Information Measures for MADM Underm-Polar Neutrosophic Environment

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Information measures for MADM under m -polar neutrosophic environment

Khalid Naeem* and Bijan Divvaz†

Abstract

Multipolar information plays a vital role in making reliable decisions in one's daily life encompassing micro to large scale decisions. These decision making problems often include imprecise and inconsistent data. This article presents some information measures including similarity, distance, correlation, divergence and Dice measures for m -polar neutrosophic sets. Desirable characteristics of these measures are also presented. The notions of angle of similarity between two m -polar neutrosophic sets, λ -similarity, entropy and less and more fuzzy are also made part of the discussion. An application of m -polar neutrosophic sets using the suggested measures in health sciences accompanied by five algorithms is also presented. Finally, comparative analysis of the proposed information measures with some existing measures is also presented.

Keywords: m -polar neutrosophic sets, Similarity measure, Correlation measure, Divergence measure, Dice measure

1 Introduction

The analytical modeling of real world problems often involve uncertainties, vagueness and imprecisions. To handle such situations, various models like probability theory, fuzzy sets [61], rough sets [41] and interval mathematics [17] etc. have been proposed by the researchers. All these theories have been recognized in favorable circumstances but unfortunately they have their own limitations. Ensuing the explorations of Zadeh, Atanassov [3, 4] introduced a new sort of sets titled intuitionistic fuzzy sets (IFSs). An IFS comprises a pair of two mappings communicating the degrees of membership and the degree of non-membership of elements of the universe. One major deficiency shared by these theories is possibly the usage of parametrization tools as indicated by Molodtsov.

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In 1999, Molodtsov [35] presented the notion of soft sets as an important mathematical tool to deal with uncertainties. Yager [57] gave notion of Pythagorean fuzzy sets. Naeem *et al.* [37, 38] rendered the notion of Pythagorean m-polar fuzzy sets and extended the notion to develop topological structure in [39]. Riaz *et al.* [47] supplemented the idea to corresponding soft sets.

Smarandache [50, 51] further expanded the notions of fuzzy sets and IFSs and brought to canvas the novel idea of neutrosophic sets. An element in the neutrosophic set has three independent components representing truth-membership, indeterminacy-membership and falsity-membership. Maji [32] expanded the notion of neutrosophic sets to neutrosophic soft sets (NSSs) and applied the notion to decision making problems (DMPs) in [31]. After the explorations of Maji, the studies on neutrosophic soft sets gained momentum and attracted vibrant researchers towards further explorations (see [5, 6, 1]). Wang *et al.* [54] presented interval valued neutrosophic sets as another expansion of neutrosophic sets. Deli [12] studied interval-valued neutrosophic soft sets and decision making. Habib *et al.* [20] presented application in life sciences under neutrosophic environment. Hashmi *et al.* [21] explored m-polar neutrosophic topology along with applications.

The idea of getting a numerical value to assess given data in the form of several numbers presents the clear scanned picture of the whole date. The use of information measures is an efficient tool to achieve this goal. Many researchers, mainly including Zadeh [62], Ansari *et al.* [2], Murthy *et al.* [36], Lin [29], Chaudhuri and Bhattacharya [8], Broumi and Smarandache [7], Rajarajeswari and Uma [45, 46], Ye [59, 60], Chiang and Lin [9], Zeng and Li [64], Mitchell [34], Garggargcc, Khan and Ansari [28], Peng *et al.* [42], Peng [43], Peng and Liu [44], Riaz *et al.* [48], Zeng *et al.* [63], Ye [58], Türkarslan *et al.* [52], Gerstenkorn and Manko [19], Singh and Ganie [49], Hong and Hwang [22], Hung and Wu [23], Mishra *et al.* [33], Hung [24], Cui and Ye [10], Deng *et al.* [11], Firozja *et al.* [13], Garg and Arora [14], Garg and Nancy [16], Lu and Ye [30], Irfan *et al.* [25], Jafar *et al.* [26], Wei [55], Jiang *et al.* [27], Nancy and Garg [40], Wang and Zhao [53], Hashmi *et al.* [21], Xue *et al.* [56], and Zulqarnain *et al.* [65, 66] worked on different sorts of information measures defined on different expansions of fuzzy sets.

There arise many situations where we need multipolar statistics. For example, in some medical tests, readings are taken before and after taking meal. We ponder time and again to invest money in some scheme or some sort of business. For matrimonial related decisions, we think again and again before finalizing it. The major aim behind this article is to suggest some novel information measures including similarity measure, distance measure, correlation measure, divergence measure and Dice measure for m-polar neutrosophic sets. The rest of the article is organized as follows: The second section gives access to some elementary notions relevant to m-polar neutrosophic sets. The next section presents five information measures i.e. similarity measure, distance measure, correlation measure, divergence measure and Dice measure for m-polar neutrosophic sets accompanied by their prime attributes and illustrations. In section four, an application of suggested measures along with a case study is given from health sector. The fifth section presents comparative analysis of

the proposed information measures with some existing measures. Section six concludes the article.

2 Preliminaries

In this section, we bring to mind some basics needed to understand the further notions.

Definition 2.1. [21] An *m-polar neutrosophic set* (*mNS*) N is typified by the maps $T_N^{(i)}$ (affiliation grades), $I_N^{(i)}$ (indeterminacy grades) and $F_N^{(i)}$ (dissociation degrees) sending elements of X to $[0, 1]$ such that $0 \leq T_N^{(i)}(\omega) + I_N^{(i)}(\omega) + F_N^{(i)}(\omega) \leq 3$, for all i .

An *mNS* is commonly expressed in different ways as

$$\begin{aligned} N &= \left\{ \langle \omega, \left((T_N^{(1)}(\omega), I_N^{(1)}(\omega), F_N^{(1)}(\omega)), \dots, (T_N^{(m)}(\omega), I_N^{(m)}(\omega), F_N^{(m)}(\omega)) \right) \rangle \right\} \\ &= \left\{ \frac{\omega}{\left((T_N^{(1)}(\omega), I_N^{(1)}(\omega), F_N^{(1)}(\omega)), \dots, (T_N^{(m)}(\omega), I_N^{(m)}(\omega), F_N^{(m)}(\omega)) \right)} \right\} \\ &= \left\{ \frac{\omega}{\left((T_N^{(i)}(\omega), I_N^{(i)}(\omega), F_N^{(i)}(\omega)) \right)} \right\} \end{aligned}$$

If size of X is r , then Table 1 yields the tabulatory array of N .

Table 1: Tabulatory array of N

N				
ω_1	$(T_N^{(1)}(\omega_1), I_N^{(1)}(\omega_1), F_N^{(1)}(\omega_1))$	$(T_N^{(2)}(\omega_1), I_N^{(2)}(\omega_1), F_N^{(2)}(\omega_1))$	$\dots$	$(T_N^{(m)}(\omega_1), I_N^{(m)}(\omega_1), F_N^{(m)}(\omega_1))$
ω_2	$(T_N^{(1)}(\omega_2), I_N^{(1)}(\omega_2), F_N^{(2)}(\omega_2))$	$(T_N^{(2)}(\omega_2), I_N^{(2)}(\omega_2), F_N^{(1)}(\omega_2))$	$\dots$	$(T_N^{(m)}(\omega_2), I_N^{(m)}(\omega_2), F_N^{(m)}(\omega_2))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
ω_r	$(T_N^{(1)}(\omega_r), I_N^{(1)}(\omega_r), F_N^{(1)}(\omega_r))$	$(T_N^{(2)}(\omega_r), I_N^{(2)}(\omega_r), F_N^{(2)}(\omega_r))$	$\dots$	$(T_N^{(m)}(\omega_r), I_N^{(m)}(\omega_r), F_N^{(m)}(\omega_r))$

The corresponding matrix format is

$$N = \begin{pmatrix} (T_N^{(1)}(\omega_1), I_N^{(1)}(\omega_1), F_N^{(1)}(\omega_1)) & (T_N^{(2)}(\omega_1), I_N^{(2)}(\omega_1), F_N^{(2)}(\omega_1)) & \dots & (T_N^{(m)}(\omega_1), I_N^{(m)}(\omega_1), F_N^{(m)}(\omega_1)) \\ (T_N^{(1)}(\omega_2), I_N^{(1)}(\omega_2), F_N^{(2)}(\omega_2)) & (T_N^{(2)}(\omega_2), I_N^{(2)}(\omega_2), F_N^{(1)}(\omega_2)) & \dots & (T_N^{(m)}(\omega_2), I_N^{(m)}(\omega_2), F_N^{(m)}(\omega_2)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_N^{(1)}(\omega_r), I_N^{(1)}(\omega_r), F_N^{(1)}(\omega_r)) & (T_N^{(2)}(\omega_r), I_N^{(2)}(\omega_r), F_N^{(2)}(\omega_r)) & \dots & (T_N^{(m)}(\omega_r), I_N^{(m)}(\omega_r), F_N^{(m)}(\omega_r)) \end{pmatrix}$$

This matrix of size $r \times m$ is titled as *mN matrix*. The aggregate of all *mNSs* defined over X is designated as *mNS(X)*.

Definition 2.2. [21] The complement of mNS

$$N = \left\{ \frac{\omega}{\left((T_N^{(i)}(\omega), I_N^{(i)}(\omega), F_N^{(i)}(\omega)) \right)} \right\}$$

over X is given by

$$N^c = \left\{ \frac{\omega}{\left((F_N^{(i)}(\omega), 1 - I_N^{(i)}(\omega), T_N^{(i)}(\omega)) \right)} \right\}.$$

Definition 2.3. [21] An mNS N_1 is called a *subset* of the mNS N_2 , written $N_1 \sqsubseteq N_2$ if $T_{N_1}^{(i)}(\omega) \leq T_{N_2}^{(i)}(\omega)$, $I_{N_1}^{(i)}(\omega) \geq I_{N_2}^{(i)}(\omega)$ and $F_{N_1}^{(i)}(\omega) \geq F_{N_2}^{(i)}(\omega)$, for all $\omega \in X$ and all i .
 N_1 and N_2 are said to be *equal* if and only if $N_1 \sqsubseteq N_2 \sqsubseteq N_1$.

Definition 2.4. [21] An mNS 0_N is called to be *null m -polar neutrosophic set* if it is in the form

$$0_N = \left\{ \frac{\omega}{((0, 1, 1), \dots, (0, 1, 1))} \right\}$$

Definition 2.5. [21] An mNS 1_N is called to be *absolute m -polar neutrosophic set* if it is in the form

$$1_N = \left\{ \frac{\omega}{((1, 0, 0), \dots, (1, 0, 0))} \right\}$$

3 Proposed Information Measures

In this section, we propose five information measures for mNS s:

- (1) Similarity measure.
- (2) Distance measure.
- (3) Correlation measure.
- (4) Divergence measure.
- (5) Dice measure.

3.1 Similarity Measure for m NSs

This subsection is devoted to propose a similarity measure for m NSs along with some of its major characteristics.

Definition 3.1. Take N to be an m NS with $N_1, N_2 \subseteq N$. A measure $Sim(N_1, N_2)$, given by the mapping $Sim(N_1, N_2) : mNS(X) \times mNS(X) \rightarrow [0, 1]$, is called a *similarity measure* if:

1. $Sim(N_1, N_2)$ is a fuzzy number i.e. $Sim(N_1, N_2) \in [0, 1]$.
2. $Sim(N_1, N_2) = 1 \Leftrightarrow N_1 = N_2$.
3. $Sim(N_1, N_2) = Sim(N_2, N_1)$.

Definition 3.2. Let $X = \{\omega_i : i = 1, \dots, n\}$. The similarity measure between m NSs $N_1 = \left\{ \frac{\omega_i}{(T_{N_1}^{(j)}(\omega_i), I_{N_1}^{(j)}(\omega_i), F_{N_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $N_2 = \left\{ \frac{\omega_i}{(T_{N_2}^{(j)}(\omega_i), I_{N_2}^{(j)}(\omega_i), F_{N_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ defined over the finite universe $X = \{\omega_i : i = 1, 2, \dots, n\}$ may be defined as

$$Sim(N_1, N_2) = \frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|}$$

where

$$\begin{aligned} \langle N_1, N_2 \rangle &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i)) \right\}, \\ \|N_1\| &= \langle N_1, N_1 \rangle = \sqrt{\sum_{j=1}^m \left\{ \sum_{i=1}^n \left((T_{N_1}^{(j)}(\omega_i))^2 + (I_{N_1}^{(j)}(\omega_i))^2 + (F_{N_1}^{(j)}(\omega_i))^2 \right) \right\}}, \\ \|N_2\| &= \langle N_2, N_2 \rangle = \sqrt{\sum_{j=1}^m \left\{ \sum_{i=1}^n \left((T_{N_2}^{(j)}(\omega_i))^2 + (I_{N_2}^{(j)}(\omega_i))^2 + (F_{N_2}^{(j)}(\omega_i))^2 \right) \right\}}. \end{aligned}$$

Proposition 3.3. The similarity measure presented in definition 3.2 observes the following requirements:

1. $0 \leq Sim(N_1, N_2) \leq 1$.
2. $Sim(N_1, N_2) = 1 \Leftrightarrow N_1 = N_2$.
3. $Sim(N_1, N_2) = Sim(N_2, N_1)$.

Proof. The first and third conditions are obvious. Suppose that $Sim(N_1, N_2) = 1$. Then, for all permissible values of i, j , we have

$$\frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|} = 1$$

which in turn yields $T_{N_1}^{(j)}(\omega_i) = T_{N_2}^{(j)}(\omega_i)$, $I_{N_1}^{(j)}(\omega_i) = I_{N_2}^{(j)}(\omega_i)$ and $F_{N_1}^{(j)}(\omega_i) = F_{N_2}^{(j)}(\omega_i)$. Hence, $N_1 = N_2$.

Conversely, assume that $N_1 = N_2$. Thus, for all i and j , we have $T_{N_1}^{(j)}(\omega_i) = T_{N_2}^{(j)}(\omega_i)$, $I_{N_1}^{(j)}(\omega_i) = I_{N_2}^{(j)}(\omega_i)$ and $F_{N_1}^{(j)}(\omega_i) = F_{N_2}^{(j)}(\omega_i)$. Thus,

$$\begin{aligned}
\langle N_1, N_2 \rangle &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i)) \right\} \\
&= \sum_{j=1}^m \left\{ \sum_{i=1}^n \left((T_{N_1}^{(j)}(\omega_i))^2 + (I_{N_1}^{(j)}(\omega_i))^2 + (F_{N_1}^{(j)}(\omega_i))^2 \right) \right\} \\
&= \|N_1\|^2 \\
&= \|N_1\| \|N_2\| \\
\therefore \text{Sim}(N_1, N_2) &= \frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|} \\
&= 1.
\end{aligned}$$

□

Example 3.4. Consider the m NSs

$$\begin{aligned}
N_1 &= \left\{ \frac{\omega_1}{(0.72, 0.75, 0.38), (0.52, 0.68, 0.79)}, \frac{\omega_2}{(0.37, 0.91, 0.48), (0.19, 0.28, 0.83)} \right\} \\
N_2 &= \left\{ \frac{\omega_1}{(0.12, 0.21, 0.89), (0.21, 0.17, 0.37)}, \frac{\omega_2}{(0.89, 0.33, 0.14), (0.71, 0.03, 0.15)} \right\}
\end{aligned}$$

defined over the crisp set $X = \{\omega_1, \omega_2\}$. Then,

$$\begin{aligned}
\langle N_1, N_2 \rangle &= \sum_{j=1}^2 \left\{ \sum_{i=1}^2 (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i)) \right\} \\
&= (0.72)(0.12) + (0.75)(0.21) + (0.38)(0.89) + \dots + (0.83)(0.15) \\
&= 2.0638,
\end{aligned}$$

$$\begin{aligned}
\|N_1\| &= \sqrt{\sum_{j=1}^2 \left\{ \sum_{i=1}^2 \left((T_{N_1}^{(j)}(\omega_i))^2 + (I_{N_1}^{(j)}(\omega_i))^2 + (F_{N_1}^{(j)}(\omega_i))^2 \right) \right\}} \\
&= \sqrt{(0.72)^2 + (0.75)^2 + \dots + (0.83)^2} \\
&= 2.1403,
\end{aligned}$$

$$\begin{aligned}
\|N_2\| &= \sqrt{\sum_{j=1}^2 \left\{ \sum_{i=1}^2 \left((T_{N_2}^{(j)}(\omega_i))^2 + (I_{N_2}^{(j)}(\omega_i))^2 + (F_{N_2}^{(j)}(\omega_i))^2 \right) \right\}} \\
&= \sqrt{(0.12)^2 + (0.21)^2 + \dots + (0.15)^2} \\
&= 1.5839
\end{aligned}$$

and hence

$$\text{Sim}(N_1, N_2) = \frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|} = 0.6088.$$

Definition 3.5. The angle θ_{N_1, N_2} defined by $\theta_{N_1, N_2} = \arccos\left(\frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|}\right)$ is called the *angle of similarity* between the m NSs N_1 and N_2 .

Example 3.6. For N_1 and N_2 , cited in Example 3.4, we have

$$\begin{aligned} \theta_{N_1, N_2} &= \arccos\left(\frac{\langle N_1, N_2 \rangle}{\|N_1\| \|N_2\|}\right) \\ &= \arccos(0.6088) \\ &= 52^\circ 30'. \end{aligned}$$

Remark 1. For finding the similarity between an NS and an m NS, we convert the NS to m NS first by multipolarizing the given NS according as the given m NS. The forthcoming example illustrates the notion.

Example 3.7. Consider the m NS

$$N_1 = \left\{ \frac{\omega_1}{(0.47, 0.21, 0.18), (0.38, 0.11, 0.54)}, \frac{\omega_2}{(0.62, 0.28, 0.13), (0.85, 0.33, 0.12)}, \frac{\omega_3}{(0.08, 0.41, 0.34), (0.61, 0.62, 0.73)} \right\}$$

and an NS

$$N = \left\{ \frac{\omega_1}{(0.74, 0.16, 0.31)}, \frac{\omega_2}{(0.59, 0.21, 0.47)}, \frac{\omega_3}{(0.59, 0.81, 0.07)} \right\}$$

defined over $X = \{\omega_1, \omega_2, \omega_3\}$. For finding the measure of similarity between N_1 and N , we make the NS N a 2NS (and call the resulting set N_2) to make it compatible with N_1 as

$$N_2 = \left\{ \frac{\omega_1}{(0.74, 0.16, 0.31), (0.74, 0.16, 0.31)}, \frac{\omega_2}{(0.59, 0.21, 0.47), (0.59, 0.21, 0.47)}, \frac{\omega_3}{(0.59, 0.81, 0.07), (0.59, 0.81, 0.07)} \right\}$$

Now,

$$\begin{aligned} \langle N_1, N_2 \rangle &= \sum_{j=1}^2 \left\{ \sum_{i=1}^3 (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i)) \right\} \\ &= (0.47)(0.74) + (0.21)(0.16) + (0.18)(0.31) + \dots + (0.73)(0.07) \\ &= 3.3326, \\ \|N_1\| &= \sqrt{\sum_{j=1}^2 \left\{ \sum_{i=1}^3 \left((T_{N_1}^{(j)}(\omega_i))^2 + (I_{N_1}^{(j)}(\omega_i))^2 + (F_{N_1}^{(j)}(\omega_i))^2 \right) \right\}} \\ &= \sqrt{(0.47)^2 + (0.21)^2 + \dots + (0.73)^2} \\ &= 1.9106, \\ \|N_2\| &= \sqrt{\sum_{j=1}^2 \left\{ \sum_{i=1}^3 \left((T_{N_2}^{(j)}(\omega_i))^2 + (I_{N_2}^{(j)}(\omega_i))^2 + (F_{N_2}^{(j)}(\omega_i))^2 \right) \right\}} \\ &= \sqrt{(0.74)^2 + (0.16)^2 + \dots + (0.07)^2} \\ &= 2.1408 \end{aligned}$$

and hence

$$Sim(N_1, N_2) = \frac{< N_1, N_2 >}{\|N_1\| \|N_2\|} = 0.8148.$$

Proposition 3.8. N is a crisp set $\Leftrightarrow Sim(N, N^c) = 1$.

Proof. The proof follows from the fact that a crisp set has the form

$$N = \left\{ \frac{\omega_\alpha}{(1, 0, 0)}, \frac{\omega_\beta}{(0, 0, 1)} \right\}$$

for all permissible values of α and β . □

Proposition 3.9. If N_1 , N_2 and N_3 are mNS s defined over X such that $N_1 \sqsubseteq N_2 \sqsubseteq N_3$, then $Sim(N_1, N_3) \leq Sim(N_1, N_2)$ and $Sim(N_1, N_3) \leq Sim(N_2, N_3)$.

Proof. The proof follows directly by definition. □

Example 3.10. Consider the mNS s

$$N_1 = \begin{pmatrix} (0.48, 0.21, 0.41) & (0.32, 0.57, 0.01) & (0.38, 0.25, 0.69) \\ (0.16, 0.74, 0.51) & (0.33, 0.12, 0.30) & (0.41, 0.45, 0.58) \\ (0.02, 0.13, 0.99) & (0.58, 0.43, 0.37) & (0.89, 0.21, 0.01) \end{pmatrix},$$

$$N_2 = \begin{pmatrix} (0.58, 0.19, 0.35) & (0.49, 0.44, 0.01) & (0.41, 0.19, 0.30) \\ (0.29, 0.36, 0.48) & (0.41, 0.01, 0.25) & (0.56, 0.32, 0.46) \\ (0.42, 0.08, 0.52) & (0.73, 0.21, 0.19) & (0.92, 0.17, 0.01) \end{pmatrix},$$

and

$$N_3 = \begin{pmatrix} (0.62, 0.04, 0.27) & (0.52, 0.35, 0.01) & (0.52, 0.02, 0.27) \\ (0.35, 0.27, 0.37) & (0.56, 0.01, 0.16) & (0.68, 0.25, 0.38) \\ (0.58, 0.07, 0.47) & (0.89, 0.19, 0.14) & (0.98, 0.11, 0.01) \end{pmatrix}$$

so that $N_1 \sqsubseteq N_2 \sqsubseteq N_3$. Here,

$$Sim(N_1, N_2) = 0.9159,$$

$$Sim(N_2, N_3) = 0.9771,$$

$$Sim(N_1, N_3) = 0.8413.$$

Thus, the results given in Proposition 3.9 are testified.

Definition 3.11. Two mNS s N_1 and N_2 are said to be λ -similar, written $N_1 \approx^\lambda N_2$, if and only if $Sim(N_1, N_2) \geq \lambda$ for some $0 < \lambda < 1$.

We observe from Example 3.10 that $\text{Sim}(N_1, N_2) \geq 0.9$ and $\text{Sim}(N_2, N_3) \geq 0.9$, but $\text{Sim}(N_1, N_3) \not\geq 0.9$ i.e. $N_1 \approx^{0.9} N_2$ and $N_2 \approx^{0.9} N_3$, but $N_1 \approx^{0.9} N_3$ is not true. It follows that being λ -similar does not follow transitive property of a relation.

Proposition 3.12. *The relation of being λ -similar is not an equivalence relation.*

Definition 3.13. The *entropy* of an mNS N is defined as $E(N) = \text{Sim}(N, N^c)$. It follows from definition that $E(N) = E(N^c)$.

Definition 3.14. Let N_1 and N_2 be two mNS s. Then, N_1 is called *less fuzzy* than N_2 if and only if $E(N_1) \leq E(N_2)$.

Example 3.15. For N_1 and N_2 given in Example 3.10, we have $E(N_1) = 0.5552$ and $E(N_2) = 0.5687$, so N_1 is less fuzzy than N_2 .

3.2 Distance Measure for mNS s

This subsection is devoted to present a distance measure for mNS s and some of its major characteristics.

Definition 3.16. Let N be an mNS and $N_1, N_2 \subseteq N$. A measure $D_m(N_1, N_2)$, given by the mapping $D_m(N_1, N_2) : mNS(X) \times mNS(X) \rightarrow [0, 1]$, is called a *distance measure* if following requirements are fulfilled:

1. $D_m(N_1, N_2)$ is a fuzzy number i.e. $D_m(N_1, N_2) \in [0, 1]$.
2. $D_m(N_1, N_2) = 0 \Leftrightarrow N_1 = N_2$.
3. $D_m(N_1, N_2) = D_m(N_2, N_1)$.

Definition 3.17. The *distance measure* between mNS s $N_1 = \left\{ \frac{\omega_i}{(T_{N_1}^{(j)}(\omega_i), I_{N_1}^{(j)}(\omega_i), F_{N_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $N_2 = \left\{ \frac{\omega_i}{(T_{N_2}^{(j)}(\omega_i), I_{N_2}^{(j)}(\omega_i), F_{N_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ defined over $X = \{\omega_i : i = 1, 2, \dots, n\}$ may be defined as

$$D_m(N_1, N_2) = \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ \left| T_{N_1}^{(j)}(\omega_i) - T_{N_2}^{(j)}(\omega_i) \right| + \left| I_{N_1}^{(j)}(\omega_i) - I_{N_2}^{(j)}(\omega_i) \right| + \left| F_{N_1}^{(j)}(\omega_i) - F_{N_2}^{(j)}(\omega_i) \right| \right\} \right\}$$

Proposition 3.18. *The distance measure D_m given in Definition 3.17 fulfills the requirements given in Definition 3.16.*

Example 3.19. The distance measure between m NSs given in Example 3.4 is

$$\begin{aligned}
D_m(N_1, N_2) &= \frac{1}{2(2)(2)} \sum_{j=1}^2 \left\{ \sum_{i=1}^2 \left\{ \left| T_{N_1}^{(j)}(\omega_i) - T_{N_2}^{(j)}(\omega_i) \right| + \left| I_{N_1}^{(j)}(\omega_i) - I_{N_2}^{(j)}(\omega_i) \right| + \left| F_{N_1}^{(j)}(\omega_i) - F_{N_2}^{(j)}(\omega_i) \right| \right\} \right\} \\
&= \frac{1}{8} \left\{ |0.72 - 0.12| + |0.75 - 0.21| + \dots + |0.83 - 0.15| \right\} \\
&= 0.7225.
\end{aligned}$$

Remark 2. In order to find the distance measure between an NS and an m NS, we proceed like Example 3.7.

Proposition 3.20. N is a crisp set $\Leftrightarrow D_m(N, N^c) = 1$.

Proof. The proof is consequence of the fact that a crisp set has the form

$$N = \left\{ \frac{\omega_\alpha}{(0, 1, 1)}, \frac{\omega_\beta}{(1, 0, 0)} \right\}$$

for all permissible values of α and β . □

Proposition 3.21. If N_1 , N_2 and N_3 are m NSs defined over X such that $N_1 \subseteq N_2 \subseteq N_3$, then $D_m(N_1, N_2) \leq D_m(N_1, N_3)$ and $D_m(N_2, N_3) \leq D_m(N_1, N_3)$.

Proof. Since $N_1 \subseteq N_2 \subseteq N_3$, so $T_{N_1}^{(j)}(\omega_i) \leq T_{N_2}^{(j)}(\omega_i) \leq T_{N_3}^{(j)}(\omega_i)$, $I_{N_1}^{(j)}(\omega_i) \geq I_{N_2}^{(j)}(\omega_i) \geq I_{N_3}^{(j)}(\omega_i)$ and $F_{N_1}^{(j)}(\omega_i) \geq F_{N_2}^{(j)}(\omega_i) \geq F_{N_3}^{(j)}(\omega_i)$ for all i and j .

Now, by definition

$$\begin{aligned}
D_m(N_1, N_2) &= \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ \left| T_{N_1}^{(j)}(\omega_i) - T_{N_2}^{(j)}(\omega_i) \right| + \left| I_{N_1}^{(j)}(\omega_i) - I_{N_2}^{(j)}(\omega_i) \right| + \left| F_{N_1}^{(j)}(\omega_i) - F_{N_2}^{(j)}(\omega_i) \right| \right\} \right\} \\
&= \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ T_{N_2}^{(j)}(\omega_i) - T_{N_1}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) - I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) - F_{N_2}^{(j)}(\omega_i) \right\} \right\}, \\
D_m(N_1, N_3) &= \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ \left| T_{N_1}^{(j)}(\omega_i) - T_{N_3}^{(j)}(\omega_i) \right| + \left| I_{N_1}^{(j)}(\omega_i) - I_{N_3}^{(j)}(\omega_i) \right| + \left| F_{N_1}^{(j)}(\omega_i) - F_{N_3}^{(j)}(\omega_i) \right| \right\} \right\} \\
&= \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ T_{N_3}^{(j)}(\omega_i) - T_{N_1}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i) - I_{N_3}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i) - F_{N_3}^{(j)}(\omega_i) \right\} \right\}.
\end{aligned}$$

On subtraction, we have

$$\begin{aligned}
D_m(N_1, N_3) - D_m(N_1, N_2) &= \frac{1}{2mn} \sum_{j=1}^m \left\{ \sum_{i=1}^n \left\{ T_{N_3}^{(j)}(\omega_i) - T_{N_2}^{(j)}(\omega_i) + I_{N_2}^{(j)}(\omega_i) - I_{N_3}^{(j)}(\omega_i) + F_{N_2}^{(j)}(\omega_i) - F_{N_3}^{(j)}(\omega_i) \right\} \right\} \\
&\geq 0 \\
\Rightarrow D_m(N_1, N_2) &\leq D_m(N_1, N_3).
\end{aligned}$$

The other result may be established analogously. $\square$

Example 3.22. For the m NSs cited in Example 3.10, we have

$$\begin{aligned} D_m(N_1, N_2) &= 0.1967, \\ D_m(N_2, N_3) &= 0.1133, \\ D_m(N_1, N_3) &= 0.3178. \end{aligned}$$

Thus, the results given in Proposition 3.21 are testified.

Remark 3. If $D_m(N_1, N_2)$ is the distance measure between two m NSs N_1 and N_2 , then $1 - D_m(N_1, N_2)$ serves as a similarity measure between N_1 and N_2 and vice versa. Similarly, $\frac{1-D_m(N_1, N_2)}{1+D_m(N_1, N_2)}$ and $\frac{e^{1-D_m(N_1, N_2)}-1}{e-1}$ are also similarity measures between N_1 and N_2 . Hence, more similarity measures may be defined utilizing the distance measure.

3.3 Correlation Measure for m NSs

This subsection proposes a correlation measure for m NSs along with some of its major characteristics.

Definition 3.23. Take N to be an m NS with $N_1, N_2 \subseteq N$. A measure $Cor(N_1, N_2)$, given by the mapping $Cor(N_1, N_2) : mNS(X) \times mNS(X) \rightarrow [0, 1]$, is called a *correlation measure* if:

1. $Cor(N_1, N_2)$ is a fuzzy number i.e. $Cor(N_1, N_2) \in [0, 1]$.
2. $Cor(N_1, N_2) = 1 \Leftrightarrow N_1 = N_2$.
3. $Cor(N_1, N_2) = Cor(N_2, N_1)$.

Definition 3.24. The *correlation measure* between m NSs $N_1 = \left\{ \frac{\omega_i}{(T_{N_1}^{(j)}(\omega_i), I_{N_1}^{(j)}(\omega_i), F_{N_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $N_2 = \left\{ \frac{\omega_i}{(T_{N_2}^{(j)}(\omega_i), I_{N_2}^{(j)}(\omega_i), F_{N_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ defined over $X = \{\omega_i : i = 1, 2, \dots, n\}$ may be defined as

$$Cor(N_1, N_2) = \frac{\eta(N_1, N_2) \{ \eta(N_1, N_1) + \eta(N_2, N_2) \}}{2\eta(N_1, N_1)\eta(N_2, N_2)}$$

where

$$\begin{aligned}
\eta(N_1, N_2) &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} \right\}, \\
\eta(N_1, N_1) &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i))^3 + (I_{N_1}^{(j)}(\omega_i))^3 + (F_{N_1}^{(j)}(\omega_i))^3 \right\}, \\
\eta(N_2, N_2) &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_2}^{(j)}(\omega_i))^3 + (I_{N_2}^{(j)}(\omega_i))^3 + (F_{N_2}^{(j)}(\omega_i))^3 \right\}.
\end{aligned}$$

Proposition 3.25. *The correlation measure $Cor(N_1, N_2)$ given in Definition 3.24 fulfills the requirements given in Definition 3.23.*

Proof. We prove (2) here. (1) and (3) follow immediately from definition.

Assume that

$$\begin{aligned}
Cor(N_1, N_2) &= 1 \\
\Rightarrow \frac{\eta(N_1, N_2) \{ \eta(N_1, N_1) + \eta(N_2, N_2) \}}{2\eta(N_1, N_1)\eta(N_2, N_2)} &= 1 \\
\Rightarrow \eta(N_1, N_2) \{ \eta(N_1, N_1) + \eta(N_2, N_2) \} &= 2\eta(N_1, N_1)\eta(N_2, N_2) \\
\Rightarrow \eta(N_1, N_1) \{ \eta(N_1, N_2) - \eta(N_2, N_2) \} + \eta(N_2, N_2) \{ \eta(N_1, N_2) - \eta(N_1, N_1) \} &= 0
\end{aligned}$$

The last equation, in general, makes sense if

$$\eta(N_1, N_2) - \eta(N_2, N_2) = 0$$

and

$$\eta(N_1, N_2) - \eta(N_1, N_1) = 0$$

i.e. if

$$\eta(N_1, N_2) = \eta(N_1, N_1) = \eta(N_2, N_2)$$

Now,

$$\begin{aligned}
& \eta(N_1, N_2) = \eta(N_1, N_1) \\
\Rightarrow & \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} \right\} \\
& = \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i))^3 + (I_{N_1}^{(j)}(\omega_i))^3 + (F_{N_1}^{(j)}(\omega_i))^3 \right\} \\
\Rightarrow & (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} = (T_{N_1}^{(j)}(\omega_i))^3 + (I_{N_1}^{(j)}(\omega_i))^3 + (F_{N_1}^{(j)}(\omega_i))^3 \\
& \Rightarrow (T_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \left\{ (T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (T_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \right\} + (I_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \left\{ (I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (I_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \right\} \\
& \quad + (F_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \left\{ (F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (F_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} \right\} = 0 \\
\Rightarrow & (T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (T_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} = 0, \quad (I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (I_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} = 0 \text{ \& } (F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} - (F_{N_1}^{(j)}(\omega_i))^{\frac{3}{2}} = 0 \\
& \Rightarrow T_{N_2}^{(j)}(\omega_i) = T_{N_1}^{(j)}(\omega_i), \quad I_{N_2}^{(j)}(\omega_i) = I_{N_1}^{(j)}(\omega_i) \text{ \& } F_{N_2}^{(j)}(\omega_i) = F_{N_1}^{(j)}(\omega_i) \\
& \Rightarrow N_1 = N_2.
\end{aligned}$$

Conversely, suppose that $N_1 = N_2$, then

$$\begin{aligned}
Cor(N_1, N_2) &= \frac{\eta(N_1, N_2) \{ \eta(N_1, N_1) + \eta(N_2, N_2) \}}{2\eta(N_1, N_1)\eta(N_2, N_2)} \\
&= \frac{\eta(N_1, N_1) \{ \eta(N_1, N_1) + \eta(N_1, N_1) \}}{2\eta(N_1, N_1)\eta(N_1, N_1)} \\
&= \frac{2\eta^2(N_1, N_1)}{2\eta^2(N_1, N_1)} \\
&= 1.
\end{aligned}$$

□

Example 3.26. The correlation measure between m NSs given in Example 3.4 is computed as

under:

$$\begin{aligned}
\eta(N_1, N_2) &= \sum_{j=1}^2 \left\{ \sum_{i=1}^2 (T_{N_1}^{(j)}(\omega_i) T_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (I_{N_1}^{(j)}(\omega_i) I_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} + (F_{N_1}^{(j)}(\omega_i) F_{N_2}^{(j)}(\omega_i))^{\frac{3}{2}} \right\} \\
&= (0.72 \times 0.12)^{\frac{3}{2}} + (0.75 \times 0.21)^{\frac{3}{2}} + \dots + (0.83 \times 0.15)^{\frac{3}{2}} \\
&= 0.9832, \\
\eta(N_1, N_1) &= \sum_{j=1}^2 \left\{ \sum_{i=1}^2 (T_{N_1}^{(j)}(\omega_i))^3 + (I_{N_1}^{(j)}(\omega_i))^3 + (F_{N_1}^{(j)}(\omega_i))^3 \right\} \\
&= (0.72)^3 + (0.75)^3 + \dots + (0.83)^3 \\
&= 2.8204, \\
\eta(N_2, N_2) &= \sum_{j=1}^2 \left\{ \sum_{i=1}^2 (T_{N_2}^{(j)}(\omega_i))^3 + (I_{N_2}^{(j)}(\omega_i))^3 + (F_{N_2}^{(j)}(\omega_i))^3 \right\} \\
&= (0.12)^3 + (0.21)^3 + \dots + (0.15)^3 \\
&= 1.8857.
\end{aligned}$$

Hence,

$$\begin{aligned}
Cor(N_1, N_2) &= \frac{\eta(N_1, N_2) \{ \eta(N_1, N_1) + \eta(N_2, N_2) \}}{2\eta(N_1, N_1)\eta(N_2, N_2)} \\
&= \frac{0.9832(2.8204 + 1.8857)}{2(2.8204)(1.8857)} \\
&= 0.4350.
\end{aligned}$$

Remark 4. For finding the correlation measure between an NS and an m NS, we proceed like Example 3.7.

3.4 Divergence Measure for m NSs

We present a divergence measure for m NSs in this subsection.

Definition 3.27. Let N be an m NS with $N_1, N_2 \subseteq N$. A measure $\delta(N_1, N_2)$, given by the mapping $\delta(N_1, N_2) : mNS(X) \times mNS(X) \rightarrow [0, 1]$, is called a *divergence measure* if:

1. $0 \leq \delta(N_1, N_2) \leq 1$.
2. $\delta(N_1, N_2) = 0 \Leftrightarrow N_1 = N_2$.
3. $\delta(N_1, N_2) = \delta(N_2, N_1)$.

Definition 3.28. The *divergence measure* between m NSs $N_1 = \left\{ \frac{\omega_i}{(T_{N_1}^{(j)}(\omega_i), I_{N_1}^{(j)}(\omega_i), F_{N_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $N_2 = \left\{ \frac{\omega_i}{(T_{N_2}^{(j)}(\omega_i), I_{N_2}^{(j)}(\omega_i), F_{N_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ defined over $X = \{\omega_i : i = 1, 2, \dots, n\}$ may be defined as

$$\delta(N_1, N_2) = \frac{\bar{\delta}(N_1, N_2) + \bar{\delta}(N_2, N_1)}{2mn}$$

where

$$\begin{aligned} \bar{\delta}(N_1, N_2) &= \exp \left\{ \sum_{j=1}^m \left\{ \sum_{i=1}^n T_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2T_{N_1}^{(j)}(\omega_i)}{T_{N_1}^{(j)}(\omega_i) + T_{N_2}^{(j)}(\omega_i)} \right\} + I_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2I_{N_1}^{(j)}(\omega_i)}{I_{N_1}^{(j)}(\omega_i) + I_{N_2}^{(j)}(\omega_i)} \right\} \right. \right. \\ &\quad \left. \left. + F_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2F_{N_1}^{(j)}(\omega_i)}{F_{N_1}^{(j)}(\omega_i) + F_{N_2}^{(j)}(\omega_i)} \right\} \right\} - 1 \right\}, \\ \bar{\delta}(N_2, N_1) &= \exp \left\{ \sum_{j=1}^m \left\{ \sum_{i=1}^n T_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2T_{N_2}^{(j)}(\omega_i)}{T_{N_1}^{(j)}(\omega_i) + T_{N_2}^{(j)}(\omega_i)} \right\} + I_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2I_{N_2}^{(j)}(\omega_i)}{I_{N_1}^{(j)}(\omega_i) + I_{N_2}^{(j)}(\omega_i)} \right\} \right. \right. \\ &\quad \left. \left. + F_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2F_{N_2}^{(j)}(\omega_i)}{F_{N_1}^{(j)}(\omega_i) + F_{N_2}^{(j)}(\omega_i)} \right\} \right\} - 1 \right\}. \end{aligned}$$

Proposition 3.29. The divergence measure $\delta(N_1, N_2)$ given in Definition 3.28 fulfills the requirements given in Definition 3.27.

Example 3.30. The divergence measure between the m NSs given in Example 3.4 is computed as under:

$$\begin{aligned} N_1 &= \left\{ \frac{\omega_1}{(0.72, 0.75, 0.38)}, \frac{\omega_2}{(0.52, 0.68, 0.79)}, \frac{\omega_3}{(0.37, 0.91, 0.48)}, \frac{\omega_4}{(0.19, 0.28, 0.83)} \right\} \\ N_2 &= \left\{ \frac{\omega_1}{(0.12, 0.21, 0.89)}, \frac{\omega_2}{(0.21, 0.17, 0.37)}, \frac{\omega_3}{(0.89, 0.33, 0.14)}, \frac{\omega_4}{(0.71, 0.03, 0.15)} \right\} \end{aligned}$$

$$\begin{aligned} \bar{\delta}(N_1, N_2) &= \exp \left\{ \sum_{j=1}^2 \left\{ \sum_{i=1}^4 T_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2T_{N_1}^{(j)}(\omega_i)}{T_{N_1}^{(j)}(\omega_i) + T_{N_2}^{(j)}(\omega_i)} \right\} + I_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2I_{N_1}^{(j)}(\omega_i)}{I_{N_1}^{(j)}(\omega_i) + I_{N_2}^{(j)}(\omega_i)} \right\} \right. \right. \\ &\quad \left. \left. + F_{N_1}^{(j)}(\omega_i) \ln \left\{ \frac{2F_{N_1}^{(j)}(\omega_i)}{F_{N_1}^{(j)}(\omega_i) + F_{N_2}^{(j)}(\omega_i)} \right\} \right\} - 1 \right\} \\ &= \exp \left\{ 0.72 \ln \left(\frac{2(0.72)}{0.72 + 0.12} \right) + 0.75 \ln \left(\frac{2(0.75)}{0.75 + 0.21} \right) + \dots + 0.83 \ln \left(\frac{2(0.83)}{0.83 + 0.15} \right) - 1 \right\} \\ &= 2.9352, \end{aligned}$$

$$\begin{aligned}
\bar{\delta}(N_2, N_1) &= \exp \left\{ \sum_{j=1}^2 \left\{ \sum_{i=1}^2 T_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2T_{N_2}^{(j)}(\omega_i)}{T_{N_1}^{(j)}(\omega_i) + T_{N_2}^{(j)}(\omega_i)} \right\} + I_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2I_{N_2}^{(j)}(\omega_i)}{I_{N_1}^{(j)}(\omega_i) + I_{N_2}^{(j)}(\omega_i)} \right\} \right. \right. \\
&\quad \left. \left. + F_{N_2}^{(j)}(\omega_i) \ln \left\{ \frac{2F_{N_2}^{(j)}(\omega_i)}{F_{N_1}^{(j)}(\omega_i) + F_{N_2}^{(j)}(\omega_i)} \right\} \right\} - 1 \right\} \\
&= \exp \left\{ 0.12 \ln \left(\frac{2(0.12)}{0.72 + 0.12} \right) + 0.21 \ln \left(\frac{2(0.21)}{0.75 + 0.21} \right) + \dots + 0.15 \ln \left(\frac{2(0.15)}{0.83 + 0.15} \right) - 1 \right\} \\
&= 0.2524.
\end{aligned}$$

Hence,

$$\begin{aligned}
\delta(N_1, N_2) &= \frac{\bar{\delta}(N_1, N_2) + \bar{\delta}(N_2, N_1)}{2mn} \\
&= \frac{2.9352 + 0.2524}{2(2)(2)} \\
&= 0.3985.
\end{aligned}$$

Remark 5. For finding the divergence measure between an NS and an m NS, we proceed like Example 3.7.

3.5 Dice Measure for m NSs

This subsection is devoted to present a Dice measure for m NSs.

Definition 3.31. Let N be an m NS and $N_1, N_2 \subseteq N$. A measure $\Delta(N_1, N_2)$, given by the mapping $\Delta(N_1, N_2) : mNS(X) \times mNS(X) \rightarrow [0, 1]$, is called a *Dice measure* if following requirements are fulfilled:

1. $\Delta(N_1, N_2)$ is a fuzzy number i.e. $\Delta(N_1, N_2) \in [0, 1]$.
2. $N_1 = N_2 \Rightarrow \Delta(N_1, N_2) = 1$.
3. $\Delta(N_1, N_2) = \Delta(N_2, N_1)$.

Definition 3.32. The *Dice measure* between m NSs $N_1 = \left\{ \frac{\omega_i}{(T_{N_1}^{(j)}(\omega_i), I_{N_1}^{(j)}(\omega_i), F_{N_1}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ and $N_2 = \left\{ \frac{\omega_i}{(T_{N_2}^{(j)}(\omega_i), I_{N_2}^{(j)}(\omega_i), F_{N_2}^{(j)}(\omega_i))} : \omega_i \in X \right\}$ defined over $X = \{\omega_i : i = 1, 2, \dots, n\}$ may be defined as

$$\Delta(N_1, N_2) = \frac{2N_1 \cdot N_2}{\|N_1\|^2 + \|N_2\|^2}$$

where

$$\begin{aligned}
N_1.N_2 &= \sum_{j=1}^m \left\{ \sum_{i=1}^n (T_{N_1}^{(j)}(\omega_i)T_{N_2}^{(j)}(\omega_i) + I_{N_1}^{(j)}(\omega_i)I_{N_2}^{(j)}(\omega_i) + F_{N_1}^{(j)}(\omega_i)F_{N_2}^{(j)}(\omega_i)) \right\}, \\
\|N_1\| &= \sqrt{\sum_{j=1}^m \left\{ \sum_{i=1}^n \left((T_{N_1}^{(j)}(\omega_i))^2 + (I_{N_1}^{(j)}(\omega_i))^2 + (F_{N_1}^{(j)}(\omega_i))^2 \right) \right\}}, \\
\|N_2\| &= \sqrt{\sum_{j=1}^m \left\{ \sum_{i=1}^n \left((T_{N_2}^{(j)}(\omega_i))^2 + (I_{N_2}^{(j)}(\omega_i))^2 + (F_{N_2}^{(j)}(\omega_i))^2 \right) \right\}}.
\end{aligned}$$

Proposition 3.33. *The Dice measure Δ given in Definition 3.32 fulfills the requirements given in Definition 3.31.*

Proof. (2) and (3) are obvious. (1) follows from the inequality $a^2 + b^2 \geq 2ab$. □

Example 3.34. The Dice measure between m NSs given in Example 3.4 is

$$\begin{aligned}
\Delta(N_1, N_2) &= \frac{2N_1.N_2}{\|N_1\|^2 + \|N_2\|^2} \\
&= \frac{2(2.0638)}{2.1403^2 + 1.5839^2} \\
&= 0.5822.
\end{aligned}$$

Remark 6. In order to find the Dice measure between an NS and an m NS, we proceed like Example 3.7.

4 Multi-Attribute Decision Making based on m NSs in Health Sciences

Decision making is a vital procedure for selecting most appropriate decision from accessible options. Employing bit by bit decision-making procedure can assist us make more purposive and intellectual decisions by verbalizing relevant data demonstrating another course of action. In this section, we discuss multi-attribute decision making (MADM) in health sciences. MADM methods decide on among a discrete set of choices assessed on multiple features and overall efficacy of the decision makers.

Case Study:

Blood disorders are conditions that influence the blood's ability to operate correctly. Most blood disarrays drop the number of cells, platelets, proteins, or nutrients in the bloodline, or obstruct with their use. A bulk of blood disorders are induced by variations in parts of specific genes and

can be handed down in families. Some medicinal conditions, medicines, and life style features can also originate blood disorders to make grow. A lot of blood disorders take their public figure from the constituent of the blood they influence. The following classifications refer to blood disorders that have a lessening in stock components or disturb their purpose:

1. Anemia:

Anemia is caused by lessened number of flowing red blood cells in the physical structure. Red blood cells are indispensable for the survival of the body. They pass on hemoglobin, which is a multifaceted protein containing iron molecules. These iron-containing molecules transfer oxygen from lungs to the rest of the body which is an essential requirement of body. This type of blood disorder is common in the worldwide population. Over and above 400 categories of anemia have been recognized. Preschool kids have the utmost menace, with a probable 47% evolving anemia, universally.

Figure 1: Normal and anemic amount of red blood cells, Courtesy: drugs.com

The most usual warning sign of all forms of anemia is a feeling of tiredness and a privation of energy. Other corporate symptoms may take account of pallor of skin, squatness of breath, chest discomfort, wild or asymmetrical heartbeat, headache and light-headedness.

2. Leukopenia:

The term leukopenia is derived from two Greek words leukos (white) and penia (deficiency). Leukopenia is a shrinkage of the quantity of white blood cells (leukocytes) present in the blood, which puts individuals at amplified jeopardy of infection. Our body fluid is made up of diverse sorts of blood cells, together with white blood cells, or leukocytes. White blood cells are a significant component of our immune organism which supports our body to hold back ailments, syndromes and infections. A person having too less white blood cells is a victim of leukopenia. Major causes of leukopenia consist of poor condition of blood cells or bone marrow, leukemia (a sort of cancer), hereditary complications, transmissible diseases (like tuberculosis and AIDS), malnutrition, autoimmune disorders (like lupus and rheumatoid arthritis), medications (such as steroids, cyclosporine, bupropion, penicillin and lamotrigine), viral infections and sarcoidosis.

Figure 2: Leukopenia Stock photo, Courtesy: depositphotos.com

Probably there are not any prominent signs of leukopenia. But if white cell counts are very low, one may have signs of infection, including fever higher than $100.5^{\circ}F$, sweating and chills.

3. Thrombocytopenia:

Thrombocytopenia is a blood disease in which one has a lesser amount of blood platelet than desired. Platelets (thrombocytes) are colorless blood cells which help blood clot. Platelets discontinue bleeding by clumping and forming plugs in blood vessel injuries. Main reasons of thrombocytopenia include cancer, chemotherapy, virus (such as chickenpox, rubella, HIV, mumps, and Epstein-Barr), platelet-lowering disease inherited from family, and aplastic anemia.

Figure 3: Body part of a thrombocytopeniac patient, Courtesy: hoacny.com

Sometimes one does not have any symptoms from thrombocytopenia. However, the main one is bleeding in the skin that looks like tiny red or purple spots on the skin, called petechiae, or bruising. One can bleed outside or inside the body. Sometimes it can be heavy or hard to stop. Some people get nosebleeds, bleeding gums, blood in urine or bowel movement and heavy menstrual periods.

Before heading towards practical utility of the suggested similarity measure, first we propose Algorithm 1 given below.

Algorithm 1

Step 1: Choose the set of patients $P = \{p_1, p_2, \dots, p_k\}$.

Step 2: Choose the set of diseases $D = \{d_1, d_2, \dots, d_i\}$ and the set of symptoms $S = \{s_1, s_2, \dots, s_j\}$.

Step 3: Construct table of NS of diseases vs symptoms & m NS of patients vs symptoms.

Step 4: Compute similarity measure between patients & diseases.

Step 5: Optimal choice is the pair with highest similarity measure between patient and disease.

Step 6: State the results in layman's language.

Example 4.1. Let $X = \{p_k : k = 1, \dots, 4\}$ be the set of patients under study, and $D = \{d_i : i = 1, 2, 3\}$ be the set of types of blood disorder, where

$$\begin{aligned} d_1 &= \text{Anemia,} \\ d_2 &= \text{Leukopenia, and} \\ d_3 &= \text{Thrombocytopenia} \end{aligned}$$

Suppose that $S = \{s_i : i = 1, 2, \dots, 4\}$ is the set of symptoms, where

$$\begin{aligned} s_1 &= \text{Skin paleness,} \\ s_2 &= \text{High fever,} \\ s_3 &= \text{Spots on skin, and} \\ s_4 &= \text{weakness} \end{aligned}$$

Table 2 gives neutrosophic numbers as symptoms vs blood disorder type. The objective is to get an appropriate diagnosis for each patient.

Table 2: NS of symptoms vs blood disorder type

	s_1	s_2	s_3	s_4
d_1	(0.84, 0.44, 0.32)	(0.53, 0.46, 0.71)	(0.50, 0.50, 0.71)	(0.92, 0.18, 0.35)
d_2	(0.33, 0.41, 0.85)	(0.81, 0.36, 0.43)	(0.27, 0.31, 0.91)	(0.31, 0.70, 0.64)
d_3	(0.59, 0.49, 0.64)	(0.13, 0.32, 0.94)	(0.86, 0.17, 0.48)	(0.45, 0.71, 0.54)

Table 3 yields information regarding patients vs symptoms in the standard form of representing m NSs. The readings are taken at three different times in order to be more specific about type of disease.

Table 3: m NS of patients vs symptoms

	s_1	s_2	s_3	s_4
p_1	(0.54, 0.68, 0.49)	(0.82, 0.44, 0.37)	(0.56, 0.51, 0.65)	(0.55, 0.79, 0.28)
	(0.42, 0.63, 0.65)	(0.77, 0.41, 0.49)	(0.40, 0.85, 0.35)	(0.54, 0.80, 0.25)
	(0.48, 0.66, 0.57)	(0.79, 0.42, 0.43)	(0.47, 0.66, 0.48)	(0.55, 0.80, 0.26)
p_2	(0.67, 0.45, 0.61)	(0.21, 0.34, 0.89)	(0.79, 0.13, 0.51)	(0.53, 0.68, 0.58)
	(0.54, 0.52, 0.59)	(0.19, 0.33, 0.88)	(0.62, 0.14, 0.39)	(0.53, 0.75, 0.61)
	(0.49, 0.48, 0.68)	(0.07, 0.29, 0.91)	(0.81, 0.23, 0.37)	(0.46, 0.80, 0.59)
p_3	(0.59, 0.02, 0.81)	(0.85, 0.12, 0.51)	(0.56, 0.44, 0.70)	(0.29, 0.34, 0.89)
	(0.53, 0.08, 0.84)	(0.71, 0.26, 0.65)	(0.56, 0.16, 0.81)	(0.12, 0.28, 0.95)
	(0.51, 0.37, 0.92)	(0.62, 0.39, 0.37)	(0.51, 0.41, 0.85)	(0.32, 0.29, 0.80)
p_4	(0.92, 0.36, 0.15)	(0.87, 0.45, 0.20)	(0.92, 0.29, 0.26)	(0.51, 0.50, 0.70)
	(0.51, 0.32, 0.80)	(0.23, 0.40, 0.89)	(0.42, 0.28, 0.86)	(0.46, 0.38, 0.80)
	(0.68, 0.27, 0.63)	(0.44, 0.41, 0.54)	(0.51, 0.44, 0.85)	(0.69, 0.21, 0.80)

Now, we compute the similarity measures between patients and type of blood disorder in Table 4. The patient p_i having maximal value of the similarity measure with d_j is likely to fall prey of the disease d_j .

Table 4: Similarity measures between patients and type of blood disorder

$Sim(p_i, d_j)$	d_1	d_2	d_3
p_1	0.8691	0.9027	0.8368
p_2	0.8480	0.8325	0.9929
p_3	0.8168	0.9321	0.8310
p_4	0.8866	0.8558	0.8640

In view of Table 4, the optimal choices are (p_1, d_2) , (p_2, d_3) , (p_3, d_2) and (p_4, d_1) . Thus, we conclude that the patients p_1 & p_3 are likely to suffer from leukopenia, p_2 from thrombocytopenia, whereas p_4 is victim of anemia.

The similarity measures of patients versus diseases are depicted in Figure 4.

Figure 4: Similarity measures of patients versus diseases

Now, we resolve Example 4.1 using proposed distance measure. But before that, first we propose Algorithm 2 as follows. Step 1 – 3 are same as Algorithm 1, so we skip them.

Algorithm 2

Step 4: Compute distance measure between patients & diseases.

Step 5: Optimal choice is the pair with least distance measure between patients & diseases.

Step 6: State the results in layman's language.

Example 4.2. The distance measures of each patient with the diseases considered are computed and tabulated in Table 5:

Table 5: Distance measures between patients and diseases

$D_m(p_i, d_j)$	d_1	d_2	d_3
p_1	0.3671	0.3125	0.3846
p_2	0.4250	0.3863	0.0850
p_3	0.4171	0.2533	0.4354
p_4	0.3304	0.2525	0.3613

Hence, keeping in view Table 5, it may be concluded that the optimal choices are (p_1, d_2) , (p_2, d_3) , (p_3, d_2) and (p_4, d_1) .

The distance measures of patients versus diseases are displayed in Figure 5.

Figure 5: Distance measures of patients versus diseases

We solve, now, Example 4.1 using proposed correlation measure. So, as earlier, first we propose Algorithm 3 as follows. Step 1 – 3 are same as Algorithm 1, so we skip them.

Algorithm 3

Step 4: Compute correlation measure between patients & diseases.

Step 5: Optimal choice is the pair with highest correlation measure between patients & diseases.

Step 6: State the results in layman's language.

Example 4.3. The correlation measures of each patient with the diseases considered are computed and tabulated in Table 6:

Table 6: Correlation measures between patients and diseases

$Cor(p_i, d_j)$	d_1	d_2	d_3
p_1	0.7520	0.8253	0.7196
p_2	0.7470	0.7068	0.9877
p_3	0.6833	0.9015	0.7225
p_4	0.7959	0.7545	0.7682

Thus, the optimal choices are (p_1, d_2) , (p_2, d_3) , (p_3, d_2) and (p_4, d_1) .

The correlation measures of patients versus diseases are given in Figure 6.

Figure 6: Correlation measures of patients versus diseases

We solve, now, Example 4.1 using proposed divergence measure. So, as earlier, first we propose Algorithm 4 as follows. Step 1 – 3 are same as Algorithm 1, so we skip them.

Algorithm 4

Step 4: Compute divergence measure between patients & diseases.

Step 5: Optimal choice is the pair with least divergence measure between patients & diseases.

Step 6: State the results in layman's language.

Example 4.4. The divergence measures of each patient with the diseases considered are computed and tabulated in Table 7:

Table 7: Divergence measures between patients and diseases

$\delta(p_i, d_j)$	d_1	d_2	d_3
p_1	0.0646	0.0583	0.0614
p_2	0.0874	0.0821	0.0329
p_3	0.1040	0.0540	0.1002
p_4	0.0606	0.0716	0.0709

In view of Table 7, the optimal pairs are (p_1, d_2) , (p_2, d_3) , (p_3, d_2) and (p_4, d_1) . The divergence measures of patients versus diseases are presented in Figure 7.

Figure 7: Divergence measures of patients versus diseases

We solve, now, Example 4.1 using proposed Dice measure. So, as earlier, first we propose Algorithm 5 as follows. Step 1 – 3 are same as Algorithm 1, so we skip them.

Algorithm 5

Step 4: Compute Dice measure between patients & diseases.

Step 5: Optimal choice is the pair with highest Dice measure between patients & diseases.

Step 6: State the results in layman’s language.

Example 4.5. The Dice measures of each patient with the diseases considered are computed and tabulated in Table 8:

Table 8: Dice measures between patients and diseases

$\Delta(p_i, d_j)$	d_1	d_2	d_3
p_1	0.8691	0.9027	0.8368
p_2	0.8477	0.8323	0.9926
p_3	0.8167	0.9321	0.8309
p_4	0.8866	0.8558	0.8640

Thus, the optimal choices are (p_1, d_2) , (p_2, d_3) , (p_3, d_2) and (p_4, d_1) .

The Dice measures of patients versus diseases are exhibited in Figure 8.

Figure 8: Dice measures of patients versus diseases

5 Comparative Analysis

Consider the $mNSs$

$$N_1 = \begin{pmatrix} (0.51, 0.31, 0.21) & (0.63, 0.43, 0.21) & (0.33, 0.61, 0.78) & (0.71, 0.35, 0.41) \\ (0.64, 0.32, 0.24) & (0.82, 0.12, 0.12) & (0.72, 0.12, 0.22) & (0.72, 0.22, 0.25) \\ (0.89, 0.21, 0.21) & (0.70, 0.19, 0.23) & (0.41, 0.72, 0.82) & (0.52, 0.42, 0.52) \end{pmatrix},$$

$$N_2 = \begin{pmatrix} (0.61, 0.21, 0.12) & (0.71, 0.32, 0.12) & (0.41, 0.51, 0.61) & (0.81, 0.21, 0.34) \\ (0.72, 0.21, 0.12) & (0.92, 0.11, 0.11) & (0.82, 0.11, 0.21) & (0.82, 0.21, 0.23) \\ (0.92, 0.12, 0.13) & (0.71, 0.12, 0.21) & (0.51, 0.61, 0.71) & (0.61, 0.32, 0.42) \end{pmatrix}$$

and

$$N_3 = \begin{pmatrix} (0.71, 0.12, 0.11) & (0.81, 0.21, 0.11) & (0.51, 0.42, 0.52) & (0.82, 0.12, 0.21) \\ (0.72, 0.12, 0.11) & (0.93, 0.11, 0.11) & (0.89, 0.11, 0.12) & (0.89, 0.18, 0.20) \\ (0.93, 0.12, 0.13) & (0.81, 0.11, 0.21) & (0.61, 0.51, 0.61) & (0.72, 0.21, 0.33) \end{pmatrix}.$$

We find the information measures, suggested in this paper, between N_k ($k = 1, 2, 3$) and

$$N = \begin{pmatrix} (0.64, 0.12, 0.11) & (0.81, 0.24, 0.12) & (0.51, 0.43, 0.51) & (0.91, 0.12, 0.24) \\ (0.74, 0.12, 0.12) & (0.92, 0.11, 0.11) & (0.89, 0.11, 0.12) & (0.84, 0.11, 0.22) \\ (0.92, 0.11, 0.12) & (0.82, 0.11, 0.21) & (0.61, 0.51, 0.62) & (0.71, 0.22, 0.32) \end{pmatrix}$$

and compare the results with some existing measures. The values computed through the proposed information measures are tabulated in Table 9.

Table 9: Values computed through the proposed information measures

Information measure	(N_1, N)	(N_2, N)	(N_3, N)
Similarity measure	0.9561	0.9899	0.9987
Distance measure	0.2013	0.0838	0.0213
Correlation measure	0.9348	0.9849	0.9976
Divergence measure	0.0527	0.0339	0.0311
Dice measure	0.9560	0.9898	0.9987

The values of the five suggested information measures are displayed by means of Figure 9.

Figure 9: Line chart showing values of the suggested measures

Comparison of proposed measures with some existing measures has been presented in Table 10. We observe that all the five suggested information measures yield the same output and that yield the same ranking when compared with some existing measures, which approves the reliability of the proposed measures.

Table 10: Comparison of proposed measures with some existing measures

Method	Ranking	Optimal Choice
Ye [58]	$N_3 \succ N_1 \succ N_2$	N_3
Wei <i>et al.</i> [55]	$N_3 \succ N_1 \succ N_2$	N_3
Hashmi <i>et al.</i> [21] (Algorithm 1)	$N_3 \succ N_1 \succ N_2$	N_3
Hashmi <i>et al.</i> [21] (Algorithm 2)	$N_3 \succ N_2 \succ N_1$	N_3
Hashmi <i>et al.</i> [21] (Algorithm 3)	$N_3 \succ N_2 \succ N_1$	N_3
Proposed Algorithm 1	$N_3 \succ N_2 \succ N_1$	N_3
Proposed Algorithm 2	$N_3 \succ N_2 \succ N_1$	N_3
Proposed Algorithm 3	$N_3 \succ N_2 \succ N_1$	N_3
Proposed Algorithm 4	$N_3 \succ N_2 \succ N_1$	N_3
Proposed Algorithm 5	$N_3 \succ N_2 \succ N_1$	N_3

6 Conclusion

Novel similarity, distance, correlation, divergence and dice measures along with five algorithms in the framework of m -polar neutrosophic environment inclusive of some of their peculiar features are proposed in this article. The proposed measures enhance the schemes for assessing the level of similarity between PmFSs. The proposed measures fluctuate from 0 to 1 which eradicates the negative similarity rating. We devised method for computing information measure between an m -polar neutrosophic set and a neutrosophic set. A case study of major blood disorders have been made part of the study. It has been observed that the five proposed information measures yield the same optimal choice. We made use of statistical charts, where necessary, to make the final rankings comprehensible for a layman. Comparative analysis of the suggested measures with some prevailing measures is also included. The information measures rendered in this article have gargantuan quiescent for more explorations in analytical beyond usance perspective. The conception may be adroitly hired in manipulating incertitude in different sectors of real life, mainly inclusive of trade and business analysis, voice recognition, economics, artificial intelligence, marketing, transportation problems, image processing, speech recognition, water management problems, agri-forming, pattern recognition, coding theory, forecasting, robotics, recruitment problems and life sciences.

Conflict of interest

The authors declare that they have no conflict of interest.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of

the authors.

Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

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References

- [1] M. Akram, S. Shahzadi and F. Smarandache, *Multi-attribute decision-making method based on neutrosophic soft rough information*, Axioms, **7**(1)(2018), 19, <https://doi.org/10.3390/axioms7010019>.
- [2] M.D. Ansari, A.R. Mishra and F.T. Ansari, *New divergence and entropy measures for intuitionistic fuzzy sets on edge detection*, International Journal of Fuzzy Systems, **20**(2018), 474-487.
- [3] K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20**(1986), 87-96.
- [4] K.T. Atanassov, *More on intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **33**(1989), 37-46.
- [5] S. Broumi, *Generalized neutrosophic soft set*, International Journal of Computer Science, Engineering and Information Technology, **3**(2)(2013), 17-30.
- [6] S. Broumi and F. Smarandache, *Intuitionistic neutrosophic soft set*, Journal of Information and Computing Science, **8**(2)(2013), 130-140.
- [7] S. Broumi and F. Smarandache, *Several similarity measures of neutrosophic sets*, Neutrosophic Sets and Systems, **1**(1)(2013), 54-62.
- [8] B.B. Chaudhuri and P. Bhattacharya, *On correlation between fuzzy sets*, Fuzzy Sets and Systems, **118**(2001), 447-456.
- [9] D.A. Chiang and N.P. Lin, *Correlation of fuzzy sets*, Fuzzy Sets and Systems, **102**(1999), 221-226.
- [10] W.H. Cui and J. Ye, *Logarithmic similarity measure of dynamic neutrosophic cubic sets and its application in medical diagnosis*, Computers in Industry, 111(2019), 198-206.

- [11] G.N. Deng, Y.L. Jiang and J.C. Fu, *Monotonic similarity measures between intuitionistic fuzzy sets and their relationship with entropy and inclusion measure*, Information Sciences, **316**(2015), 348-369.
- [12] I. Deli, *Interval-valued neutrosophic soft sets and decision making*, International Journal of Machine Learning & Cybernetics, **8**(2)(2017), 665-676.
- [13] M.A. Firozja, B. Agheli and E.B. Jamkhaneh, *A new similarity measure for Pythagorean fuzzy sets*, Complex & Intelligent Systems, **6**(2020), 67-74.
- [14] H. Garg and R. Arora, *Distance and similarity measures for dual hesitant fuzzy soft sets and their applications in multicriteria decision-making problem*, International Journal for Uncertainty Quantification **7**(3)(2017), 229-248.
- [15] H. Garg, *A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision making processes*, International Journal of Intelligent Systems, **31**(12)(2016), 1234-1252.
- [16] H. Garg and Nancy, *Algorithms for single-valued neutrosophic decision making based on TOPSIS and clustering methods with new distance measure*, AIMS Mathematics, **5**(3)(2020), 2671-2693.
- [17] M.B. Gorzalzany, *A method of inference in approximate reasoning based on interval-valued fuzzy sets*, Fuzzy Sets and Systems, **21**(1)(1987), 1-17.
- [18] A. Guleria and R.K. Bajaj, *On Pythagorean fuzzy soft matrices, operations and their applications in decision making and medical diagnosis*, Soft Computing, (2018), DOI.org/10.1007/s00500-018-3419-z.
- [19] T. Gerstenkorn and J. Manko, *Correlation of intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **44**(1991), 39-43.
- [20] S. Habib, W. Salam, M.A. Butt, M. Akram and F. Smarandache, *A neutrosophic clinical decision-making system for cardiovascular diseases risk analysis*, Journal of Intelligent and Fuzzy Systems, **39**(5)(2020), 7807-7829.
- [21] M.R. Hashmi, M. Riaz and F. Smarandache, *m-polar neutrosophic topology with applications to multicriteria decision-making in medical diagnosis and clustering analysis*, International Journal of Fuzzy Systems, **22**(1)(2020), 273-292.
- [22] D.H. Hong and S.Y. Hwang, *Correlation of intuitionistic fuzzy sets in probability spaces*, Fuzzy Sets and Systems, **75**(1995), 77-81.

- [23] W.L. Hung and J.W. Wu, *Correlation of intuitionistic fuzzy sets by centroid method*, Information Sciences, **144**(2002), 219-225.
- [24] W.L. Hung, *Using statistical viewpoint in developing correlation of intuitionistic fuzzy sets*, International Journal of Uncertainty Fuzziness Knowledge Based Systems, **9**(2001), 509-516.
- [25] M. Irfan, M. Rani, M. Saqlain and M. Saeed, *Tangent, cosine, and Ye similarity measures of m-polar neutrosophic hypersoft sets*, In book: Theory and Application of Hypersoft Set Edition, Chapter: 8, Pons Publishing House Brussels, (2021), DOI: 10.5281/zenodo.4788269
- [26] N. Jafar, M. Saeed and M. Saqlain, *Trigonometric similarity measures for neutrosophic hypersoft sets with application to renewable energy source selection*, IEEE Acces, **9**(2021), 129178-129187.
- [27] G.-J. Jiang, H.-X. Chen, H.-H. Sun, M. Yazdi, A. Nedjati and K.A. Adesina, *An improved multi-criteria emergency decision-making method in environmental disasters*, Soft Computing, **25**(2021), 10351-10379.
- [28] M. Khan and M.D. Ansari, *Multi-criteria software quality model selection based on divergence measure and score function*, Journal of Intelligent & Fuzzy Systems, **38**(1)(2020), 1-10.
- [29] J. Lin, *Divergence measures based on the Shannon entropy*, IEEE Transactions on Information Theory, **37**(1)(1991), 145-151.
- [30] Z. Lu and J. Ye, *Cosine measures of neutrosophic cubic sets for multiple criteria decision making*, Symmetry, **9**(7)(2017), 121.
- [31] P.K. Maji, *A neutrosophic soft set approach to a decision making problem*, Annals of Fuzzy Mathematics & Informatics, **3**(2)(2012), 313-319.
- [32] P.K. Maji, *Neutrosophic soft set*, Comput Math Appl, **45**(2013), 555-562.
- [33] A.R. Mishra, D. Jain and D.S. Hooda, *Exponential intuitionistic fuzzy information measure with assessment of service quality*, International Journal of Fuzzy Systems, **19**(3)(2017), 788-798.
- [34] H.B. Mitchell, *A correlation coefficient for intuitionistic fuzzy sets*, International Journal of Intelligent Systems, **19**(2004), 483-490.
- [35] D. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, **37**(4-5)(1999), 19-31.
- [36] C.A. Murthy, S.K. Pal and D.D. Majumder, *Correlation between two fuzzy membership functions*, Fuzzy Sets and Systems, **17**(1985), 23-38.

- [37] K. Naeem, M. Riaz and D. Afzal, *Pythagorean m-polar fuzzy sets and TOPSIS method for the selection of advertisement mode*, Journal of Intelligent & Fuzzy Systems, **37**(6)(2019), 84418458.
- [38] K. Naeem, M. Riaz and F. Karaaslan, *Some novel features of Pythagorean m-polar fuzzy sets with applications*, Complex & Intelligent Systems, (2020), <https://doi.org/10.1007/s40747-020-00219-3>.
- [39] K. Naeem, M. Riaz, X. Peng and D. Afzal, *Pythagorean m-polar fuzzy topology with TOPSIS approach in exploring most effectual method for curing from COVID-19*, International Journal of Biomathematics, **13**(8)(2020), <https://doi.org/10.1142/S1793524520500758>
- [40] Nancy and H. Garg, *A novel divergence measure and its based TOPSIS method for multi criteria decision-making under single-valued neutrosophic environment*, Journal of Intelligent & Fuzzy Systems, **36**(1)(2019), 101-115.
- [41] Z. Pawlak, *Rough sets*, International Journal of Computer and Information Science, **11**(1982), 341-356.
- [42] X. Peng, H.Y. Yuan and Y. Yang, *Pythagorean fuzzy information measures and their applications*, International Journal of Intelligent Systems, **32**(10)(2017), 9911029.
- [43] X. Peng, *New similarity measure and distance measure for Pythagorean fuzzy set*, Complex & Intelligent Systems, (2018), DOI.org/10.1007/s40747-018-0084-x.
- [44] X. Peng and L. Liu, *Information measures for q-rung orthopair fuzzy sets*, International Journal of Intelligent Systems, **34**(8)(2019), 1795-1834.
- [45] P. Rajarajeswari and N. Uma, *Intuitionistic fuzzy multi similarity measure based on cotangent function*, International Journal of Engineering Research & Technology, **2**(11)(2013), 1323-1329.
- [46] P. Rajarajeswari and N. Uma, *Correlation measure for intuitionistic fuzzy multi sets*, International Journal of Research in Engineering and Technology, **3**(1)(2014), 611-617.
- [47] M. Riaz, K. Naeem and D. Afzal, *Pythagorean m-polar fuzzy soft sets with TOPSIS method for MCGDM*, Punjab University Journal of mathematics, **52**(3)(2020), 21-46.
- [48] M. Riaz, K. Naeem and D. Afzal, *A similarity measure under Pythagorean fuzzy soft environment with applications*, Computational and Applied Mathematics, **269**(39)(2020), 1-17, <https://doi.org/10.1007/s40314-020-01321-5>
- [49] S. Singh and A.H. Ganie, *On some correlation coefficients in Pythagorean fuzzy environment with applications*, International Journal of Intelligent Systems, (2020) <https://doi.org/10.1002/int.22222>

- [50] F. Smarandache, *Neutrosophy: Neutrosophic probability, set and logic*, American Research Press, Rehoboth, (1998).
- [51] F. Smarandache, *Neutrosophic set, a generalisation of the intuitionistic fuzzy sets*, International Journal of Pure and Applied Mathematics, **24**(3)(2005), 287-297.
- [52] E. Türkarslan, M. Olgun, M. Ünver and Ş. Yardimci, *Some trigonometric similarity measures based on the Choquet integral for Pythagorean fuzzy sets and applications to pattern recognition*, Springer, (2021), 82-106, https://doi.org/10.1007/978-981-16-1989-2_4
- [53] F. Wang and X. Zhao, *Similarity and Pythagorean reliability measures of multivalued neutrosophic cubic set and its application to multiple-criteria decision-making*, International Journal of Intelligent Systems, (2021), <https://doi.org/10.1002/int.22618>
- [54] H. Wang, F. Smarandache, Y-Q. Zhang and R. Sundrraman, *Single valued neutrosophic sets*, Multispace and Multistructure, **4**(2010), 410-413.
- [55] G.W. Wei, *Some similarity measures for picture fuzzy sets and their applications*, Iranian Journal of Fuzzy Systems, **15**(1)(2018), 77-89.
- [56] H. Xue, M.R. Yu and C.F. Chen, *Research on novel correlation coefficient of neutrosophic cubic sets and its applications*. Mathematical Problems in Engineering, (2019), 2019:7453025.
- [57] R.R. Yager, *Pythagorean fuzzy subsets*, IFSA World Congress and NAFIPS Annual Meeting, 2013 Joint, Edmonton, Canada, IEEE (2013), 57-61.
- [58] J. Ye, *Cosine similarity measures for intuitionistic fuzzy sets and their applications*, Information and Control, **53**(1)(2011), 91-97.
- [59] J. Ye, *Generalized Dice measures for multiple attribute decision making under intuitionistic and interval-valued intuitionistic fuzzy environments*, Neural Computing and Applications, **30**(2018), 3623-3632.
- [60] J. Ye, *Multicriteria decision-making method using the Dice similarity measure based on the reduct intuitionistic fuzzy sets of interval-valued intuitionistic fuzzy sets*, Applied Mathematical Modelling, **36**(9)(2012), 4466-4472.
- [61] L.A. Zadeh, *Fuzzy sets*, Information and Control, **8**(1965), 338-353.
- [62] L.A. Zadeh, *Similarity relations and fuzzy orderings*, Information Sciences, **3**(2)(1971), 177-200.
- [63] W. Zeng, D. Li and Q. Yin, *Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision making*, International Journal of Intelligent Systems, **33**(11)(2018), 2236-2254.

- [64] W. Zeng, H.X. Li, *Correlation coefficient of intuitionistic fuzzy sets*, Journal of Industrial Engineering International, **3**(5)(2007), 33-40.
- [65] R.M. Zulqarnain, H. Garg, I. Siddique, R. Ali, A. Alsubie, N.N. Hamadneh and I. Khan, *Algorithms for a generalized multipolar neutrosophic soft set with information measures to solve medical diagnoses and decision-making problems*, Journal of Mathematics, (2021), <https://doi.org/10.1155/2021/6654657>
- [66] R.M. Zulqarnain, I. Siddique, A. Iampan and E. Bonyah, *Algorithms for multipolar interval-valued neutrosophic soft set with information measures to solve multicriteria decision-making problem*, Computational Intelligence and Neuroscience, (2021), <https://doi.org/10.1155/2021/7211399>