Optimal Unambiguous Discrimination And Quantum Nonlocality Without Entanglement: Locking And Unlocking By Post-Measurement Information

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Optimal unambiguous discrimination and quantum nonlocality without entanglement: locking and unlocking by post-measurement information

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ABSTRACT

The phenomenon of nonlocality without entanglement(NLWE) arises in discriminating multi-party quantum separable states. Recently, it has been found that the post-measurement information about the prepared subensemble can lock or unlock NLWE in minimum-error discrimination of non-orthogonal separable states. Thus it is natrual to ask whether the availability of the post-measurement information can influence on the occurrence of NLWE even in other state-discrimination stratigies. Here, we show that the post-measurement information can be used to lock as well as unlock the occurence of NLWE in terms of optimal nambiguous discrimination. Our results can provide a useful application for hiding or sharing information based on non-orthogonal separable states.

Introduction

Quantum nonlocality is of central importance in multi-party quantum systems. A typical phenomenon of quantum nonlocality is quantum entanglement which is a useful resource for multi-party quantum communication[1]. Quantum entanglement is the correlation that cannot be shared among multiple parties using only *local operations and classical communication*(LOCC)[1–3]. However, it is also known that some nonlocal phenomena in multi-party quantum systems are still possible even in the absence of quantum entanglement.

Nonlocality without entanglement(NLWE) is another nonlocal phenomenon that arises in discriminating non-entangled states of multi-party quantum systems[4,5]. NLWE occurs when what can be achieved with global measurement in discriminating non-entangled states cannot be achieved only by LOCC. In the case of discriminating orthogonal non-entangled states, NLWE occurs when the perfect discrimination cannot be implemented by LOCC[5–9]. On the other hand, in the case of discriminating non-orthogonal non-entangled states, NLWE occurs when the globally optimal discriminations such as *minimum-error discrimination*(ME)[10–13] or *optimal unambiguous discrimination*(OUD)[14–17] cannot be implemented by LOCC[18–21]. We also note that some non-local phenomena without entanglement can occur in the generalized probabilistic theories beyond quantum theory[22].

In quantum state discrimination[23–26], orthogonal states can be perfectly discriminated, whereas non-orthogonal states cannot. However, some non-orthogonal states can be perfectly discriminated when the *post-measurement information*(PI) about the prepared subensemble is available[27]. Nevertheless, some non-orthogonal states cannot be perfectly discriminated even the PI about the prepared subensemble is provided[28–30]. Therefore, in optimal discriminations with the PI about the prepared subensemble, the NLWE phenomenon arises when the globally optimal discrimination cannot be implemented by LOCC with the help of PI. Recently, it was shown that the availability of PI can lock or unlock NLWE in terms of ME[31], therefore it is natural to ask whether the PI affects the occurrence of NLWE in terms of state-discrimination strategies other than ME.

Here, we show that even in OUD, the availability of the PI about the prepared subensemble can affect the occurrence of NLWE. We first provide an ensemble of two-qubit product states having NLWE in terms of OUD, and show that the availability of PI about the prepared subensemble vanishes the occurrence of NLWE, therefore *locking NLWE in terms of OUD by PI*. We further provide another ensemble of two-qubit product states that does not have NLWE in terms of OUD, and show that NLWE in the OUD can be released when the PI about the prepared subensemble is provided. Thus *unlocking NLWE in terms of OUD by PI*.

This paper is organized as follows. First, we present the form of two-qubit product state ensemble to be considered. In the "Methods" Section, we review the definitions and properties with respect to OUD without and with PI and provide some useful lemmas in optimal local discrimination. As a main result of this paper, we provide a quantum state ensemble consisting of four two-qubit product states and show the occurrence of NLWE in terms of OUD. With the same ensemble, we further show that NLWE does not occur in the OUD with the PI about the prepared subensemble is available. As another main result of this paper, we provide another quantum state ensemble consisting of four two-qubit product states and show the non-occurrence of NLWE in terms of OUD. With the same ensemble, we further show that NLWE occurs in the OUD with the PI about the prepared subensemble.

Results

Throughout this paper, we only consider the situation of unambiguously discriminating four states from the quantum state ensemble,

$$\mathscr{E}=\{\eta_i,\rho_i\}_{i\in\Lambda},\ \Lambda=\{0,1,+,-\}, \tag{1}$$

where ρ_i is a $2\otimes 2$ non-entangled pure state,

$$\rho_i=|\varphi_i\rangle\langle\varphi_i| \ \text{ for each } i\in\Lambda, \tag{2}$$

and $\{|\varphi_i\rangle\}_{i\in\Lambda}$ is a product basis of $\mathscr{H}$. Each η_i is the probability that the state ρ_i is prepared.

The ensemble $\mathscr{E}$ can be seen as an ensemble consisting of two subensembles,

$$\begin{aligned}&\mathscr{E}_0=\{\eta_i/\textstyle\sum_{j\in\mathsf{A}_0}\eta_j,\rho_i\}_{i\in\mathsf{A}_0}, \quad \mathsf{A}_0=\{0,1\},\\ &\mathscr{E}_1=\{\eta_i/\textstyle\sum_{j\in\mathsf{A}_1}\eta_j,\rho_i\}_{i\in\mathsf{A}_1}, \quad \mathsf{A}_1=\{+,-\},\end{aligned} \tag{3}$$

where $\mathscr{E}_0$ and $\mathscr{E}_1$ are prepared with probabilities $\sum_{j\in\mathsf{A}_0}\eta_j$ and $\sum_{j\in\mathsf{A}_1}\eta_j$, respectively. The definitions and properties related to OUD of $\mathscr{E}$ without and with PI are provided in the "Methods" Section.

Locking NLWE by PI in OUD

In this section, we consider a situation where PI about the prepared subensemble $\mathscr{E}_b$ *locks* NLWE in terms of OUD. We first provide a specific example of a state ensemble $\mathscr{E}$ and show that NLWE in terms of OUD occurs. With the same ensemble, we further show that the occurrence of NLWE in terms of OUD can be vanished when PI is provided, thus locking NLWE by PI.

Example 1[31]**.** *Let us consider the ensemble $\mathscr{E}$ in Eq. (1) with*

$$\begin{array}{lllllllll}\eta_0&=&\frac{\gamma}{2(1+\gamma)},&\rho_0&=&|\varphi_0\rangle\langle\varphi_0|,&|\varphi_0\rangle&=&|0\rangle\otimes|0\rangle,\\ \eta_1&=&\frac{\gamma}{2(1+\gamma)},&\rho_1&=&|\varphi_1\rangle\langle\varphi_1|,&|\varphi_1\rangle&=&|0\rangle\otimes|1\rangle,\\ \eta_+&=&\frac{1}{2(1+\gamma)},&\rho_+&=&|\varphi_+\rangle\langle\varphi_+|,&|\varphi_+\rangle&=&|+\rangle\otimes|+\rangle,\\ \eta_-&=&\frac{1}{2(1+\gamma)},&\rho_-&=&|\varphi_-\rangle\langle\varphi_-|,&|\varphi_-\rangle&=&|-\rangle\otimes|-\rangle,\end{array} \tag{4}$$

where $2\leqslant\gamma<\infty$, $\{|0\rangle,|1\rangle\}$ is the standard basis in one-qubit system, and $|\pm\rangle=\frac{1}{\sqrt{2}}(|0\rangle\pm|1\rangle)$. In this case, the subensembles in Eq. (3) become

$$\begin{aligned}&\mathscr{E}_0=\{\tfrac{1}{2},|0\rangle\langle 0|\otimes|0\rangle\langle 0|,\ \tfrac{1}{2},|0\rangle\langle 0|\otimes|1\rangle\langle 1|\},\\ &\mathscr{E}_1=\{\tfrac{1}{2},|+\rangle\langle +|\otimes|+\rangle\langle +|,\ \tfrac{1}{2},|-\rangle\langle -|\otimes|-\rangle\langle -|\},\end{aligned} \tag{5}$$

with the probabilities of preparation $\frac{\gamma}{1+\gamma}$ and $\frac{1}{1+\gamma}$, respectively.

To show the occurrence of NLWE in terms of OUD about the ensemble $\mathscr{E}$ in Example 1, we first evaluate the optimal success probability $p_{\mathrm{G}}(\mathscr{E})$ defined in Eq. (42) of the "Methods" Section. The reciprocal vectors $\{|\tilde{\varphi}_i\rangle\}_{i\in\Lambda}$ corresponding to $\{|\varphi_i\rangle\}_{i\in\Lambda}$ defined in Eq. (4) are

$$\begin{aligned}&|\tilde{\varphi}_0\rangle=\sqrt{2}|\Phi_-\rangle,\ |\tilde{\varphi}_+\rangle=\sqrt{2}|1+\rangle,\\ &|\tilde{\varphi}_1\rangle=\sqrt{2}|\Psi_-\rangle,\ |\tilde{\varphi}_-\rangle=-\sqrt{2}|1-\rangle.\end{aligned} \tag{6}$$

where

$$|\Phi_\pm\rangle=\tfrac{1}{\sqrt{2}}|00\rangle\pm\tfrac{1}{\sqrt{2}}|11\rangle,\ |\Psi_\pm\rangle=\tfrac{1}{\sqrt{2}}|01\rangle\pm\tfrac{1}{\sqrt{2}}|10\rangle. \tag{7}$$

We can easily verify that the following $\{M_i\}_{i\in\overline{\Lambda}}$ is an unambiguous POVM satisfying the error-free condition in Eq. (41):

$$\begin{aligned}
&M_0 = |\Phi_-\rangle\langle\Phi_-|,\ M_+ = 0,\\
&M_1 = |\Psi_-\rangle\langle\Psi_-|,\ M_- = 0,\\
&M_? = \mathbb{1} - |\Phi_+\rangle\langle\Phi_+| - |\Psi_+\rangle\langle\Psi_+|,
\end{aligned} \tag{8}$$

Also, it is optimal because Condition (43) holds for this unambiguous POVM along with a positive-semidefinite operator

$$K = \tfrac{\gamma}{4(1+\gamma)}(|\Phi_-\rangle\langle\Phi_-| + |\Psi_-\rangle\langle\Psi_-|). \tag{9}$$

Thus, the optimality of the POVM $\{M_i\}_{i\in\overline{\Lambda}}$ in Eq. (8) and the definition of $p_{\mathrm{G}}(\mathscr{E})$ lead us to

$$p_{\mathrm{G}}(\mathscr{E}) = \mathrm{Tr}K = \tfrac{\gamma}{2(1+\gamma)} = \eta_0. \tag{10}$$

In order to obtain the maximum success probability $p_{\mathrm{L}}(\mathscr{E})$ defined in Eq. (45) of the "Methods" Section, we consider lower and upper bounds of $p_{\mathrm{L}}(\mathscr{E})$. A lower bound of $p_{\mathrm{L}}(\mathscr{E})$ can be obtained from the following unambiguous measurement $\{M_i\}_{i\in\overline{\Lambda}}$,

$$\begin{aligned}
&M_0 = 0,\ M_+ = |1\rangle\langle 1| \otimes |+\rangle\langle +|,\\
&M_1 = 0,\ M_- = |1\rangle\langle 1| \otimes |-\rangle\langle -|,\\
&M_? = |0\rangle\langle 0| \otimes (|+\rangle\langle +| + |-\rangle\langle -|),
\end{aligned} \tag{11}$$

which can be implemented by finite-round LOCC because it can be realized by performing local measurements $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$ and $\{|+\rangle\langle +|, |-\rangle\langle -|\}$ on first and second subsystems, respectively. As we can easily verify that the success probability for the unambiguous LOCC measurement in Eq. (11) is $\frac{1}{2(1+\gamma)}$, the success probability is obviously a lower bound of $p_{\mathrm{L}}(\mathscr{E})$,

$$p_{\mathrm{L}}(\mathscr{E}) \geqslant \tfrac{1}{2(1+\gamma)} = \eta_+. \tag{12}$$

To obtain an upper bound of $p_{\mathrm{L}}(\mathscr{E})$, let us consider a positive-semidefinite operator

$$H = \tfrac{1}{4(1+\gamma)}|1\rangle\langle 1| \otimes (|+\rangle\langle +| + |-\rangle\langle -|) \tag{13}$$

with

$$\langle\tilde{\varphi}_+|H|\tilde{\varphi}_+\rangle = \eta_+ = \eta_- = \langle\tilde{\varphi}_-|H|\tilde{\varphi}_-\rangle. \tag{14}$$

Lemma 1 in the "Methods" Section leads us to

$$p_{\mathrm{L}}(\mathscr{E}) \leqslant \mathrm{Tr}H = \tfrac{1}{2(1+\gamma)} = \eta_+. \tag{15}$$

Inequalities (12) and (15) imply

$$p_{\mathrm{L}}(\mathscr{E}) = \eta_+. \tag{16}$$

From Eqs. (10) and (16), we note that there exists a nonzero gap between $p_{\mathrm{G}}(\mathscr{E})$ and $p_{\mathrm{L}}(\mathscr{E})$,

$$p_{\mathrm{L}}(\mathscr{E}) = \eta_+ < \eta_0 = p_{\mathrm{G}}(\mathscr{E}), \tag{17}$$

thus NLWE occurs in terms of OUD in discriminating the states of the ensemble $\mathscr{E}$ in Example 1.

Now, we show that the availability of PI about the prepared subensemble vanishes the occurrence of NLWE in Inequality (17). To show it, we use the fact that the states of $\mathscr{E}$ in Example 1 can be unambiguously discriminated without inconclusive results using LOCC when the PI about the prepared subensemble is available[31], or equivalently,

$$p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E}) \geqslant 1. \tag{18}$$

From the definitions of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ and $p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E})$, we note that

$$p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E}) \geqslant p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E}). \tag{19}$$

As both $p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E})$ and $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ are bound above by 1, we have

$$p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E}) = p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E}) = 1. \tag{20}$$

Thus, NLWE does not occur in terms OUD in discriminating the states of the ensemble $\mathscr{E}$ in Example 1 when the PI about the prepared subensemble is available.

Inequality (17) shows that NLWE occurs in terms of OUD about the ensemble $\mathscr{E}$ in Example 1, whereas Eq. (20) shows that NLWE does not occur when PI is available. Figure 1 illustrates the relative order of $p_{\mathrm{G}}(\mathscr{E})$, $p_{\mathrm{L}}(\mathscr{E})$, $p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E})$, and $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ for the range of $\frac{1}{3} \leqslant \eta_0 < \frac{1}{2}$.

Theorem 1. *For OUD of the ensemble $\mathscr{E}$ in Example 1, the PI about the prepared subensemble locks NLWE.*

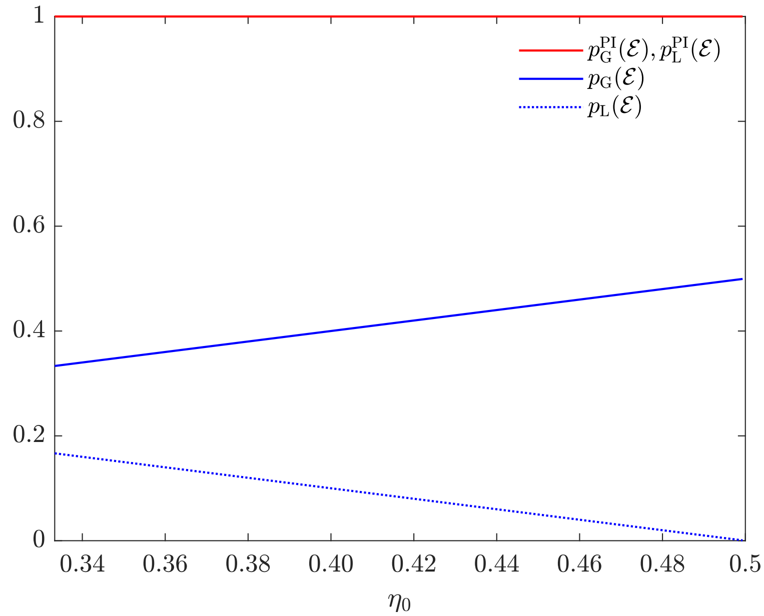

Figure 1. Locking NLWE by PI in terms of OUD. For all $\eta_0 \in [\frac{1}{3}, \frac{1}{2})$, $p_L(\mathscr{E})$(dashed blue) is less than $p_G(\mathscr{E})$(solid blue), but $p_L^{PI}(\mathscr{E})$(red) is equal to $p_G^{PI}(\mathscr{E})$(red).

Unlocking NLWE by PI in OUD

In this section, we consider the opposite situation to the previous section; the PI about the prepared subensemble $\mathscr{E}_b$ in Eq. (3) *unlocks* NLWE. After providing an example of a state ensemble $\mathscr{E}$, we first show that NLWE in terms of OUD does not occur in discriminating the states of the ensemble. With the same ensemble, we further show the occurrence of NLWE in terms of OUD in the state discrimination with the help of PI, thus unlocking NLWE by PI.

Example 2[31]**.** *Let us consider the ensemble $\mathscr{E}$ in Eq. (1) with*

$$\begin{array}{llllll} \eta_0 &= \frac{\gamma}{2(1+\gamma)}, & \rho_0 &= |\varphi_0\rangle\langle\varphi_0|, & |\varphi_0\rangle &= |0\rangle\otimes|0\rangle, \\ \eta_1 &= \frac{\gamma}{2(1+\gamma)}, & \rho_1 &= |\varphi_1\rangle\langle\varphi_1|, & |\varphi_1\rangle &= |0\rangle\otimes|1\rangle, \\ \eta_+ &= \frac{1}{2(1+\gamma)}, & \rho_+ &= |\varphi_+\rangle\langle\varphi_+|, & |\varphi_+\rangle &= |+\rangle\otimes|+\rangle, \\ \eta_- &= \frac{1}{2(1+\gamma)}, & \rho_- &= |\varphi_-\rangle\langle\varphi_-|, & |\varphi_-\rangle &= |+\rangle\otimes|-\rangle, \end{array} \tag{21}$$

where $2 \leqslant \gamma < \infty$. In this case, the subensembles in Eq. (3) become

$$\begin{array}{l} \mathscr{E}_0 = \{\frac{1}{2}, |0\rangle\langle 0|\otimes|0\rangle\langle 0|,\ \frac{1}{2}, |0\rangle\langle 0|\otimes|1\rangle\langle 1|\}, \\ \mathscr{E}_1 = \{\frac{1}{2}, |+\rangle\langle +|\otimes|+\rangle\langle +|,\ \frac{1}{2}, |+\rangle\langle +|\otimes|-\rangle\langle -|\}, \end{array} \tag{22}$$

with the probabilities of preparation $\frac{\gamma}{1+\gamma}$ and $\frac{1}{1+\gamma}$, respectively.

To show the non-occurrence of NLWE in terms of OUD about the ensemble $\mathscr{E}$ in Example 2, we first evaluate the optimal success probability $p_G(\mathscr{E})$ defined in Eq. (42) of the "Methods" Section. Since the reciprocal vectors $\{|\tilde{\varphi}_i\rangle\}_{i\in\Lambda}$ corresponding to $\{|\varphi_i\rangle\}_{i\in\Lambda}$ defined in Eq. (21) are

$$\begin{array}{l} |\tilde{\varphi}_0\rangle = \sqrt{2}|-\rangle\otimes|0\rangle,\ |\tilde{\varphi}_+\rangle = \sqrt{2}|1\rangle\otimes|+\rangle, \\ |\tilde{\varphi}_1\rangle = \sqrt{2}|-\rangle\otimes|1\rangle,\ |\tilde{\varphi}_-\rangle = \sqrt{2}|1\rangle\otimes|-\rangle, \end{array} \tag{23}$$

the following POVM $\{M_i\}_{i\in\overline{\Lambda}}$ satisfies the error-free condition in Eq. (41),

$$\begin{array}{l} M_0 = |-\rangle\langle -|\otimes|0\rangle\langle 0|,\ M_+ = 0, \\ M_1 = |-\rangle\langle -|\otimes|1\rangle\langle 1|,\ M_- = 0, \\ M_? = |+\rangle\langle +|\otimes(|0\rangle\langle 0| + |1\rangle\langle 1|). \end{array} \tag{24}$$

Moreover, the unambiguous measurement is optimal because Condition (43) holds for this unambiguous POVM along with the following positive-semidefinite operator

$$K = \tfrac{\gamma}{4(1+\gamma)}|-\rangle\langle -|\otimes(|0\rangle\langle 0|+|1\rangle\langle 1|). \tag{25}$$

Thus, the optimality of the POVM $\{M_i\}_{i\in\overline{\Lambda}}$ in Eq. (24) and the definition of $p_{\mathrm{G}}(\mathscr{E})$ lead us to

$$p_{\mathrm{G}}(\mathscr{E}) = \mathrm{Tr}K = \tfrac{\gamma}{2(1+\gamma)} = \eta_0. \tag{26}$$

The POVM given in Eq. (24) can be perforemed using finite-round LOCC; two local measurements $\{|+\rangle\langle +|, |-\rangle\langle -|\}$ and $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$ are perforemed on first and second subsystems, respectively. Thus, the success probability for the unambiguous LOCC measurement in Eq. (24) is a lower bound of $p_{\mathrm{L}}(\mathscr{E})$ defined in Eq. (45), therefore

$$p_{\mathrm{L}}(\mathscr{E}) \geqslant \eta_0, \tag{27}$$

Moreover, from the definition of $p_{\mathrm{G}}(\mathscr{E})$ and $p_{\mathrm{L}}(\mathscr{E})$ in Eqs. (42) and (45), respectively, we have

$$p_{\mathrm{G}}(\mathscr{E}) \geqslant p_{\mathrm{L}}(\mathscr{E}). \tag{28}$$

Inequalities (27) and (28) lead us to

$$p_{\mathrm{L}}(\mathscr{E}) = p_{\mathrm{G}}(\mathscr{E}) = \eta_0. \tag{29}$$

Thus, NLWE does not occur in terms of OUD in discriminating the states of the ensemble $\mathscr{E}$ in Example 2.

Now, we show that NLWE in terms of OUD occurs when the PI about the prepared subensemble is available. To show it, we use the fact that the states of $\mathscr{E}$ in Example 2 can be unambiguously discriminated without inconclusive results when the PI about the prepared subensemble is available[31], or equivalently,

$$p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E}) \geqslant 1. \tag{30}$$

As $p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E})$ is bound above by 1, we have

$$p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E}) = 1. \tag{31}$$

To obtain the maximum success probability $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ in Eq. (52) of the "Methods" Section, we consider lower and upper bounds of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$. For a lower bound of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$, let us first consider the following measurement $\{M_{\vec{\omega}}\}_{\vec{\omega}\in\Omega}$,

$$\begin{aligned}
&M_{(0,?)} = |\nu_-\rangle\langle\nu_-|\otimes|0\rangle\langle 0|,\ M_{(?,+)} = |\nu_+\rangle\langle\nu_+|\otimes|+\rangle\langle +|,\\
&M_{(1,?)} = |\nu_-\rangle\langle\nu_-|\otimes|1\rangle\langle 1|,\ M_{(?,-)} = |\nu_+\rangle\langle\nu_+|\otimes|-\rangle\langle -|,\\
&M_{\vec{\omega}} = 0\ \forall\vec{\omega}\in\{(0,+),(0,-),(1,+),(1,-),(?,?)\},
\end{aligned} \tag{32}$$

where

$$|\nu_\pm\rangle = \sqrt{\tfrac{1}{2}\mp\tfrac{\gamma}{2\sqrt{1+\gamma^2}}}|0\rangle \pm \sqrt{\tfrac{1}{2}\pm\tfrac{\gamma}{2\sqrt{1+\gamma^2}}}|1\rangle. \tag{33}$$

The POVM given in Eq. (32) is unambiguous because it satisfies the error-free condition in Eq. (50). Moreover, this measurement can be performed with finite-round LOCC; we first measure $\{|\nu_+\rangle\langle\nu_+|, |\nu_-\rangle\langle\nu_-|\}$ on first subsystem, and then measure $\{|+\rangle\langle +|, |-\rangle\langle -|\}$ or $\{|0\rangle\langle 0|, |1\rangle\langle 1|\}$ on second subsystem depending on the first measurement result $|\nu_+\rangle\langle\nu_+|$ or $|\nu_-\rangle\langle\nu_-|$. As we can verify from a straightforward calculation that the success probability for the unambiguous LOCC measurement in Eq. (32) is

$$\sum_{b\in\{0,1\}}\sum_{i\in\mathsf{A}_b}\eta_i\mathrm{Tr}\Big[\rho_i\sum_{\substack{\vec{\omega}\in\Omega\\ \omega_b=i}}M_{\vec{\omega}}\Big] = \sum_{i\in\mathsf{A}_0}\eta_i\mathrm{Tr}(\rho_i M_{(i,?)}) + \sum_{j\in\mathsf{A}_1}\eta_j\mathrm{Tr}(\rho_j M_{(?,j)}) = \frac{1}{2}\Big(1+\frac{\sqrt{1+\gamma^2}}{1+\gamma}\Big), \tag{34}$$

thus the definition of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ lead us to

$$p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E}) \geqslant \tfrac{1}{2}\Big(1+\tfrac{\sqrt{1+\gamma^2}}{1+\gamma}\Big). \tag{35}$$

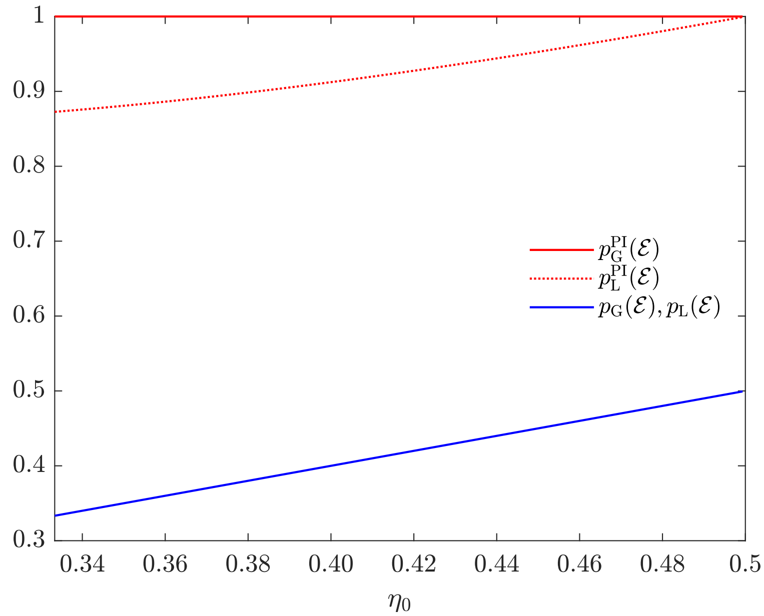

Figure 2. Unlocking NLWE by PI in terms of OUD. For all $\eta_0 \in [\frac{1}{3}, \frac{1}{2})$, $p_{\rm L}(\mathscr{E})$(blue) is equal to $p_{\rm G}(\mathscr{E})$(blue), but $p_{\rm L}^{\rm PI}(\mathscr{E})$(dashed red) is less than $p_{\rm G}^{\rm PI}(\mathscr{E})$(solid red).

We also note that the measurement in Eq. (32) yields $p_{\rm guess}(\mathscr{E})$ defined in Eq. (56) of the "Methods" Section when considering $M_0 = M_{(0,?)}$, $M_1 = M_{(1,?)}$, $M_+ = M_{(?,+)}$, and $M_- = M_{(?,-)}$[31], that is,

$$p_{\rm guess}(\mathscr{E}) = \tfrac{1}{2}\left(1 + \tfrac{\sqrt{1+\gamma^2}}{1+\gamma}\right). \tag{36}$$

In order to obtain an upper bound of $p_{\rm L}^{\rm PI}(\mathscr{E})$, let us consider the assumption of Lemma 2 in the "Methods" Section. For each $(\omega_0, \omega_1) \in \mathsf{A}_0 \times \mathsf{A}_1$, there does not exist any nonzero product vector $|v\rangle = |a\rangle \otimes |b\rangle$ satisfying Condition (57); otherwise, $|a\rangle$ is not orthogonal to both $|0\rangle$ and $|+\rangle$. At the same time, $|b\rangle$ is orthogonal to the $|k\rangle$'s with $k \in \Lambda \setminus \{\omega_0, \omega_1\}$, which leads us a contradiction. Thus, the guessing probability of $\mathscr{E}$ is also an upper bound of $p_{\rm L}^{\rm PI}(\mathscr{E})$ due to Lemma 2 in the "Methods" Section, that is,

$$p_{\rm L}^{\rm PI}(\mathscr{E}) \leqslant p_{\rm guess}(\mathscr{E}) = \tfrac{1}{2}\left(1 + \tfrac{\sqrt{1+\gamma^2}}{1+\gamma}\right). \tag{37}$$

Inequalities (35) and (37) imply

$$p_{\rm L}^{\rm PI}(\mathscr{E}) = \tfrac{1}{2}\left(1 + \tfrac{\sqrt{1+\gamma^2}}{1+\gamma}\right). \tag{38}$$

From Eqs. (31) and (38), we note that there exists a nonzero gap between $p_{\rm G}^{\rm PI}(\mathscr{E})$ and $p_{\rm L}^{\rm PI}(\mathscr{E})$,

$$p_{\rm L}^{\rm PI}(\mathscr{E}) = \tfrac{1}{2}\left(1 + \tfrac{\sqrt{1+\gamma^2}}{1+\gamma}\right) < 1 = p_{\rm G}^{\rm PI}(\mathscr{E}). \tag{39}$$

Thus, NLWE occurs in terms of OUD when the PI about the prepared subensemble is available.

Equation (29) shows that NLWE in terms of OUD does not occur in discriminating the states of the ensemble $\mathscr{E}$ in Example 2, whereas Inequality (39) shows that NLWE occurs when PI is available. Figure 2 illustrates the relative order of $p_{\rm G}(\mathscr{E})$, $p_{\rm L}(\mathscr{E})$, $p_{\rm G}^{\rm PI}(\mathscr{E})$, and $p_{\rm L}^{\rm PI}(\mathscr{E})$ for the range of $\frac{1}{3} \leqslant \eta_0 < \frac{1}{2}$.

Theorem 2. *For OUD of the ensemble $\mathscr{E}$ in Example 2, the PI about the prepared subensemble unlocks NLWE.*

Discussion

We have shown that the PI about the prepared subensemble can lock or unlock NLWE in terms of OUD. We have provided a quantum state ensemble consisting of four $2 \otimes 2$ non-entangled pure states (Example 1) and shown the occurrence of NLWE in terms of OUD with respect to the ensemble. With the same state ensemble, we have further shown that the availability of PI about the prepared subensemble vanishes the occurrence of NLWE, thus locking NLWE in terms of OUD by PI (Theorem 1). Moreover, we have provided another quantum state ensemble consisting of four $2\otimes 2$ non-entangled pure states (Example 2) and shown the non-occurrence of NLWE in terms of OUD with respect to the ensemble. With the same state ensemble, we have further shown the occurrence of NLWE in the OUD with the PI about the prepared subensemble, thus unlocking NLWE in terms of OUD by PI (Theorem 2).

We remark that the two state ensembles of this paper can also be used to demonstrate locking and unlocking NLWE in terms of ME[31]. Thus, it is a natural future work to investigate locking and unlocking NLWE even in generalized state discrimination strategies such as an optimal discrimination with a fixed rate of inconclusive results[32–36].

Our results can also provide us with a useful application in quantum cryptography. Whereas the existing quantum data hiding and secret sharing schemes are based on orthogonal states[37–41], our results can extend those schemes to improved ones using non-orthogonal states. In Example 1, the availability of the PI about the prepared subensemble makes the globally hidden information accessible locally. On the other hand, in Example 2, the PI makes locally accessible information hidden locally but accessible globally. Finally, it is an interesting task to investigate if locking or unlocking NLWE by the PI about the prepared subensemble can depend on nonzero prior probabilities.

Methods

In two-qubit (or $2\otimes 2$) systems, a state and a measurement are expressed by a density operator and a positive operator valued measure(POVM), respectively, acting on a two-party complex Hilbert space $\mathbb{C}^2\otimes\mathbb{C}^2$. A density operator ρ is a positive-semidefinite operator $\rho\succeq 0$ with unit trace $\mathrm{Tr}\rho=1$ and a POVM $\{M_i\}_i$ is a set of positive-semidefinite operators $M_i\succeq 0$ satisfying $\sum_i M_i=\mathbb{1}$, where $\mathbb{1}$ is the identity operator on $\mathbb{C}^2\otimes\mathbb{C}^2$. The probability of obtaining the measurement outcome corresponding to M_i is $\mathrm{Tr}(\rho M_i)$ when $\{M_i\}_i$ is performed on a quantum system prepared with ρ.

A positive-semidefinite operator is called *separable*(or *non-entangled*) if it is a sum of positive-semidefinite product operators; otherwise, it is said to be *entangled*. Also, a POVM is called *separable* if all elements are separable. In particular, a *LOCC measurement* that can be realized by LOCC is a separable measurement[2].

Optimal unambiguous discrimination

Let us consider the unambiguous discrimination of the states in $\mathscr{E}$ of Eq. (1) using a measurement $\{M_i\}_{i\in\overline{\Lambda}}$, where

$$\overline{\Lambda}=\Lambda\cup\{?\}=\{0,1,+,-,?\}. \tag{40}$$

For each $i\in\Lambda$, M_i is to detect ρ_i, and $M_?$ gives inconclusive results: "I don't know what state is prepared." The measurement $\{M_i\}_{i\in\overline{\Lambda}}$ can be expressed as

$$M_i=s_i|\tilde{\varphi}_i\rangle\langle\tilde{\varphi}_i|\ \ \forall i\in\Lambda,\ M_?=\mathbb{1}-\textstyle\sum_{j\in\Lambda}s_j|\tilde{\varphi}_j\rangle\langle\tilde{\varphi}_j|, \tag{41}$$

where $\{s_i\}_{i\in\Lambda}$ is a non-negative number set and $\{|\tilde{\varphi}_i\rangle\}_{i\in\Lambda}$ is the set of reciprocal vectors corresponding to $\{|\varphi_i\rangle\}_{i\in\Lambda}$ in Eq. (2) such that $\langle\varphi_i|\tilde{\varphi}_j\rangle=\delta_{ij}$[42]. We say a POVM $\{M_i\}_{i\in\overline{\Lambda}}$ is *unambiguous* if it satisfies the error-free condition in Eq. (41).

OUD of $\mathscr{E}$ is to minimize the probability of obtaining inconclusive results. Equivalently, OUD of $\mathscr{E}$ is to maximize the average probability of unambiguously discriminating states in $\mathscr{E}$;

$$p_{\mathrm{G}}(\mathscr{E})=\max_{\text{Eq.(41)}}\sum_{i\in\Lambda}\eta_i\mathrm{Tr}(\rho_iM_i) \tag{42}$$

where the maximum is taken over all possible unambiguous POVMs satisfying the error-free condition in Eq. (41). It is known that an unambiguous POVM $\{M_i\}_{i\in\overline{\Lambda}}$ is optimal if and only if there is a positive-semidefinite operator K satisfying the following condition[21,43–45],

$$\langle\tilde{\varphi}_i|K|\tilde{\varphi}_i\rangle\geqslant\eta_i\ \forall i\in\Lambda,\ \mathrm{Tr}[M_i(K-\eta_i\rho_i)]=0\ \forall i\in\Lambda,\ \mathrm{Tr}(M_?K)=0. \tag{43}$$

In this case, we have

$$p_{\mathrm{G}}(\mathscr{E})=\sum_{i\in\Lambda}\eta_i\mathrm{Tr}(\rho_iM_i)=\mathrm{Tr}K \tag{44}$$

if an unambiguous POVM $\{M_i\}_{i\in\overline{\Lambda}}$ and a positive-semidefinite operator K satisfy Condition (43)[21,43–45].

When the available measurements are restricted to unambiguous LOCC measurements, we denote the maximum success probability by

$$p_{\rm L}(\mathscr{E}) = \max_{\substack{\text{Eq.(41)} \\ \text{LOCC}}} \sum_{i\in\Lambda} \eta_i \mathrm{Tr}(\rho_i M_i). \tag{45}$$

Because the states of $\mathscr{E}$ are non-entangled, NLWE occurs in terms of OUD if and only if OUD of $\mathscr{E}$ cannot be achieved using LOCC, that is,

$$p_{\rm L}(\mathscr{E}) < p_{\rm G}(\mathscr{E}). \tag{46}$$

In the following lemma, we provide an upper bound of $p_{\rm L}(\mathscr{E})$.

Lemma 1. *If H is a positive-semidefinite operator satisfying*

$$\langle\tilde{\varphi}_i|H|\tilde{\varphi}_i\rangle \geqslant \eta_i \tag{47}$$

for all reciprocal vectors $|\tilde{\varphi}_i\rangle$ that is a product vector, then $\mathrm{Tr}H$ *is an upper bound of* $p_{\rm L}(\mathscr{E})$.

Proof. Let us suppose that $\{M_i\}_{i\in\overline{\Lambda}}$ is an unambiguous LOCC measurement and χ is the set of all $i\in\Lambda$ such that $|\tilde{\varphi}_i\rangle$ is a product vector. Since every LOCC measurement is separable, M_i is separable for all $i\in\overline{\Lambda}$. For all $i\in\Lambda$ with $i\notin\chi$, $M_i=0$ because M_i is proportional to entangled $|\tilde{\varphi}_i\rangle\langle\tilde{\varphi}_i|$. Thus, the success probability is

$$\textstyle\sum_{i\in\chi}\eta_i\mathrm{Tr}(\rho_iM_i) \leqslant \sum_{i\in\chi}\eta_i\mathrm{Tr}(\rho_iM_i)+\sum_{i\in\chi}\mathrm{Tr}[(H-\eta_i\rho_i)M_i]+\mathrm{Tr}(HM_?) = \mathrm{Tr}H, \tag{48}$$

where the inequality is due to the assumption of Inequality (47) and the positive-semidefiniteness of H and $M_?$, and the equality is from $M_? = \mathbb{1}-\sum_{i\in\chi}M_i$. As Inequality (48) is true for any unambiguous LOCC measurement $\{M_i\}_{i\in\overline{\Lambda}}$, $\mathrm{Tr}H$ is an upper bound of $p_{\rm L}(\mathscr{E})$. □

Optimal unambiguous discrimination with postmeasurement information

Let us consider the situation of unambiguously discriminating the two-qubit states of $\mathscr{E}$ in Eq. (1) when the classical information $b\in\{0,1\}$ about the prepared subensemble $\mathscr{E}_b$ defined in Eq. (3) is given after performing a measurement. We use a POVM $\{M_{\vec{\omega}}\}_{\vec{\omega}\in\Omega}$ to unambiguously discriminate the states of $\mathscr{E}$ in Eq. (1), where Ω is the Cartesian product of two outcome sets $\mathsf{A}_0\cup\{?\}$ and $\mathsf{A}_1\cup\{?\}$ with inconclusive results,

$$\begin{aligned}\Omega &= \{(\omega_0,\omega_1)\,|\,\omega_0\in\mathsf{A}_0\cup\{?\},\,\omega_1\in\mathsf{A}_1\cup\{?\}\}\\ &= \{(0,+),(0,-),(1,+),(1,-),(0,?),(1,?),(?,+),(?,-),(?,?)\}.\end{aligned} \tag{49}$$

For each $(\omega_0,\omega_1)\in\Omega$, $M_{(\omega_0,\omega_1)}$ detects a state in $\mathscr{E}$ unambiguously or gives inconclusive results depending on PI $b\in\{0,1\}$. If $\omega_b\neq ?$, the state ρ_{ω_b} is detected unambiguously, that is, the POVM $\{M_{\vec{\omega}}\}_{\vec{\omega}\in\Omega}$ satisfies

$$\begin{aligned}&\mathrm{Tr}[\rho_-M_{(i,+)}]=\mathrm{Tr}[\rho_+M_{(i,-)}]=0\quad\forall i\in\mathsf{A}_0\cup\{?\},\\ &\mathrm{Tr}[\rho_1M_{(0,j)}]=\mathrm{Tr}[\rho_0M_{(1,j)}]=0\quad\forall j\in\mathsf{A}_1\cup\{?\}.\end{aligned} \tag{50}$$

However, if $\omega_b = ?$, inconclusive results are obtained. We say that a POVM $\{M_{\vec{\omega}}\}_{\vec{\omega}\in\Omega}$ is *unambiguous* if it satisfies the error-free condition in Eq. (50).

OUD of $\mathscr{E}$ with PI is to minimize the probability of obtaining inconclusive results. Equivalently, OUD of $\mathscr{E}$ with PI is to maximize the average probability of unambiguously discriminating states where the optimal success probability is defined as

$$p_{\rm G}^{\rm PI}(\mathscr{E}) = \max_{\text{Eq.(50)}} \sum_{b\in\{0,1\}}\sum_{i\in\mathsf{A}_b}\eta_i\mathrm{Tr}\Big[\rho_i\sum_{\substack{\vec{\omega}\in\Omega\\ \omega_b=i}}M_{\vec{\omega}}\Big] \tag{51}$$

over all possible unambiguous measurements in Eq. (50).

When the available measurements are limited to unambiguous LOCC measurements, we denote the maximum success probability by

$$p_{\rm L}^{\rm PI}(\mathscr{E}) \;=\; \max_{\substack{\text{Eq.(50)}\\ \text{LOCC}}} \sum_{b\in\{0,1\}}\sum_{i\in\mathsf{A}_b}\eta_i\mathrm{Tr}\Big[\rho_i\sum_{\substack{\vec{\omega}\in\Omega\\ \omega_b=i}}M_{\vec{\omega}}\Big]. \tag{52}$$

We note that $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ in Eq. (52) can also be rewritten as

$$p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})=\max_{\substack{\text{Eq.(50)}\\ \text{LOCC}}}\left[\sum_{\vec{\omega}\in\mathsf{A}_0\times\mathsf{A}_1}\tilde{\eta}_{\vec{\omega}}\mathrm{Tr}(\tilde{\rho}_{\vec{\omega}}M_{\vec{\omega}})+\sum_{i\in\mathsf{A}_0}\eta_i\mathrm{Tr}(\rho_iM_{(i,?)})+\sum_{j\in\mathsf{A}_1}\eta_j\mathrm{Tr}(\rho_jM_{(?,j)})\right],\tag{53}$$

where

$$\tilde{\eta}_{\vec{\omega}}=\tfrac{1}{2}\textstyle\sum_{b\in\{0,1\}}\eta_{w_b},\ \tilde{\rho}_{\vec{\omega}}=\frac{\sum_{b\in\{0,1\}}\eta_{w_b}\rho_{\omega_b}}{\sum_{b'\in\{0,1\}}\eta_{w_{b'}}}.\tag{54}$$

Since the states of $\mathscr{E}$ are non-entangled, NLWE occurs in terms of OUD with PI if and only if OUD of $\mathscr{E}$ with PI cannot be achieved using LOCC, that is,

$$p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})<p_{\mathrm{G}}^{\mathrm{PI}}(\mathscr{E}).\tag{55}$$

For an upper bound of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$, let us consider the following quantity,

$$p_{\mathrm{guess}}(\mathscr{E})=\max_{\substack{\{M_i\}_{i\in\Lambda}:\\ \text{POVM}}}\sum_{i\in\Lambda}\eta_i\mathrm{Tr}(\rho_iM_i),\tag{56}$$

which is the maximum average probability of correct guessing the prepared state when the available measurements are limited to LOCC measurements without inconclusive results[10–13]. The following lemma shows that $p_{\mathrm{guess}}(\mathscr{E})$ can be used as an upper bound of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$.

Lemma 2. *For each $(\omega_0,\omega_1)\in\mathsf{A}_0\times\mathsf{A}_1$, if there is no nonzero product vector $|v\rangle$ satisfying*

$$\langle\varphi_i|v\rangle\neq0\ \forall i\in\{\omega_0,\omega_1\},\ \langle\varphi_j|v\rangle=0\ \forall j\in\Lambda\setminus\{\omega_0,\omega_1\},\tag{57}$$

then $p_{\mathrm{guess}}(\mathscr{E})$ is an upper bound of $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$.

Proof. The assumption in (57) implies that for each $\vec{\omega}=(\omega_0,\omega_1)\in\mathsf{A}_0\times\mathsf{A}_1$, there does not exist any nonzero separable $M_{\vec{\omega}}\succeq0$ that unambiguously detects the state ρ_{ω_0} or ρ_{ω_1} depending on PI $b=0$ or 1, respectively. Then, the term $\sum_{\vec{\omega}\in\mathsf{A}_0\times\mathsf{A}_1}\tilde{\eta}_{\vec{\omega}}\mathrm{Tr}(\tilde{\rho}_{\vec{\omega}}M_{\vec{\omega}})$ in Eq. (52) disappears. Thus, we have

$$\begin{aligned}p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E}) &= \max_{\substack{\text{Eq.(50)}\\ \text{LOCC}}}\left[\sum_{i\in\mathsf{A}_0}\eta_i\mathrm{Tr}(\rho_iM_{(i,?)})+\sum_{j\in\mathsf{A}_1}\eta_j\mathrm{Tr}(\rho_jM_{(?,j)})\right]\\ &\leq \max_{\substack{\{M_i\}_{i\in\Lambda}:\\ \text{LOCC}}}\left[\sum_{i\in\mathsf{A}_0}\eta_i\mathrm{Tr}(\rho_iM_i)+\sum_{j\in\mathsf{A}_1}\eta_j\mathrm{Tr}(\rho_jM_j)\right]=p_{\mathrm{guess}}(\mathscr{E}),\end{aligned}\tag{58}$$

where the inequality is from the fact that $p_{\mathrm{guess}}(\mathscr{E})$ is the maximum obtained from measurements without any constraint, whereas $p_{\mathrm{L}}^{\mathrm{PI}}(\mathscr{E})$ is the maximum obtained from unambiguous LOCC measurements. ☐

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D.H. and J.S.K. contributed to design the ideas, perform the calculations, analyse the results and write the manuscript.

Competing Interests

The authors declare no competing interests.