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Numerical and Experimental Investigations on a Friction Ring Damper for a Flywheel

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Abstract: A damping strategy using a friction ring damper for an industrial flywheel was numerically and experimentally investigated. The friction ring damper, located on the arms of the flywheel, was experimentally found to effectively reduce the vibration amplitude of the flywheel. The vibration energy is dissipated when relative motions occur at the friction contact interfaces. Nonlinear dynamic analysis based on a lumped-parameter model of a flywheel equipped with a friction ring damper was conducted. A dimensionless parameter, κ , defined as the ratio of the critical friction force to the amplitude of harmonic force, was used to evaluate the damping performance. For several values of κ , steady-state responses under harmonic excitation and nonlinear modes were obtained using the harmonic balance method (HBM) combined with the alternating frequency–time domain method (AFT). The forced response analysis proved the existence of an optimal value of κ , which can minimize the vibration amplitude of the flywheel. The nonlinear modal analysis showed that all the damping ratio–frequency curves are completely coincident even for different κ , and the frequency corresponding to the maximum damping ratio is equal to the frequency at the intersection of the forced response curves under the fully

slip and the fully stick states of the friction contact interface. By analyzing the behaviors of the friction contact interface, it is shown that the friction contact interface provides damping in the combined stick–slip state. The forced response under random excitation was calculated using the Runge–Kutta method and the friction interface behaviors were analyzed. Finally, spectral testing was conducted to verify the numerical results.

Keywords: flywheel, friction ring damper, nonlinear dynamics, HBM, spectral testing

1. Introduction

This study focused on the vibration suppression of flywheels. Flywheels are widely used to control the attitude and maintain the stability of satellites. In the launch phase of the rocket, the flywheel is excited to a violent resonance under severe random vibrations. According to estimations based on the resonance intensity of the flywheel, the force acting on the bearing caused by the vibration amplification of the wheel body greatly exceeds the static load capacity of the bearing, putting the bearing at risk of damage. Meanwhile, the inherent damping in the flywheel is very low, and there is nearly no source of energy dissipation. Therefore, the flywheel’s ability to withstand these vibrations mainly depends on the external inputs of damping; therefore, it is necessary to develop new methods and devices to damp the vibrations of the flywheel.

In previous studies, the use of a flexible dynamic vibration absorber (DVA) was proposed to suppress the high-frequency micro vibrations of the flywheel [1-3], in this method, rubber pads were used to increase the stiffness and damping of the DVA. In this study, friction pads made of epoxy phenolic glass cloth were used to replace the rubber pads in the damping ring, the elastic modulus of the friction pad is 1 GPa. Thus, the original flexible DVA became a dry friction damper mounted on the arms of the flywheel, in which contact is permanently maintained by the preload of the bolts. Friction thus occurs at the friction contact interfaces when relative motions occur between the friction pads and arms of the flywheel to dissipate the vibration energy. The friction at the contact interface is the most widely used

source of external damping in bladed disk assemblies [4] and blisks (integrally bladed disks) [5], which include shroud contact in blades, platform dampers, and friction rings. Our work drew upon numerous methods of design and analysis of these devices reported in the literature. The HBM [6] is the most widely used method for the nonlinear steady-state response prediction of bladed disk assemblies and blisks with friction dampers [7-11], which is a computationally efficient and accurate method to perform dynamic analyses quantitatively. The nonlinear modal analysis of the bladed disks with friction contact interfaces [12, 13] is based on the concept of complex nonlinear modes [14] and the frequency domain formulation of the dynamical system's equations of motion [15]. The AFT method [16] is used for calculating the implicit nonlinear friction forces. Recently, the HBM combined with the AFT to calculate the friction force in the time domain and the arc-length continuation technique to improve the convergence of the method [17] were used when performing nonlinear forced response [18, 19] and nonlinear modal analysis [20].

Although the bladed disk assemblies and blisks with friction dampers have been extensively studied, little numerical or experimental research has been performed on the friction ring damper for flywheels. Motivated by this gap in research, in this study, an equivalent lumped-parameter model of the flywheel with friction contact interfaces was established to study the dynamic behavior of the system. The vibration suppression performance of the damper is discussed based on nonlinear forced response and nonlinear modal analysis. In addition, spectral testing was conducted to verify the numerical results.

The remainder of this paper is organized into five sections: the lumped-parameter model of the friction ring damper-flywheel-bearing system is described and the governing equation of the system presented in Section 2. The numerical analysis (nonlinear modal and nonlinear forced response analysis based on HBM combined with AFT) is described in Section 3. The numerical results and related discussions are presented in Section 4. The spectral testing on the flywheel conducted with and without the friction ring damper to verify the numerical results is discussed in Section 5. The conclusions are presented in Section 6.

2. Dynamic modelling

As shown in Figure 1, the characteristic mode of the flywheel obtained from the finite element model is an umbrella-shaped flapping mode, which is dominated by the axial movement of the wheel body and the bearing resting on the disc spring [1]. The flapping mode is the main contributor to the axial force acting on the bearing owing to a unit force applied at the rim of the flywheel. The aim of this study is to reduce the vibration amplitude of the flywheel and mitigate the axial force acting on the bearing.

Fig. 1 Umbrella-shaped flapping mode of the flywheel.

As shown in Figure 2(a), the friction ring damper is attached to the arms of the flywheel. When relative motions occur at the contact interfaces between the friction pads and arms, the vibration energy of the flywheel is dissipated by friction damping. An equivalent 11-degree of freedom (DOF) lumped-parameter model of the flywheel-bearing system equipped with the friction ring damper is developed as shown in Figure 2(b). Where $m_{f,i} = m_f$ ($i = 1, 2, \dots, 5$) and k_f denote the masses and stiffness of the flywheel, respectively; k_{f0} denotes the connection stiffness; m_b and k_b denote the mass and stiffness of the bearing, respectively; $m_{r,i} = m_r$ ($i = 1, 2, \dots, 5$) and k_{r0} denote the masses and the connection stiffness of the damping ring, respectively; c_r denotes the viscous damping; and k_g denotes the connection stiffness to the ground. Five external forces, $F = 0.2\cos(\omega t)$, were uniformly applied at the five flywheel masses. The numerical values of the parameters of the model are listed in Table 1.

Fig. 2 Flywheel equipped with the friction ring damper (a) and the 11-DOF lumped-parameter model (b).

Table 1 Parameter values of the lumped-parameter model.

m_f	k_f	k_{f0}	m_b	k_b
0.9105 kg	2.824×10^6 N/m	1×10^7 N/m	1.0710 kg	1.6150×10^7 N/m
m_r	k_g	k_{r0}	c_r	—
0.0990 kg	1×10^{10} N/m	1×10^6 N/m	0	—

The friction contact interfaces between the flywheel and the damping ring were modelled based on the Coulomb friction model, the Coulomb friction force is expressed as [21, 18]

$$f_{nl,i} = \begin{cases} k_t(z_i - x_{d,i}) & \text{stick} \\ f_c \operatorname{sgn}(\dot{x}_{d,i}) & \text{slip} \end{cases}, \quad (1)$$

where k_t is the tangential stiffness of the friction contact interface, $z_i = x_{f,i} - x_{r,i}$ is the relative displacement between the flywheel and the damping ring, $x_{d,i}$ is the displacement of the friction contact point, $f_c = \mu_f N$ is the critical friction force where μ_f is the coefficient of friction, and N is the normal load.

The governing equation of the equivalent 11-DOF lumped-parameter model of the system is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{F}_{\text{nl}}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{F}_{\text{ext}}, \quad (2)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass matrix, damping matrix, and stiffness matrix, respectively; $\mathbf{x}$, $\dot{\mathbf{x}}$, and $\ddot{\mathbf{x}}$ are the displacement vector, velocity vector, and acceleration vector, respectively; and $\mathbf{F}_{\text{nl}}$ and $\mathbf{F}_{\text{ext}}$ are the nonlinear friction force vector and external force vector, respectively.

The matrices and vectors in Eq. (2) are given by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_r & & \\ & \mathbf{M}_f & \\ & & m_b \end{bmatrix}, \quad \mathbf{M}_r = \text{diag}\{m_r\}_{5 \times 5}, \quad \mathbf{M}_f = \text{diag}\{m_f + m_r\}_{5 \times 5}, \quad (3)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_r & -\mathbf{C}_r & \\ -\mathbf{C}_r & \mathbf{C}_r & \\ & & 0 \end{bmatrix}, \quad \mathbf{C}_r = \text{diag}\{c_r\}_{5 \times 5}, \quad (4)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{rr} & \mathbf{0}_{5 \times 5} & \mathbf{0}_{5 \times 1} \\ \mathbf{0}_{5 \times 5} & \mathbf{K}_{ff} & \{-k_f\}_{5 \times 1} \\ \mathbf{0}_{1 \times 5} & \{-k_f\}_{1 \times 5} & 5k_f + k_b \end{bmatrix}, \quad (5)$$

$$\mathbf{K}_{rr} = \begin{bmatrix} 2k_{r0} + k_g & -k_{r0} & 0 & 0 & -k_{r0} \\ -k_{r0} & \cdot & \cdot & 0 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & -k_{r0} \\ -k_{r0} & 0 & 0 & -k_{r0} & 2k_{r0} \end{bmatrix}, \quad \mathbf{K}_{ff} = \begin{bmatrix} k_f + 2k_{f0} & -k_{f0} & 0 & 0 & -k_{f0} \\ -k_{f0} & \cdot & \cdot & 0 & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & -k_{f0} \\ -k_{f0} & 0 & 0 & -k_{f0} & k_f + 2k_{f0} \end{bmatrix}, \quad (6)$$

$$\mathbf{x} = \left(\{x_{r,i}\}_{1 \times 5} \quad \{x_{f,i}\}_{1 \times 5} \quad x_b \right)^T, \quad \dot{\mathbf{x}} = \left(\{\dot{x}_{r,i}\}_{1 \times 5} \quad \{\dot{x}_{f,i}\}_{1 \times 5} \quad \dot{x}_b \right)^T, \quad \ddot{\mathbf{x}} = \left(\{\ddot{x}_{r,i}\}_{1 \times 5} \quad \{\ddot{x}_{f,i}\}_{1 \times 5} \quad \ddot{x}_b \right)^T, \quad (7)$$

$$\mathbf{F}_{\text{nl}} = \left(\{1\}_{1 \times 5} \quad \{-1\}_{1 \times 5} \quad 0 \right)^T \cdot f_{\text{nl}}, \quad \mathbf{F}_{\text{ext}} = \left(\{0\}_{1 \times 5} \quad \{1\}_{1 \times 5} \quad 0 \right)^T \cdot F. \quad (8)$$

3. Numerical analysis

In this section, the nonlinear mode and nonlinear forced response of the flywheel equipped with the friction ring damper are derived using HBM [11] combined with AFT [16]. Without loss of generality,

the HBM–AFT method is presented for a general multi-DOF dynamic system with a general nonlinear force.

3.1. Nonlinear modal analysis

Nonlinear modal analysis is performed based on the HBM for $\mathbf{F}_{\text{ext}} = \mathbf{0}$ in Eq. (2) by assuming that

$$\mathbf{x}(t) = \mathbf{Q}^0 + \sum_{k=1}^N \left(\mathbf{Q}^{k,c} \cos k\omega t + \mathbf{Q}^{k,s} \sin k\omega t \right), \quad (9)$$

$$\mathbf{F}_{\text{nl}}(t) = \mathbf{F}_{\text{nl}}^0 + \sum_{k=1}^N \left(\mathbf{F}_{\text{nl}}^{k,c} \cos k\omega t + \mathbf{F}_{\text{nl}}^{k,s} \sin k\omega t \right), \quad (10)$$

where N is the retained number of harmonics.

The vectors of the Fourier coefficients in Eqs. (9) and (10) are

$$\mathbf{Q} = \left\{ \mathbf{Q}^0 \quad \mathbf{Q}^{1,c} \quad \mathbf{Q}^{1,s} \quad \dots \quad \mathbf{Q}^{N,c} \quad \mathbf{Q}^{N,s} \right\}^T, \quad (11)$$

$$\mathbf{F}_{\text{nl}} = \left\{ \mathbf{F}_{\text{nl}}^0 \quad \mathbf{F}_{\text{nl}}^{1,c} \quad \mathbf{F}_{\text{nl}}^{1,s} \quad \dots \quad \mathbf{F}_{\text{nl}}^{N,c} \quad \mathbf{F}_{\text{nl}}^{N,s} \right\}^T, \quad (12)$$

where

$$\mathbf{F}_{\text{nl}}^0 = \frac{1}{T} \int_{-T/2}^{T/2} \mathbf{F}_{\text{nl}}(\mathbf{x}, \mathbf{x}) dt, \quad \mathbf{F}_{\text{nl}}^{k,c} = \frac{2}{T} \int_{-T/2}^{T/2} \mathbf{F}_{\text{nl}}(\mathbf{x}, \mathbf{x}) \cos k\omega t dt, \quad \mathbf{F}_{\text{nl}}^{k,s} = \frac{2}{T} \int_{-T/2}^{T/2} \mathbf{F}_{\text{nl}}(\mathbf{x}, \mathbf{x}) \sin k\omega t dt. \quad (13)$$

Then Eq. (2) can be expressed as

$$\mathbf{M} \otimes \mathbf{I}_{2N+1} \mathbf{B}^2 + \mathbf{C} \otimes \mathbf{I}_{2N+1} \mathbf{B} + \mathbf{K} \otimes \mathbf{I}_{2N+1} + \mathbf{F}_{\text{nl}}(\mathbf{x}, \mathbf{x}) = \mathbf{0}, \quad (14)$$

where $\otimes$ is the Kronecker product, and

$$\mathbf{B} = \text{diag}(\mathbf{0} \quad \Lambda_1 \quad \Lambda_2 \quad \dots \quad \Lambda_{2N+1}), \quad \Lambda_k = k \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}. \quad (15)$$

The AFT procedure is employed to obtain the frequency-domain components of the nonlinear friction force

$$\mathbf{Q} \rightarrow \mathbf{x}(\omega) \xrightarrow{\text{IFFT}} \mathbf{x}(t) \xrightarrow{\mathbf{B}} \mathbf{x}(t) \rightarrow \mathbf{F}_{\text{nl}}(\mathbf{x}, \mathbf{x}) \xrightarrow{\text{FFT}} \mathbf{F}_{\text{nl}}(\mathbf{Q}, \omega). \quad (16)$$

This gives the following set of the nonlinear algebraic equations

$$\mathbf{H}(\mathbf{Q}, \omega) = \mathbf{M} \otimes \mathbf{I}_{2N+1} \mathbf{B}^2 + \mathbf{C} \otimes \mathbf{I}_{2N+1} \mathbf{B} + \mathbf{K} \otimes \mathbf{I}_{2N+1} - \mathbf{F}_{\text{nl}}(\mathbf{Q}, \omega) = \mathbf{0}. \quad (17)$$

Then, the Newton–Raphson method is employed to solve Eq. (17) and obtain the nonlinear modes.

3.2. Forced response analysis

The forced responses are also obtained using the Newton–Raphson method to solve the nonlinear algebraic equation

$$\mathbf{H}(\mathbf{Q}, \omega) = \mathbf{M} \otimes \mathbf{I}_{2N+1} \mathbf{B}^2 + \mathbf{C} \otimes \mathbf{I}_{2N+1} \mathbf{B} + \mathbf{K} \otimes \mathbf{I}_{2N+1} - \mathbf{F}_{\text{nl}}(\mathbf{Q}, \omega) - \mathbf{F}_{\text{ext}}(\mathbf{Q}, \omega) = \mathbf{0}, \quad (18)$$

where the external force vector $\mathbf{F}_{\text{ext}}(\mathbf{Q}, \omega)$ can be expressed in terms of a truncated Fourier series

$$\mathbf{F}_{\text{ext}}(t) = \mathbf{F}_{\text{ext}}^0 + \sum_{k=1}^N \left(\mathbf{F}_{\text{ext}}^{k,c} \cos k\omega t + \mathbf{F}_{\text{ext}}^{k,s} \sin k\omega t \right). \quad (19)$$

The vector of the Fourier coefficients of the external force is

$$\mathbf{F}_{\text{ext}} = \left\{ \mathbf{F}_{\text{ext}}^0 \quad \mathbf{F}_{\text{ext}}^{1,c} \quad \mathbf{F}_{\text{ext}}^{1,s} \quad \dots \quad \mathbf{F}_{\text{ext}}^{N,c} \quad \mathbf{F}_{\text{ext}}^{N,s} \right\}^T. \quad (20)$$

This was solved using an arc-length continuation-based prediction-correction algorithm [17-19].

4. Results and discussions

4.1. Nonlinear forced responses

When the friction contact interface is in the fully stick state ($N = \infty$), the only connector between the flywheel and damping ring is the tangential stiffness spring k_t . The nonlinear forced responses of the flywheel under harmonic excitation for several values of k_t were obtained and plotted, as shown in Figure 3. It can be seen that when the friction contact interface is in the fully stick state, the frequency increases monotonically as k_t increases. A modal test was performed on the system to determine the value of k_t , and the experimental results showed that the frequency of the flapping mode was 200 Hz. Thus, k_t was taken as 2×10^4 N/m in the lumped-parameter model, and the frequency obtained from simulation is 200.7 Hz when the friction contact interface is in the fully stick state ($N = \infty$) and 199.4 Hz when it is in the fully slip state ($k_t = 0$).

Fig. 3 Effect of k_t on the frequency of the flywheel.

We define the ratio of the critical friction force $\mu_t N$ to the amplitude of harmonic force F as a dimensionless parameter $\kappa = \mu_t N / F$. The nonlinear forced responses of the flywheel were obtained with several values of κ , as shown in Figure 4 (Figure 4(b) is an enlarged view of Figure 4(a)). It can be seen that as κ gradually increases from 0 to ∞ , the vibration amplitude of the flywheel first decreases and then increases, and the friction contact interface undergoes a transition from the fully slip ($\kappa = 0$, $N = 0$) to the fully stick state ($\kappa = \infty$, $N = \infty$). The qualitative results are similar to those for bladed disks with underplatform dampers or friction rings [6]. When κ is in the range of 4–15, the friction ring damper greatly and effectively reduces the vibration amplitude of the flywheel. When κ is in the range of 5–8, an optimal value of κ exists that can minimize the vibration amplitude.

Fig. 4 Effect of κ on the vibration amplitude of the flywheel.

4.2. Nonlinear modes and behaviors of the friction interface

Figures 5(a) and (b) show the frequency–modal amplitude and damping ratio–modal amplitude diagrams, respectively, for several values of κ . As κ increases, the nonlinear modal curves translate in the direction in which the modal amplitude becomes larger. The damping ratio–frequency diagrams at the modal amplitude shown in Figures 5(a) and (b) are presented in Figure 5(c). It can be seen from Figure 5(d) that all the damping ratio–frequency curves are fully coincident even for different κ , and the frequency at the maximum damping ratio is equal to the intersection frequency of the forced response curves for $\kappa = 0$ (fully slip state) and $\kappa = \infty$ (fully stick state).

Fig. 5 Frequency–modal amplitude diagram (a), damping ratio–modal amplitude diagram (b), damping ratio–frequency diagram (c), and intersection of the forced response curves for $\kappa = 0$ and $\kappa = \infty$ (d).

The optimal value of κ was found to be 6.07 by employing a genetic algorithm [3]. The nonlinear forced responses of the flywheel at $\kappa = 0$, $\kappa = \infty$, and $\kappa = \text{opt}$ (the optimal value of κ) are shown in Figure 6(a), where f_{slip} and f_{stick} are the frequencies when the friction contact interface is in the fully slip and fully stick state, respectively. It can be seen that the vibration amplitude of the flywheel is the smallest at $\kappa = \text{opt}$, and the frequency at the maximum vibration amplitude is equal to the frequency at the intersection of the response curves for $\kappa = 0$ and $\kappa = \infty$.

The nonlinear modes at $\kappa = \text{opt}$ are showed in Figure 6(b), where some feature points on the nonlinear modal curves are marked out using small squares (points a–f). As shown in Figure 6(b), when the vibration amplitude of the flywheel is less than the modal amplitude at point a, the friction contact interface is in the fully stick state. Point a is a state transition point, where the state of the friction contact interface begins to change from the fully stick to the combined stick–slip state as the modal amplitude increases. At points b and c, the friction contact interface is in the combined stick–slip state, i.e., the stick and slip states coexist. At points d and e, the combined stick–slip state gradually approaches the fully slip state. When the vibration amplitude of the flywheel exceeds the modal amplitude at point f, the friction contact interface is in a fully slip state. In the state transition of the friction contact interface from the fully stick to the fully slip state, the frequency decreases monotonously as the modal amplitude increases from point a to f. Correspondingly, the damping ratio increases rapidly from 0 (the fully stick state at point a) to the maximum value (the combined stick–slip state at point c) and then slowly decreases to 0 (the fully slip state at point f).

The nonlinear forced response curve and nonlinear modal curve at $\kappa = \text{opt}$ (Figures 6(a) and (b)) are plotted on the same graph in Figure 6(c). The vibration amplitudes of the flywheel at point 1 and 5 on the nonlinear forced response curve are the same as the modal amplitude at point a on the nonlinear modal curve. As discussed earlier, the friction contact interface is in the fully stick state at points 1 and 5. Point 3 is the peak of the nonlinear forced response curve, at which point the vibration amplitude is the same as the modal amplitude at point c on the nonlinear modal curve. The frequency range between

f_s at point 1 and f_e at point 5 is the non-coinciding section for the forced response curves at $\kappa = \text{opt}$ and $\kappa = \infty$. The vibration amplitudes of this section lie between the modal amplitudes at points a and c. Therefore, in the non-coinciding section (Figure 6(b)), the friction contact interface is in the combined stick–slip state, and the friction contact interface is able to provide damping.

Fig. 6 Nonlinear forced responses of the flywheel (a), nonlinear modes (b), and comparison of the nonlinear forced response and the nonlinear mode (c) at the optimal value of κ .

The time history of the friction force and the corresponding hysteresis loop at points 1–5 in Figure 6(c) are showed in Figures 7(a) and (b), respectively. The time history of the displacement of the friction contact point and slip ratio are displayed in Figures 7(c) and (d), respectively.

As shown in Figures 7(a) and (b), the horizontal and inclined sections of the friction force curves correspond to the slip and the stick states, respectively. In contrast, for the displacement curves of the friction contact point, the horizontal and inclined sections correspond to the stick and slip states, respectively (Figure 7(c)).

In Figure 7(d), the slip ratio is defined as the ratio of the time that the friction contact point is in the fully slip state to the total period of a vibration; the slip ratio can be observed to be in the range of 10–45%. At points 1 and 5, the friction contact interface is in the fully stick state, i.e., no slip occurs. Therefore, the slip ratios, areas of the hysteresis loops, and displacements of the friction contact point are all zero, and the time histories of the friction forces are harmonic.

As shown in Figure 7, as the frequency increases from point 1 to 5, the area of the hysteresis loop, displacement of the friction contact point, and slip ratio of the friction contact interface increase rapidly from zero and then decrease slowly to zero. This observation is consistent with the law of damping ratio change with modal amplitude (Figure 6(b)).

Fig. 7 Time history (a) and hysteresis loop (b) of the friction force, time history of the displacement of the friction contact point (c), and slip ratio of the friction contact interface (d) at different frequencies.

5. Experimental validation

Spectral testing of the flywheel was conducted with and without the friction ring damper to confirm its vibration suppression effect. Photographs of the spectral testing site are shown in Figure 8. Random excitations were produced through the vibrating table (Figure 8(a)), and acceleration data measured using the accelerometers (Figure 8(b)).

Fig. 8 Spectral testing of the flywheel (a) and layout of the accelerometers (b).

The experimental results are presented in Figure 9. Figures 9(a) and (b) show the time history of the accelerations of the rim of the flywheel and the corresponding accelerations of the vibrating table (named

as ‘base’), respectively, without and with the friction ring damper. Figure 9(c) shows the acceleration power spectral density (PSD) of the vibrating table. Figure 9(d) compares the acceleration PSD of the flywheel rim without and with the friction ring damper. According to Figure 9(d), the maximum acceleration PSD of the flywheel without the friction ring damper is $2.706 \text{ g}^2/\text{Hz}$ at the frequency 206.8 Hz , and the maximum acceleration PSD of the flywheel with the friction ring damper is $1.659 \text{ g}^2/\text{Hz}$ at the frequency 200.7 Hz . The maximum acceleration PSD of the flywheel is reduced by 38.69% with the damper as compared to that without the damper.

Fig. 9 Time history of the acceleration of the flywheel and the vibrating table without (a) and with (b) the friction ring damper, acceleration PSD of the vibrating table (c), and comparison of the acceleration PSD without and with the friction ring damper (d).

The measured acceleration of the vibrating table and acceleration PSD of the flywheel without and with the friction ring damper, shown in Figure 9, were used to verify the numerical results. In the lumped-parameter model, we removed the harmonic excitation force F and replaced the fixed foundation under the bearing stiffness with the random excitation of the vibrating table shown in Figures 9(a) and (b). The fourth-order Runge–Kutta method was used to calculate the acceleration PSD of the flywheel. In this way, the vibration characteristics of the flywheel without and with the friction ring damper with actual random excitation of the vibrating table were obtained. The comparison between the numerical results and the experimental results of the acceleration PSD are shown in Figures 10(a) and (b), respectively. It can be seen that the numerical results were consistent with the experimental results.

Fig. 10 Comparison of the acceleration PSD of the flywheel obtained by the Runge–Kutta method and from the spectral testing without (a) and with (b) the friction ring damper.

The friction forces under the random excitation of the vibrating table are shown in Figure 11. As shown in Figure 11(a), the friction force is no longer periodic. Figure 11(b) shows that when the spring force F_{kt} of the tangential stiffness spring k_t exceeds the critical friction force $\mu_t N$, slip occurs at the friction contact interface. As shown in Figure 11(c), the friction curve is no longer a closed loop; instead, it consists of a series of combined stick–slip fold lines. The fast Fourier transform was performed on the friction force to obtain the friction force in the frequency domain, as shown in Figure 11(d), where it can

be seen that the friction force peaks at the resonance frequency (198.8 Hz) of the flywheel and at its double (397.5 Hz) and triple resonance frequencies (596.3 Hz).

Fig. 11 Time history of the friction force (a)(b), friction force–displacement curve of the flywheel (c), and friction force in the frequency domain (d) under the random excitation.

The time history of the displacement of the friction contact point under the random excitation of the vibrating table was obtained and is shown in Figure 12(a). The displacement curve of the friction contact point shows a staircase shape, where the horizontal and inclined sections correspond to the stick and slip states of the friction contact interface, respectively. In each step of the stairs, we define the duration of the inclined section divided by the sum of the durations of the horizontal and the inclined sections as the slip ratio in a step. The slip ratios of all the steps were obtained, as shown in Figure 12(b). It can be seen

that the slip ratios of most stair steps are in the range of 10–50%, which is the same as the range of the slip ratio under the optimal value of κ shown in Figure 7(d).

Fig. 12 Time history of the displacement of the friction contact point (a) and slip ratio of the friction contact interface (b) under random excitation.

6. Conclusions

A damping strategy for an industrial flywheel using a friction ring damper was investigated numerically and experimentally. The vibration reduction performance of the damper was predicted based on the established lumped-parameter model of the flywheel with friction contact interfaces. The model was numerically analyzed using the HBM combined with AFT to obtain the nonlinear forced responses under harmonic excitation, nonlinear modes of the flywheel equipped with the friction ring damper, and the behaviors of the friction contact interface. The damping performance of the friction ring damper is discussed based on numerical results. The results show that when the friction contact interface is in the fully stick state, the frequency increases monotonically as the tangential stiffness increases. A dimensionless parameter κ is defined to evaluate the damping performance, and it is shown that an optimal value of κ exists, which can minimize the vibration amplitude of the flywheel. In addition, all the damping ratio–frequency curves are completely coincident even with different values of κ , and the frequency at the maximum damping ratio is equal to that at the intersection of the forced response curves

under the fully slip and the fully stick states. When the friction contact interface is in the combined stick–slip state, the friction contact interface is able to provide damping. The friction interface behaviors under random excitation were also analyzed. The slip ratio is defined as the ratio of the time when the friction contact point is in the fully slip state to the total period of a vibration, which is shown to be in the range of 10–45% at the optimal value of κ . Finally, spectral testing was conducted to confirm the vibration suppression effect of the friction ring damper and experimentally verify the numerical results.

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Declaration of conflicting interests

The authors declare no conflict of interests in preparing this article.

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Statements & Declarations

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The datasets generated during the current study are available from the corresponding author on reasonable request.

Table 1 Parameter values of the lumped-parameter model.

m_f	k_f	k_{f0}	m_b	k_b
0.9105 kg	2.824×10^6 N/m	1×10^7 N/m	1.0710 kg	1.6150×10^7 N/m
m_r	k_g	k_{r0}	c_r	—
0.0990 kg	1×10^{10} N/m	1×10^6 N/m	0	—

Figure Captions

Fig. 1 Umbrella-shaped flapping mode of the flywheel.

Fig. 2 Flywheel equipped with the friction ring damper (a) and the 11-DOF lumped-parameter model (b).

Fig. 3 Effect of k_t on the frequency of the flywheel.

Fig. 4 Effect of κ on the vibration amplitude of the flywheel.

Fig. 5 Frequency–modal amplitude diagram (a), damping ratio–modal amplitude diagram (b), damping ratio–frequency diagram (c), and intersection of the forced response curves for $\kappa = 0$ and $\kappa = \infty$ (d).

Fig. 6 Nonlinear forced responses of the flywheel (a), nonlinear modes (b), and comparison of the nonlinear forced response and the nonlinear mode (c) at the optimal value of κ .

Fig. 7 Time history (a) and hysteresis loop (b) of the friction force, time history of the displacement of the friction contact point (c), and slip ratio of the friction contact interface (d) at different frequencies.

Fig. 8 Spectral testing of the flywheel (a) and layout of the accelerometers (b).

Fig. 9 Time history of the acceleration of the flywheel and the vibrating table without (a) and with (b) the friction ring damper, acceleration PSD of the vibrating table (c), and comparison of the acceleration PSD without and with the friction ring damper (d).

Fig. 10 Comparison of the acceleration PSD of the flywheel obtained by the Runge–Kutta method and from the spectral testing without (a) and with (b) the friction ring damper.

Fig. 11 Time history of the friction force (a)(b), friction force–displacement curve of the flywheel (c), and friction force in the frequency domain (d) under the random excitation.

Fig. 12 Time history of the displacement of the friction contact point (a) and slip ratio of the friction contact interface (b) under random excitation.

Fig. 1 Umbrella-shaped flapping mode of the flywheel.

(a)

(b)

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(a)

(b)

(c)

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(a)

(b)

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(a)

(b)

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