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Quantum harmonic free energies for biomolecules and nanomaterials

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Abstract

Obtaining the free energy of large molecules from quantum mechanical energy functions is a longstanding challenge. We describe a method that allows us to estimate, at the quantum mechanical level, the harmonic contributions to the thermodynamics of molecular systems of unprecedented size, with modest cost. Using this approach, we compute the vibrational thermodynamics of a series of diamond nanocrystals, and show that the error per atom decreases with system size in the limit of large systems. We further show that we can obtain the vibrational contributions to the binding free energies of prototypical protein-ligand complexes where the exact computation is too expensive to be practical. Our work raises the possibility of routine quantum mechanical estimates of thermodynamic quantities in complex systems.

The contributions to the free energy from atomic motion are critically important to the thermodynamics and kinetics of biological, chemical, and materials systems. Changes in such contributions govern processes ranging from the affinity of drug binding to structural phase transitions in crystals. When the internal energy is computed at the quantum mechanical level, a harmonic approximation is often the only feasible option to describe atomic motion. However, for large systems such as nanostructures and biomolecules, computing free energy contributions is expensive even within the harmonic approximation. For a system of N atoms, the Hessian matrix which describes the vibrations requires $O(3N)$ gradient calculations, or $O(3N)$ times the cost of computing the internal energy. This is clearly prohibitive when N

is large, making free-energy computation in large systems with quantum mechanical methods a major contemporary challenge [1].

There are many possible strategies to speed up harmonic vibrational analysis, including methods based on a partial Hessian[2, 3], iterative diagonalization [4], and Hessian-free methods that use molecular dynamics to approximate the the harmonic problem[5, 6]. Here we describe a different strategy where we estimate vibrational thermodynamic quantities directly without ever computing the full Hessian or taking advantage of any local structure.

The starting point is to express each harmonic thermodynamic quantity as a matrix function trace. Then, our technique contains three elements. First, we sample the matrix trace operation using random vectors and stochastic Lanczos quadrature [7]. Second, we compute the Hessian-vector product at the same cost as the gradient from the difference of gradients at displaced geometries, bypassing the Hessian construction entirely. Third, we ameliorate the stochastic error, especially for free energy differences, through a form of correlated sampling. Related stochastic methods have been used for anharmonic corrections to the harmonic free energy[8, 9], as well as in stochastic electronic structure[10], but to our knowledge this is the first time these ideas have been brought to bear on the harmonic thermodynamic quantities themselves. As we demonstrate, this allows us to compute at the quantum mechanical level and with modest cost, vibrational free energy contributions for nanocrystals with more than 600 atoms, and free energy differences in protein-ligand complexes with more than 3000 atoms.

Theory

We first express the harmonic thermochemical quantities as traces of matrix functions. In particular, we are interested in the zero-point energy (ZPE),

$$\text{ZPE} = \sum_I \frac{\omega_I}{2} = \sum_I \frac{\sqrt{\omega_I^2}}{2} = \text{Tr} \left[\frac{\sqrt{\mathbf{D}}}{2} \right] \quad (1)$$

the thermal contribution to the enthalpy,

$$H_{\text{vib}} = \sum_I \omega_I \left(\frac{e^{-\beta\omega_I}}{1 - e^{-\beta\omega_I}} \right) = \text{Tr} \left[\frac{\sqrt{\mathbf{D}} \exp(-\beta\sqrt{\mathbf{D}})}{1 - \exp(-\beta\sqrt{\mathbf{D}})} \right] \quad (2)$$

and the entropy,

$$\begin{aligned} S_{\text{vib}}/k_B &= \sum_I \left[\beta\omega_I \frac{e^{-\beta\omega_I}}{1 - e^{-\beta\omega_I}} - \ln(1 - e^{-\beta\omega_I}) \right] \\ &= \text{Tr} \left[\beta \frac{\sqrt{\mathbf{D}} \exp(-\beta\sqrt{\mathbf{D}})}{1 - \exp(-\beta\sqrt{\mathbf{D}})} - \ln(1 - \exp(-\beta\sqrt{\mathbf{D}})) \right] \end{aligned} \quad (3)$$

58 Here β is the inverse temperature, $\{\omega_I\}$ is the set of normal mode frequencies, and
 59 \mathbf{D} is the mass-weighted Hessian matrix.

60 The above expressions have the form $\text{Tr} f(\mathbf{D})$. We now employ a stochastic esti-
 61 mator of the trace. The simplest version writes $\text{Tr} f(\mathbf{D}) \approx \frac{M}{n} \sum_{l=1}^n \mathbf{v}_l^T f(\mathbf{D}) \mathbf{v}_l$
 62 where \mathbf{v}_l are a set of n random vectors with zero mean and unit covariance, and
 63 $M = 3N - 6$ is the dimension of \mathbf{D} . This direct stochastic evaluation requires a
 64 polynomial approximation of $f(\mathbf{D})$, which is typically carried out using a Cheby-
 65 shev expansion [11]. A closely related idea, which we use in this work, is stochastic
 66 Lanczos quadrature[7]. In this technique, the polynomial approximation is generated
 67 for the scalar $\mathbf{v}_l^T f(\mathbf{D}) \mathbf{v}_l$ rather than globally for the function $f(\mathbf{D})$. We have found
 68 the stochastic Lanczos method to be slightly superior to the Chebyshev polynomial
 69 approach for the quantities in this work (see the supporting information for further
 70 comparison). Within the polynomial expansion, the main operation is the matrix-
 71 vector product $\mathbf{D}\mathbf{x}$, where \mathbf{x} is in the stochastic Lanczos space. This can be computed
 72 from the difference of gradients, displaced by $\delta\mathbf{x}$ in mass-weighted coordinates, for
 73 small δ . Thus no Hessian is needed at all in this approach (further details are provided
 74 in the Methods section).

75 Within the above scheme, there are two sources of error. The first is from the order
 76 of the Lanczos quadrature, m . This vanishes when m is greater than or equal to the
 77 matrix dimension. The second is the sampling error, which decreases like $1/\sqrt{n}$ for
 78 n random vectors. For a quadrature order of m and n samples, the cost of the method
 79 is equal to $O(mn)$ gradient calculations. To reduce the statistical error, we use a
 80 form of correlated sampling. For the absolute thermodynamic quantities computed
 81 for the diamond nanocrystals we employ a high-level quantum mechanical approach
 82 as well as a cheaper low-level method (for example, a force-field, or semi-empirical
 83 quantum-mechanical approach) where the exact computation of the harmonic ther-
 84 modynamic quantity X_{low} is possible. Then, the free energy contribution for the
 85 high-level method is obtained as

$$X_{\text{high}} = X_{\text{low}} + \Delta \quad (4)$$

86 where Δ is computed by applying the stochastic Lanczos quadrature to the difference
 87 of high-level and low-level methods. In the case of protein(P)-ligand(L) binding, we
 88 are interested in the difference between the holo (ligand-bound) state and the apo
 89 (ligand-free) state, i.e.

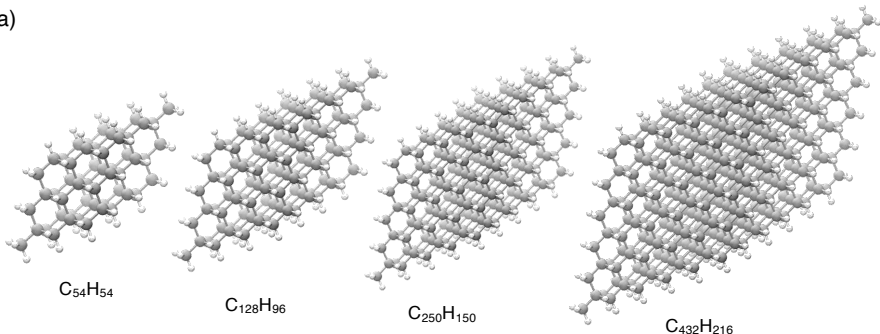
$$X_{\text{bind}} = X_{\text{P+L}} - X_{\text{P}} - X_{\text{L}} \quad (5)$$

90 where X_{bind} represents the ligand binding free energy, enthalpy, entropy, etc. In
 91 this case, we perform correlated sampling by using the same random vectors in the
 92 stochastic Lanczos treatment of the P+L, P, L systems (zeroing out elements for P
 93 and L respectively).

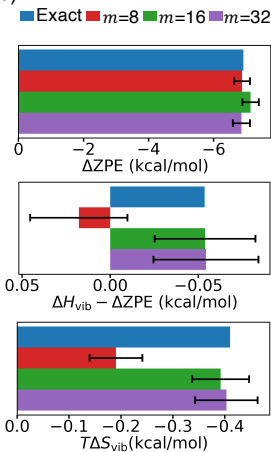
94 Results

95 As a first application, we take diamond nanocrystals (Figure 1a) as prototypical nano-
 96 material, and compute the free energies as a function of size. We employ the high-

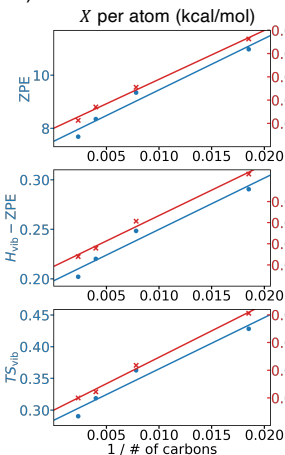
a)



b)



c)



d)

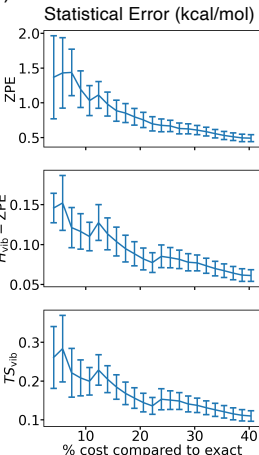


Fig. 1 a) Structures and chemical formulae of the diamond nanocrystals used in the calculations. b) Δ values estimated with different Lanczos quadrature orders (m) for the smallest system ($C_{54}H_{54}$) compared to the exact values. Error bars were estimated using the standard error of 50 samples. c) Per-atom quantity values and errors as a function of system size for a fixed number of samples (50 samples). The solid lines correspond to linear fits to inverse size. d) Statistical errors (for $m = 16$ quadrature) in absolute thermodynamic quantities of $C_{432}H_{216}$ as a function of % of computational cost of the exact calculation (error bars denote error of error).

97 and low-level correlated sampling approach described above, with Kohn-Sham density
 98 functional theory (DFT) with the PBE functional [12] as the high-level method
 99 and the semi-empirical extended tight-binding (XTB) method [13] as the low-level
 100 method. For the smallest system ($C_{54}H_{54}$), we can compute the Hessian explicitly to
 101 provide an exact reference. Figure 1b shows Δ quantities for the zero-point energy,
 102 the thermal enthalpy, and the entropy respectively, as a function of quadrature order
 103 m . The error bar indicates the statistical error for 50 samples. A stochastic quadra-
 104 ture level of $m = 8$ does not provide sufficient accuracy, so we choose $m = 16$
 105 for further calculations. Figure 1c shows the value and stochastic error per atom
 106 (estimated as one standard deviation) for the ZPE, thermal enthalpy, and entropy
 107 respectively. We note the error decreases with the size of the system, which is evi-
 108 dence of “self-averaging” due to the large system size. Thus if one is interested in

per-atom quantities, as is often the case for thermodynamics, for example to locate phase transitions, our stochastic approach becomes increasingly more efficient in a large system. The difference between our largest simulation and the extrapolated thermodynamic limits for the per-atom ZPE, enthalpy, and entropy is only 0.2 kcal/mol, 0.004 kcal/mol, and 0.006 kcal/mol respectively; statistical errors with 50 samples are about 0.001 kcal/mol or less. In fact, with a *single* sample, one can estimate the per-atom quantities with a statistical error of less than 0.01 kcal/mol at a 120-fold speedup relative to the exact Hessian calculation.

If one is instead interested in the absolute values, the method can still be cheaper than the full computation of the Hessian. Figure 1d shows the stochastic error for the largest carbon system ($C_{432}H_{216}$) as a function of computational cost. At less than 20% of the exact calculation’s computational effort, the error estimate is well within 1 kcal/mol. Less precise estimates can be obtained even more cheaply; a 20 times speedup is possible if 2 kcal/mol of error in the absolute quantities is tolerable.

Vibrational contributions to the free energy are also central to the study of biomolecules. For protein-ligand interactions in particular, where the aggregate binding is often only 5-10 kcal/mol, the vibrational contribution to binding free energies can be significant. Although the harmonic approximation is not necessarily a faithful approximation in these systems, the harmonic contributions nonetheless provide a first estimate of the thermal and entropic contributions. For these quantities, we use the fixed random vector correlated sampling approach described above, and employ the semi-empirical extended tight binding method as the quantum mechanical theory. We do not include explicit water molecules in the simulation: in principle, these could be included at additional cost, or the desolvation contribution to the free energy can be separately estimated by standard continuum methods [14]. We first study a system which is just small enough that exact results can be obtained at a high computational cost: a cutout (~ 1600 atoms) of the human tankyrase 2 (TNKS2) protein with a bound ligand shown in Figure 2a (see Methods for more information). In Figure 2b we show the thermal contributions to the binding enthalpy, entropy and free energy for stochastic Lanczos quadrature order $m = 16$. We do not separately plot the zero-point energy in this case because the change in the zero-point energy due to binding is small and not easily resolvable with a small number of random samples. For each of the above quantities, the error due to the Lanczos order is less than 1 kcal/mol. A significant physical quantity is the thermal contribution to the binding free energy ($\Delta G_{\text{vib}} - \Delta \text{ZPE}$). From Figure 2c we see that this can be estimated with a statistical error of less than 1 kcal/mol with a cost of roughly 10% of the exact Hessian calculation.

Finally, we apply our approach to HIV protease bound to a small molecule (JE-2147)[16] (Figure 3a). This system contains over 3000 atoms including hydrogens. The number of gradient calculations required to compute the full Hessian in this case ($\sim 10,000$) is so large that the exact computation is expensive even at the level of semi-empirical quantum mechanics (and would be completely impractical at the level of full density functional theory). In Table 1 we show our stochastic estimates of the harmonic contributions to the thermodynamic binding quantities, using a quadrature order of $m = 20$. Interestingly, the thermal contributions ($\Delta H_{\text{vib}} - \text{ZPE}$ and $T S_{\text{vib}}$)

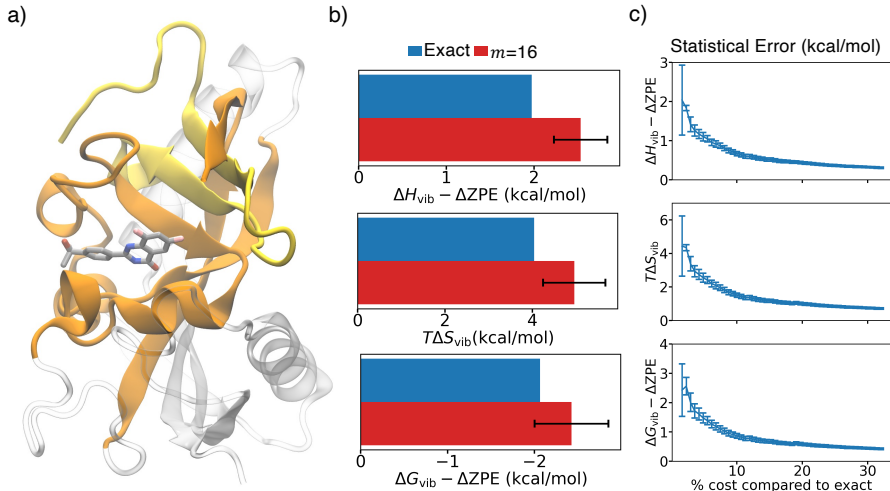


Fig. 2 a) The TNKS2 complex. The truncated part of the protein is shown as transparent, while the remaining two protein chains and the ligand are colored orange, yellow and grey, respectively. Image rendered by VMD[15]. b) The thermal enthalpy and entropy, and free energy of binding for the TNKS2 system for Lanczos order ($m = 16$). The error bars represent \pm one standard error from 100 random samples. c) Statistical errors in binding free energy quantities as a function of % of cost of the exact calculation (error bars denote error of error).

Quantity	value	error
$\Delta H_{\text{vib}} - \Delta ZPE$	1.88	0.72
$T\Delta S_{\text{vib}}$	1.69	1.79
$\Delta G_{\text{vib}} - \Delta ZPE$	0.19	1.13

Table 1 Contributions to the binding free energy from thermal enthalpy and entropy at 298.15 K for the JE-2147-HIV protease system. The error estimate in the final column is one standard error from 50 random samples (corresponding to 10% of exact cost). All values are given in kcal/mol.

154 to the free energy are both similar, small, and of opposite sign, meaning that the total
 155 thermal free energy contribution is almost zero. Nonetheless, estimating $\Delta G_{\text{vib}} - ZPE$
 156 to an accuracy of 1 kcal/mol is clearly feasible within our scheme at roughly 10% of
 157 the estimated cost of the exact calculation (Figure 3b).

158 Discussion

159 We have presented results which demonstrate the feasibility of computing harmonic
 160 contributions to the free energy at the quantum mechanical level for systems of sev-
 161 eral thousand atoms. The cost is greatly reduced from that needed to compute the
 162 Hessian of the system. This is particularly true when one is interested in intensive
 163 (or “per-atom”) quantities, where self-averaging behavior shows that in large sys-
 164 tems, we may estimate the quantities at a cost comparable to that of a few energy
 165 evaluations. This holds promise in evaluating thermodynamic transitions in materi-
 166 als involving large unit cells, for example, those associated with alloys and disorder.
 167 In the case of free energy differences, the correlated sampling technique employed

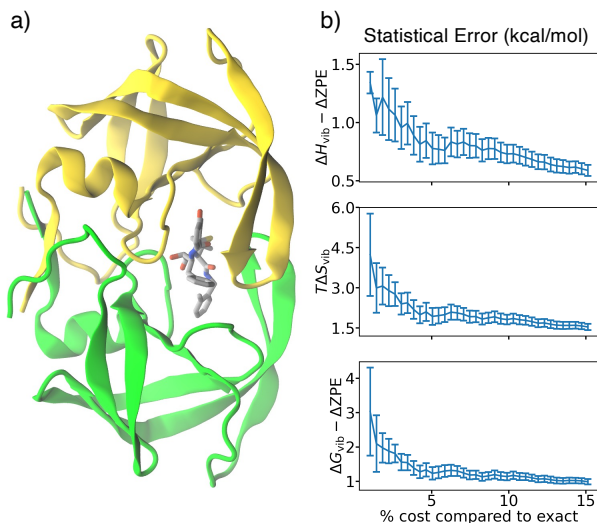


Fig. 3 a) An image of the JE-2147-HIV protease complex. The two protein chains and the ligand are shown in yellow, green, and grey respectively. Image rendered by VMD[15]. b) Statistical errors in binding free energy quantities as a function of % of cost of the exact calculation (error bars denote error of error).

168 here makes the evaluation of even small thermal free energy differences, as found in
 169 protein-ligand complexes, feasible at the level of 1 kcal/mol.

170 An additional advantage of the current approach is that the cost may be contin-
 171 ually tuned. This is relevant to new applications, for example in the computational
 172 screening for therapeutics [1, 17], where less precise estimates are an acceptable
 173 tradeoff for speed. Although the harmonic approximation is not necessarily quanti-
 174 tative in biomolecular systems, the increased facility to obtain harmonic estimates
 175 also raises the possibility for new approaches to compute anharmonic contributions
 176 to free energies. In summary, the technique presented here suggests that the estima-
 177 tion of free energy effects at the quantum mechanical level for systems with hundreds
 178 or thousands of atoms need not be considered a future challenge [1], but one which
 179 can be begin to be addressed today.

180 Methods

181 Stochastic Lanczos quadrature

182 The stochastic Lanczos method is a numerical method which has been employed
 183 in different contexts (see e.g. Ref. [18] for an early application in quantum many-
 184 body systems). We follow the general mathematical formulation in Ref. [7]. The
 185 Lanczos iterations were performed starting from a vector randomly selected from
 186 a Rademacher distribution. The Lanczos iterations require the action of the mass-
 187 weighted Hessian matrix on this random vector. We compute this matrix-vector

188 product from finite difference gradient calculations:

$$\mathbf{D}\mathbf{v} = \mathbf{M}^{-1/2} \frac{\mathbf{g}(\delta\mathbf{v}) - \mathbf{g}(-\delta\mathbf{v})}{2\delta}. \quad (6)$$

189 Here, \mathbf{D} as the mass-weighted hessian matrix, \mathbf{g} is the gradient, \mathbf{M} is the diagonal
190 matrix of masses. The displacement is given by

$$\delta\mathbf{v} = \mathbf{M}^{-1/2}\mathbf{v} \quad (7)$$

191 where the factor of $\mathbf{M}^{-1/2}$ accounts for the mass-weighting. The value of δ is chosen
192 based on the norm of the random vector so that the average displacement per atom is
193 0.0012 Å.

194 Additionally, we also implemented and tested a Chebyshev fitting method as an
195 alternative to the stochastic Lanczos quadrature. We found that often a higher order
196 Chebyshev fit was required making it a slightly more expensive alternative to Lanc-
197 zos quadrature. However, there may situations where Chebyshev fitting is preferable.
198 Results from the Chebyshev fitting method are presented in the supplementary
199 information.

200 **Calculations on diamond nanocrystals**

201 Diamond nanocrystals were constructed by creating supercells of the bulk diamond
202 unit cell and then capping with hydrogens. The resulting structure was optimized
203 using the PBE functional and the def2-SV(P) basis set. All DFT calculations were
204 performed with the ORCA program[19, 20]. The structure was also optimized using
205 the second generation extended tight-binding (GFN2-xTB) method[21] as imple-
206 mented in the Semiempirical Extended Tight-Binding (xTB) program package[22].

207 **Calculations on protein-ligand systems**

208 All calculations on protein-ligand systems used the second generation extended tight-
209 binding (GFN2-xTB) method[21] as implemented in the Semiempirical Extended
210 Tight-Binding (xTB) program package[22]. Generalized Born, solvent-accessible area
211 (GBSA) solvation was used to mimic an aqueous environment for all calculations.

212 The truncated TNKS2 protein was constructed from the the ligands/protein-
213 structure obtained from Ref. [23]. The entire protein was minimized using Amber-
214 Tools, using the Generalized Born implicit, (igb=5), the Amber 14 force field[24], and
215 the general AMBER force field (GAFF)[25] for ligands, assigned using Antecham-
216 ber from AmberTools[26]. Following minimization, truncation and capping of the
217 terminals were carried out using PyMol [27]. Truncation was performed to remove
218 all protein atoms beyond 3-4Å around the ligand. Truncated ends were capped using
219 ACE/NME terminal patches. The ligand bound to the protein is one of the many
220 inhibitors identified in Ref. [28] whose structure is available in the Protein Data Bank
221 [29](PDB: JKN).

222 The structure of the JE-2147-HIV protease complex was obtained from PDB
223 1KZK. Hydrogens were added using UCSF Chimera[30] and the structure was

224 optimized first using the GFN-FF force field as implemented in the xTB pro-
225 gram package[22] and finally with the GFN2-xTB method[21] ultimately used for
226 harmonic vibrational analysis.

227 **Supplementary information.** Supplementary figure comparing stochastic Lanc-
228 zos and Chebyshev methods.

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230 **Declarations**

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- 235 • Availability of data and materials: Data is available from the authors upon
236 reasonable request.
- 237 • Code availability: Code is available from the authors upon reasonable request.
- 238 • Authors' contributions: AFW and GKC conceived the project. AFW, CL carried
239 out the work. All authors contributed to the writing of the paper.

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