS}=-\frac{r^2\left\{A^2\left(\begin{array}{c}9r^{12}+60r^{11}-148r^{10}-1792r^9+2912r^8+24064r^7-48896r^6\\-161792r^5+344064r^4+491520r^3-1015808r^2-524288r+1048576\end{array}\right)\right\}}{4(r^2-8)^2(7r^2-16)^2(r^3+4r^2-16r-32)^2}<0.$$

(3) Comparison of different competition modes:

$$({x_1}^{CSI}-{q_1}^{CSI})>({x_1}^{SSI}-{q_1}^{SSI})=\frac{r^2[(4-r)A-2kr]}{r^4-32r^2+128}>0;$$

$$({x_2}^{CSI}-{q_2}^{CSI})<({x_2}^{SSI}-{q_2}^{SSI})=\frac{r^3\left[(r^3-4r^2-8r+32)A+2kr(r^2-16)\right]}{8(r^2-4)(r^4-32r^2+128)}<0;$$

$${\pi_1}^{CSI}-{\pi_1}^{SSI}=\frac{((r-4)A+2kr)^2}{16(r^2-4)(r^4-32r^2+128)}<0;$$

$${\pi_2}^{CSI}-{\pi_2}^{SSI}=-\frac{\begin{array}{c}r^3\{A^2(r^9+8r^8-128r^7-64r^6+2880r^5-2048r^4-21504r^3+24576r^2+49152r-65536)\\+4Ak(r^9+4r^8-90r^7-32r^6+1344r^5-1024r^4-8192r^3+12288r^2+16384r-32768)+\\4k^2(r^9-90r^7+1344r^5-8192r^3+16384r)\}\end{array}}{32(r^4-32r^2+128)^2(r^2-4)^2}>0;$$

$$CS^{CSI}-CS^{SSI}=\frac{\begin{array}{c}A^2\left(\begin{array}{c}9r^{12}-56r^{11}-304r^{10}+1600r^9+6208r^8-13312r^7-99328r^6+57344r^5\\+770048r^4-229376r^3-2490368r^2+52428r+2621440\end{array}\right)+\\4Ak\left(\begin{array}{c}9r^{12}-28r^{11}-416r^{10}+800r^9+8256r^8-6656r^7-78336r^6\\+28672r^5+368640r^4-114688r^3-786432r^2+262144r+524288\end{array}\right)+\\4k^2(9r^{12}-416r^{10}+8256r^8-78336r^6+368640r^4-786432r^2+524288)\end{array}}{-64(r^4-32r^2+128)^2(r^2-4)^2}<0;$$

$$SW^{CSI}-SW^{SSI}=-\frac{\begin{array}{c}A^2\left(\begin{array}{c}7r^{12}-8r^{11}-480r^{10}+320r^9+13248r^8-9216r^7-156672r^6+90112r^5\\-901120r^4-360448r^3-2490368r^2+52428r+2621440\end{array}\right)+\\4Ak\left(\begin{array}{c}7r^{12}-4r^{11}-432r^{10}+160r^9+9920r^8-4608r^7-92672r^6+45056r^5\\+401408r^4-180224r^3-786432r^2+262144r+524288\end{array}\right)+\\4k^2(7r^{12}-432r^{10}+9920r^8-92672r^6+401408r^4-786432r^2+524288)\end{array}}{64(r^4-32r^2+128)^2(r^2-4)^2}<0.$$

$$({x_1}^{SSP}-{q_1}^{SSP})-({x_1}^{CSP}-{q_1}^{CSP})=-\frac{r^2[32A+(r^2-16)r(B+2k)]}{128(r^2-8)}>0;$$

$$({x_2}^{SSP}-{q_2}^{SSP})-({x_2}^{CSP}-{q_2}^{CSP})=\frac{r^3\left[32(r^2-16)A+(r^4-56r^2+320)r(B+2k)\right]}{256(r^2-8)(r^2-16)}<0;$$

$${\pi_1}^{CSP}-{\pi_1}^{SSP}=\frac{r^4\left\{\begin{array}{c}1024A^2(r-16)^2+(r^6-80r^4+1408r^2-6144)A(B+2k)r+\\(r^8-96r^6+3712r^4-40960r^2+131072)(B+2k)^2r^2)\end{array}\right\}}{8192(r^2-8)(r^2-16)(r^4-24r^2+128)}>0;$$

$$\pi_2^{CSP} - \pi_2^{SSP} = -\frac{r^3\{64(r-16)^2A(B+2k)+(r^6-88r^4+1792r^2-8192)(B+2k)^2r\}}{512(r^2-8)(r^2-16)^2} > 0;$$

$$CS^{CSP} - CS^{SSP} = \frac{r^2\left\{\begin{matrix}1024A^2(r^6-64r^4+1280r^2-8192)+64r^3(3r^6-208r^4+3584r^2-16384)A(B+2k)r- \\ (7r^{10}-624r^8+23040r^6-304128r^4+1572864r^2-2621440)(B+2k)^2r^2\end{matrix}\right\}}{32768(r^2-16)^2(r^2-8)^2} < 0;$$

$$SW^{CSP} - SW^{SSP} =$$

$$\frac{r^2\left\{\begin{matrix}1024A^2(5r^6-192r^4+2304r^2-8192)+64(r^8-176r^6+4608r^4-40960r^2+131072)A(B+2k)r- \\ (3r^{10}-176r^8+2048r^6+19456r^4-393216r^2+1572864)(B+2k)^2r^2\end{matrix}\right\}}{32768(r^2-16)^2(r^2-8)^2} < 0.$$

Declarations

Compliance with ethical standards

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Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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