The Fast Fixed-Time Bipartite Synchronization of Coupled Delayed Neural Networks with Signed Graphs

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The fast fixed-time bipartite synchronization of coupled delayed neural networks with signed graphs

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Abstract In this paper, the fixed-time bipartite synchronization problem for coupled delayed neural networks with signed graphs is discussed. Different from traditional neural networks, the interactions between nodes of delayed neural networks can be either collaborative or antagonistic. Furthermore, compared with the initial-condition based finite-time synchronization, the settling time is bounded by a constant within fixed-time regardless of the initial condition. It is worth noting that the fixed-time stable network for bipartite synchronization in this paper achieves more faster convergence than most existing publications. By applying constructing comparison system method, Lyapunov stability theory and inequality techniques, some sufficient criteria for fixed-time bipartite synchronization are obtained. Finally, two numerical examples are granted to display the performance of the obtained results.

Keywords Delayed neural networks · fast fixed-time bipartite synchronization · signed graphs

1 Introduction

As we know, the control problem of neural networks plays an important role due to its broad applications, which has been capturing considerable attention from various fields, such as parallel computation, signal and image processing, nonlinear optimization, pattern recognition, among many others [1–8]. Generally speaking, according to different interactions between nodes of neural networks, the networks are classified into two types: unsigned networks and signed networks. If the interaction digraphs are assumed to be unsigned, then we get an unsigned networks, where all the communication links are positive. If the network with signed interaction digraphs, where the communication links between neighboring nodes of network can be either positive or negative, then it is a signed network. The pioneering works in [9–12] have been investigated many dynamical behaviors of unsigned networks. However, the analysis method is difficult to be applied to signed networks because these proofs depend on the symmetry of the communication topology. Yet different from the traditional unsigned neural networks, the signed neural networks can exhibit an interesting synchronization phenomenon named as bipartite synchronization. Bipartite means the nodes of these networks can be divided into two disjoint nonempty sets such that every edge only connects a pair of nodes which belong to different sets. Many real-world networks can be described by bipartite networks with signed graphs, such as the papers-scientists networks [13], producer-consumer networks [14], the actors-films networks [15] and so on.

Over the past years, the synchronization problem have been investigated by applying different control methods, such as adaptive synchronization [16], impulsive synchronization [17], pinning synchronization [18] and so on. Recently, the researchers pay more attentions to investigate the bipartite synchronization for some networks. For example, Bian investigated the adaptive synchronization of bipartite dynamical networks with distributed delays and nonlinear derivative coupling by constructing effective adaptive feedback controllers and update laws in [19]. In [20], the authors study the bipartite synchronization in a network of nonlinear systems with collaborative and antagonistic interactions by using contraction theory to obtain some sufficient conditions. Liu investigated the bipartite synchronization in coupled delayed neural networks by utilizing the pinning control strategy and M-matrix theory in [21]. Guo derived the bipartite consensus for multi-agent

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systems with antagonistic interactions and communication delays in [22], and Zhang researched the bipartite consensus problems for continuous-time multi-agent system over signed graphs by establishing state feedback and output feedback control approaches in [23].

With the developing of investigation, Polyakov proposed the concept of fixed-time stability in [24], where the estimation of settling time for the linear control systems is not related to initial conditions. This means one most important advantage of fixed-time control is that the settling time can be determined by controller's parameters and system's parameters. Thus, the convergence time can be flexibly adjusted if suitable control parameters are chosen. Further investigations of fixed-time control problems have been discussed in recent years. Based on the new fixed-time stability theorem, the fixed-time synchronization of neural networks is investigated by designing feedback controller [25]. The authors focus on the cluster synchronization problem of coupled complex networks with uncertain disturbances under an adaptive fixed-time control strategy [26]. By proposing a delay-dependent controller, the problem of fixed-time outer synchronization over double-layered neuron-based networks with uncertain parameters in nonlinear nodes dynamics and delayed coupling among the nodes is studied in [27]. By constructing Lyapunov functionals, using the properties of the Wiener process as well as applying a designed comparison system, the authors investigate the fixed-time synchronization (FDTS) of complex networks with stochastic perturbations in [28]. In [29], by designing a new Lyapunov function and constructing comparison systems, the authors investigate the fixed-time synchronization of complex networks with synchronizing and desynchronizing impulses via nonchattering control. However, the convergence time of these fixed-time controller is not an optimal one. Therefore, many researchers try to design new controller to optimize the settling time, that is fast fixed-time control schemes. For example, based on fixed-time stability theory, the authors proposed a novel fixed-time stable system, and then derived a fast fixed-time nonsingular terminal sliding mode control method in [30]. The synchronization control of a class of delayed neural networks using a fast fixed-time control theory has been discussed in [31]. Motivated by above discussions, the fast fixed-time bipartite synchronization problems for delayed neural networks still remain open, we will consider its fast fixed-time bipartite synchronization of coupled delayed neural networks with signed graphs by extending the preliminary reported in [19–23, 25–31].

To our best knowledge, the main contributions of this paper can be listed as follows: a) In contrast to the traditional neural networks, the directed signed network is discussed, which implies that the interactions between neighboring nodes can be either collaborative or antagonistic; b) to formulate some sufficient criteria for fixed-time bipartite synchronization by applying constructing comparison system method, while the traditional fixed-time synchronization problems are not used; c) In this paper, the network with directed signed graph is considered, it means the analysis and mathematics proofs are not depend on the symmetry of the communication topology; d) The settling times of fixed-time bipartite synchronization do not based on the initial values of the system, and the fixed-time bipartite synchronize network presented in this paper achieves more faster convergence than the previous works [25–30].

This paper is organized as follows: In the following section, some definitions, lemmas, and assumptions are presented. In Section 3, we design the control protocol for fast fixed-time bipartite synchronization, and validity of proposed protocol is rigorously proved. The examples are carried out to demonstrate the effectiveness of the obtained results in Section 4. Finally, the conclusion and discussions about future work are presented in Section 5.

Notations: $\mathbb{R}$ is the set of real numbers. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the n -dimensional Euclidean space and the set of $n \times m$ matrices. $\mathcal{A} = (a_{ij})_{n \times n}$ represents a matrix of $n \times n$ -dimension. $\mathcal{A}^s = \frac{1}{2}(\mathcal{A} + \mathcal{A}^T)$. $\mathcal{A} \geq 0$ ($\mathcal{A} \leq 0$) implies $\mathcal{A}$ is symmetric and semi-positive (semi-negative) definite matrix. $\|\cdot\|$ is the 2-norm of a vector or a matrix. I_N stands for N -dimension identity matrix. $\text{sign}(\cdot)$ is sign function and $\text{sign}(s) = (\text{sign}(s_1), \text{sign}(s_2), \dots, \text{sign}(s_n))^T$ for $s = (s_1, s_2, \dots, s_n)^T \in \mathbb{R}^n$.

2 Preliminaries

In this section, we will formulate the considered problem in this paper and give some useful mathematical preliminaries.

This paper investigates the following coupled delayed neural networks:

$$\dot{z}_i(t) = -Cz_i(t) + Af(z_i(t)) + Bf(z_i(t - \tau)) - \sigma \sum_{j=1}^N |\ell_{ij}|(z_i(t) - \text{sign}(\ell_{ij})z_j(t)) + u_i(t), \quad (1)$$

where $i \in \mathcal{N} = \{1, 2, \dots, N\}$, $z_i(t) = (z_{i1}(t), z_{i2}(t), \dots, z_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the i th node; $C = \text{diag}(c_1, c_2, \dots, c_n) > 0$, and $A = (a_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$, $B = (b_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$ represent the weight and delayed weight matrices for the i th neural-network node, respectively; $f(z_i(t)) = (f_1(z_{i1}(t)), f_2(z_{i2}(t)), \dots, f_n(z_{in}(t)))^T \in \mathbb{R}^n$ is continuous vector function, $f_j(\cdot)$, $j = 1, 2, \dots, n$ is odd function, that is $f_j(-y) = -f_j(y)$, $\tau \geq 0$ is the node-delay; the constant $\sigma > 0$ is the coupling strength, $\mathcal{L} = (\ell_{ij})_{N \times N}$ is the adjacency matrix associated with the signed graph G of the network (1), satisfying $\ell_{ii} = 0$ for $i \in \mathcal{N}$; $\ell_{ij} \neq 0$ ($i \neq j$) if there is

a directed communication link from nodes j to i , otherwise $\ell_{ij} = 0$. If $\ell_{ij} > 0$, the coupling term is presented by $\ell_{ij}(z_i(t) - z_j(t))$, in this case, the interaction between nodes i and j is cooperative; if $\ell_{ij} < 0$, the coupling term becomes $-\ell_{ij}(z_i(t) + z_j(t))$ implying that the nodes i and j are competitive. $u_i(t)$ is a controller that we will design later. $z_i(0) \in \mathbb{R}^n$, $i \in \mathcal{N}$ is the initial value of system.

The goal node of network (1) is presented by

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \tau)), \quad (2)$$

where $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in \mathbb{R}^n$. The other parameters are same as those in system (1).

To derive the main results of this paper, we will make the following assumptions:

Assumption 1 The signed graph G of network(1) is structurally balanced if it admits a bipartition of the nodes $\{V_1, V_2\}$, $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = V$, such that $\ell_{ij} \geq 0$ for $v_i, v_j \in V_k$ ($k \in \{1, 2\}$) and $\ell_{ij} \leq 0$ for $v_i \in V_k, v_j \in V_l$ ($k \neq l, k, l \in \{1, 2\}$), where $V = \{v_1, v_2, \dots, v_n\}$ denotes a set which concludes all the nodes of the signed graph G .

Assumption 2 In network (1), Nonlinearity $f(\cdot)$ is an odd function satisfied a Lipschitz condition, that is, there exists an unknown constant η such that

$$\|f(z) - f(y)\| \leq \eta \|z - y\|, \forall z, y \in \mathbb{R}^n.$$

Under Assumption (1), define a diagonal matrix $W = \text{diag}(\omega_1, \omega_2, \dots, \omega_N)$, where $\omega_i \in \{-1, 1\}$. In this paper, if $v_i \in V_1$, $\omega_i = 1$, otherwise $\omega_i = -1$, for $i \in \mathcal{N}$. Next, the definition of the fixed-time bipartite synchronization is proposed.

Definition 1 The network (1) is said to achieve fixed-time bipartite synchronization with (2) if there exists a constant $t_1 > 0$ (t_1 is independent of $z(0) = (z_1(0), z_2(0)), \dots, z_N(0))^T$ and y_0) such that

$$\lim_{t \rightarrow t_1} \|z_i(t) - \omega_i y(t)\| = 0 \quad \text{and} \quad \|z_i(t) - \omega_i y(t)\| \equiv 0 \quad \text{for} \quad t > t_1, i \in \mathcal{N}.$$

There is no doubt that when $W = I_N$, the fixed-time bipartite synchronization reduces to the traditional fixed-time synchronization. Therefore, this paper can improve some existing fixed-time synchronization.

Lemma 1 (Young's inequality) Suppose a, b, u, v are all positive scalars, and $\frac{1}{u} + \frac{1}{v} = 1$, $u > 1$, $v > 1$, then

$$ab \leq \frac{a^u}{u} + \frac{b^v}{v},$$

where “=” holds if and only if $a^u + b^v = 1$.

Lemma 2 [32] Consider the following differential equation:

$$\dot{y} = -\alpha y^{\frac{m}{n}} - \beta y^{\frac{p}{q}}, \quad (3)$$

where $\alpha > 0, \beta > 0$ and m, n, p, q are positive odd integers, $m > n, p < q$. Then the system (3) is fixed time stable

$$T_1 < \frac{1}{\alpha} \frac{n}{m-n} + \frac{1}{\beta} \frac{q}{q-p}.$$

Lemma 3 [31] Consider the following differential equation:

$$\dot{y} = -\alpha y^{1+\frac{m}{n} \text{sign}(|y|)-1} - \beta y^{\frac{p}{q}}, \quad (4)$$

where $\alpha > 0, \beta > 0$ and m, n, p, q are positive odd integers, $m > n, p < q$. Then the system (4) is fixed time stable

$$T_2 < \frac{1}{\alpha} \frac{n}{m} + \frac{q}{q-p} \frac{1}{\alpha} \ln(1 + \frac{\alpha}{\beta}).$$

Remark 1 Since the inequalities $\frac{n}{m} < \frac{n}{m-n}$ and $\ln(1 + \frac{\alpha}{\beta}) \leq (\alpha/\beta)$ hold, then the inequality $\frac{1}{\alpha} \frac{n}{m} + \frac{q}{q-p} \frac{1}{\alpha} \ln(1 + \frac{\alpha}{\beta}) < \frac{1}{\alpha} \frac{n}{m-n} + \frac{1}{\beta} \frac{q}{q-p}$. So $T_2 < T_1$, that is, the fixed-time stable system presented in [31] achieves faster convergence than Lemma 4 in [32].

Lemma 4 [33] Let $\eta_1, \eta_2, \dots, \eta_N \geq 0$, $0 < \alpha \leq 1$, $\beta > 1$. Then, one can derive

$$\sum_{i=1}^N \eta_i^\alpha \geq \left(\sum_{i=1}^N \eta_i \right)^\alpha, \quad \sum_{i=1}^N \eta_i^\beta \geq N^{1-\beta} \left(\sum_{i=1}^N \eta_i \right)^\beta.$$

Lemma 5 [34] Let $a_1, a_2, \dots, a_N \geq 0$ and $0 < r < p$. Then

$$\left(\sum_{i=1}^N a_i^p \right)^{1/p} \leq \left(\sum_{i=1}^N a_i^r \right)^{1/r}.$$

3 Fixed-time bipartite synchronization of the delayed neural networks

In this section, we focus on studying the fixed-time bipartite synchronization of complex networks by constructing comparison system. Under Assumption 1, we design the following controller $u_i(t)$ to achieve bipartite synchronization.

$$u_i(t) = -\lambda_i(z_i(t) - \omega_i y(t)) - \theta_i \frac{z_i(t) - \omega_i y(t)}{\|z_i(t) - \omega_i y(t)\|^2} (z_i(t - \tau) - \omega_i y(t - \tau))^T (z_i(t - \tau) - \omega_i y(t - \tau)) \\ - \gamma_1 (z_i(t) - \omega_i y(t))^{1 + \frac{c_1}{c_2} \text{sign}(\Delta(t) - 1)} - \gamma_2 (z_i(t) - \omega_i y(t))^{\frac{c_3}{c_4}}, \quad (5)$$

where $\lambda_i, \theta_i > 0$ is constant to be determined, $\gamma_1 > 0$ and $\gamma_2 > 0$ are tunable constants. $c_1 > c_2 > 0$ and $0 < c_3 < c_4$ are odd integers, $\Delta(t) = \sum_{i=1}^N (z_i(t) - \omega_i y(t))^T (z_i(t) - \omega_i y(t))$.

Remark 2 In the designed controller, ω_i is brought to overcome the difficulties induced by signed graphs.

Let L denote the Laplacian matrix of signed graph G , then we have $l_{ij} = -\ell_{ij}$, $i \neq j$ and $l_{ii} = \sum_{j=1, j \neq i}^N |\ell_{ij}|$.

Clearly, if there is a $\ell_{ij} < 0$, L can not guarantee the sum of row is zero, which means the Laplacian matrix is different from this generated by unsigned graph. Furthermore, one can derive that

$$\dot{z}_i(t) = -Cz_i(t) + Af(z_i(t)) + Bf(z_i(t - \tau)) - \sigma \sum_{j=1}^N l_{ij} z_j(t) + u_i(t). \quad (6)$$

Now we define $\hat{\mathcal{L}} = (\hat{\ell}_{ij})_{N \times N} = (|\ell_{ij}|)_{N \times N}$. Let $\hat{G}$ and $\hat{L} = (\hat{l}_{ij})_{N \times N}$ represent the graph and Laplacian matrix associated with $\hat{\mathcal{L}}$. Obviously, $\hat{G}$ is an unsigned graph and $\hat{L}$ is a zero-row-sum matrix. Take $\hat{z}_i(t) = \omega_i z_i(t)$, $i \in \mathcal{N}$, then one can obtain $\omega_i f(z_i(t)) = f(\hat{z}_i(t))$ and by considering the fact that $f_j(\cdot)$ is odd function. Furthermore, we can generate that

$$\dot{\hat{z}}_i(t) = \begin{cases} -C\hat{z}_i(t) + Af(\hat{z}_i(t)) + Bf(\hat{z}_i(t - \tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} \hat{z}_j(t) - \lambda_i(\hat{z}_i(t) - y(t)) \\ \quad - \theta_i \frac{\hat{z}_i(t) - y(t)}{\|\hat{z}_i(t) - y(t)\|^2} (\hat{z}_i(t - \tau) - y(t - \tau))^T (\hat{z}_i(t - \tau) - y(t - \tau)) \\ \quad - \gamma_1 (\hat{z}_i(t) - y(t))^{1 + \frac{c_1}{c_2}} - \gamma_2 (\hat{z}_i(t) - y(t))^{\frac{c_3}{c_4}} & \Delta(t) \geq 1, \\ -C\hat{z}_i(t) + Af(\hat{z}_i(t)) + Bf(\hat{z}_i(t - \tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} \hat{z}_j(t) - \lambda_i(\hat{z}_i(t) - y(t)) \\ \quad - \theta_i \frac{\hat{z}_i(t) - y(t)}{\|\hat{z}_i(t) - y(t)\|^2} (\hat{z}_i(t - \tau) - y(t - \tau))^T (\hat{z}_i(t - \tau) - y(t - \tau)) \\ \quad - \gamma_1 (\hat{z}_i(t) - y(t))^{1 - \frac{c_1}{c_2}} - \gamma_2 (\hat{z}_i(t) - y(t))^{\frac{c_3}{c_4}} & \Delta(t) < 1. \end{cases} \quad (7)$$

Let $e_i(t) = \hat{z}_i(t) - y(t)$, $f(e_i(t)) = f(\hat{z}_i(t)) - f(y(t))$, then we obtain

$$\dot{e}_i(t) = -Ce_i(t) + Af(e_i(t)) + Bf(e_i(t - \tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} e_j(t) - \lambda_i e_i(t) \\ - \theta_i \frac{e_i(t)}{\|e_i(t)\|^2} e_i^T(t - \tau) e_i(t - \tau) - \gamma_1 e_i^{1 + \frac{c_1}{c_2} \text{sign}(\Delta(t) - 1)}(t) - \gamma_2 e_i^{\frac{c_3}{c_4}}(t) \quad (8)$$

That is

$$\dot{e}_i(t) = \begin{cases} -Ce_i(t) + Af(e_i(t)) + Bf(e_i(t - \tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} e_j(t) - \lambda_i e_i(t) \\ \quad - \theta_i \frac{e_i(t)}{\|e_i(t)\|^2} e_i^T(t - \tau) e_i(t - \tau) - \gamma_1 e_i^{1 + \frac{c_1}{c_2}}(t) - \gamma_2 e_i^{\frac{c_3}{c_4}}(t) & \Delta(t) \geq 1, \\ -Ce_i(t) + Af(e_i(t)) + Bf(e_i(t - \tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} e_j(t) - \lambda_i e_i(t) \\ \quad - \theta_i \frac{e_i(t)}{\|e_i(t)\|^2} e_i^T(t - \tau) e_i(t - \tau) - \gamma_1 e_i^{1 - \frac{c_1}{c_2}}(t) - \gamma_2 e_i^{\frac{c_3}{c_4}}(t) & \Delta(t) < 1. \end{cases} \quad (9)$$

Therefore, if there exists a $T > 0$ such that $e_i(T) = 0$ and $e_i(t) \equiv 0$ for $t \geq T$, then according to $\omega_i^2 = 1$, it follows that $\lim_{t \rightarrow T} \|z_i(t) - \omega_i y(t)\| = 0$ and $\|z_i(t) - \omega_i y(t)\| \equiv 0$, for $t > T$. This implies the fast fixed-time bipartite synchronization is realized.

With the help of controller (5), the following theorem presents some conditions for achieving fast fixed-time bipartite synchronization in the signed network (1).

Theorem 1 Let Assumptions 1 and 2 hold, Under the control protocol defined by (5), if the control gains λ_i, θ_i satisfy the following conditions

$$\Lambda \geq \|C\|I_N + \eta\|A\|I_N + \frac{1}{2}\eta^2\|B\|^2I_N + \sigma\Phi^s, \quad (10)$$

$$\Theta \geq \frac{1}{2}I_N, \quad (11)$$

where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$, $\Theta = \text{diag}(\theta_1, \theta_2, \dots, \theta_N)$, $\Phi = (\phi_{ij})_{N \times N}$ and $\phi_{ii} = -\hat{l}_{ii}$, $\phi_{ij} = |\hat{l}_{ij}|$ for $i, j \in \mathcal{N}$ and $j \neq i$. Then the delayed neural network (1) can achieve fast fixed-time bipartite synchronization with (2). Moreover, the settling time is estimated as

$$T = \frac{1}{\varrho} \frac{c_1}{c_2} + \frac{c_4}{c_4 - c_3} \frac{1}{\varrho} \ln(1 + \frac{\varrho}{\gamma_2}), \quad (12)$$

where $\varrho = \gamma_1(nN)^{-\frac{c_1}{2c_2}}$.

Proof Consider the following Lyapunov function:

$$V(t) = \sum_{i=1}^N e_i^T(t) e_i(t).$$

By calculating $\dot{V}(t)$ along the trajectory of system (8) as follows:

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) [-Ce_i(t) + Af(e_i(t)) + Bf(e_i(t-\tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} e_j(t) - \lambda_i e_i(t) \\ & - \theta_i \frac{e_i(t)}{\|e_i(t)\|^2} e_i^T(t-\tau) e_i(t-\tau) - \gamma_1 e_i^{1+\frac{c_1}{c_2} \text{sign}(\Delta(t)-1)}(t) - \gamma_2 e_i^{\frac{c_3}{c_4}}(t)] \end{aligned} \quad (13)$$

From Assumption 2, we can derive

$$\begin{aligned} e_i^T(t) Af(e_i(t)) &\leq \eta\|A\| \cdot \|e_i(t)\|^2 \\ e_i^T(t) Bf(e_i(t-\tau)) &\leq \eta\|B\| \cdot \|e_i^T(t)\| \cdot \|e_i(t-\tau)\| \end{aligned} \quad (14)$$

Considering Lemma 1, one gets

$$\begin{aligned} e_i^T(t) Bf(e_i(t-\tau)) &\leq \eta\|B\| \cdot \|e_i^T(t)\| \cdot \|e_i(t-\tau)\| \\ &\leq \frac{1}{2}\eta^2\|B\|^2 \cdot \|e_i(t)\|^2 + \frac{1}{2}\|e_i(t-\tau)\|^2 \end{aligned} \quad (15)$$

Substituting inequalities (14)-(15) into equality (13), it is easy to obtain that

$$\begin{aligned} \dot{V}(t) = & 2 \sum_{i=1}^N e_i^T(t) [-Ce_i(t) + Af(e_i(t)) + Bf(e_i(t-\tau)) - \sigma \sum_{j=1}^N \hat{l}_{ij} e_j(t) - \lambda_i e_i(t) \\ & - \theta_i \frac{e_i(t)}{\|e_i(t)\|^2} e_i^T(t-\tau) e_i(t-\tau) - \gamma_1 e_i^{1+\frac{c_1}{c_2} \text{sign}(\Delta(t)-1)}(t) - \gamma_2 e_i^{\frac{c_3}{c_4}}(t)] \\ \leq & 2 \sum_{i=1}^N [\|C\| \cdot \|e_i(t)\|^2 + \eta\|A\| \cdot \|e_i(t)\|^2 + \frac{1}{2}\eta^2\|B\|^2 \cdot \|e_i(t)\|^2 + \frac{1}{2}\|e_i(t-\tau)\|^2 - \sigma \hat{l}_{ii} \|e_i(t)\|^2 \\ & + \sigma \sum_{j=1, j \neq i}^N |\hat{l}_{ij}| \|e_i(t)\| \cdot \|e_j(t)\| - \lambda_i \|e_i(t)\|^2 - \theta_i e_i^T(t-\tau) e_i(t-\tau)] - 2\Gamma(t) \\ = & 2\hat{e}^T(t) [\|C\|I_N + \eta\|A\|I_N + \frac{1}{2}\eta^2\|B\|^2I_N + \sigma\Phi - \Lambda] \hat{e}(t) \\ & + 2\hat{e}^T(t-\tau) [\frac{1}{2}I_N - \Theta] \hat{e}(t-\tau) - 2\Gamma(t) \\ = & 2\hat{e}^T(t) [\|C\|I_N + \eta\|A\|I_N + \frac{1}{2}\eta^2\|B\|^2I_N + \sigma\Phi^s - \Lambda] \hat{e}(t) \\ & + 2\hat{e}^T(t-\tau) [\frac{1}{2}I_N - \Theta] \hat{e}(t-\tau) - 2\Gamma(t), \end{aligned} \quad (16)$$

where $\hat{e}(t) = (\|e_1(t)\|, \|e_2(t)\|, \dots, \|e_N(t)\|)^T$, $\hat{e}(t - \tau) = (\|e_1(t - \tau)\|, \|e_2(t - \tau)\|, \dots, \|e_N(t - \tau)\|)^T$, $\Gamma(t) = \gamma_1 \sum_{i=1}^N \sum_{j=1}^n e_{ij}^{2+\frac{c_1}{c_2} \text{sign}(\Delta(t)-1)}(t) + \gamma_2 \sum_{i=1}^N \sum_{j=1}^n e_i^{1+\frac{c_3}{c_4}}(t)$.

Next, $\Gamma(t)$ will be separately discussed for two cases.

Case 1: when $\sum_{i=1}^N \|e_i(t)\|^2 \geq 1$, i.e. $V(t) \geq 1$, by Lemma 4, it generates

$$\begin{aligned} \Gamma(t) &= \gamma_1 \sum_{i=1}^N \sum_{j=1}^n e_{ij}^{2+\frac{c_1}{c_2}}(t) + \gamma_2 \sum_{i=1}^N \sum_{j=1}^n e_i^{1+\frac{c_3}{c_4}}(t), \\ &\geq \gamma_1 (nN)^{-\frac{c_1}{2c_2}} \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1+\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, \end{aligned} \quad (17)$$

Case 2: when $\sum_{i=1}^N \|e_i(t)\|^2 < 1$, i.e. $V(t) < 1$, from Lemma 4 and Lemma 5, one derives

$$\begin{aligned} \Gamma(t) &= \gamma_1 \sum_{i=1}^N \sum_{j=1}^n e_{ij}^{2-\frac{c_1}{c_2}}(t) + \gamma_2 \sum_{i=1}^N \sum_{j=1}^n e_i^{1+\frac{c_3}{c_4}}(t), \\ &\geq \gamma_1 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1-\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, \end{aligned} \quad (18)$$

That is

$$\begin{aligned} \Gamma(t) &\geq \begin{cases} \gamma_1 (nN)^{-\frac{c_1}{2c_2}} \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1+\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, & \Delta(t) \geq 1, \\ \gamma_1 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1-\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, & \Delta(t) < 1, \end{cases} \\ &\geq \begin{cases} \gamma_1 (nN)^{-\frac{c_1}{2c_2}} \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1+\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, & \Delta(t) \geq 1, \\ \gamma_1 (nN)^{-\frac{c_1}{2c_2}} \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{1-\frac{c_1}{2c_2}} + \gamma_2 \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{\frac{c_3+c_4}{2c_4}}, & \Delta(t) < 1, \end{cases} \end{aligned} \quad (19)$$

Note that $V(t) = \Delta(t)$, by the conditions (10) and (11), inequalities (17)–(19), it yields

$$\begin{aligned} \dot{V}(t) &\leq -2\Gamma(t) \\ &\leq \begin{cases} -2\gamma_1 (nN)^{-\frac{c_1}{2c_2}} V^{1+\frac{c_1}{2c_2}}(t) - 2\gamma_2 V^{\frac{c_3+c_4}{2c_4}}(t), & V(t) \geq 1, \\ -2\gamma_1 (nN)^{-\frac{c_1}{2c_2}} V^{1-\frac{c_1}{2c_2}}(t) - 2\gamma_2 V^{\frac{c_3+c_4}{2c_4}}(t), & V(t) < 1, \end{cases} \end{aligned}$$

By using comparison analysis method, we establish the following system for comparison purpose:

$$\begin{cases} \dot{\nu}(t) = \begin{cases} -2\gamma_1 (nN)^{-\frac{c_1}{2c_2}} \nu^{1+\frac{c_1}{2c_2}}(t) - 2\gamma_2 \nu^{\frac{c_3+c_4}{2c_4}}(t), & \nu(t) \geq 1, \\ -2\gamma_1 (nN)^{-\frac{c_1}{2c_2}} \nu^{1-\frac{c_1}{2c_2}}(t) - 2\gamma_2 \nu^{\frac{c_3+c_4}{2c_4}}(t), & 0 < \nu(t) < 1, \\ 0, & \nu(t) = 0, \end{cases} \\ \nu(0) = \sum_{i=1}^N e_i^T(0) e_i(0). \end{cases}$$

Obviously, if there is a constant T such that $\nu(t) \equiv 0$, for any $t \geq T$, then it also hold $V(t) \equiv 0$, for any $t \geq T$.

According to the similar analysis of Lemma 1 in [30] and Theorem 1 in [31], let $\chi(t) = \nu^{1-\frac{c_3+c_4}{2c_4}}(t)$, we can get $\frac{2c_4}{c_4-c_3} \dot{\chi}(t) \nu^{\frac{c_3+c_4}{2c_4}}(t) = \dot{\nu}(t)$, then

$$\dot{\chi}(t) + \frac{c_4 - c_3}{c_4} \varrho \chi^{\frac{c_4}{c_4-c_3}} (1 + \frac{c_1}{c_2} - \frac{c_3}{c_4})(t) + \frac{c_4 - c_3}{c_4} \gamma_2 = 0, \quad \chi(t) \geq 1$$

and

$$\dot{\chi}(t) + \frac{c_4 - c_3}{c_4} \varrho \chi^{1-\frac{c_4}{c_4-c_3} \frac{c_1}{c_2}}(t) + \frac{c_4 - c_3}{c_4} \gamma_2 = 0, \quad \chi(t) < 1$$

By use the similar calculation with Lemma 1 in [30] and Theorem 1 in [31], we will obtain the estimation of settling time is

$$T = \frac{1}{\varrho} \frac{c_1}{c_2} + \frac{c_4}{c_4 - c_3} \frac{1}{\varrho} \ln(1 + \frac{\varrho}{\gamma_2}),$$

where $\varrho = \gamma_1(nN)^{-\frac{c_1}{2c_2}}$.

Therefore, the control algorithm (5) solves the bipartite synchronization problem of signed network (1). that is, the coupled delayed neural network (1) is fixed-time bipartite synchronization with system (2) in T , which can be described by (1). The proof is completed.

Remark 3 The results reported in [25–29] have been discussed the fixed-time control problems, the convergence rate in this paper is faster than most existing fixed-time results since $\frac{1}{\alpha} \frac{n}{m} + \frac{q}{q-p} \frac{1}{\alpha} \ln(1 + \frac{\alpha}{\beta}) < \frac{1}{\alpha} \frac{n}{m-n} + \frac{1}{\beta} \frac{q}{q-p}$, which is presented in [30,31]. In addition, the estimation of settling time (12) is also more accurate than the previous works in [25–29], where the settling time of fixed-time synchronization is independent of system's initial value and it is related to parameters of system and controllers.

Remark 4 In the previous investigation, the fast fixed-time synchronization of delayed neural networks have been discussed in [31], but this paper have not considered the fast fixed-time bipartite synchronization of delayed neural network with signed graphs. In addition, although the authors have considered the fast fixed-time synchronization in [25, 26], but the fast fixed-time bipartite synchronization problem is much more general, which is still challenging.

In the following, we will discuss some special cases by applying Theorem 1.

When there is no delays in neural networks, system (1) reduces to

$$\dot{z}_i(t) = -Cz_i(t) + Af(z_i(t)) - \sigma \sum_{j=1}^N |\ell_{ij}|(z_i(t) - \text{sign}(\ell_{ij})z_j(t)) + u_i(t), \quad (20)$$

The goal node of network (20) is presented by

$$\dot{y}(t) = -Cy(t) + Af(y(t)), \quad (21)$$

where $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T \in \mathbb{R}^n$. The definitions of other parameters are similar with system (1).

Then the control law $u_i(t)$ is given

$$u_i(t) = -\tilde{\lambda}_i(z_i(t) - \omega_i y(t)) - \tilde{\gamma}_1(z_i(t) - \omega_i y(t))^{1+\frac{\tilde{c}_1}{\tilde{c}_2} \text{sign}(\Delta(t)-1)} - \tilde{\gamma}_2(z_i(t) - \omega_i y(t))^{\frac{\tilde{c}_3}{\tilde{c}_4}}, \quad (22)$$

We can obtain the following corollary by applying the similar analysis from Theorem 1.

Corollary 1 Let Assumptions 1 and 2 hold, Under the control protocol defined by (22), if the control gain $\tilde{\lambda}_i$ satisfy the following conditions

$$\Lambda \geq \|C\|I_N + \eta\|A\|I_N + \sigma\Phi^s, \quad (23)$$

where the definitions of corresponding parameters are similar with Theorem 1. Then the neural network (20) can achieve fast fixed-time bipartite synchronization with (21). Moreover, the settling time is estimated as

$$\tilde{T} = \frac{1}{\tilde{\varrho}} \frac{\tilde{c}_1}{\tilde{c}_2} + \frac{\tilde{c}_4}{\tilde{c}_4 - \tilde{c}_3} \frac{1}{\tilde{\varrho}} \ln(1 + \frac{\tilde{\varrho}}{\tilde{\gamma}_2}), \quad (24)$$

where $\tilde{\varrho} = \tilde{\gamma}_1(nN)^{-\frac{\tilde{c}_1}{2\tilde{c}_2}}$.

Proof Consider the following Lyapunov function:

$$V(t) = \sum_{i=1}^N e_i^T(t) e_i(t).$$

One can derive that

$$\begin{aligned} \dot{V}(t) &\leq 2\hat{e}^T(t) [\|C\|I_N + \eta\|A\|I_N + \sigma\Phi^s - \Lambda] \hat{e}(t) \\ &\quad - 2\tilde{\gamma}_1 \sum_{i=1}^N \sum_{j=1}^n e_{ij}^{2+\frac{\tilde{c}_1}{\tilde{c}_2} \text{sign}(\Delta(t)-1)}(t) - 2\tilde{\gamma}_2 \sum_{i=1}^N \sum_{j=1}^n e_i^{1+\frac{\tilde{c}_3}{\tilde{c}_4}}(t) \end{aligned} \quad (25)$$

According to the similar analysis with Theorem 1, we can construct the following comparison system

$$\begin{cases} \dot{\tilde{\nu}}(t) = \begin{cases} -2\tilde{\gamma}_1(nN)^{-\frac{\tilde{c}_1}{2\tilde{c}_2}}\tilde{\nu}^{1+\frac{\tilde{c}_1}{2\tilde{c}_2}}(t) - 2\tilde{\gamma}_2\tilde{\nu}^{\frac{\tilde{c}_3+\tilde{c}_4}{2\tilde{c}_4}}(t), & \tilde{\nu}(t) \geq 1, \\ -2\tilde{\gamma}_1(nN)^{-\frac{\tilde{c}_1}{2\tilde{c}_2}}\tilde{\nu}^{1-\frac{\tilde{c}_1}{2\tilde{c}_2}}(t) - 2\tilde{\gamma}_2\tilde{\nu}^{\frac{\tilde{c}_3+\tilde{c}_4}{2\tilde{c}_4}}(t), & 0 < \tilde{\nu}(t) < 1, \\ 0, & \tilde{\nu}(t) = 0, \end{cases} \\ \tilde{\nu}(0) = \sum_{i=1}^N e_i^T(0)e_i(0). \end{cases}$$

Using the similar discussion with Theorem 1, if the constant $\tilde{T}$ satisfies $\tilde{\nu}(t) \equiv 0$, for any $t \geq \tilde{T}$, then it also hold $V(t) \equiv 0$, for any $t \geq \tilde{T}$. Therefore, there is no difficult to derive that when

$$t \geq \frac{1}{\tilde{\varrho}} \frac{\tilde{c}_1}{\tilde{c}_2} + \frac{\tilde{c}_4}{\tilde{c}_4 - \tilde{c}_3} \frac{1}{\tilde{\varrho}} \ln\left(1 + \frac{\tilde{\varrho}}{\tilde{\gamma}_2}\right).$$

Therefore, the control algorithm (22) solves the bipartite synchronization problem of signed network (20). that is, the coupled neural network (20) is fixed-time bipartite synchronization with system (21) in $\tilde{T}$.

Remark 5 From the controllers (5) and (22), one can see that $-\theta_i \frac{z_i(t) - \omega_i y(t)}{\|z_i(t) - \omega_i y(t)\|^2} (z_i(t - \tau) - \omega_i y(t - \tau))^T (z_i(t - \tau) - \omega_i y(t - \tau))$ is used to deal with the delays introduced in (1).

4 Numerical Simulations

In this section, two numerical examples will be proposed to demonstrate the correctness of the bipartite synchronization criteria. Considering signed network (1) including nine nodes, which is shown in Fig. 1. It can be seen from Fig. 1 that the topology of the signed network (1) is structurally balanced by taking $V_1 = \{1, 2, \dots, 5\}$ and $V_2 = \{6, 7, 8, 9\}$. Hence, Assumption 1 is satisfied with $V = \text{diag}\{1, 1, 1, 1, 1, -1, -1, -1, -1\}$. From Fig. 1, one can get its adjacency matrix is

Fig. 1 The considered signed graph with nine nodes.

$$\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Example 1. Consider a single delayed neural network is described as

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \tau)), \quad (26)$$

where $y(t) = (y_1(t), y_2(t))^T$, $f(y(t)) = (\tanh(y_1(t)), \tanh(y_2(t)))^T$, the parameters are given as follows:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & -0.1 \\ -5 & 1.5 \end{bmatrix}, B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1 \end{bmatrix},$$

Fig. 2 Chaotic trajectory of system (26) with $y(0) = (0.5, 0.6)^T$.

Obviously, $f(\cdot)$ is odd function, Assumption 2 is satisfied with $\eta = 1$. To illustrate the effectiveness of the proposed method, let node-delay $\tau = 1$ and the initial value is $y(0) = (0.5, 0.6)^T$, Fig. 2 shows the the chaotic trajectory of goal node (2).

The following neural network with signed graphs which are coupled by chaotic system (26) described by

$$\dot{z}_i(t) = -Cz_i(t) + Af(z_i(t)) + Bf(z_i(t - \tau)) - \sigma \sum_{j=1}^9 |\ell_{ij}|(z_i(t) - \text{sign}(\ell_{ij})z_j(t)), i = 1, 2, \dots, 9. \quad (27)$$

According to Theorem 1, one can get that if $\lambda_i \geq \|C\| + \eta\|A\| + \frac{1}{2}\eta^2\|B\|^2 + \sigma\lambda_{\max}(\Phi^s)$, $\theta_{\min} \geq \frac{1}{2}$, where $\lambda_{\max}$ is the maximum eigenvalue of Φ^s , $\theta_{\min} = \min\{\theta_1, \theta_2, \dots, \theta_N\}$, $i = 1, 2, \dots, 9$, then the equalities (10) and (11) can be satisfied.

The values of the parameters of the fixed-time control laws are designed by $\lambda_i = 8$, $\theta_i = 1$, $\gamma_1 = 10$, $\gamma_2 = 5$, $c_1 = 5$, $c_2 = 3$, $c_3 = 1$, $c_4 = 3$, and the coupling strength $\sigma = 0.2$ which satisfy the conditions in Theorem 1. By simple computation, we can get the settling time is about 1.2176. Obviously, the settling time is less than the theoretical estimate in this control protocol as illustrated in Fig. 3. The trajectories of $y(t)$ and $z_i(t)$, $i = 1, 2, \dots, 9$ are presented in Fig. 3, where the red line denotes the trajectories of $y_1(t)$, $y_2(t)$, and the blue line represents the trajectories of $z_{i1}(t)$, $z_{i2}(t)$, from which it can be observed that the fixed-time bipartite synchronization can be achieved for the delayed neural network, which verifies the theoretical result in Theorem 1 well.

Fig. 3 State trajectories of systems (26) and (27) under the controller (5).

Example 2. Consider the following single delayed neural networks:

$$\dot{y}(t) = -Cy(t) + Af(y(t)) + Bf(y(t - \tau)), \quad (28)$$

where $y(t) = (y_1(t), y_2(t))^T$,

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & 1.5 \\ 0.1 & 1.8 \end{bmatrix}, B = \begin{bmatrix} -1.5 & 0.1 \\ 0.1 & -1.3 \end{bmatrix},$$

The activation functions are assumed to be

$$f(y(t)) = 0.5(|y(t) + 1| - |y(t) - 1|). \quad (29)$$

Obviously, the function $f(\cdot)$ satisfies Assumption 2 with $\eta = 0.5$. Similarly, the chaotic trajectory of system (28) with $\tau = 1$ and the initial value $y(0) = (0.2, 0.1)^T$ is shown in Fig. 4.

Fig. 4 Chaotic trajectory of system (28) with $y(0) = (0.2, 0.1)^T$.

Considering the following network with signed graphs which are coupled by chaotic system (28) described by

$$\dot{z}_i(t) = -Cz_i(t) + Af(z_i(t)) + Bf(z_i(t - \tau)) - \sigma \sum_{j=1}^9 |\ell_{ij}|(z_i(t) - \text{sign}(\ell_{ij})z_j(t)), i = 1, 2, \dots, 9. \quad (30)$$

In this example, if $\lambda_i \geq \|C\| + \eta\|A\| + \frac{1}{2}\eta^2\|B\|^2 + \sigma\lambda_{\max}(\Phi^s)$, $\theta_{\min} \geq \frac{1}{2}$, where $\lambda_{\max}$ is the maximum eigenvalue of Φ^s , $\theta_{\min} = \min\{\theta_1, \theta_2, \dots, \theta_N\}$, $i = 1, 2, \dots, 9$, then the Theorem 1 can be satisfied.

Fig. 5 State trajectories of systems (28) and (30) under the controller (5).

For numerical simulation, the parameters of the controller are selected by $\lambda_i = 4$, $\theta_i = 0.5$, $\gamma_1 = 20$, $\gamma_2 = 10$, $c_1 = 5$, $c_2 = 3$, $c_3 = 1$, $c_4 = 3$, and the coupling strength $\sigma = 0.2$, which satisfy the conditions in Theorem 1. By simple computation, we can get the settling time is about 0.5853. Obviously, the settling time is less than the theoretical estimate in this control protocol as illustrated in Fig. 5. The trajectories of $y(t)$ and $z_i(t)$, $i = 1, 2, \dots, 9$ are presented in Fig. 5, where the red line denotes the trajectories of $y_1(t)$, $y_2(t)$, and the blue line represents the trajectories of $z_{i1}(t)$, $z_{i2}(t)$, from which it can be observed that the fixed-time bipartite synchronization can be achieved for the delayed neural network, which verifies the theoretical result in Theorem 1 well.

5 Conclusions

In this paper, the fast fixed-time bipartite synchronization problem for delayed neural networks with signed graphs has been investigated. By using algebraic graph theory and Lyapunov theory, an efficacious controller has been designed to ensure the fixed-time bipartite synchronization. The settling time of fixed-time bipartite synchronization is proposed accurately, which is more faster convergence sufficient condition than the previous results [25–30]. Some numerical examples are given to validate the correctness of the our obtained theoretical results. Note that node-delays in neural networks are always affected by time-varying, it is expected to extend the presented method to time-varying delayed systems. In addition, impulsive effects always exist in complex networks, thus the bipartite synchronization of complex networks under impulsive control will be considered in our future works.

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Declarations

Ethical Approval This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of Interest The authors declare that they have no conflict of interest.

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