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Finishing of micro-aspheric tungsten carbide mold using small viscoelastic tool

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Abstract: Tungsten carbide is widely used as the material of replication mold to produce small aspheric optics, and the polishing process determines the precision of the mold. However, for micro-aspheric tungsten carbide mold, the existing polishing methods are difficult to realize the form error modification during the polishing because the polishing tool is always larger than small mold. Therefore, a polishing tool which using polyester fiber cloth to wrap small-size rigid ball is used in this paper. In order to predict the tool influence function (TIF) of this polishing tool, a series of theoretical analysis and experimental verification are carried out in this paper. Firstly, by analyzing the structural and viscoelastic characteristics of the fiber cloth, the pressure distribution in the polishing contact area is determined. And the polishing speed distribution is obtained by analyzing the kinematic movement of the polishing tool; Then, combined with Preston equation, the tool influence function is derived; Afterward, through a series of single point polishing experiments, it is verified that the volume error between the theoretical removal model and the experimental removal is less than 10.8%; Finally, the tool influence function is applied to the form error corrective polishing of small size symmetric aspheric tungsten carbide mold. After one form error corrective polishing, the PV value (Peak to Valley) of form error is decreased from 0.405 μm to 0.068 μm , which verifies the effectiveness of the polishing method of small size tungsten carbide mold in form error correction.

Keywords: Tungsten carbide, micro-aspheric mold, viscoelastic tool, deterministic polishing

1 Introduction

Aspheric lens has the advantages of reducing the volume of optical system, improving imaging quality and avoiding the imaging error of spherical lens. It is widely used in mobile phone lens, vehicle imaging system, infrared optical imaging system, etc[1]. In the fabrication of aspheric lens, small aperture aspheric lens is mainly manufactured by glass aspheric molding for low-cost. The aspherical lens is formed by replicate the surface profile of the aspheric mold. Thus, the form accuracy and surface roughness of the aspheric mold determine the accuracy of the aspheric lens after molding[2].

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In the glass aspherical molding technology, tungsten carbide, silicon carbide and other cemented carbide materials with stable performance at molding temperature are usually used to manufacture aspherical molds[3]. This kind of cemented carbide mold needs to be polished after grinding[4]. The polishing process determines the final accuracy and surface roughness of aspheric mold and the tool influence function (TIF) is essential to form error corrective polishing to reduce the form error of aspheric profile.

At present, aspheric polishing is based on Preston equation[5], and the TIF is established by analyzing the pressure distribution, polishing speed distribution of the polishing contact area and the removal constant related to the workpiece material and polishing particle size.

There are mainly two methods to obtain the tool influence function. One is measuring it by experiment, and

the other is predicting it by analyzing the stress distribution and velocity distribution in the polishing contact area. Due to the different curvature of aspheric surface, tool influence function will change with polishing position and time. Thus, a large amount of work needs to be done if measuring TIF by experiment. Therefore, predicting the TIF of aspheric surface is an efficient choice to save time and labor. However, the change of TIF with time and polishing position and the relevancy of other process parameters make it difficult to predict TIF precisely.

In the past decades, many researchers have done a lot of research on TIF of aspheric polishing. In the 1980s, Aspden et al, researched the removal mechanism of workpiece materials and proposed a polishing removal model based on pressure, polishing speed and time[5]. Subsequently, Jones et al^[6], proposed a dwell time solution model based on Aspden's research. The model uses the convolution iteration method to calculate the dwell time of the polishing tool at each position of the workpiece, and achieves the purpose of reducing the form error of the workpiece by controlling the polishing dwell time. The research work of Aspden and Jones constructed a theoretical foundation for computer-controlled optical surfacing. However, the research work of Aspden and Jones has a certain limit of application, thus, other researchers improve the aspheric polishing theory and put forward new polishing methods under their research framework.

Different polishing methods have also appeared in the field of aspheric cemented carbide mold polishing. In the 1990s, Prokhorov[7] invented a new polishing technology called magnetorheological polishing (MRF). Under the high intensity gradient magnetic field, the magnetorheological fluid hardens in a short time and turns into a viscoplastic medium. When the polishing head moves relative to the workpiece, this medium has shear force on the workpiece surface, resulting in material removal[8]. In order to adapt to the polishing of small diameter cemented carbide mold and avoid interference between polishing tool and concave mold, Yin[9] et al, proposed inclined axis magnetorheological polishing technology to avoid interference by controlling the angle of polishing tool. Using this method, the tungsten carbide mold with 8mm diameter can be polished. However, for concave mold with smaller curvature radius, inclined axis magnetorheological polishing method is also difficult to carry out. Magnetorheological polishing technology can

polish tungsten carbide molds and obtain high surface quality, but its polishing efficiency is low and limited to molds with large radius of curvature.

In 2001, Walker[10] proposed the bonnet polishing technology. It is reported that the bonnet polishing technology can achieve a removal rate of 0.025-120mm³/min and achieve a smoother surface quality[11]. Due to the high material removal rate, bonnet polishing technology is widely used in the polishing of optical elements of brittle and hard materials[12]. For bonnet polishing technology, an accurate TIF is also essential to the polishing accuracy. Therefore, Kim[13] et al., Wang[14] et al, Cheung[15] et al, based on Preston equation[16], combined with Hertz contact theory and geometric motion of bonnet polishing head, established TIF on the perspective of macro removal. Cao [17, 18] et al, analyzed the removal mechanism of single free abrasive particles in bonnet polishing, and established the TIF by calculating the removal of all abrasive particles. Lin[19] et al, analyzed the influence of the surface morphology of polyurethane polishing pad on the TIF through simulation. In the above research about bonnet polishing, it is assumed that the bonnet is elastic deformation and its viscoelastic properties are ignored. Although bonnet polishing technology can effectively remove cemented carbide materials, but it is limited by the size of bonnet tool and easy to interfere with small concave mold. Bonnet polishing is difficult to realize the form error modification during the polishing for small-diameter tungsten carbide mold.

Using above polishing methods, medium and large aperture aspheric surfaces can be polished to obtain high quality surface. However, the polishing methods mentioned above, all have large size of polishing tools, which resulting in large polishing spots. Thus, they are not suitable for form error corrective polishing of micro-aspheric tungsten carbide molds (especially < 5mm). Therefore, this paper uses a rigid ball as a small-size polishing tool. The surface of the tool is wrapped with a layer of cloth which made of polyurethane and nylon. Combined with the viscoelastic properties and geometric structure of the polishing cloth, the polishing pressure distribution function is established, the velocity distribution of the polishing contact area is analyzed, and the TIF is established. Finally, a series of experiments are carried out to verify the accuracy of TIF and the effectiveness of form error corrective polishing.

2. Material removal mechanism and model of tool influence function

In this paper, the relative position of the polishing tool and workpiece is shown in Figure 1. According to Preston's empirical formula, modeling the TIF of the small tool needs to analyze the pressure distribution in the contact area and the relative velocity distribution between the workpiece surface and the polishing tool surface in the contact area. On one hand, the pressure distribution in the contact area is determined by the viscoelastic properties and the structure of the polishing cloth. On the other hand, the speed distribution is determined by the relative movement of the polishing tool and the workpiece. In order to simplify the modeling of tool influence function, in the subsequent analysis, we make the following assumptions:

- (1) Before polishing starts, the polishing tool wrapped with cloth is regarded as a standard ball.
- (2) The polishing tool and the workpiece are regarded as rigid bodies.
- (3) The polishing slurry and particles are uniformly attached to the surface of the polishing cloth, and the final effect on the polishing removal is reflected as removal constant k_p .
- (4) The height of the high-frequency unevenness on the surface of the polishing cloth obeys the normal distribution, and the height of the intermediate-frequency unevenness obeys the sinusoidal distribution.
- (5) The polishing cloth is a non-uniform viscoelastic medium. During the compression process of the cloth, the probability of the silk participating in the compression deformation obeys the exponential probability distribution.

Fig 1 Graphical illustration of polishing tool and workpiece

As well known, the TIF are mostly based on the Preston equation. During the polishing process, the removal depth of material is proportional to the pressure, the relative velocity and the dwell time in contact area. The relationship between these factors can express as:

$$dh = k_p p_c v_s dT \quad (1)$$

In formula (1), dh is the amount of removal at the one point of the workpiece surface, k_p is the removal constant, p_c is the pressure exerted by the polishing tool at the workpiece, v_s is the relative speed of the polishing tool surface and the workpiece surface at the contact area, dT is the dwell time of the polishing tool on the workpiece.

As we known, the accuracy of the TIF is essential to the result of form error corrective polishing process. Thus, in order to achieve the deterministic removal of surface material in the polishing process, the TIF should be predictable precisely. Based on the Preston equation, the TIF can be expressed as:

$$R(x, y) = \frac{k_p}{T} \int_0^T p(x, y) v(x, y) dt \quad (2)$$

In formula (2): k_p is the remove constant, T is the total polishing time, $p(x, y)$ is the pressure distribution function, $v(x, y)$ is the relative velocity distribution function.

For contact polishing, the shape of the contact area, the pressure distribution and the relative velocity distribution in the contact area need to be predicted, and finally TIF is obtained. In this paper, the process of predicting the TIF is shown in Figure 2.

Fig 2 Flowchart of predicting the TIF

2.1 Structural simulation and viscoelastic properties of polishing cloth

2.1.1 Structural modeling of polishing cloth

In this paper, a small rigid ball which wrapped with a layer of cloth is used as a polishing tool. The cloth is woven from polyurethane and nylon composite twisted core yarns, and has a mid-frequency and high-frequency profile in structure, as shown in Figure 3. The structure and viscoelastic properties of the polishing cloth determine the stress distribution under a certain deformation, so it is necessary to explore the structure of the polishing cloth. In order to simplify the mathematical model of the three-dimensional structure of the polishing cloth, we assume that the structure of the polishing cloth is composed of a base layer and a rough layer. The cross-sectional morphology of the polishing cloth is shown in Figure 4. According to the textile structure of the polishing cloth, the height deviation of the rough layer of the polishing cloth is superimposed by two types of periodic height ,that is intermediate frequency periodic height and high frequency periodic height. We assume that the height deviation of the intermediate frequency periodic height conforms to the sinusoidal distribution, and the initial phase of the sinusoidal distribution conforms to the Normal distribution $N(\mu_\theta, \delta_\theta)$, and the height deviation of the high frequency height conforms to the normal distribution $N(\mu_z, \delta_z)$. Fig. 4 shows the height distribution diagram of the YZ section. In order to get the three-dimensional structure of the cloth, we extend the two-dimensional structure diagram of the YZ section along the X direction. As shown in Figure 5, it is the structure simulation process of the polishing cloth, and the height deviation of the rough layer is recorded as $\delta(x, y)$.

Fig 3 Sketch of polishing tool and polishing cloth **a** Physical picture of polishing tool head **b** Physical picture of polished cloth **c** Surface structure diagram of polished cloth

Fig 4 Sketch of layered simulation structure of polished cloth

Fig 5 Structure simulation process of polishing cloth

2.1.2 Viscoelastic properties of polishing cloth

The silk of the polishing cloth is made of polyurethane and nylon composite twist core, which has the viscoelastic properties of polymer materials. Therefore, a four-parameter viscoelastic model is used to describe the viscoelastic properties of the polishing cloth. As shown in Figure 6, it is a schematic diagram of the deformation and deformation recovery of the polishing cloth under a certain pressure, Where δ is the stress, η_1 and η_2 are the viscous coefficients of viscous elements 1 and 2 respectively, E_1 and E_2 are the elastic coefficients of elastic elements 1 and 2 respectively, ε_1 is the initial deformation, ε_2 is maximum compression, ε_3 is the unrecoverable deformation. When we give a deformation to the cloth, the cloth will appear an unrecoverable deformation. The unrecoverable deformation is mainly caused by viscous element 1, and it causes the shape of the polished contact

area to change from a circle to an ellipse.

(b)

Fig 6 Four-parameter model of polishing cloth

In order to explore the relationship between initial deformation and unrecoverable deformation, a test compression experiment of compression ε_2 and unrecoverable deformation ε_3 is designed as shown in Figure 7(a). Firstly, we give a compression ε_2 by exert a Force. Then, we release the Force and measure the unrecoverable deformation ε_3 . The experimental results are shown in Figure 7(b). From the experimental results, it can be seen that there is a linear relationship between the amount of compression and the unrecoverable deformation variable, and the linear relationship between the two can be expressed by a proportional coefficient k_a .

(a)

Fig 7 Compression test diagram of polishing cloth **a** Sketch of polishing cloth compression **b** Compression test results

Through compression experiments, we found that when carry out the same amount of compression second time after the first compression, the unrecoverable deformation is almost the same. Therefore, when we analyze the polishing pressure distribution after the first compression, the unrecoverable deformation are constant, and the viscoelastic model of the polishing cloth is simplified to a three-parameter viscoelastic model, as shown in Figure 8. According to the theory of viscoelasticity, the stress relaxation equation of the three-parameter model is expressed as follow:

$$\sigma(t) = E_1 \varepsilon + E_2 \varepsilon \times e^{-\frac{E_2}{\eta_2} t} \quad (3)$$

Where, σ is the total stress, t is the time, ε is the total strain, E_1 is the elastic modulus of elastic element 1, E_2 is the elastic modulus of elastic element 2, η_2 is the viscosity coefficient of the viscous element.

A compression test that exert a constant deformation to the cloth and measure the stress release with time was performed on the polishing cloth. Combined with formula (3), we can obtain the elastic modulus E_1 , E_2 and viscosity coefficient η_2 by fitting the test data. The compression test parameters and results are shown in Table 1 and Figure 9.

Fig 8 Three-parameter model

Fig 9 Stress relaxation test

Table 1 Experimental parameters and results

Experimental parameters	value
Compressive strain ε (mm)	0.02948
Relaxation time t (S)	0.9
Elastic Modulus E_1 (N/mm)	60.11
Elastic Modulus E_2 (N/mm)	6.511
Coefficient of Viscosity η_2	1.759

During the polishing process, the deformation of the polishing cloth at different points in the polishing contact area will change with the rotation of the polishing tool, as shown in Figure 10. Since the cloth is the sparse and non-uniform elastic medium composed of silk threads, it is assumed that the proportion of silk threads participating in the compression conforms to the exponential probability distribution. According to Boltzmann superposition principle[20] and the probability and statistics theory, the relationship between stress and strain for a point at the cloth in contact area can express as follow:

$$\sigma(t) = D(t) * d\varepsilon(t) \times F(\varepsilon(t))$$

$$= \frac{1}{E_0} (1 + \beta(1 - e^{-\lambda t})) * d\varepsilon(t) \times (1 - e^{-\mu \varepsilon(t)}) \quad (4)$$

$$\beta = \frac{E_1}{E_2} \quad (5)$$

$$\lambda = \frac{E_2}{\eta_2} \quad (6)$$

$$E_0 = E_1 \quad (7)$$

Where σ is the stress, D is the relaxation modulus of the cloth, ε is the deformation of the cloth, F is the probability distribution relate to ε , t is the force deformation time of the cloth, $d\varepsilon$ is the derivative of the cloth deformation, E_1 is the elastic modulus of the elastic element 1, and E_2 is the elastic modulus of the elastic element 2. , η_2 is the viscosity coefficient of the viscous element, μ is the probability related constant, $*$ is the convolution operator.

In order to determine the value of the probability-related constant, a compression test of the cloth was performed, and the stress-strain curve was fitted by formula 4 to obtain the value of the probability-related constant. The test results are shown in Figure 11. Through the test results, we can know that the stress-strain relationship of the cloth is nonlinear.

Fig 10 Sketch of the change in compression

Fig 11 Stress-strain compression test results

2.2 Pressure distribution modeling

In the previous studies on TIF, most of the analysis of pressure is based on the Hertzian contact theory, and the applicable object of the Hertzian contact theory is a uniform elastic medium with a smooth surface. In this paper, a rigid ball wrapped with cloth is used as the polishing tool, and the polishing cloth is not a uniform elastic medium with a smooth surface. Therefore, the Hertzian contact theory is not applicable to the analysis of the pressure distribution in the polished contact area. Thus, we calculate the pressure distribution of the polishing contact area based on the compression deformation of each

point in the polishing contact area.

The contact between the polishing tool and the surface of the workpiece can be divided into two stages, as shown in Figure 12. In the first stage, it is assumed that the polishing tool is a standard ball, and the contact area between the ball and the surface of the workpiece is a circle. In the second stage, when the polishing tool rotates once, the contact area between the polishing tool and the surface of the workpiece becomes elliptical due to the irreversible deformation of the polishing cloth after it is compressed. Since most of the polishing time is in the second stage where the contact area between the polishing tool and the surface of the workpiece is elliptical. Thus, we only analyze the pressure distribution in the second stage. After the irreversible deformation occurs, the compression distribution of the cloth in the polishing area is expressed as:

$$\begin{aligned} z(x, y) &= \sqrt{r^2 - x^2 - y^2} - (r - d) \\ &- k_a \times (\sqrt{r^2 - y^2} - (r - d)) + \delta(x, y) \times (1 - k_a) \end{aligned} \quad (8)$$

In formula (8), r is the radius of the cloth-covered ball head, d is the maximum compression, k_a is the proportional constant between the compression and the irreversible deformation mentioned earlier, and σ is the height difference of the polishing cloth surface mentioned earlier. Combining formula (4) and formula (8), the pressure distribution of the polishing contact area can be obtained as:

$$\begin{aligned} P(x, y) &= \sigma(x, y, t) \\ &= D(x, y, t) * d\varepsilon(x, y, t) \times F(\varepsilon(x, y)) \\ &= \frac{1}{E_0} (1 + \beta(1 - e^{-\lambda t})) * dz(x, y, t) \times (1 - e^{-\mu z(x, y)}) \end{aligned} \quad (9)$$

Fig 12 Change of contact area shape

2.3 Modeling the velocity distribution in the contact area of the small tool

Figure 13 shows the geometric positional relationship between the tool and the contact area of the workpiece. In the polishing process, the tool head rotates around the axis $\vec{Z}_t$, the angular velocity of rotation is ω_p , and the angular velocity vector is in the form of $\vec{\omega}$. $\vec{Z}_t$ is a unit vector, $\vec{\omega}$ is expressed as:

$$\vec{\omega} = \omega_p \vec{Z}_t = \omega_p (0, \sin \sigma y, \cos \sigma z) \quad (10)$$

In formula (10), $\omega_p = |\omega|$. In Figure 13, point $P(x_p, y_p, 0)$ is any point in the polishing area, and $\vec{r}$ is the vector from $O_t = (0, 0, R_p)$ to point P, that is expressed as:

$$\vec{r} = \vec{O_t P} = (x_p, y_p, R_p) \quad (11)$$

In formula (11), $R_p = R - d$, where R is the radius of the polishing tool and d is the maximum compression. Since the linear velocity generated by the rotation of the polishing tool in the contact area is v_r^p , it can be expressed as the cross product of the angular velocity vector $\vec{\omega}$ and the vector $\vec{r}$ like that:

$$v_r^p = \vec{\omega} \times \vec{r} = \omega_p \begin{bmatrix} -R_p \sin \sigma - \cos \sigma y_p \\ \cos \sigma x_p \\ -\sin \sigma x_p \end{bmatrix} \quad (12)$$

When polishing with a spherical tool, the amount of material removal is only related to the moving linear velocity in the x and y directions, and is not related to the velocity of the component in the z direction, so only the x and y directions need to be considered. The combined velocity of a point $p(x_p, y_p, z_p)$ in a contact area can be expressed as the following formula:

$$\begin{aligned} v(x_p, y_p) &= \sqrt{((v_{rx}^p)^2 + (v_{ry}^p)^2)} \\ &= \sqrt{((-R_p \omega_p \sin \sigma - \cos \sigma \omega_p y_p)^2 + (\omega_p \cos \sigma x_p + \omega_p R_p)^2)} \end{aligned} \quad (13)$$

Fig 13 Sketch of the relative linear velocity distribution in the contact area

2.4 Modeling of tool influence function

After the above analysis, combined with formulas (2), (8), (9), (13), the TIF can be expressed as:

$$R(x, y) = \frac{k_p}{T} \int_0^T p(x, y) v(x, y) dt$$

$$= \frac{k_p}{T} \int_0^T \frac{1}{E_0} (1 + \beta(1 - e^{-\lambda})) * dz(x, y, t) \times (1 - e^{-\mu z(x, y)}) \times v(x, y) dt$$

(14)

From the tool influence function formula (14), it can be seen that the polishing removal rate of the material is proportional to the compression of the polishing cloth, and proportional to the relative speed distribution of the contact area.

3 Experimental verification of tool influence function

3.1 Experimental conditions and results

In order to verify the effectiveness of the theoretical TIF, a series of single-point polishing experiments were carried out. In the experiment, the tungsten carbide plane was used as the polished workpiece, the diamond with a diameter of 1um was used as the polishing abrasive grain, the cloth was used as the polishing cloth, and the white light interferometer Bruker contourGT was used to measure the three-dimensional removal topography. The polishing equipment used is shown in Figure 14, and the characteristics of the polishing spots are explored by setting the experimental parameters shown in Table 2. These processing parameters that affect the polishing spot tool influence function include polishing dwell time, polishing tool rotation speed, and applied polishing pressure. The polishing result of single-point polishing is shown in Figure 15.

Fig 14 Sketch of polishing equipment **a** Sketch of the movement axis of the polishing equipment **b** Sketch of polishing process

Table 2 Single-point polishing experiment parameters

Spot number	time (S)	speed(rpm)	pressure (N)
1	20	180	1.5
2	40	180	1.5
3	60	180	1.5
4	80	180	1.5
5	100	180	1.5
6	40	120	1.5
7	40	180	1.5
8	40	240	1.5
9	40	300	1.5
10	40	360	1.5
11	40	180	0.3
12	40	180	0.8
13	40	180	1.2
14	40	180	1.7
15	40	180	2.2

Fig 15 Sketch of polishing spots

3.2 The influence of processing parameters on the removal rate

In order to explore the relationship between polishing time and removal volume, experiments with serial numbers 1 to 5 were carried out. The experimental results are shown in Figure 16. A linear fit was performed on the removal volume and the dwell time, and the goodness of fit R2 was 0.9945. It can be seen that the removal rate has a linear relationship with the dwell time.

In order to explore the influence of polishing tool rotation speed on polishing removal rate, experiments with

serial numbers 6 to 10 were carried out. The experimental results are shown in Figure 17. The polishing removal rate is linearly fitted to the rotation speed of the polishing tool, and the fitting R^2 is 0.9921. It can be seen that the rotation speed of the polishing tool has a linear relationship with the polishing removal rate.

In order to explore the influence of polishing pressure on polishing removal rate, experiments with serial numbers 11-15 were carried out. The experimental results are shown in Figure 18. When the polishing pressure is low, the polishing removal rate is in a linear relationship with the polishing pressure. When the pressure increases to a certain level, it starts to deviate from the linear relationship. This is because a part of the abrasive particles is embedded in the polishing fiber cloth and part is embedded in the workpiece. When the force of the fiber from the polishing cloth increases, the pressure increases, and the depth of the abrasive particles pressing into the surface of the workpiece also begins to increase. But when the pressure increases to a certain extent, most of the increased pressure of the polishing cloth fiber directly acts on the surface of the workpiece, and because the hardness of the polishing fiber is much smaller than that of the workpiece, the impact on the depth of the abrasive particles pressed into the surface of the workpiece is reduced. In order to simplify the analysis, this article only discusses the relationship between the polishing pressure and the linear range of the removal rate.

By analyzing the experimental results of the influence of processing parameters and removal rate, it can be seen that the polishing time, polishing tool speed, and polishing pressure are in a linear relationship with the polishing removal rate per unit time within a certain range. From formula (14), the value of removing the constant can be obtained:

$$k_p = \frac{R(x, y)}{p(x, y)v(x, y)} = \frac{\int R(x, y)dx dy}{\int p(x, y)v(x, y)dx dy} = \frac{V}{Fv} \quad (15)$$

Where V is the volume removed per unit time, F is the pressure of the polishing tool on the workpiece, and v is the average speed of the contact area.

Fig 16 Linear fit between removal volume and polishing time

Fig 17: Linear fit between volume removal rate and polishing tool rotation speed

Fig 18 Linear fit between volume removal rate and pressure

3.3 Single-point polishing removal prediction

The experimental parameters and experimental results are shown in Table 3 and Figure 15. Among them, Figure 15 (a) is a three-dimensional diagram drawn by MATLAB that processing the three-dimensional data measured by the white light

interferometer Bruker contourGT, Figure 15 (b) is the three-dimensional structure diagram of the simulation TIF calculated by the above theory, and 15 (c) is the cross-sectional contour drawing of the three-dimensional diagram on the YZ plane, 15(d) is the cross-sectional contour drawing of the three-dimensional figure on the XZ plane. From the figure, it can be seen that the theoretically predicted removal model is consistent well with the actual processed three-dimensional removal amount, which proves the effectiveness of the TIF.

Table 3 Experimental parameters of single-point polishing

Experimental parameters	symbol	value
Elastic Modulus (N/mm)	E1	60.11
Elastic Modulus (N/mm)	E2	6.511
Coefficient of Viscosity	η_2	1.759
Maximum compression (mm)	d	0.07
Diamond abrasive grain size (μm)	D	1
Polishing time (S)	T	25
Polishing shaft speed (rpm)	ω_p	180
Polishing ball radius (mm)	r	1.5
Removal constant	k_p	0.0131
Polishing cloth thickness (mm)	h	0.35
Probability correlation constant	μ	0.03276
Irrecoverable deformation proportional constant	k_a	0.75

Fig 19 Comparison of single-point polishing experiment and simulation results **a** Sketch of measured polishing three-dimensional removal **b** Sketch of predictive polishing three-dimensional removal **c** Sketch of Y-Z section profile **d** Sketch of X-Z section profile

In order to verify the applicable scope of the removal model, and according to the tool influence function obtained by formula (14), combined with formula (15), we substitute the polishing processing parameters in Table 2 to predict the polishing

removal volume. As shown in Figure 20, it is a comparison chart of the actual polishing removal volume and the predicted volume. The maximum removal error rate is 10.8%, and the variance of the removal error rate is 7.7%. It can be seen that the

theoretical removal model can greatly match the actual processing effect.

Figure 20 Measured removal volume and predicted removal volume

4. Form error corrective polishing experiment

4.1 Solving polishing dwell time

According to the above-obtained TIF, a small-diameter aspheric tungsten carbide mold is subjected to form error corrective polishing. The aspheric profile formula is shown in equation (16).

$$g(x, y) = \frac{(x^2 + y^2) * c}{1 + \sqrt{1 - c * (k+1)(x^2 + y^2)}} + \sum_{j=1}^{10} A_{2j} \times (x^2 + y^2)^j \quad (16)$$

In the formula, c is the curvature of the apex of the aspheric surface, k is the quadratic constant, and A_j is the coefficient of the higher-order term of the aspheric surface.

The aspheric profile in formula (16) is a rotationally symmetrical aspherical surface. The sketch of polishing and staying on a rotationally symmetrical aspherical mold is shown in Figure 21. In the figure 21, $O_0X_0Y_0Z_0$ is the coordinate system of the aspheric mold profile, and $O_iX_iY_iZ_i$ is the coordinate system of the tool influence function when the polishing tool dwell on points $P(x_i, y_i, z_i)$ on the aspheric surface. The Z axis of the tool influence function coordinate system coincides with the normal direction of point P .

Fig 21 Sketch of removing function residency

During the polishing process, the polishing tool is fixed. By controlling the rotation and translation of the aspheric mold, the location of the polishing dwell point is controlled. As shown in Figure 22 (a) and (b), the equidistant concentric circle path is adopted as the polishing path of the rotationally symmetric aspheric mold, and the polishing tool is fed along the concentric circle path through the rotation of the mold.

When the polishing tool performs polishing along the circular arc track with the ring radius r_i , the mold needs to rotate around the y axis from the initial position and translate the aspheric mold along the xz plane. Let the point $p(x_i, 0, z_i)$ be an intersection point of the aspherical meridian passing through the center of the circle and the arc trajectory, as shown in Figure 22(c). When polishing, it is necessary to move the point $p(x_i, z_i)$ to the origin O after rotation and translation, as shown in Figure 22 (d). After rotation and translation of the mold, the coordinate system $O_iX_iY_iZ_i$ and $O_0X_0Y_0Z_0$ are coincidence.

The coordinate change of any point on the aspherical mold after rotation and translation has the following relationship:

$$\begin{cases} x' = z \sin \alpha_i + (x - x_i) \cos \alpha_i \\ y' = y \\ z' = (z - z_i) \cos \alpha_i - (x - x_i) \sin \alpha_i \end{cases} \quad (17)$$

Where x_i and z_i are the coordinates of point P in the coordinate system $O_0X_0Y_0Z_0$, (x', y', z') are the coordinate values of the aspherical surface in the $O_iX_iY_iZ_i$ coordinate system after the rotation and translation of mold, and (x, y, z) are the coordinate values of the aspherical surface in the

$O_0X_0Y_0Z_0$ coordinate system.

Combining formula (16) and (17), the height of the aspherical mold after rotation and translation in the Z-axis direction of the polishing contact area is:

$$z' = g'(x, y) = [g(x - x_i, y) - z_i] \times \cos \alpha_i - (x - x_i) \times \sin \alpha_i \quad (18)$$

In the formula, x_i and z_i are the coordinates of the dwell point P, and α_i is the tangent angle of the dwell point P on the meridian. Combining formula (18) and formula (8), the compression of the polishing cloth in the polishing area is:

$$\begin{aligned} z'(x, y) = & \sqrt{r^2 - x'^2 - y'^2} - (r - d) + g'(x, y) \\ & - k_a \times (\sqrt{r^2 - y'^2} - (r - d) + g'(0, y)) \\ & + \delta(x, y) \times (1 - k_a) \end{aligned} \quad (19)$$

Combining formulas (14) and (19), the tool influence function at the dwell point $p(x_i, 0, z_i)$ is:

$$\begin{cases} R(x, y) = \frac{k_p}{T} \int_0^T p(x, y) v(x', y') dt \\ = \frac{k_p}{T} \int_0^T \frac{1}{E_0} (1 + \beta(1 - e^{-\lambda})) * dz'(x, y, t) \times (1 - e^{-\mu z'(x, y)}) \times v(x', y') dt \\ x' = g(x, y) \times \sin \alpha_i + (x - x_i) \cos \alpha_i \\ y' = y \end{cases} \quad (20)$$

(a) Sketch of polishing tool feed

(b) Sketch of polishing contact area

Fig 22 Polishing Sketch **a** Sketch of polishing tool feed **b** Sketch of polishing contact area **c** The initial position of the aspherical cross-section profile **d** Polishing position of aspheric cross-section profile

Due to the aspheric surface has the characteristics of rotational symmetry, this article only analyzes the removal of the cross-sectional profile on the meridian of the aspheric mold. When the tool influence function of formula (20) is fed along the arc trajectory of radius r_i , the removal amount on the meridian is:

$$R_i(\rho) = \sum R(x, y) \times dt = \int_L R(x, y) \frac{ds}{\omega \rho} \quad (21)$$

In the formula, ρ is the distance from the aspherical meridian point to the axis of symmetry, dt is the time, $R(x, y)$ is the unit time removal, ds is the arc differential, and ω is the angular velocity of the aspherical mold, L is the circular path.

The polishing tool has different tool influence functions on different arc trajectories, which are discrete and superimposed, as shown in Figure 23. Formula (21) can discretize to removal control point matrix as:

$$R_i = [0, 0, \dots, h_{\rho_1}, h_{\rho_2}, \dots, h_{\rho_m}, 0, \dots, 0] \quad (22)$$

The discretized tool influence functions on different arc trajectories can be superimposed to form the overall tool influence function matrix on the entire meridian:

$$\bar{R} = [R_1^T, R_2^T, \dots, R_n^T] \quad (23)$$

Suppose the dwell time matrix of the polishing tool at each ring belt position is:

$$\bar{T} = [T_1 \cdots T_i \cdots T_n]^T \quad (24)$$

Fig 23 Sketch of superposition of discrete tool influence function at each sampling point

Suppose the form error on this meridian is H , and the dwell time matrix satisfying the following formula can be obtained by using the non-negative least square method:

$$\min \|\bar{R}\bar{T} - H\|_2 \quad T \geq 0 \quad (25)$$

4.2 Surface form error corrective polishing experiment

In order to verify the surface error corrective polishing effect of the mold, a small-diameter mold shown in Figure 24 was polished. The aspheric profile coefficient of the mold is shown in Table 4. According to the polishing parameters shown in Table 5, TIF on the aspheric meridian section is calculated, combined with the formula (25) and the surface error data of the mold, the dwell time after compensation is calculated by the least square method.

Fig 24 Sketch of tungsten carbide aspheric mold

Table 4 Aspheric profile parameters

Parameter symbol	value
c	0.0487
k	21.5252
A2	0
A4	-1.2107676E-4
A6	-9.5469562E-06
A8	-1.0769511E-06
A10	1.3118654E-07
A12	-6.9768705E-09
A14	0
A16	0
A18	0
A20	0

Table 5 Form error corrective polishing experiment parameters

Experimental parameters	Value
Maximum compression of polishing cloth (mm)	0.07
Diamond abrasive grain size (μm)	1
Polishing tool speed (rpm)	180
Polishing ball radius (mm)	1.5
Total polishing time (min)	7.2
Polishing cloth thickness (mm)	0.35

Use Taylor profiler PGI2440 to measure the meridian of the mold after form error corrective polishing. The measurement result is shown in figure 25. Before polishing, the PV value of the mold is 0.405μm. After one cycle polishing, the PV value of the mold is reduced to 0.068μm. And the surface roughness measure by white light interferometer is shown as figure 26. Observed from the polishing results, the surface accuracy of the experimental mold has been improved.

Fig 25 Sketch of comparison before and after polishing

Fig 26 Surface roughness after polishing

5 Conclusion

For the polishing of small-size aspheric tungsten carbide molds, the existing polishing methods are limited by the size of the polishing tool, and is difficult to perform form error corrective polishing on small-size molds well. In this paper, a small-size rigid ball wrapped with polyester fiber cloth is used as a polishing tool to adapt to the curvature of the small-diameter aspheric mold. The main findings of this paper are summarized as follows:

(1) In order to predict the pressure distribution of the polishing base area and the contact area, this paper carried out structural simulation analysis, compression non-recoverability experiment and viscoelasticity characteristic measurement experiment on the polyester fiber cloth used in the polishing tool. From the above analysis, it can be seen that the structure of the polyester fiber cloth causes the fluctuation of the polishing pressure; the irreversible deformation of the polyester fiber cloth

causes the polishing contact area to be approximately elliptical; and the relationship between the polishing contact pressure and the deformation is nonlinear. Combining the above analysis of polyester fiber cloth, the pressure distribution of polyester fiber cloth under a certain amount of deformation is obtained.

(2) Perform geometric motion analysis on the position and rotation of the polishing tool in contact with the workpiece, and obtain the velocity distribution of the polishing contact area. Combined with pressure distribution and velocity distribution, the tool influence function of polishing in the form of Preston equation is derived.

(3) In this paper, a series of single-point polishing experiments and simulations of tool influence function are carried out. It is found in the experiment that the polishing dwell time, the relative speed of the workpiece and the polishing tool are directly proportional to the material removal rate, and the polishing pressure is proportional to the material removal rate when the polishing pressure is small. Tool influence function is simulated according to the experimental parameters, and the maximum error between the simulation result and the removal volume rate of the actual polishing tool influence function is less than 10.8%. The accuracy of the theoretical prediction of the tool influence function is verified.

(4) On the basis of obtaining tool influence function, we analyze the surface correction and polishing of the rotationally symmetric small-diameter aspheric mold. The removal of the three-dimensional space is transformed into a two-dimensional tool influence function on the position meridian, and the polishing dwell time is calculated by combining the non-negative least square method. Finally, through the form error corrective polishing test of the rotationally symmetrical aspheric mold, the surface error PV value was reduced from 0.405 μm before polishing to 0.068 μm after polishing, which verified the effectiveness of TIF for micro-aspheric mold in this paper.

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Data availability The data sets supporting the results of this article are included within the article and its additional files.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication We would like to submit the manuscript entitled “Finishing of micro-aspheric tungsten carbide mold using small viscoelastic tool” for your consideration for publication in the International Journal of Advanced Manufacturing Technology. No conflict of interest exists in the submission of this manuscript, and the manuscript is approved by all authors for publication. On behalf of the co-authors, we declare that the work described was original review work that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

Competing interests The authors declare no competing interests.

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