A Novel Dynamic of Localized Solitary Waves for the Hirota-Maccari System

Pei Xia

Zhejiang Normal University

Yi Zhang (✉ zy2836@163.com)

Zhejiang Normal University <https://orcid.org/0000-0002-8483-4349>

Heyan Zhang

Zhejiang Normal University

Yindong Zhuang

Zhejiang Normal University

Research Article

Keywords: Hirota-Maccari system, KP hierarchy reduction method, semi-rational solutions, lumps, solitons

Posted Date: November 29th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1105870/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

A novel dynamic of localized solitary waves for the Hirota-Maccari system

Pei Xia^a, Yi Zhang^{a,*}, Heyan Zhang^a, Yindong Zhuang^a

^a Department of Mathematics, Zhejiang Normal University, Jinhua 321004, PR China

Abstract

This paper investigates a particular family of semi-rational solutions in determinant form by using the KP hierarchy reduction method, which describe resonant collisions among lumps or resemble line rogue waves and dark solitons in the Hirota-Maccari system. Due to the resonant collisions, the line resemble rogue waves are generated and attenuated in the background of dark solitons with line profiles of finite length, it takes a short time for the lumps to appear from and disappear into the dark solitons background. These novel dynamic of localized solitary waves may be help to understand some physical phenomena of nonlinear localized waves propagation in many physical settings.

Keywords: Hirota-Maccari system, KP hierarchy reduction method, semi-rational solutions, lumps, solitons

1. Introduction

It is a hot topic to investigate the dynamics of nonlinear-wave interactions, which lead to rich dynamics in many fields of nonlinear physical systems such as fluids, plasmas, nonlinear optics, field optics and so on. Seeking the coherent solutions of nonlinear-wave interactions are fast becoming a key in properly understanding their theoretical description and experimental observations in a variety of physical settings. In the past years, many nonlinear evolution equations (NEEs) can be used to describe the rather complicated interactions of the nonlinear waves, which have been analyzed and numerically studied. For the analytic solutions for the NEEs, certain methods such as the inverse scattering transformation [1], the bilinear method [2], Darboux transformation [3], Bäcklund transformation [4] are proposed. Some hybrid solutions which exhibit the interactions of the soliton, breather, rogue wave and lump are studied for nonlinear Schrödinger equation [5, 6], Davey-Stewartson equations [7, 8], Mel'nikov equation [9, 10], Yajima-Oikawa systems [11, 12], KP equation [13, 14], etc. It is interesting that the lump solutions can be transformed into rogue waves under certain condition [15–17]. Rogue waves are waves of enormous energy that unpredictably arise transient and its appearing time are very short-lived. Actually, for most of the two-dimensional equations, the rogue waves are mostly manifested as line waves with profiles of infinite length generated by the wave background, and then rapidly attenuate to the same background. In this case, the rogue waves are divided as the line rogue waves that are not localized in the two-dimensional space but in time.

*Corresponding author
Email address: zy2836@163.com (Yi Zhang)

In this paper, we investigate a novel dynamic of localized solitary waves for the Hirota-Maccari (HM) system [18]:

$$\begin{aligned} iu_t + u_{xy} + i\beta u_{xxx} + uv - i\beta|u|^2 u_x &= 0, \\ 3v_x + (|u|^2)_y &= 0. \end{aligned} \quad (1)$$

By using various methods and techniques soliton solutions, periodic waves solutions and exact travelling wave solutions of the Eq. (1) have been obtained [19–21]. Besides, we are devoted to the study of nonlinear-wave interactions of the Eq. (1) in [22], which capture a resonant interaction of lumps with dark solitons. Recently, Rao has derived the so-called doubly localized two-dimensional rogue waves, which described by semi-rational type solutions are doubly localized in two-dimensional space and time [23–25]. To our knowledge, these waveform structures have not been investigated in the HM system. Thus one motive of our work is to get doubly localized two-dimensional rogue waves to the HM system. We consider a new type of semi-rational solutions of the HM system which are expressed in determinant form through KP reduction technique, which describes resonant collisions among lumps or resemble line rogue waves and dark solitons. Compared with the previous results that the interaction between lumps and dark solitons in [22], the lumps studied in the present paper lead to a intriguing soliton behaviours: the lumps are generated from the dark soliton background, and then decay to the dark soliton background equally quickly. Namely that the lumps have the characteristics of two-dimensional spatial localization and one-dimensional temporal localization.

The organization of this paper is as follows. In Section 2, we obtain a new family of semi-rational solutions to the HM system by means of KP hierarchy reduction method. In Section 3, we investigate a novel dynamic of localized solitary waves for the HM system (1), which displays the interaction between three types of localized solitary waves and dark line solitons. Our conclusions will be given in Section 4.

2. Semi-rational solutions to the HM system

It is cleared that the HM system (1) can be rewritten in the following form by using the coordinate transformation $x \rightarrow ix, y \rightarrow -y, t \rightarrow -it$:

$$\begin{aligned} u_t - iu_{xy} + \beta u_{xxx} + uv + \beta|u|^2 u_x &= 0, \\ 3iv_x - (|u|^2)_y &= 0. \end{aligned} \quad (2)$$

In order to construct a family of semi-rational solutions of the HM system (2) by utilizing the KP hierarchy reduction method. We employ the variable transformation

$$u = \frac{g}{f}, \quad v = -2i(\ln(f))_{xy}, \quad (3)$$

then (2) can be transformed into the bilinear form

$$\begin{aligned} (D_t - iD_x D_y + \beta D_x^3 + \beta D_x)g \cdot f &= 0, \\ (3D_x^2 + 1)f \cdot f &= gg^*, \end{aligned} \quad (4)$$

where D is Hirota operator and $*$ represents the complex conjugation. Consider the conversion criteria $(\partial_x - \partial_s)f = cf, h = g^*$ and c is a arbitrary constant. Then (4) can be generalized as a $(3+1)$ -dimensional

system

$$\begin{aligned} (D_t - iD_x D_y + \beta D_x^3 + \beta D_x)g \cdot f &= 0, \\ (3D_x D_s + 1)f \cdot f &= gh, \end{aligned} \quad (5)$$

where s is an auxiliary variable. Now, we begin with characterizing the following result on the KP hierarchy [26–28]:

$$\tau_n = \det_{1 \leq r, j \leq N+1} (m_{rj}^{(n)}), \quad (6)$$

where the elements of matrix $m_{rj}^{(n)}$ are

$$\begin{aligned} m_{rj}^{(n)} &= \int^{x_1} \phi_r^n \psi_j^n dx_1, \quad \phi_r^n = A_r p_n e^{\xi_r}, \quad \psi_j^n = B_j (-q)^{-n} e^{\eta_j}, \\ A_r &= \sum_{k=0}^{N+1-s} c_{rk} (p \frac{\partial}{\partial p})^{N+1-r-k}, \quad B_j = \sum_{l=0}^{N+1-j} d_{jl} (q \frac{\partial}{\partial q})^{N+1-j-l}, \\ \xi_r &= \frac{1}{p} x_{-1} + p x_1 + p^2 x_2 + p^3 x_3 + \bar{\xi}_r, \quad \eta_j = \frac{1}{p} x_{-1} + q x_1 - q^2 x_2 + p^3 x_3 + \bar{\eta}_j. \end{aligned} \quad (7)$$

The following bilinear equations in the KP hierarchy are satisfied by these tau functions:

$$\begin{aligned} (D_{x_1}^3 + 3D_{x_1} D_{x_2} - 4D_{x_3}) \tau_{n+1} \cdot \tau_n &= 0, \\ (D_{x_1} D_{x_{-1}} - 2) \tau_n \cdot \tau_n &= -2\tau_{n+1} \tau_{n-1}. \end{aligned} \quad (8)$$

Assuming x_{-1}, x_1, x_3 are real and x_2 is pure imaginary. Considering parametric constraints $q = p^*$, $d_{jl} = c_{jl}^*$, $(\bar{\xi}_j)^* = \bar{\eta}_j$, then we can obtain

$$\xi_j = \eta_j^*, \quad \xi_j' = (\eta_j')^*, \quad (m_{rj}^{(n)})^* = m_{rj}^{(-n)}, \quad \tau_n^* = \tau_{-n}.$$

Change the variable to $x_{-1} = -\frac{1}{\sqrt{6}}s$, $x_1 = \frac{1}{\sqrt{6}}x$, $x_2 = \frac{1}{2}i\beta y$, $x_3 = -\frac{\sqrt{6}}{9}\beta t - \frac{\sqrt{6}}{9}x$ and treat x_{-1} just as a parameter that can be substituted any value, in particular, the value here is zero. Then Eq. (8) can be reduced to the bilinear equations (4) of the HM system for $\tau_0 = f$, $\tau_1 = g$, $\tau_{-1} = g^*$, and the tau functions (6) would be turned into the solution of Eq. (4). Hence, we obtain a new family of semi-rational solutions to the HM system (1) by the following theorem.

Theorem 1. *The HM system (1) has semi-rational solution (3) with f , g , g^* given by*

$$f = \det_{1 \leq r, j \leq N+1} (m_{rj}^{(0)}), \quad g = \det_{1 \leq r, j \leq N+1} (m_{rj}^{(1)}), \quad g^* = \det_{1 \leq r, j \leq N+1} (m_{rj}^{(-1)}), \quad (9)$$

where the matrix elements are presented by

$$\begin{aligned} m_{rj}^{(n)} &= \delta_{rj} + \left(-\frac{p}{p^*}\right)^n e^{\xi + \xi^* + \bar{\xi}_r + (\bar{\xi}_j)^*} \\ &\quad \sum_{k=0}^{N+1-r} c_{rk} (p \partial p + \xi' + n)^{N+1-r-k} \times \sum_{l=0}^{N+1-j} c_{jl}^* (p^* \partial p^* + (\xi')^* - n)^{N+1-j-l} \frac{1}{p + p^*}, \end{aligned} \quad (10)$$

with

$$\xi = \left(\frac{\sqrt{6}}{6}p - \frac{\sqrt{6}}{9}p^3\right)x + \frac{1}{2}ip^2y - \frac{\sqrt{6}}{9}\beta p^3t, \quad \xi' = \left(\frac{\sqrt{6}}{6}p - \frac{\sqrt{6}}{3}p^3\right)x + ip^2y - \frac{\sqrt{6}}{3}\beta p^3t, \quad (11)$$

and $p, c_{rk}, \bar{\xi}_r$ are complex parameters, N is an arbitrary positive integer and δ_{rj} is the Kronecker delta.

Remark 1. The imaginary parts of parameters $\bar{\xi}_j$ ($j = 1, 2, \dots, N$) not affect the wave structures and dynamics of the solutions in Theorem 1. Thereafter, $\bar{\xi}_j$ can be taken as real parameters. Without loss of generality, we set $c_{r0} = 1$, $c_{rj} = 0$ ($r, j = 1, 2, \dots, N$).

3. A novel dynamic of localized solitary waves and dark line solitons

The semi-rational solutions in Theorem 1 illustrate the interaction between N th-order lumps or resemble line rogue waves and $(N + 1)$ dark line solitons, which lead to the true localization of N th-order lumps or resemble line rogue waves in two-dimensional space and time. In this section, we study in detail the novel dynamics of these doubly localized solitary waves and dark line solitons for the HM system.

3.1. The interaction between fundamental lump or resemble line rogue wave and two dark line solitons

Taking $N = 1$ in Theorem 1, we have the first-order semi-rational solution for the HM system as

$$u = \frac{g}{f}, \quad f = \begin{vmatrix} m_{11}^{(0)} & m_{12}^{(0)} \\ m_{21}^{(0)} & m_{22}^{(0)} \end{vmatrix}, \quad g = \begin{vmatrix} m_{11}^{(1)} & m_{12}^{(1)} \\ m_{21}^{(1)} & m_{22}^{(1)} \end{vmatrix}, \quad (12)$$

where $m_{rj}^{(0)}, m_{rj}^{(1)}$ are given by (10). After simple algebra calculations, the final form of the first-order semi-rational solution is given as

$$u = \frac{1 - \frac{p}{p^*} \frac{e^{2(\xi_R + \bar{\xi}_1)}}{p + p^*} (G + e^{2(\bar{\xi}_2 - \bar{\xi}_1)}) + \frac{pp^*}{(p + p^*)^4} e^{4\xi_R + 2\bar{\xi}_1 + 2\bar{\xi}_2}}{1 + \frac{e^{2(\xi_R + \bar{\xi}_1)}}{p + p^*} (F + e^{2(\bar{\xi}_2 - \bar{\xi}_1)}) + \frac{pp^*}{(p + p^*)^4} e^{4\xi_R + 2\bar{\xi}_1 + 2\bar{\xi}_2}}, \quad (13)$$

where

$$\begin{aligned} F &= (\xi' - \frac{p}{p + p^*})((\xi')^* - \frac{p^*}{p + p^*}) + \frac{pp^*}{(p + p^*)^2}, \\ G &= (\xi' + 1 - \frac{p}{p + p^*})((\xi')^* - 1 - \frac{p^*}{p + p^*}) + \frac{pp^*}{(p + p^*)^2}, \\ \xi_R &= (\frac{\sqrt{6}}{6} p_R - \frac{\sqrt{6}}{9} (p_R^3 - 3p_R p_I^2))x - \beta p_R p_I y - \frac{\sqrt{6}}{9} \beta (p_R^3 - 3p_R p_I^2)t, \quad p = p_R + ip_I. \end{aligned} \quad (14)$$

It should be emphasized that $u = (\ln(F))_x$ is the first-order rational solution for the HM system, which is described as the first-order resemble line rogue wave when $p_I = 0$ or fundamental lump when $p_I \neq 0$. Hence, the semi-rational solution of Eq. (1) also has two different dynamics when $p_I = 0$ and $p_I \neq 0$, respectively.

When $p_I = 0$, the semi-rational solution of Eq. (1) is composed of a resemble line rogue wave and two dark line solitons, where the resemble line rogue wave and two dark line solitons have resonance collisions. As studied in Ref. [22], the resemble line rogue wave in the HM system with features is similar to line rogue waves, which appears and disappears with profiles of infinite length in (x, t) planes. That means it is not localized in (x, t) planes. However, owing to the resonant collision, we derive a new type of localized resonant wave that is localized in temporal-spatial. Thus the localized resonant wave is intrinsically different from the resemble line rogue waves that can be treated as resemble line segment rogue waves in this work. Fig. 1 exhibits the evolution of the resemble line segment rogue waves. We can observe that the resemble line segment rogue waves with profiles of finite length firstly appears from the background of two dark solitons, and lastly quickly fades into this background again (see the region of $-10 \leq y \leq 10$).

When $p_I \neq 0$, the semi-rational solution of Eq. (1) contains a lump and two dark line solitons, in which the lump is localized not only in two-dimensional space but also in time. We note that the fusion or fission of the lumps and solitons occur in the long-wave-short-wave resonance interaction system, KP equation, Fokas system [29–31] and, etc. It is known that the lumps in this kind of wave structure persistently being for

Figure 1: The three-dimensional plots of the resonant collision between resemble line segment rogue waves and two dark line solitons with the following parameters $p = 1$, $\beta = 2$, $\bar{\xi}_1 = 2\pi$, $\bar{\xi}_2 = -2\pi$.

$t \rightarrow -\infty$ or $t \rightarrow +\infty$. That is to say the lumps are not localized in time. We investigate a novel dynamic of a lump and two dark line solitons for the HM system, which is described as the resonant interaction among a lump with two dark line solitons. Fig. 2 exhibits the dynamics of evolution of this type of collision. At the initial stage of the evolution, it can be seen that the lump not arises, and the resonant solutions only describe two dark line solitons (see the panel at $t = -15$). As times go, the lump splits out of one dark line soliton at the lower right corner of the image (see the panel at $t = -10$). Then, it temporarily exists in the background of two dark line solitons (see the panels at $t = 0, 5, 10$). At the end of the evolution, the lump dissolves into the other dark line soliton (see the panel at $t = 15$). It is an interesting feature: the lump splits from one dark line soliton and then fades into other dark line soliton, this process only takes place in a very short time. This shows the particular lump in (12) is localized in time. In contrast to usual lumps, the lumps in this resonant solution are close to the two-dimensional rogue waves in physics, therefore it can be regarded as a lump-type rogue wave on a background of dark line solitons.

3.2. The interaction between non-fundamental lump or resemble line rogue wave and three dark line solitons

By taking $N = 2$, Theorem 1 generates the second-order semi-rational solution

$$u = \frac{g}{f}, \quad f = \begin{vmatrix} m_{11}^{(0)} & m_{12}^{(0)} & m_{13}^{(0)} \\ m_{21}^{(0)} & m_{22}^{(0)} & m_{23}^{(0)} \\ m_{31}^{(0)} & m_{32}^{(0)} & m_{33}^{(0)} \end{vmatrix}, \quad g = \begin{vmatrix} m_{11}^{(1)} & m_{12}^{(1)} & m_{13}^{(1)} \\ m_{21}^{(1)} & m_{22}^{(1)} & m_{23}^{(1)} \\ m_{31}^{(1)} & m_{32}^{(1)} & m_{33}^{(1)} \end{vmatrix}, \quad (15)$$

where $m_{rj}^{(0)}$, $m_{rj}^{(1)}$ are given by (10). This semi-rational solution also exhibits two different dynamical scenarios depended on $p_I = 0$ and $p_I \neq 0$, respectively.

When $p_I = 0$, the second-order semi-rational solution (15) yields to more rich wave patterns. According to the three choices of relations between phase parameters $\bar{\xi}_j (j = 1, 2, 3)$ choices, we get two types dynamics expressed by the resonant collision among localized solitary waves and three dark line solitons.

Type I: When $\bar{\xi}_1 \gg 0$, $\bar{\xi}_1 \gg \bar{\xi}_2 \gg \bar{\xi}_3$, $\bar{\xi}_3 \ll 0$, the corresponding solution (15) describes two localized solitary waves separates from the leftmost and rightmost two dark line solitons at the same time, then collide with the middle dark line soliton. Finally, the two localized solitary waves disappear into the rightmost and leftmost two dark line solitons. Noting that the special localized solitary wave only lives in the background of three dark line solitons at a very short region of y . Because the wave pattern of this

Figure 2: The three-dimensional plots of the resonant collisions among a lump and two dark line solitons with the following parameters $p = 1 + \frac{1}{3}i$, $\beta = 2$, $\bar{\xi}_1 = 2\pi$, $\bar{\xi}_2 = -2\pi$.

localized solitary waves is very similar to that of lump-type rogue waves shown in Fig. 2 except it appears in different planes, we also call this type of localized solitary waves as resemble lump-type rogue waves. In order to demonstrate the result of this interaction by graphs, we take parameters as $p = 1$, $\beta = 2$. As inferred from Fig. 3, we see that at the initial stage, the resemble lump-type rogue waves do not exist, there only appears two dark line solitons (see the panel at $y = -10$). As times go, two resemble lump-type rogue waves separate from the leftmost and rightmost two different dark line solitons at the same time and keep relative movement (see the panels at $y = -2$), then collision with the middle dark line soliton. It exists in the background of three dark line solitons at a region of y (see the panels at $y = -2, 0, 2$). At the end of the evolution, the resemble lump-type rogue waves fuse in the background (see the panel at $y = 10$).

Figure 3: Type I : the waveform structure of the two resemble lump-type rogue and three dark line solitons with $p = 1$, $\beta = 2$, $\bar{\xi}_1 = 4\pi$, $\bar{\xi}_2 = 0$, $\bar{\xi}_3 = -4\pi$.

(a) Case 1: the waveform structures of the mixed-type resemble rogue waves and three dark line solitons with $\bar{\xi}_1 = 4\pi$, $\bar{\xi}_2 = -4\pi$, $\bar{\xi}_3 = 0$.

(b) Case 2: the waveform structures of the mixed-type resemble rogue waves and three dark line solitons with $\bar{\xi}_1 = 0$, $\bar{\xi}_2 = 4\pi$, $\bar{\xi}_3 = -4\pi$.

Figure 4: Type II: two symmetric waveform structures of mixed-type resemble rogue waves and three dark line solitons for the HM system with the following parameters $p = 1$, $\beta = 2$ and different values of $\bar{\xi}_j (j = 1, 2, 3)$. (a) Case 1: $\bar{\xi}_1 = 4\pi$, $\bar{\xi}_2 = -4\pi$, $\bar{\xi}_3 = 0$; (b) Case 2: $\bar{\xi}_1 = 0$, $\bar{\xi}_2 = 4\pi$, $\bar{\xi}_3 = -4\pi$.

Type II: In this situation, the semi-rational solutions are consisted of a resemble lump-type rogue wave, a resemble line segment rogue wave and three dark line solitons. Firstly, a resemble lump-type rogue wave detach from a leftmost (rightmost) dark line soliton and move towards the soliton in the middle. Meanwhile, a resemble line segment rogue wave appears from the background of middle dark line soliton and rightmost (leftmost) dark line soliton with profiles of finite length. The amplitude of resemble line segment rogue wave amplitude reaches its maximum at the intermediate stage. Afterwards, the resemble line segment rogue wave rapidly fades into this background again. The resemble lump-type rogue wave backward motions following the original trajectory after the collision and finally rapidly disappears into leftmost (rightmost) dark line soliton again. It is noted that the position of $\xi_j = 0 (j = 1, 3)$, where it appears determine whether the resemble lump-type rogue wave splits from the leftmost or rightmost dark line soliton in the above evolution process. We will discuss it in two cases. Case 1, when $\bar{\xi}_1 \gg 0$, $\bar{\xi}_1 \gg \bar{\xi}_3 \gg \bar{\xi}_2$, $\bar{\xi}_2 \ll 0$, $\bar{\xi}_3 = 0$, the resemble lump-type rogue wave first separates from a leftmost dark line soliton shown in Fig. 4(a). Case 2 is a fully symmetric structure with case 1. When $\bar{\xi}_2 \gg 0$, $\bar{\xi}_2 \gg \bar{\xi}_1 \gg \bar{\xi}_3$, $\bar{\xi}_3 \ll 0$, $\bar{\xi}_1 = 0$, the resemble lump-type rogue wave first separates from a rightmost dark line soliton shown in Fig. 4(b).

When $p_I \neq 0$, we obtain two lump-type rogue waves on a background of three dark line solitons. Consider the key features of the resonant collision between two lump-type rogue waves and three dark line solitons. Below, we focus on the dynamics of resonant solution, which show more complicated wave patterns. The semi-rational solution (15) has three different dynamics are determined by different relations about parameter $\bar{\xi}_j (j = 1, 2, 3)$, in which the two lump-type rogue waves arise from the three dark line solitons and finally fuse into the three dark line solitons again. As observed in Fig. 4: **Type I:** Two lump-type rogue waves fissure from the outermost (innermost) of dark line soliton and after that the lump-type rogue waves collides with the middle dark line soliton, finally fuse into the innermost (outermost) dark line soliton when $\bar{\xi}_1 \gg 0$, $\bar{\xi}_1 \gg \bar{\xi}_2 \gg \bar{\xi}_3$, $\bar{\xi}_3 \ll 0$. **Type II:** The lump-type rogue wave separates from two different dark line solitons and then together merges with another dark soliton when $\bar{\xi}_2 \gg 0$, $\bar{\xi}_2 \gg \bar{\xi}_1 \gg \bar{\xi}_3$, $\bar{\xi}_3 \ll 0$. **Type III:** Two lump-type rogue waves detach from a same dark line soliton and get annihilated one by one into two different dark line solitons when $\bar{\xi}_1 \gg 0$, $\bar{\xi}_1 \gg \bar{\xi}_3 \gg \bar{\xi}_2$, $\bar{\xi}_2 \ll 0$.

With the increase of $N (N > 2)$, the higher-order semi-rational solution is the superposition of several first-order semi-rational solutions with more complex dynamics.

4. Conclusion

In this paper, we have investigated a novel dynamic of localized solitary waves for the HM system. By using the KP hierarchy reduction method, a new family of semi-rational solutions to the HM system has been constructed, which are described by the resonant interaction among lumps or resemble line rogue waves and dark line solitons. For the first-order semi-rational solution, compared with the line rogue waves with profiles of infinite length, the resemble line segment rogue wave with profiles of finite length arises from the background of dark line solitons and finally rapidly fuses into this background again are shown in Fig. 1, which are localized in temporal-spatial. Due to the resonant collision, the lump-type rogue wave splits from the dark line solitons and lastly fade into the other dark line soliton, and this process only lasts a short time are presented in Fig. 2. For the second-order semi-rational solution, it will produce more complex waveform

Type I: the three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons

Type II: the three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons

Type III: The three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons

Figure 5: Time evolution of three kinds of the resonant collision among two lump-type rogue waves and three dark line solitons for the HM system with parameters $p = 1 + \frac{1}{3}i$, $\beta = 2$ and different values of $\bar{\xi}_j$, ($j = 1, 2, 3$): Type I: the three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons $\bar{\xi}_1 = 4\pi$, $\bar{\xi}_2 = 0$, $\bar{\xi}_3 = -4\pi$; Type II: the three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons $\bar{\xi}_1 = 4\pi$, $\bar{\xi}_2 = -4\pi$, $\bar{\xi}_3 = 0$; Type III: the three-dimensional plots of the resonant collision among two lump-type rogue waves and three dark line solitons $\bar{\xi}_1 = 0$, $\bar{\xi}_2 = 4\pi$, $\bar{\xi}_3 = -4\pi$.

structures. It is interesting that we also unearth a new type of localized resonant wave that is localized in temporal-spatial, which termed as resemble lump-type rogue waves. Comparing with lump-type rogue waves, the wave pattern of resemble lump-type rogue waves is very similar, but appear in different planes. We have shown the dynamics of the two resemble lump-type rogue waves and three dark line solitons in Fig. 3 and the mixed solution of a resemble lump-type rogue wave and a resemble line segment rogue wave with two symmetric structures in Fig. 4. Three different types of resonant structures of the lump-type rogue waves and three dark line solitons are exhibited in Fig. 5. The results of this paper might help to understand some physical phenomena of nonlinear localized nonlinear waves propagation.

Acknowledge

This work is supported by the National Natural Science Foundation of China (No. 11371326 and No. 11975145).

Data Availability

The data that support the findings of this article are available from the corresponding author, upon reasonable request.

Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest.

References

- [1] Gardner C.S., Greene J.M., Kruskal M.D., Miura R.M.: Method for solving the korteweg-devries equation. *Phys. Rev. Lett.* **19**(19), 1095(1967).
- [2] Hirota R.: *The Direct Method in Soliton Theory*. Cambridge University Press, Cambridge(2004).
- [3] Matveev V.B., Salle M.A.: *Darboux Transformations and Solitons*, Springer, Berlin(1991).
- [4] Weiss J.: Bäcklund transformations and the Painlevé property. *J. Math. Phys.* **27**(5), 1293-1305(1986).
- [5] Peng W., Tian S., Zhang T., et al.: Rational and semi-rational solutions of a nonlocal (2+1)-dimensional nonlinear Schrödinger equation. *Math. Method. Appl. Sci.* **42**(18), 6865-6877(2019).
- [6] Cao Y., Malomed B. A., He J.: Two (2+1)-dimensional integrable nonlocal nonlinear Schrödinger equations: breather, rational and semi-rational solutions. *Chaos, Soliton. Fract.* **114**, 99-107(2018).
- [7] Fokas A. S., Pelinovsky D. E., Sulem C.: Interaction of lumps with a line soliton for the DSII equation. *Physica D* **152**, 189-198(2002).
- [8] Rao J., Zhang Y., Fokas A. S., et al.: Rogue waves of the nonlocal Davey-Stewartson I equation. *Nonlinearity* **31**(9), 4090(2018).
- [9] Rao J., Malomed B. A., Cheng Y., et al.: Dynamics of interaction between lumps and solitons in the Mel'nikov equation. *Commun. Nonlinear Sci. Numer. Simul.* **91**, 105429(2020).
- [10] Xu Y., Mihalache D., He J.: Resonant collisions among two-dimensional localized waves in the Mel'nikov equation. *Nonlinear Dyn.* **106**, 2431-2448(2021)
- [11] Chen J., Chen Y., Feng B., et al.: Rational solutions to two-and one-dimensional multicomponent Yajima-Oikawa systems. *Phys. Lett. A* **379**(24-25), 1510-1519(2015).

- [12] Chen J., Chen Y., Feng B., et al.: General mixed multi-soliton solutions to one-dimensional multicomponent Yajima-Oikawa system. *J. Phys. Soc. Jpn.* **84** (7), 074001(2015).
- [13] Yuan F., Cheng Y., He J.: Degeneration of breathers in the Kadomtsev-Petviashvili I equation. *Commun. Nonlinear Sci. Numer. Simul.* **83**, 105027(2020).
- [14] Yang J., Ma W.: Abundant interaction solutions of the KP equation. *Nonlinear Dyn.* **89**(2), 1539-1544(2017).
- [15] Kundu A., Mukherjee A., Naskar T.: Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents. *P. Roy. Soc. A-Math. Phys.* **470**(2164), 20130576(2014).
- [16] Estévez P. G., Díaz E., Domínguez-Adame F., et al.: Lump solitons in a higher-order nonlinear equation in 2+1 dimensions. *Phys. Rev. E* **93**(6), 062219(2016).
- [17] Rao J., Cheng Y., He J.: Rational and semirational solutions of the nonlocal Davey-Stewartson equations. *Stud. Appl. Math.* **139**(4), 568-598(2017).
- [18] Maccari A.: A generalized Hirota equation in 2+1 dimensions. *J. Math. Phys.* **39**(12), 6547-6551(1998).
- [19] Wazwaz A. M.: Abundant soliton and periodic wave solutions for the coupled Higgs field equation, the Maccari system and the Hirota-Maccari system. *Phys. Scripta* **85**(6), 065011(2012).
- [20] Demiray S. T., Pandir Y., Bulut H.: All exact travelling wave solutions of Hirota equation and Hirota-Maccari system. *Optik* **127**(4), 1848-1859(2016).
- [21] Yu X., Gao Y., Sun Z., et al.: N-soliton solutions for the (2+1)-dimensional Hirota-Maccari equation in fluids, plasmas and optical fibers. *J. Math. Anal. Appl.* **378**(2), 519-527(2011).
- [22] Wang R., Zhang Y., Chen X., et al.: The rational and semi-rational solutions to the Hirota-Maccari system. *Nonlinear Dyn.* **100**(3), 2767-2778(2020).
- [23] Rao J., He J., Mihalache D.: Doubly localized rogue waves on a background of dark solitons for the Fokas system. *Appl. Math. Lett.* **121**, 107435(2021).
- [24] Rao J., Fokas A.S., He J.: Doubly localized two-dimensional rogue waves in the Davey-Stewartson I equation. *J. Nonlinear Sci.* **31**(4), 1-44(2021).
- [25] Jiang Y., Rao J., Mihalache D., et al.: Rogue breathers and rogue lumps on a background of dark line solitons for the Maccari system. *Commun. Nonlinear Sci. Numer. Simul.* **102**, 105943(2021).
- [26] Date E., Jimbo M., Kashiwara M., et al.: Transformation groups for soliton equations-Euclidean Lie algebras and reduction of the KP hierarchy. *Publ. Res. I. Math. Sci.* **18**(3), 1077-1110(1982).
- [27] Ohta Y., Yang J.: General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation. *P. Roy. Soc. A-Math. Phys.* **468** (2142), 1716-1740(2012).
- [28] Ohta Y., Yang J.: Rogue waves in the Davey-Stewartson I equation. *Phys. Rev. E* **86**(3), 036604(2012).
- [29] Rao J., Porsezian K., He J., et al.: Dynamics of lumps and dark-dark solitons in the multi-component long-wave-short-wave resonance interaction system. *P. Roy. Soc. A-Math. Phys.* **474**(2209), 20170627(2018).
- [30] Zhang X., Chen Y., Tang X.: Rogue wave and a pair of resonance stripe solitons to KP equation. *Comput. Math. Appl.* **76**(8), 1938-1949(2018).
- [31] Rao J., Mihalache D., Cheng Y., et al.: Lump-soliton solutions to the Fokas system. *Phys. Lett. A* **383**(11), 1138-1142(2019).