Fixed-Time Adaptive Neural Network Tracking Control for Output-Constrained High-Order Systems Using Command Filtered Strategy

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Abstract This paper proposes a fixed time adaptive neural command filtered controller for a category of high-order systems based on adding a power integrator technique. Different from existing research, the presented controller has the following distinguishing advantages: i) fixed-time control framework is extended to the tracking control problem of high-order systems. ii) the error compensation mechanism eliminates filter errors that arise from dynamic controllers. iii) growth assumptions about unknown functions are relaxed with the help of adaptive neural networks. iv) more general systems: the developed controller can apply to high-order systems subject to uncertain dynamics, unknown gain functions and asymmetric constraints. Stability analysis shows that all states are semi-globally bounded, and the convergence rate of tracking error is independent of initial conditions. The main innovation of our work is that the presented controller considers simultaneously filter errors, fixed-time convergence, growth conditions and asymmetric output constraint for the tracking control issue of high-order systems. Finally, the simulation results validate the advantages and efficacy of the developed control scheme.

Keywords Fixed-time stability · Output constraints · Command Filtered control · High-order systems · Neural network control

1 Introduction

This paper focuses the following class of nonlinear systems [1]:

$$\begin{aligned}\dot{x}_i &= \rho_i(\bar{x}_i)x_{i+1}^{a_i} + \varphi_i(\bar{x}_i), \quad 1 \leq i \leq n-1, \\ \dot{x}_n &= \rho_n(\bar{x}_n)u^{a_n} + \varphi_n(\bar{x}_n), \\ y &= x_1\end{aligned}\tag{1}$$

where x_i, x_n represent system states, and they constitute these state vectors $\bar{x}_i = [x_1, x_2, \dots, x_i]$, $\bar{x}_n = [x_1, x_2, \dots, x_n]$; $\rho_i(\bar{x}_i)$, $\rho_n(\bar{x}_n)$, $\varphi_i(\bar{x}_i)$, $\varphi_n(\bar{x}_n)$ are unknown bounded nonlinear functions, u denotes the final control signal; a_i and a_n are positive odd integers and at least one of their values is greater than 1, which leads to (1) being a high-order system [2]. It can be observed that when both a_i and a_n are equal to 1, (1) is a standard strict feedback system [3]. The high-order nonlinear system can be viewed as a general form of strict feedback systems, so it has a wider range of applications. x_{i+1} and u can be regarded as affine variables of the i -th subsystem and the n -th subsystem, respectively. Due to the existence of affine variables whose powers greater than 1, the Jacobian linearization of higher-order systems may be uncontrollable. As a result, how to develop a viable controller for higher-order systems has become a challenging task. An effective tool called a power integrator [4, 5] was adopted to analyze the stability of higher-order systems, which promotes the development of controllers for this type of system. For example, a large number of controllers with different convergence rates (asymptotical convergence rate [6, 7], finite-time convergence rate [8, 9] and fixed time convergence rate [10, 11]) are proposed and used to solve the stability control problem of high-order systems.

Different from asymptotical convergence control schemes [12], finite-time controllers have distinct advantages, including faster response speeds, higher tracking accuracy, and

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stronger anti-disturbance ability [1, 13]. The finite-time stability framework has been widely employed to solve the control problems of high-order systems because of its excellent characteristics in recent years [1, 8, 14, 9, 15, 16, 13, 17]. Among them, Cai et al. proposed a finite-time stabilization controller for a class of uncertain high-order systems and considered various practical difficulties (dead-zone, saturation and unknown control coefficients) that the controller may encounter [14]. Sun et al. extended the finite-time theory to a fast finite-time theorem [15, 16] that is able to less conservatively estimate the convergent time of state variables, and then the theorem was used to address the stabilization control issue of high-order systems that suffer from asymmetric output constraint [17]. It should be emphasized that the convergence time of closed-loop systems contain the information of initial states under the finite-time stable framework [18].

Hence, when the initial states are unavailable or far from the equilibrium point [17], the transient characteristics of controllers will be seriously affected [19]. Fortunately, fixed-time stable theorems have gained remarkable attention because they ensure that the convergence speed of system states is only associated with the main control parameters [20, 21]. That is, the initial state information no longer determines the convergence time of closed-loop systems. For example, the fixed-time stable criterion has begun to deal with the stabilization control problem of linear systems [22] and high-order systems [10]. In addition, Sun et al. expanded the application range of traditional fixed-time convergence theories such that they can cope with the tracking control problem of nonlinear systems with unknown dynamics [23]. It can be found that these researches in [4, 6, 10, 14, 15, 17] concentrate on the stable control of high-order systems. Therefore, how to extend the fixed-time stable theory to the tracking control problem of high-order systems containing unknown dynamics is still unclear and open [24].

Fuzzy logic systems [25], neural networks [26–28] or state observers [11, 29] are viewed as promising approximation tools and widely employed to identify and deal with the unknown dynamic that is contained in practical engineering [30]. For instance, fuzzy logic systems were utilized to overcome growth assumptions imposed on unknown functions contained by high-order systems [31]; an adaptive neural backstepping controller was presented to achieve the position tracking of a linear induction motor where the recurrent neural network is mainly used to model uncertain friction force [32]. Notice that the backstepping technologies [13] exist differential explosion problem [30]. Therefore, the backstepping-based controller may be extremely complex when the order of practical dynamic systems are more than three [33, 34]. To simplify the structure of backstepping controllers, dynamic surface control (DSC) approaches appeared. Recently the dynamic surface controller has been

utilized to solve the tracking control issue of output-constrained high-order systems [35].

It should be pointed out that limiting the output of control systems is of great significance in many practical control environments, for example, industrial robots [36], unmanned vehicles [12, 37] and permanent magnet synchronous motors [38]. Because of physical limitations, each joint of the manipulators should rotate within a prescribed angle range [39, 40]. Unmanned vehicles must drive in the specified lane to avoid collisions [30, 41]. Ling et al. proposed an adaptive DSC algorithm for a 4-order under-actuated high-order systems with asymmetric output constraint [42], which can be regarded as a pioneering work. It is noteworthy that the DSC algorithm mainly uses the output of filters to approximate derivatives of virtual control signals such that the repeated derivative operations of backstepping controllers are avoided [43], however the DSC does not consider filtering errors. A command filtering technique [33] is used to conquer the shortcoming of the DSC methods because the technology provides a filtering compensation mechanism to eliminate filtering errors. By applying the command filtered scheme [26] an adaptive backstepping controller was presented for a class of n -order nonlinear systems [11, 44]. Thereafter, a fixed-time command-filtered controller was presented to deal with the tracking control of surface vehicles, which proves the practical application prospects of the command-filtered technology [45]. Because the power contained in high-order systems is difficult to handle, the command filtering technology has not been applied to the control design of high-order systems, which is a strong motivation of this paper.

As far as our knowledge goes, filter errors, the fixed-time convergence, growth condition assumptions and asymmetric output constraints are not considered in the control field of uncertain high-order systems simultaneously [35]. Therefore, this paper aims to handle the trajectory tracking problem of high-order systems. The error compensation system is employed to overcome the problems of differentiation explosion and filter error. With the help of the fixed-time convergence theory, the fixed-time stability analysis of the closed-loop system is completed. Adaptive neural networks are utilized to remove growth conditions imposed on uncertain functions [31]. A coordinate transformation composed of newly defined output and reference trajectory is used to conquer the difficulty of asymmetric output constraints. Unlike the available research achievements, the proposed controller has the following novelties:

- 1 This paper proposes a fixed time adaptive command filtered control approach for high-order systems with unknown dynamic knowledge and asymmetric output constraints. The main challenge of our work is how to take filter errors, the fixed-time convergence, growth condition hypotheses and asymmetric output constraints into consideration simultaneously during the process of track-

ing controller design for high-order systems in comparison with available researches [9, 39].

- 2 Compared with the existing literature [42, 35], a new error compensation system and fixed-time stable theory are introduced into the controller design, so the presented controller not only overcomes the weaknesses of dynamic control schemes but also improves the transient performance of itself.
- 3 Different from the refs [8, 10, 46], the developed controller is also applicable even though high-order systems contain unknown functions that may not satisfy nonlinear growth conditions.

The rest of this paper is outlined as follows. Section 2 provides preliminaries, including Lemmas, Assumptions, and a brief description of a class of neural networks. The design of the proposed controller and its fixed-time stable analysis are given in Section 3. A comparison simulation example is provided to elaborate on the superiority of the developed control method in Section 4. Section 5 summarizes the current research results and gives the future development direction of the presented control scheme.

2 Preliminaries

This section first introduces the basic concept of a radial basis function neural network (RBFNN) and then provides some necessary assumptions and lemmas. These tools play an important role in the design process of the controller.

For the convenience of description, we give the definition of these symbols: $\mathfrak{R}$ stands for the real number, $\mathfrak{R}^+$ stands for the positive real number, and $\mathfrak{R}_i$ is i -dimensional real space throughout this paper. Some variables about functions will be omitted if no confusion arises.

2.1 Brief description of neural networks

This paper utilizes a RBFNN [47] to identify some unknown bounded function $\varphi(\bar{x}) \in \mathfrak{R}^1$, and it is mainly composed of an input layer, a hidden layer and an output layer. Assume that $\bar{x} = [x_1, x_2, \dots, x_{N_a}]^T \in \mathfrak{R}^{N_a}$, so the total number of nodes of the input layer is N_a . A brief definition of the hidden layer is described as follows [48]:

$$F_o(\bar{x}) = \bar{\eta}^T \bar{M}(\bar{x}) \quad (2)$$

where $F_o(\bar{x})$ is the output of the output layer. It is a approximation value of $\varphi(\bar{x}) \in \mathfrak{R}^1$; $\bar{\eta} = [\eta_1, \eta_2, \dots, \eta_{L_a}]^T \in \mathfrak{R}^{L_a}$ denotes positive weight vector, and L_a is also equal to the total number of neurons contained in the hidden layer; $\bar{M}(\bar{x}) = [M_1(\bar{x}), M_2(\bar{x}), \dots, M_{L_a}(\bar{x})]^T \in \mathfrak{R}^{L_a}$ stands for a activation function vector. $M_j, j = 1, 2, \dots, L_a$ can be viewed as a

neuron model, and its detailed definition is given as follows

$$M_j(\bar{x}) = \exp\left(-\frac{(\bar{x} - \bar{c}_j)^T (\bar{x} - \bar{c}_j)}{b_j}\right) \quad (3)$$

where $b_j \in \mathfrak{R}^+$ presents the Gaussian width; $\bar{c}_j \in \mathfrak{R}^{N_a}$ denotes a center position vector. All center position vectors form the following matrix:

$$\mathbf{c} = \begin{bmatrix} \bar{c}_1^T \\ \vdots \\ \bar{c}_{L_a}^T \end{bmatrix} = \begin{bmatrix} c_{1,1} & \cdots & c_{1,N_a} \\ \vdots & \ddots & \vdots \\ c_{L_a,1} & \cdots & c_{L_a,N_a} \end{bmatrix} \quad (4)$$

where $c_{i,j}, i = 1, 2, \dots, L_a; j = 1, 2, \dots, N_a$ is a designed positive constant.

2.2 Lemmas and Assumptions

Lemma 1[49]. For arbitrary small positive constant δ , then their is a neural network such that

$$\max \left| \varphi(\bar{x}) - (\bar{\eta}^*)^T \bar{M}(\bar{x}) \right| \leq \delta \quad (5)$$

where $\bar{\eta}^*$ denotes the optimal weight vector. **Lemma 1** implies that the RBFNN in this paper is able to approximate a unknown bounded function $\varphi(\bar{x}) \in \mathfrak{R}^1$ with arbitrary accuracy if we choose suitable weight parameters.

Lemma 2[13]. Let $p \geq 1$ be a odd integer, and then for given real functions g_1, g_2 , their is the following inequality:

$$|g_1^p - g_2^p| \leq p|g_1 - g_2|(|g_1|^{p-1} + |g_2|^{p-1}) \quad (6)$$

Lemma 3[14]. Let $p \in [0, +\infty)$. For any real variables $g_1, g_2, \dots, g_n$, one obtains the following results:

$$\begin{cases} \left| \sum_{j=1}^n g_j \right|^p \leq n^{p-1}(|g_1|^p + |g_2|^p), p \geq 1 \\ \left| \sum_{j=1}^n g_j \right|^p \leq (|g_1|^p + |g_2|^p), \text{else} \end{cases} \quad (7)$$

Lemma 4[50]. Let p_1, p_2 be positive constants. There are real functions g_1, g_2 such that

$$|g_1|^{p_1} |g_2|^{p_2} \leq \frac{p_1}{p_1 + p_2} \delta |g_1|^{p_1 + p_2} + \frac{p_2}{p_1 + p_2} \delta^{-\frac{p_1}{p_2}} |g_2|^{p_1 + p_2} \quad (8)$$

where $\delta > 0$ denotes a specific real number.

Lemma 5[51]. Let g_1, g_2 be real variables, For given real numbers $p_1 > 1, p_2 > 1$, there exists a Young's inequality:

$$g_1 g_2 \leq \frac{\delta^{p_1}}{p_1} |g_1|^{p_1} + \frac{1}{p_2 \delta^{p_2}} |g_2|^{p_2} \quad (9)$$

where δ is a positive constant, and p_1 and p_2 additionally satisfy the equality: $(p_1 - 1)(p_2 - 1) = 1$.

Lemmas 2–5 are employed to extend application range of related inequalities.

Lemma 6 [23]. Let $\tau_1, \tau_2, \tau_3, \alpha_1 \in (0, 1), \alpha_2 \in (1, +\infty)$ be positive numbers. For any dynamic system $\dot{e} = \varphi(e, t), e(0) = 0$, if there is a positive definite function $L(e(t))$ which has a first-order time derivative such that

$$\dot{L}(e(t)) \leq -\tau_1 L(e(t))^{\alpha_1} - \tau_2 L(e(t))^{\alpha_2} + \tau_3 \quad (10)$$

then, two conclusions can be gained. The first conclusion is that the state trajectory $e(t)$ practically fixed-time stable. The second conclusion is that the dynamic system $\dot{e} = \varphi(e, t)$ will enter a stable state after the total time T_a :

$$T_a = \frac{100}{\tau_1(1-\alpha_1)} + \frac{100}{\tau_2(\alpha_2-1)}. \quad (11)$$

Lemma 6 help us to analysis the stability of the proposed controller.

Lemma 7 [41]. Let $B_{L,1}$ and $B_{R,1}$ be designed positive smooth functions. For any real variable g , a new barrier function g_1 can be defined as

$$g_1 = \frac{g B_{L,1} B_{R,1}}{(B_{L,1} + g)(B_{R,1} - g)} \quad (12)$$

where g belongs to a compact set

$$\Xi_g = \{g \in \mathfrak{R} : -B_{L,1} < g < B_{R,1}\}. \quad (13)$$

Lemma 7, as an unified tool, can be used to tackle with the output constraint problem of dynamics systems.

Lemma 8 [52]. Let g be a real function. For given positive constant p , there is a inequality

$$|g| - g \tanh\left(\frac{g}{p}\right) \leq 0.2785p \quad (14)$$

where $\tanh(\bullet)$ stands for the hyperbolic tangent function. **Lemma 8** is utilized to deal with the derivative of filter output $\dot{v}_{i,3}, i = 1, 2, \dots, n-1$.

Lemma 9 [23]. Let $g_1 \in \mathfrak{R}^+$ and $g_2 \in \mathfrak{R}^+$ be real functions. For any positive constant p , the following inequality holds:

$$g_2(g_1 - g_2)^p \leq \frac{P}{p+1} (g_1^{p+1} - g_2^{p+1}) \quad (15)$$

where g_1 is greater than or equal to g_2 .

Lemma 9 will be employed to simplify inequalities related to adaptive parameter θ .

Assumption 1 [46]. The control gain function $\rho_i(\bar{x}_i), i = 1, 2, \dots, n$ included in the dynamic system (1) not only ensures strictly positive or strictly negative, but also satisfies the following inequality:

$$\rho_{i,\min} \leq |\rho_i(\bar{x}_i)| \leq \rho_{i,\max} \quad (16)$$

where $\rho_{i,\min}$ and $\rho_{i,\max}$ stand for positive parameters. Assumption 1 guarantees that the control signal always acts

on the dynamic system (1). For the convenience of stability analysis, suppose that $\rho_i(\bar{x}_i)$ always positive.

Assumption 2 [53]. The designed signal $y_{r,1}$ and its time derivative $\dot{y}_{r,1}$ are measured and known.

Assumption 3. The parameters $a_1, a_2, \dots, a_n$ in the dynamic system (1) satisfy the inequality $(a - a_i + 1) \leq \frac{\alpha_1}{(1-\alpha_1)}$, $i = 1, 2, \dots, n$, where $\alpha_1 \in (0, 1)$ is a real number; a denotes an odd integer and satisfies inequality $a > \max\{a_1, a_2, \dots, a_n\}$.

Remark 1. Assumption 3 can help us avoid the singularity problem of controllers caused by zero tracking error. It can be inferred from Assumption 3 that the minimum value of α_1 is 0.5. Different from Assumption 3 of reference [42], the parameter α_1 can be determined flexibly by ourselves such that Assumption 3 of this paper is less strict. As a result, the proposed controller is more suitable than Ling's controller [42] in a real industry environment.

3 Main results

This section first gives the main framework of the new fixed-time adaptive neural command filtered controller, and then provides the stability analysis of the developed controller with the aid of the fixed-time convergence theory [23] and the adding one power integrator tool [4].

3.1 Design of a fixed-time controller

Before designing the controller, the following preparations are required.

Preparation 1: According to **Lemma 7**, the output x_1 and reference signal $y_{r,1}$ can be redefined as

$$\begin{aligned} e_{1,1} &= \frac{B_{L,1} B_{R,1} x_1}{(B_{L,1} + x_1)(B_{R,1} - x_1)}, \quad x_1 \in \Xi_{x_1} \\ e_{1,2} &= \frac{B_{L,1} B_{R,1} y_{r,1}}{(B_{L,1} + y_{r,1})(B_{R,1} - y_{r,1})}, \quad y_{r,1} \in \Xi_{y_{r,1}} \end{aligned} \quad (17)$$

where $B_{L,1} \in \mathfrak{R}^+$ and $B_{R,1} \in \mathfrak{R}^+$ denote smooth bounded functions which can be designed by ourselves; Ξ_{x_1} and $\Xi_{y_{r,1}}$ are compact sets, and their detailed definition are given as follows [42]:

$$\begin{aligned} \Xi_{x_1} &= \{x_1 \in \mathfrak{R} : -B_{L,1} < x_1 < B_{R,1}\} \\ \Xi_{y_{r,1}} &= \{y_{r,1} \in \mathfrak{R} : -B_{L,1} < y_{r,1} < B_{R,1}\} \end{aligned} \quad (18)$$

Remark 2. Compared with existing log-type barrier function (BLF) [35] and tan-type barrier function [1, 54], the designed barrier functions (17) obtain the following strengths simultaneously: i) $B_{L,1}$ is used to constrain the lower bound of x_1 and $y_{r,1}$, and $B_{R,1}$ is used to constrain the upper bound of x_1 and $y_{r,1}$. Therefore the asymmetric output constraint can be achieved by designing $B_{L,1}$ and $B_{R,1}$ respectively; ii) the barrier functions (17) unify the fixed-time controller

design for high-order system with or without output constraints.

The time derivative of (17) can be expressed as

$$\begin{aligned} \dot{e}_{1,1} &= \partial_{B_{L,1}}^{e_{1,1}} \dot{B}_{L,1} + \partial_{B_{R,1}}^{e_{1,1}} \dot{B}_{R,1} + \partial_{x_1}^{e_{1,1}} \dot{x}_1 \\ \dot{e}_{1,2} &= \partial_{B_{L,1}}^{e_{1,2}} \dot{B}_{L,1} + \partial_{B_{R,1}}^{e_{1,2}} \dot{B}_{R,1} + \partial_{y_{r,1}}^{e_{1,2}} \dot{y}_{r,1} \end{aligned} \quad (19)$$

where

$$\begin{aligned} \partial_{B_{L,1}}^{e_{1,1}} &= \frac{B_{R,1} x_1^2}{(B_{L,1} + x_1)^2 (B_{R,1} - x_1)}, \\ \partial_{B_{R,1}}^{e_{1,1}} &= \frac{-B_{L,1} x_1^2}{(B_{L,1} + x_1) (B_{R,1} - x_1)^2}, \\ \partial_{x_1}^{e_{1,1}} &= \frac{B_{L,1} B_{R,1} (B_{L,1} B_{R,1} + x_1^2)}{(B_{L,1} + x_1)^2 (B_{R,1} - x_1)^2}, \\ \partial_{B_{L,1}}^{e_{1,2}} &= \frac{B_{R,1} y_{r,1}^2}{(B_{L,1} + y_{r,1})^2 (B_{R,1} - y_{r,1})}, \\ \partial_{B_{R,1}}^{e_{1,2}} &= \frac{-B_{L,1} y_{r,1}^2}{(B_{L,1} + y_{r,1}) (B_{R,1} - y_{r,1})^2}, \\ \partial_{y_{r,1}}^{e_{1,1}} &= \frac{B_{L,1} B_{R,1} (B_{L,1} B_{R,1} + y_{r,1}^2)}{(B_{L,1} + y_{r,1})^2 (B_{R,1} - y_{r,1})^2}. \end{aligned} \quad (20)$$

Preparation 2: define some coordinate transformations

$$\begin{aligned} e_1 &= e_{1,1} - e_{1,2}, e_k = x_k - v_{k-1,3} \\ w_i &= v_{i,3} - v_{i,2}, k = 2, 3, \dots, n, i = 1, 2, \dots, n-1. \end{aligned} \quad (21)$$

where e_1 and e_k denote tracking errors; w_i stands for a filter error; $v_{i,2}$ is a virtual control law designed by ourselves, and $v_{i,3}$ is a output of the filter (22).

Preparation 3: this paper utilizes the following command filter [43] to approximate the virtual control signal of every subsystem:

$$\begin{aligned} \dot{v}_{i,3} &= \mu_{i,1} \overset{\leftrightarrow}{v}_{i,3} \\ \overset{\leftrightarrow}{v}_{i,3} &= -2\mu_{i,2} \mu_{i,1} \overset{\leftrightarrow}{v}_{i,3} - \mu_{i,2} (v_{i,3} - v_{i,2}) \end{aligned} \quad (22)$$

where $\mu_{i,1}, \mu_{i,2} \in (0, 1)$ present designed positive parameters, and $v_{i,2}$ can be viewed as the input signal of the command filter. $v_{i,3}$ and $\overset{\leftrightarrow}{v}_{i,3}$ can replace $v_{i,2}$ and $\dot{v}_{i,2}$ respectively, if $\mu_{i,1}$ and $\mu_{i,2}$ are selected appropriately.

Under Assumptions 1-3, a fixed-time adaptive neural command filtered controller is constructed. The proposed controller mainly consists of the following three parts:

Part 1: the control signal corresponding to the j -th subsystem, $j = 1, 2, \dots, n$, can be designed as follows:

$$v_{j,2} = - \left(\frac{v_{j,1} + d_j + \frac{e_j^{a-j+1} \hat{\theta} M_j^T(\bar{x}_j) M_1(\bar{x}_j)}{2c_1^2}}{\phi_j} \right)^{\frac{1}{a_j}} \quad (23)$$

where $v_{1,1} = \frac{e_1^{a-1+1} (\partial_{x_1}^{e_{1,1}})^2}{2} + b_{1,1} \frac{e_1^{(\alpha_1(a-1+2)-(a-1+1))}}{(a-1+2)^{\alpha_1}} + e_1^{a_1} + b_{1,2} \frac{e_1^{(\alpha_2(a-1+2)-(a-1+1))}}{(a-1+2)^{\alpha_2}}, v_{m,1} = \frac{e_m^{a_m} + \frac{e_m^{a-m+1}}{2}}{(a-a_m+2)^{\alpha_1}} + b_{m,1} \frac{e_m^{(\alpha_1(a-a_m+2)-(a-a_m+1))}}{(a-a_m+2)^{\alpha_1}} + b_{m,2} \frac{e_m^{(\alpha_2(a-a_m+2)-(a-a_m+1))}}{(a-a_m+2)^{\alpha_2}} + e_{m-1} w_{m-1} + \dot{v}_{m-1,3} \tanh \left(\frac{e_m^{a-m+1} \dot{v}_{m-1,3}}{h_m} \right), m = 2, 3, \dots, n-1, v_{n,1} = \frac{e_n^{a-n+1}}{2} + b_{n,1} \frac{e_n^{(\alpha_1(a-a_n+2)-(a-a_n+1))}}{(a-a_n+2)^{\alpha_1}} + b_{n,2} \frac{e_n^{(\alpha_2(a-a_n+2)-(a-a_n+1))}}{(a-a_n+2)^{\alpha_2}} + \dot{v}_{n-1,3} \tanh \left(\frac{e_n^{a-n+1} \dot{v}_{n-1,3}}{h_n} \right) + e_{n-1} w_{n-1}, d_1 = \partial_{B_{L,1}}^{e_{1,1}} \dot{B}_{L,1} + \partial_{B_{R,1}}^{e_{1,1}} \dot{B}_{R,1} - \dot{e}_{1,2}, \phi_1 = \partial_{x_1}^{e_{1,1}} \rho_{1,\min}, \phi_1^* = \partial_{x_1}^{e_{1,1}} \rho_{1,\max}, k = 2, 3, \dots, n, d_k = (2^{a_{k-1}-1} \phi_{k-1}^* a_{k-1})^{\frac{(a+1)}{a_{k-1}}} \left(\frac{a-a_{k-1}+1}{a+1} \right)^{\frac{(a-a_{k-1}+1)}{a_{k-1}}} \frac{a_{k-1}}{a+1} e_k^{a_k} + \left(v_{k-1,3}^{a_{k-1}-1} + \frac{v_{k-1,2}^{a_{k-1}-1}}{2^{a_{k-1}-1}} \right) 2^{a_{k-1}-1} \rho_{k-1,\max} a_{k-1} \frac{1}{a-a_k+2} e_k, \phi_k^* = \rho_{k,\max}, \phi_k = \rho_{k,\min}; b_{j,1}, b_{j,2}, c_j, h_k, \alpha_1 \in (0, 1), \alpha_2 \in (1, +\infty) are positive design parameters; $\hat{\theta}$ denotes the estimation value of the adaptive positive parameter θ . $M_j(\bar{x}_j)$ represents the activation function vector of the j -th RBFNN, and the input information of the RBFNN is $\bar{x}_j$. If the control signal (23) is further classified, $v_{i,2}, i = 1, 2, \dots, n-1$ is a virtual control signal, and $v_{n,2}$ is the actual control signal u required by the dynamics system (1).$

Part 2: an adaptive law $\dot{\hat{\theta}}$ can update the weight parameters of the RBFNN automatically, and

$$\dot{\hat{\theta}} = \xi_1 \sum_{j=1}^n \frac{e_j^{2(a-a_j+1)} M_j^T(\bar{x}_j) M_j(\bar{x}_j)}{2c_j^2} - \xi_2 \hat{\theta} - \xi_3 \hat{\theta}^{(2\alpha_2-1)} \quad (24)$$

where $\xi_1 \in \mathbb{R}^+, \xi_2 \in \mathbb{R}^+, \xi_3 \in \mathbb{R}^+$ are design constants.

Part 3: a new filter compensation system that is employed to reduce filter errors is designed as

$$\begin{aligned} \dot{\chi}_{k-1} &= -\kappa_{k-1,1} \frac{\chi_{k-1}^{(2\alpha_1-1)}}{2\alpha_1} - \kappa_{k-1,2} \frac{\chi_{k-1}^{(2\alpha_2-1)}}{2\alpha_2} \\ &\quad - \frac{d_{k,2}}{\chi_{k-1}} + \frac{e_k^{a-a_k+1} e_{k-1} w_{k-1}}{\chi_{k-1}} \end{aligned} \quad (25)$$

where $k = 2, 3, \dots, n, \kappa_{k-1,1} \in \mathbb{R}^+, \kappa_{k-1,2} \in \mathbb{R}^+$ are constants;

$$\begin{aligned} d_{k,2} &= \phi_{k-1}^* a_{k-1} 2^{a_{k-1}-1} \left(v_{k-1,3}^{a_{k-1}-1} + \frac{v_{k-1,2}^{a_{k-1}-1}}{2^{a_{k-1}-1}} \right) \\ &\quad \frac{a-a_{k-1}+1}{a-a_k+2} |e_{k-1}|^{\frac{(a-a_{k-1}+1)(a-a_k+2)}{a-a_k+1}} + a_{k-1} \phi_{k-1}^* |e_{k-1}^{a-a_{k-1}+1}| \\ &\quad |w_{k-1}| \left(x_k^{a_{k-1}-1} + v_{k-1,2}^{a_{k-1}-1} \right). \end{aligned}$$

Remark 3. The proposed controller not only avoids the “explosion of complexity” of backstepping controllers but also further considers filter errors arise from dynamic surface controllers. Since the filter compensation system (25) is introduced into existing dynamic surface controllers [42,

35] for high-order systems, it is theoretically guaranteed that the developed controller can obtain a smaller $|x_1 - y_{r,1}|$ compared with these research results [42, 35].

First, **Theorem 1** is provided to illustrate the property of the developed controller.

Theorem 1: Assume that the high-order dynamic system (1) satisfies Assumptions 1-3 and the output constraints (17). If a set of control parameters are chosen properly, the new controller composed of (23), (24) and (25) obtain the following properties: i) all closed-loop signals $(\chi_1, \chi_2, \dots, \chi_{n-1}, x_1, x_2, \dots, x_n, \hat{\theta})$ are practically fixed-time stable. ii) the tracking error approaches a neighborhood of origin after the fixed time T_a independent of initial conditions. iii) the output is constrained in the given interval $(-B_{L,1}, B_{R,1})$.

Then, the rigorous proof of **Theorem 1** can be performed in n steps.

Step 1: we design a Lyapunov candidate function $L_1 = \frac{e_1^{a-a_1+2}}{a-a_1+2} + \frac{\chi_1^2}{2}$, and then its first derivative can be computed as

$$\dot{L}_1 = e_1^{a-a_1+1} \dot{e}_1 + \chi_1 \dot{\chi}_1. \quad (26)$$

With the help of $\dot{x}_1 = \rho_1(\bar{x}_1)x_2^{a_1} + \varphi_1(\bar{x}_1, t)$, introduced items $\partial_{x_1}^{e_1,1} \rho_1(\bar{x}_1)v_{1,2}^{a_1} - \partial_{x_1}^{e_1,1} \rho_1(\bar{x}_1)v_{1,2}^{a_1}$ and coordinate transformations (21), $\dot{L}_1$ can be further described as

$$\begin{aligned} \dot{L}_1 = & e_1^{a-a_1+1} \partial_{x_1}^{e_1,1} \left(\rho_1(\bar{x}_1) \left(x_2^{a_1} - v_{1,2}^{a_1} \right) + \rho_1(\bar{x}_1) v_{1,2}^{a_1} \right) \\ & + e_1^{a-a_1+1} \left(d_1 + \partial_{x_1}^{e_1,1} \varphi_1(\bar{x}_1) \right) + \chi_1 \dot{\chi}_1. \end{aligned} \quad (27)$$

Lemma 2, Lemma 3, Lemma 4 and **Lemma 5** are applied to $e_1^{a-a_1+1} \partial_{x_1}^{e_1,1} \rho_1(\bar{x}_1) \left(x_2^{a_1} - v_{1,2}^{a_1} \right)$, and then one gains

$$e_1^{a-a_1+1} \partial_{x_1}^{e_1,1} \rho_1(\bar{x}_1) \left(x_2^{a_1} - v_{1,2}^{a_1} \right) \leq e_1^{a+1} + d_2 e_2^{a-a_2+1} + d_{2,2}. \quad (28)$$

Remark 4. Since the residual term d_2 contains the information of $v_{1,2}$, directly using $v_{1,2}$ to eliminate d_2 will cause algebraic loop problems. So we try to use the control signal of the next subsystem to cancel d_2 . Different from existing research results [35, 42], the error compensation system will further handle the filtering error contained in $d_{2,2}$, which will help improve the tracking accuracy of the controller.

In accordance with **Lemma 1**, a RBFNN $\eta_1^{*T} M_1(\bar{x}_1)$ is able to achieve the following approximation:

$$\varphi_1(\bar{x}_1) = \eta_1^{*T} M_1(\bar{x}_1) + \delta_1(\bar{x}_1) \quad (29)$$

where the absolute value of the error function $\delta_1(\bar{x}_1)$ is less than or equal to the given accuracy $\delta_{1,\max} \in \mathfrak{R}^+$.

Remark 5. The related control algorithms [8, 10, 46] require that unknown function $\varphi_1(\bar{x}_1)$ must be bounded and satisfy the growth assumption. It can be inferred from **Lemma 1** that if $\varphi_1(\bar{x}_1)$ is a bounded function, then there exists a ideal neural network to approximate it with given accuracy.

The proposed controller is applicable even though $\varphi_1(\bar{x}_1)$ does not satisfies growth assumption such that it is more suitable in practical control filed.

According to the Young's inequality (**Lemma 5**) and (29), $e_1^{a-a_1+1} \partial_{x_1}^{e_1,1} \varphi_1(\bar{x}_1)$ can be further transformed into

$$\begin{aligned} e_1^{a-a_1+1} \partial_{x_1}^{e_1,1} \varphi_1(\bar{x}_1) \leq & \frac{e_1^{2(a-a_1+1)} \left(\partial_{x_1}^{e_1,1} \right)^2 \left\| \eta_1^{*T} M_1(\bar{x}_1) \right\|^2}{2c_1^2} \\ & + \frac{c_1^2}{2} + \frac{\left(e_1^{(a-a_1+1)} \partial_{x_1}^{e_1,1} \right)^2}{2} + \frac{(\delta_{1,\max})^2}{2}. \end{aligned} \quad (30)$$

Substituting (28), (30), the first virtual control law (23) and the first filter compensation signal (25) into (27) results in

$$\begin{aligned} \dot{L}_1 \leq & -b_{1,1} \left(\frac{e_1^{a-a_1+2}}{a-a_1+2} \right)^{\alpha_1} - b_{1,2} \left(\frac{e_1^{a-a_1+2}}{a-a_1+2} \right)^{\alpha_2} \\ & + \frac{e_1^{2(a-a_1+1)} \|M_1(\bar{x}_1)\|^2}{2c_1^2} \left(\left(\partial_{x_1}^{e_1,1} \right)^2 \|\eta_1^*\|^2 - \hat{\theta} \right) \\ & + d_2 e_2^{a-a_2+1} + \frac{c_1^2 + (\delta_{1,\max})^2}{2} - \kappa_{1,1} \left(\frac{\chi_1^2}{2} \right)^{\alpha_1} \\ & - \kappa_{1,2} \left(\frac{\chi_1^2}{2} \right)^{\alpha_2} + e_2^{a-a_2+1} e_1 w_1. \end{aligned} \quad (31)$$

Step m ($m = 2, 3, \dots, n-1$): construct a positive definite power-based function $L_m = \frac{e_m^{a-a_m+2}}{a-a_m+2} + \frac{\chi_m^2}{2}$. In view of $\dot{x}_m = \rho_m(\bar{x}_m)x_{m+1}^{a_m} + \varphi_m(\bar{x}_m)$, $e_m^{a-a_m+1} \rho_m(\bar{x}_m)v_{m,2}^{a_m} - e_m^{a-a_m+1} \rho_m(\bar{x}_m)v_{m,2}^{a_m}$ and $\dot{e}_m = \dot{x}_m - \dot{v}_{m-1,3}$, $\dot{L}_m$ is further rephrased as

$$\begin{aligned} \dot{L}_m = & e_m^{a-a_m+1} \rho_m(\bar{x}_m) \left(x_{m+1}^{a_m} - v_{m,2}^{a_m} \right) \\ & + e_m^{a-a_m+1} \rho_m(\bar{x}_m) v_{m,2}^{a_m} + e_m^{a-a_m+1} \varphi_m(\bar{x}_m) \\ & - e_m^{a-a_m+1} \dot{v}_{m-1,3} + \chi_m \dot{\chi}_m. \end{aligned} \quad (32)$$

Based on a similar analysis in (28)–(31), a tedious but key result is obtained:

$$\begin{aligned} \dot{L}_m \leq & -b_{m,1} \left(\frac{e_m^{(a-a_m+2)}}{a-a_m+2} \right)^{\alpha_1} - b_{m,2} \left(\frac{e_m^{(a-a_m+2)}}{a-a_m+2} \right)^{\alpha_2} \\ & + \frac{e_m^{2(a-a_m+1)} \|M_m(\bar{x}_m)\|^2}{2c_m^2} \left(\|\eta_m^*\|^2 - \hat{\theta} \right) \\ & + (d_{m+1} + e_m w_m) e_m^{a-a_m+1} - \kappa_{m,2} \left(\frac{\chi_m^2}{2} \right)^{\alpha_2} \\ & - e_m^{a-a_m+1} \dot{v}_{m-1,3} \left(\tanh \left(\frac{e_m^{a-a_m+1} \dot{v}_{m-1,3}}{h_m} \right) + 1 \right) \\ & + \frac{c_m^2 + (\delta_{m,\max})^2}{2} - e_m^{a-a_m+1} (e_{m-1} w_{m-1} + d_m) \\ & - \kappa_{m,1} \left(\frac{\chi_m^2}{2} \right)^{\alpha_1} \end{aligned} \quad (33)$$

where $\delta_{m,\max}$ denotes a small positive constant. By applying

Lemma 8, $-e_m^{a-a_m+1}\dot{v}_{m-1,3} - e_m^{a-a_m+1}\dot{v}_{m-1,3}$
 $\tanh\left(\frac{e_m^{a-a_m+1}\dot{v}_{m-1,3}}{h_m}\right) \leq 0.2785h_m$, and then $\dot{L}_m$ can be further expressed as

$$\begin{aligned} \dot{L}_m \leq & -b_{m,1} \left(\frac{e_2^{(a-a_m+2)}}{a-a_m+2} \right)^{\alpha_1} - b_{m,2} \left(\frac{e_m^{(a-a_m+2)}}{a-a_m+2} \right)^{\alpha_2} \\ & + \frac{e_m^{2(a-a_m+1)} \|M_m(\bar{x}_m)\|^2}{2c_m^2} \left(\|\eta_m^*\|^2 - \hat{\theta} \right) \\ & + (d_{m+1} + e_m w_m) e_{m+1}^{a-a_m+1} - \kappa_{m,2} \left(\frac{\chi_m^2}{2} \right)^{\alpha_2} \\ & + 0.2785h_m - e_m^{a-a_m+1} (e_{m-1} w_{m-1} + d_m) \\ & + \frac{c_m^2 + (\delta_{m,\max})^2}{2} - \kappa_{m,1} \left(\frac{\chi_m^2}{2} \right)^{\alpha_1}. \end{aligned} \quad (34)$$

Step n : Consider a positive definite power-based function $L_n = \frac{e_n^{a-a_n+2}}{a-a_n+2} + \frac{\chi_n^2}{2}$. From the definition of $\dot{x}_n = \rho_n(\bar{x}_n)u^{a_n} + \varphi_n(\bar{x}_n, t)$ and $\dot{e}_n = \dot{x}_n - \dot{v}_{n-1,3}$, $\dot{L}_n$ is further calculated as

$$\dot{L}_n = e_n^{a-a_n+1} (\rho_n(\bar{x}_n)u^{a_n} + \varphi_n(\bar{x}_n) - \dot{v}_{n-1,3}). \quad (35)$$

First, the n -th RBFNN is used to deal with $\varphi_n(\bar{x}_n)$ (refer to the derivation procedure (29)–(30)). Then, $-e_n^{a-a_n+1}\dot{v}_{n-1,3}$ $\left(1 + \tanh\left(\frac{e_n^{a-a_n+1}\dot{v}_{n-1,3}}{h_n}\right)\right)$ will appear after substituting the actual control signal u into (35). Finally, according to **Lemma 8**, $\dot{L}_n$ can be further simplified as

$$\begin{aligned} \dot{L}_n \leq & -b_{n,1} \left(\frac{e_n^{(a-a_n+2)}}{a-a_n+2} \right)^{\alpha_1} - b_{n,2} \left(\frac{e_n^{(a-a_n+2)}}{a-a_n+2} \right)^{\alpha_2} \\ & + \frac{e_n^{2(a-a_n+1)} \|M_n(\bar{x}_n)\|^2}{2c_n^2} \left(\|\eta_n^*\|^2 - \hat{\theta} \right) \\ & - e_n^{a-a_n+1} e_{n-1} w_{n-1} - d_n e_n^{a-a_n+1} \\ & + \frac{c_n^2 + (\delta_{n,\max})^2}{2} + 0.2785h_n \end{aligned} \quad (36)$$

Construct a overall Lyapunov function $L = \sum_{j=1}^n \frac{e_j^{a-a_j+2}}{a-a_j+2} + \sum_{i=1}^{n-1} \frac{\chi_i^2}{2} + \frac{\tilde{\theta}^2}{2\xi_1}$ where $\tilde{\theta} = \theta - \hat{\theta}$ denotes the estimation error of θ . After then, differentiating L produces $\dot{L} = \sum_{j=1}^n e_j^{a-a_j+2} \dot{e}_j + \sum_{i=1}^{n-1} \chi_i \dot{\chi}_i - (\tilde{\theta} \dot{\tilde{\theta}})/\xi_1$. According to these conclusions (inequalities (31), (34) and (36)) of previous derivation, $\dot{L}$ can be rewritten as

$$\begin{aligned} \dot{L} \leq & -\sum_{j=1}^n b_{j,1} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_1} - \sum_{j=1}^n b_{j,2} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_2} \\ & + \frac{e_1^{2(a-a_1+1)} \|M_1(\bar{x}_1)\|^2}{2c_1^2} \left(\left(\partial_{x_1}^{e_{1,1}} \right)^2 \|\eta_1^*\|^2 - \hat{\theta} \right) \end{aligned}$$

$$\begin{aligned} & + \sum_{k=2}^n \frac{e_k^{2(a-a_k+1)} \|M_k(\bar{x}_k)\|^2}{2c_k^2} \left(\|\eta_k^*\|^2 - \hat{\theta} \right) \\ & - \sum_{i=1}^{n-1} \kappa_{i,1} \left(\frac{\chi_i^2}{2} \right)^{\alpha_1} - \sum_{i=1}^{n-1} \kappa_{i,2} \left(\frac{\chi_i^2}{2} \right)^{\alpha_2} - \frac{\tilde{\theta} \dot{\tilde{\theta}}}{\xi_1} \\ & + \sum_{j=1}^n \frac{c_j^2 + (\delta_{j,\max})^2}{2} + \sum_{k=2}^n 0.2785h_k. \end{aligned} \quad (37)$$

Assume that

$$\theta = \max \left\{ \left(\partial_{x_1}^{e_{1,1}} \right)^2 \|\eta_1^*\|^2, \|\eta_2^*\|^2, \|\eta_3^*\|^2, \dots, \|\eta_n^*\|^2 \right\}. \quad (38)$$

Substituting (24) and (38) into (37) produces

$$\begin{aligned} \dot{L} \leq & -\sum_{j=1}^n b_{j,1} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_1} - \sum_{j=1}^n b_{j,2} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_2} \\ & - \sum_{i=1}^{n-1} \kappa_{i,1} \left(\frac{\chi_i^2}{2} \right)^{\alpha_1} - \sum_{i=1}^{n-1} \kappa_{i,2} \left(\frac{\chi_i^2}{2} \right)^{\alpha_2} + \frac{\xi_2 \tilde{\theta} \dot{\tilde{\theta}}}{\xi_1} \\ & + \sum_{j=1}^n \frac{c_j^2 + (\delta_{j,\max})^2}{2} + \sum_{k=2}^n 0.2785h_k + \frac{\xi_3 \tilde{\theta} \dot{\tilde{\theta}} (2\alpha_2-1)}{\xi_1}. \end{aligned} \quad (39)$$

Let $\Theta = \frac{\xi_2 \tilde{\theta} \dot{\tilde{\theta}}}{\xi_1} + \frac{\xi_3 \tilde{\theta} \dot{\tilde{\theta}} (2\alpha_2-1)}{\xi_1}$, and then with the aid of **Lemma 4** and **Lemma 9**, one has

$$\begin{aligned} \Theta \leq & -\xi_2^{\alpha_1} \left(\frac{\tilde{\theta}^2}{2\xi_1} \right)^{\alpha_1} - \frac{\xi_3 (2\alpha_2-1) (2\xi_1)^{\alpha_2}}{2\xi_1 \alpha_2} \left(\frac{\tilde{\theta}^2}{2\xi_1} \right)^{\alpha_2} \\ & + \frac{\xi_2}{2\xi_1} \theta^2 + \frac{\xi_3 (2\alpha_2-1) \theta^{2\alpha_2}}{\xi_1 2\alpha_2} + (1-\alpha_1) e^{\frac{\alpha_1 \ln(\alpha_1)}{(1-\alpha_1)}}. \end{aligned} \quad (40)$$

Substituting (40) into (39) results in

$$\begin{aligned} \dot{L} \leq & -\sum_{j=1}^n b_{j,1} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_1} - \sum_{j=1}^n b_{j,2} \left(\frac{e_j^{a-a_j+2}}{a-a_j+2} \right)^{\alpha_2} \\ & - \sum_{i=1}^{n-1} \kappa_{i,1} \left(\frac{\chi_i^2}{2} \right)^{\alpha_1} - \sum_{i=1}^{n-1} \kappa_{i,2} \left(\frac{\chi_i^2}{2} \right)^{\alpha_2} \\ & - \xi_2^{\alpha_1} \left(\frac{\tilde{\theta}^2}{2\xi_1} \right)^{\alpha_1} - \frac{\xi_3 (2\alpha_2-1) (2\xi_1)^{\alpha_2}}{2\xi_1 \alpha_2} \left(\frac{\tilde{\theta}^2}{2\xi_1} \right)^{\alpha_2} \\ & + \frac{\xi_2}{2\xi_1} \theta^2 + \frac{\xi_3 (2\alpha_2-1) \theta^{2\alpha_2}}{\xi_1 2\alpha_2} + (1-\alpha_1) e^{\frac{\alpha_1 \ln(\alpha_1)}{(1-\alpha_1)}} \\ & + \sum_{j=1}^n \frac{c_j^2 + (\delta_{j,\max})^2}{2} + \sum_{k=2}^n 0.2785h_k. \end{aligned} \quad (41)$$

Assuming τ_1 is the minimum of these positive constants $(b_{1,1}, b_{2,1}, \dots, b_{n,1}, \kappa_{1,1}, \kappa_{2,1}, \dots, \kappa_{n-1,1}, \xi_2^{\alpha_1})$. Suppose that τ_2

is the minimum of these positive parameters ($\frac{b_{1,2}}{2n}, \frac{b_{2,2}}{2n}, \dots, \frac{b_{n,2}}{2n}, \frac{\kappa_{1,:}}{2n}$). Letting

$$\begin{aligned} \tau_3 = & \frac{\xi_2}{2\xi_1} \theta^2 + \frac{\xi_3 (2\alpha_2 - 1) \theta^{2\alpha_2}}{\xi_1 2\alpha_2} + (1 - \alpha_1) e^{\frac{\alpha_1 \ln(\alpha_1)}{(1-\alpha_1)}} \\ & + \sum_{j=1}^n \frac{c_j^2 + (\delta_{j,\max})^2}{2} + \sum_{k=2}^n 0.2785 h_k \end{aligned} \quad (42)$$

and then, according to **Lemma 3**, we obtain

$$\dot{L} \leq -\tau_1 L^{\alpha_1} - \tau_2 L^{\alpha_2} + \tau_3. \quad (43)$$

Remark 6. Referring to **Lemma 6** we can get the conclusion: $e_1, e_2, \dots, e_n, \chi_1, \chi_2, \dots, \chi_{n-1}, \hat{\theta}$ are practically bounded near the zero after the fixed time $T_a = \frac{100}{\tau_1(1-\alpha_1)} + \frac{100}{\tau_2(\alpha_2-1)}$. Notice that the value of T_a is only related to the main design parameters. It is easy to find an open set $(-B_{L,1}, B_{R,1})$ because $B_{L,1}, B_{R,1}$ and $y_{r,1}$ are designed by ourselves. $e_1 \rightarrow 0$ implies that x_1 tends to $y_{r,1}$. If $x_1(0)$ belongs to $(-B_{L,1}, B_{R,1})$, then the output can not escape from the set $(-B_{L,1}, B_{R,1})$. Because x_1 tends to the bounded function $y_{r,1}$, x_1 is a bounded signal. $\tilde{\theta} = \theta - \hat{\theta}$, as a result $\hat{\theta}$ is a bounded signal. Since the virtual control law $v_{1,2}$ a continuous function with bounded variables $(B_{L,1}, \dot{B}_{L,1}, B_{R,1}, \dot{B}_{R,1}, y_{r,1}, \dot{y}_{r,1}, \partial_{x_1}^{e_{1,1}}, e_1, \hat{\theta})$, $v_{1,2}$ is bounded. Based on $e_2 = x_2 - v_{1,2}$, x_2 is also a bounded signal. Following the similar analysis, $x_3, x_4, \dots, x_n$ are all bounded signals. In summary, **Theorem 1** has been rigorously proved.

Remark 7. Many existing researches [14, 1, 9, 40, 2, 16, 15] are concentrate on the finite-time stable control problem for high-order systems. It is hard for finite-time controller to obtain the accurate convergence time, since its convergence time linked to initial states and tracking error of close loop systems. Stability control can be regarded as a special case of tracking control. In contrary, this paper use the fixed-time convergence theory to design a tracking controller for high-order systems. Therefore, the controller designed here is an upgraded version of these controllers [14, 1, 9, 40, 2, 16, 15].

4 Simulation Verification

This section illustrates the efficacy and superiority of the proposed controller by a comparative simulation.

Assume that the proposed controller is applied to a 2-order high-order nonlinear system [13]:

$$\begin{aligned} \dot{x}_1 &= x_1 + 0.1 \sin(x_1), \\ \dot{x}_2 &= u^3 + 0.1 \cos(x_1) \sin(x_2), \\ y &= x_1 \end{aligned} \quad (44)$$

Assume that the proposed controller tracks the reference signal $y_{r,1} = \sin(0.2\pi t + \pi/6) + 0.4 \cos(0.4\pi t)$ and constraint functions $B_{L,1} = 2.1e^{-5t} + 1.9$, $B_{R,1} = 1.8e^{-5t} + 1.7$. It can be observed from (44) that gain functions $\rho_1(\bar{x}_1) = \rho_2(\bar{x}_2) =$

Fig. 1 System tracking under different initial conditions

Fig. 2 Other close-loop signals under different initial conditions.

Fig. 3 Actual control signal under different initial conditions.

1; Power values $\alpha_1 = 1, \alpha_2 = 3$; unknown nonlinear functions $\varphi_1(\bar{x}_1) = 0.1 \sin(x_1)$, $\varphi_2(\bar{x}_2) = 0.1 \cos(x_1) \sin(x_2)$.

Since the dynamic system (44) contains two unknown functions, two RBFNNs $(\bar{\eta}_1^*)^T \bar{M}_1(\bar{x}_1)$ and $(\bar{\eta}_2^*)^T \bar{M}_2(\bar{x}_2)$ are required to approximate $\varphi_1(\bar{x}_1)$ and $\varphi_2(\bar{x}_2)$ respectively. Because the weight parameters $(\bar{\eta}_1^*, \bar{\eta}_2^*)$ of the two RBFNNs are automatically adjusted by the adaptive rate $\hat{\theta}$, only the parameters related to the activation function vectors $(\bar{M}_1(\bar{x}_1),$

$\bar{M}_2(\bar{x}_2)$) need to be determined by ourselves. According to a trade-off between approximation ability and computation burden, the dimensions and Gaussian width of the two activation function vectors are set to 11 and 1.5 respectively. For the center position vector parameters of the two RBFNNs, they can be given by the following matrix:

$$c_1 = \begin{bmatrix} -3 \\ -2.4 \\ \vdots \\ 3 \end{bmatrix}, c_2 = \begin{bmatrix} -3 & -4 \\ -2.4 & -3.2 \\ \vdots & \vdots \\ 3 & 4 \end{bmatrix} \quad (45)$$

The control parameters in virtual control signal $v_{1,2}$, actual control signal u , filter compensation signal $\hat{\chi}_1$ and adaptive updating law $\hat{\theta}$ (The detailed controller design can refer to Section 3.1 of this paper) are selected as $a = 5$, $\rho_{1,\min} = 0.85$, $\rho_{2,\min} = 0.9$, $\rho_{1,\max} = 1.4$, $\rho_{2,\max} = 1$, $c_1 = 4$, $c_2 = 1$, $b_{1,1} = b_{2,1} = 20$, $b_{1,2} = b_{2,2} = 15$, $\alpha_1 = 0.96$, $\alpha_2 = 2$, $\kappa_{1,1} = \kappa_{2,1} = 12$, $\kappa_{1,2} = 5$, $\kappa_{2,2} = 50$, $h_2 = 15$, $\mu_{1,1} = 20$, $\mu_{1,2} = 0.6$, $\xi_1 = \xi_3 = 5$, $\xi_2 = 3$.

The simulation focuses on the tracking control, so these finite-time stabilization controllers [14, 1, 9, 40, 2, 16, 15] fail to suitable. To further illustrate the advantages of the fixed-time control framework, two different initial conditions are considered:

Condition 1: $x_1(0) = 0.2, x_2(0) = 0.25, v_{1,3}(0) = 0.1, \hat{v}_{1,3}(0) = 0, \chi_1(0) = 0.6, \hat{\theta}(0) = 0.8$.

Condition 2: $x_1(0) = 0.4, x_2(0) = 0.5, v_{1,3}(0) = \hat{v}_{1,3}(0) = 0, \chi_1(0) = 0.3, \hat{\theta}(0) = 1.5$.

Figs. 1-3 show that all close-loop signal are bounded, and Fig. 1 implies that the output signals under different initial conditions all enter a stable state after 0.44 seconds. Therefore, simulation results validate the effectiveness and superiority of the presented controller, compared with the existing finite-time stabilization controllers.

5 Conclusion

This paper has investigated the adaptive tracking control problem for a category of high-order nonlinear systems. Both the output constrain and the uncertainty of dynamics systems are taken into consideration. Adaptive RBFNNs are used to identify unknown functions that may not satisfy the growth conditions. A new error compensation system is employed to overcome filter errors. As a result, a fixed time adaptive neural command filtered controller is proposed. Simulation results validate the effectiveness and superiority of the presented controller, compared with the existing finite-time stabilization controllers. Future research will further consider the impact of the actual environment on the output signal of the proposed controller, such as input saturation, dead zone, and control channel bandwidth constraints.

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About Data Availability Statements: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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