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Modeling and analyzing the propagation of COVID-19 in Wuhan based on game theory: quarantine or not?

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ABSTRACT

The isolation strategy and quarantine strategy played a crucial role in the prevention of the Corona Virus Disease 2019 in China. This paper establishes a two-layer network model that couples epidemics and the behavior of individuals based on game theory. We calculated the basic reproduction number of the infectious disease, and analyzed the existence and stability of the positive equilibrium point in the behavioral dynamic model. Through simulation, we adjusted the behavior parameters to fit the actual data, and then analyzed the sensitivity of each parameter to the system. The contradiction between national strategy and individual behavior was found in the simulation process. The simulation results show that increasing the awareness of people can accelerate changes in behavior, and improving the efficiency of working at home can reduce the relative loss of isolation, all of which can reduce the severity of the infectious disease.

Introduction

The recent outbreak of coronavirus disease 2019 (COVID-19) in many countries has aroused great attention from governments and researchers¹⁻³. Under strong government measures, the spread of COVID-19 in China has been controlled. Looking back at the entire outbreak, human behavior has played an important role in containing the spread of the epidemic^{4,5}. Through domestic and foreign comparison, it is found that wearing masks, isolation and other non-drug measures are effective in curbing the epidemic^{6,7}.

There has been a lot of papers on COVID-19⁸. Oliver N et al.⁹ summarized the important role of different types of mobile phone data during a pandemic. Different types of mobile phone data can be used to study population estimates and mobility information to better understand trends in COVID-19. At the same time, mobile phone data can help determine the effectiveness of implementing different measures to contain the spread of COVID-19. Studies have shown that isolation is an effective control measure for COVID-19. Chinazzi M et al.¹⁰ uses a disease transmission model to evaluate the impact of travel restrictions on the domestic and international transmission of COVID-19. The results show that, regardless of domestic or international, travel restrictions have delayed the COVID-19 pandemic. Kraemer M U G et al.¹¹ used mobility data from Wuhan and case data to elucidate the role of case importation in transmission of COVID-19 and to figure out the impact of control measures, the results shows that the control measures implemented in China substantially mitigated the spread of COVID-19. Giordano G et al.¹² have proposed a SIDARTHE model to predict the course of the epidemic and help develop effective control strategies in Italy. The results suggest combining limiting social distance with extensive testing and contact tracing to contain the COVID-19 pandemic. These studies used mobile phone data, travel data, and case data to assess the effectiveness of total lockdowns of region and countries during the COVID-19 epidemic.

In addition to travel restrictions, isolation, face masks⁶, and eye protection are also key measures to prevent the transmission COVID-19¹³. Muhammad Altaf Khan et al.¹⁴ formulated a model for the dynamics of COVID-19 with quarantine and isolation, in which isolation is one of the states (Q). A novel model in line with the process of current epidemic and control measures was proposed in¹⁵, in which quarantined susceptible (Sq) and quarantined suspected individuals (B) are considered, the quarantined suspected compartment consists of exposed infectious individuals resulting from contact tracing and individuals with common fever needing clinical medication. Shilei Zhao et al.¹⁶ developed a Susceptible, Un-quarantined infected, Quarantined infected, Confirmed infected (SUQC) model to characterize the dynamics of COVID-19. The results show that the quarantine and control measures are effective in preventing the spread of COVID-19. In these papers, isolation is represented as a state that changes at a fixed rate, reducing the randomness of human behavior. Shi Zhao et al.¹⁷ developed a compartmental epidemic model based on the classic SEIR model and behavioral imitation through a game theoretical decision-making process to study and project the dynamics of the COVID-19 outbreak in Wuhan, China. the results show that increasing sensitivity to take infection prevention actions and the effectiveness of infection prevention measures are likely to mitigate the COVID-19 outbreak in Wuhan.

The application of game theory to the coupling model of epidemic disease and human behavior has been studied for a long time¹⁸. Zhang et. al.¹⁹ proposed an evolutionary epidemic model coupled with human behaviors. Individuals have three strategies: vaccination, self-protection and laissez faire. they found that increasing the successful rate of self-protection does not necessarily reduce the epidemic size or improve the system payoff. Arefin et. al.²⁰ built a mean-field vaccination game scheme to analyze the effect of an imperfect vaccine on a two-strain epidemic spreading taking into account vaccination behavior of individuals. Lim et.al.²¹ experimentally investigated vaccination choices of people in the context of a nonlinear public good game. Ning et. al.²² studied the epidemic transmission process using an evolutionary game model of complex networks. In this article, we propose a model that couples human behavior and epidemics, in which human behavior is described by game theory.

During the entire process of the COVID-19 epidemic, WHO collected various case data and organized national strategies to provide assistance for future research on the disease. The domestic epidemic has ended, but the international epidemic continues. In the future, similar epidemics may occur, so the study of defense strategies is of great research significance.

Results

According to the case data of the new crown pneumonia virus, we first fit the model parameters, use the fitted parameters as the baseline value, and then analyze the sensitivity of a single parameter and multiple parameters. By analyzing the sensitivity of parameters and their impact on epidemics and human behavior, we will give many suggestions on the defense of new coronary pneumonia. Using baseline parameters, we replaced the scale-free network with a homogeneous hybrid network, simulated the model in this paper, and found that the network played a great role in the defense of epidemics.

Parameter estimation

The Chinese Center for Disease Control and Prevention collected key data during the epidemic, including new confirmed cases, new deaths, new cured and discharged cases, cumulative confirmed cases, etc. We use the cumulative confirmed cases to subtract the cumulative cured cases and deaths due to illness, and use the results to fit the value of $I(t)$.

The total population of Wuhan is more than 10 million. It is very difficult to construct such a network. Therefore, we use the population percentage to express the dynamics of the epidemic. Based on this, we have established a scale-free network with 1,000 nodes. When constructing a scale-free network, there are 3 initial nodes, a new node is added, and two edges are added between the new node and the old network. The average degree of the generated network is 3.99. Then calculate Θ based on the generated network.

On January 23, Wuhan blocked all traffic. On January 30, the WHO declared the new crown pneumonia epidemic as a public health emergency of international concern. Due to the serious shortage of medical staff and nucleic acid detection and treatment capabilities in the early stage of the epidemic, there were late reporting, under-reporting and false-reporting phenomena, so there was a gap between the data of early case reports and the real situation²³. We select data from February 2 to April 15 to estimate the parameters of the model. After April 15, the epidemic has been brought under control and there are no more new cases.

The incubation period of new coronary pneumonia is 5.1 days on average. It takes approximately 7 days for asymptomatic individuals to recover. The average hospital stay for infection is 14 days⁶. Based on this, the values of α , γ_1 , and γ_2 are 0.2, 0.1429, 0.0714 respectively. According to Reference⁶, we set the infection rate to 0.583. The values of these four parameters depend on the transmission characteristics of the new crown pneumonia virus. Choose appropriate values for other parameters, and use the model to fit the actual cases of infection. The fitting result is shown in Figure 1.

By fitting the proportion of infected persons with the actual number of confirmed diagnoses, we determined that the value of ε is 0.3, the value of q_1 is 0.005, and the value of δ is 60. The value of the above parameters is defined as the baseline value. Next we will analyze the impact of parameters on the spread of epidemics.

Table 1 summarizes the meaning and value of each parameter in this article.

Table 1. The parameters and their values in this article

symbol	definition	value
β	Infection rates	0.583
α	Transfer rate from latent to infected persons	0.2
γ_1	Latent recovery rate	0.1429
γ_2	Recovery rate of infected persons	0.0714
q_1	The relative benefits of isolation	0.005
ε	People's sensitivity to the return difference	0.3
δ	The relative speed of people's behavior changes	60

Figure 1. Fitting result(fit the proportion of infected persons in the model with the proportion of confirmed individuals in Wuhan). The circles indicate the proportion of the number of confirmed cases in the total population in Wuhan. The dotted line represents the fitting result of the proportion of infected persons in the model.

Sensitivity analysis of parameters

Among the parameters in this article, α , and γ_1 are not controllable, γ_2 can be improved by improving medical standards and increasing medical staff, but it is not within the scope of this article. other parameters can be controlled through prevention and control strategies. Among the controllable parameters, β is directly related to the epidemic. We first analyze the impact of infection rate β on epidemics and human behavior. Based on the baseline values of other parameters, by changing β , we obtain the corresponding time series of the proportion of infected persons and the proportion of quarantined persons, as shown in Figure 2.

Figure 2. Time series of epidemics and human behavior under different β ($\beta = 0.383$; $\beta = 0.583$; $\beta = 0.783$). The figure on the left shows the time series of the proportion of infected persons. The figure on the right shows the time series of the proportion of people in quarantine.

From the Figure 2, we can find that as β increases, the peak of I also increases, and X will increase faster. The infection rate increases, the number of people infected will increase, and the coverage of the epidemic will be greater. We need to reduce the possibility of infection by reducing exposure. From Figure 2 we also find that as the epidemic becomes more serious, the speed at which people take quarantine measures will increase.

δ represents the adjusting factor of the speed of human behavior change. q_1 represents the loss of the quarantined person relative to the infected person. ε represents the sensitivity of people to the difference in payoffs. δ , q_1 and ε are all direct factors affecting human behavior, which in turn affect the spread of epidemics. The analysis of their impact on epidemics and human behavior is shown in Figure 3.

The upper three graphs of Figure 3 show the time series of the proportion of infected persons under the influence of the three parameters, and the three graphs below show the corresponding time series of the proportion of quarantined people. From

Figure 3. Time series of epidemics and human behavior under different q_1 , δ and ε ($\delta = 40, \delta = 60, \delta = 80$; $q_1 = 0.03, q_1 = 0.05, q_1 = 0.07$; $\varepsilon = 0, \varepsilon = 0.3, \varepsilon = 0.5$). The above three figures respectively represent the time series of the proportion of infected persons under the influence of three parameters. The following three figures show the time series of the proportion of quarantined persons under the influence of three parameters.

Figure 3, We found that as δ increases, the peak of I will decrease, and X will increase faster, but the situation of parameter q_1 and ε are just the opposite. As the relative losses of the quarantined people increase, from an economic perspective, the willingness of completely rational people to quarantine will decrease, and the spread of epidemics will be relatively serious. δ directly affects the rate of change of human behavior, thereby controlling the spread of epidemics. We can speed up the rate of change of human behavior by improving human safety awareness. Therefore, improving human safety awareness and reducing the relative loss of quarantine have a great effect on the suppression of epidemics. Corresponding measures to raise human awareness include: Internet and broadcast propaganda on the hazards and defenses of epidemics, transparency of epidemic-related data, school education on epidemic-related knowledge, etc. Corresponding measures to reduce the loss of quarantine include online office, online teaching, discovering their various interests, etc.

Interestingly, as human sensitivity to the difference in returns increases, the peak of the epidemic has increased and the rate of change in human behavior has slowed down. Through analysis, we found that the reason for this phenomenon is: because the country has already strictly controlled people during the epidemic, even if a person is not isolated, the possibility of him being infected is very small, so the loss of not being quarantined will be very small, resulting in the more rational people are, the less they will voluntarily quarantine. This phenomenon reflects the contradiction between national policy and individual rational response. Increasing human sensitivity to income cannot make people more voluntarily isolated under strict national policies.

The above analysis can understand the influence of the parameters on the epidemic and human behavior in the entire time interval, but only the influence of some local parameters can be understood. Next, we will analyze the influence of parameters on epidemic peaks, time to peaks, and human behavior through heat maps. β directly affects the spread of epidemics, which in turn affects the dynamics of human behavior. Other parameters affect the spread of epidemics by changing the dynamics of human behavior. Therefore, we divide the parameters into whether to include A to discuss. Figure 4 shows the impact of β and other parameters on epidemics and human behavior.

From the previous simulation, we can see that the larger the peak of I , the more serious the epidemic. Therefore, we use the peak of I to denote the severity of the epidemic. The first column of Figure 4 describes the change of the peak value of I with the change of β and other parameters. The earlier the infection peak is reached, the more timely and effective the control strategy. We use T to indicate the time when the proportion of infected people reaches its peak. The second column of Figure 4 shows the time to peak value of the epidemic as each two parameters change. It can be seen from the previous simulation that the proportion of people who choose the quarantine strategy is monotonically increasing after 15 days, so we choose the value of X at 20 days to measure the impact of the parameters on human behavior. The last column of Figure 4 shows the effect of each two parameters on the proportion of people who choose the quarantine strategy. The larger the value of $X(20)$, the faster human behavior changes.

Figure 4. Heat map of the impact of β and other parameters on epidemics and human behavior.

From the first column of Figure 4, we can see that the influence of the parameters on the peak value of I is monotonous, which is consistent with the previous simulation results. At the same time, we can see that q_1 has the greatest impact on the peak value. As can be seen from the last row of Figure 4, the effects of β and ε on epidemics and human behavior are monotonous. As β and ε increase, the epidemic will become more and more serious. But as β increases and ε decreases, people will take preventive measures faster. The time for I to reach the peak value is consistent with human behavior. The faster human behavior changes, the sooner I reaches the peak value. The reaction of ε on epidemics and human behavior is caused by the contradiction between national control and personal payoff maximization. The first row of Figure 4 shows that the effect of q_1 on human behavior is non-monotonous. When β is small, q_1 has a great influence on the speed of human behavior, and also has a great influence on the time when I reaches peak value. The results of these two figures are relatively consistent. The impact of β and δ on the time I reaches the peak is a bit complicated. When δ is very small, the result is more sensitive to β . The time decreases monotonically with δ . We need to increase δ to improve the timeliness of the strategy.

We analyzed the impact of β , directly related to the epidemic, on the epidemic and human behavior, and then we will analyze the impact of other parameters on them.

Figure 5 is about the impact of δ , ε and q_1 on epidemics and human behavior. Similarly, the first column of Figure 5 indicates the peak value of I with the change of the two parameters, the second column of Figure 5 indicates the time to reach the peak value, and the last column of Figure 5 indicates the quarantine proportion on the 20th day. From the first column of Figure 5, it can be seen that the effects of parameters on the peak proportion of infected persons are monotonically increasing, contrary to the previous simulation conclusion about δ . This result shows that ε and q_1 will change the impact of δ on the epidemic. It can be seen from the last two columns of Figure 5 that the time for the epidemic to reach its peak is consistent with the speed of behavior change, that is, the faster human behavior changes, the faster the epidemic will reach its peak. When q_1 changes, the influence of other parameters on human behavior will not be monotonous. When q_1 is larger, as human sensitivity to the income difference increases, the speed of human behavior changes will increase, that is, when the relative loss of quarantine is large, the contradiction between national strategy and human rationality will disappear.

In addition to the impact of human behavior on epidemics, the Internet also has a great impact on the spread of epidemics. In this paper, we analyze the impact of the network on the epidemic by simulating the epidemic on the scale-free network and the uniformly mixed network respectively. We first simulate system (3) on the homogeneous mixed network, and obtain the time series of the proportion of infected persons as shown in Figure 6a. We also simulate system (3) on the uniformly mixed network, and plot the time series of the epidemic, as shown in Figure 6b. To verify the accuracy of the results, we use cellular automata to simulate the epidemic on the homogeneously mixed network, as shown in Figure 6c. The monte carlo is used to control the spread of epidemics.

It can be seen from figure above that under the same average degree, the proportion of infected people on a homogeneous

Figure 5. Heat map of the impact of ε , q_1 and δ on epidemics and human behavior.

Figure 6. Epidemic time series on scale-free network and homogeneous hybrid network. Other parameters are the same as before. The dotted line in Figure (a) is a time series of the proportion of infected persons on the scale-free network. The dotted line in Figure (b) is a time series of the proportion of infected persons on a homogeneous mixed network. The dotted line in Figure (c) is a time series of the proportion of infected persons on the grid network.

mixed network is much smaller than that on a scale-free network. From Figure 6a and 6b, we can conclude that preventing people from running around can effectively prevent the spread of epidemics. It can be seen from Figure 6b and 6c that the results of the model solution are consistent with the simulation results.

Discussion

In this article, we propose a coupled model of the epidemic of new coronary pneumonia and human behavior. The epidemic is a SEIR model based on a scale-free network. Unlike other models, the latent person is infectious, but the infected person is not infectious due to hospitalization and isolation treatment. Human behavior is a game model based on a homogeneous mixed network. According to disease status and whether isolated, people are divided into: healthy and quarantine, healthy without quarantine, and infection and quarantine. We calculated the basic reproduction number of the epidemic. We simulated and analyzed the impact of various parameters in the epidemic on the epidemic and human behavior. We also analyzed the impact of the Internet on the epidemic, and found that the epidemic on the scale-free network is more serious than the epidemic on the homogeneous mixed network. Therefore, we recommend that in the control strategy of the epidemic, focus on controlling people with a large number of contacts.

Methods

In this article, we propose a coupled model of epidemics and human behavior based on a two-layer network. COVID-19 can only spread through contact, so we simulate the spread of the epidemic on the actual contact network and assume that the actual contact network is a scale-free network. Whether people choose to self-quarantine at home is affected by information about epidemics on the Internet, including social network information and self-media networks, etc. There are different opinions on the Internet, but the information about epidemic data is consistent, so we assume that human behavior changes are based on a fully connected network.

Figure 7. Network diagram of the model in this article.

The network structure diagram is shown in Figure 7. The two-layer network has the same number of nodes, and there is a one-to-one correspondence between nodes. The spontaneous isolation of people influences the spread of epidemics, and the spread of epidemics also influences the change of human behavior. Nodes within a single-layer network affect each other, and nodes between two-layer networks also affect each other.

The epidemic model

In the models describing COVID-19, people were divided into multiple compartments, including susceptible, latent, symptomatically infected, asymptotically infected, hospitalized, recovered, etc. In this article, we only divide people into susceptibility (S), latency (E), infection (I) and removal (R) compartments. The latent state includes all undetected individuals such as latent persons, asymptomatic infected persons, etc. Infected status refers to people who are infected and hospitalized. Due to the national importance and rapid action, there are no individuals who have been detected and have not been hospitalized after February. Removed compartments include those who recovered and those who died due to illness. New coronavirus is infectious during incubation period. Figure 8 shows a schematic diagram of the epidemic.

In response to the outbreak of infectious diseases, China established a hospital within a few days and established many sheltered hospitals to receive mild patients. Due to the strong support of the state, on February 5th, the number of beds for receiving infected persons was sufficient. Due to the harmfulness of the disease and the high attention paid by the state and doctors, once an infected person is found, he/she will be immediately isolated and treated, that is, the confirmed infected persons

Figure 8. Schematic diagram of epidemic status transition.

are isolated and have no ability to spread the disease. Considering the population mobility of Wuhan, we use a scale-free network to model the epidemic. In the context described above, we will build the following epidemic model:

$$\begin{cases} \dot{S}_k(t) = -\beta k S_k(t) \Theta(t), \\ \dot{E}_k(t) = \beta k S_k(t) \Theta(t) - (\alpha + \gamma_1) E_k(t), \\ \dot{I}_k(t) = \alpha E_k(t) - \gamma_2 I_k(t), \\ \dot{R}_k(t) = \gamma_1 E_k(t) + \gamma_2 I_k(t). \end{cases} \quad (1)$$

Here $\Theta(t) = \sum_k k p(k) E_k(t) / \langle k \rangle$, is the probability that any edge points to the infected node. $S_k(t)$, $E_k(t)$, $I_k(t)$, $R_k(t)$ respectively represent the proportion of susceptible, latent, infected, and removing nodes in the node with degree k . $S_k(t) + E_k(t) + I_k(t) + R_k(t) = 1$. The distribution of node degrees in the network is $p(k)$. β is the infection rate. α represents the rate of undiagnosed cases to confirmed cases. γ_1 represents the rate of recovery of undiagnosed individuals. γ_2 indicates the rate of recovery of the diagnosed and hospitalized individual.

Model with quarantine

In the epidemic prevention and control, travel restrictions are of great significance to reduce the number of infected people and control the spread of the virus. When an epidemic spreads, quarantine can protect not only yourself, but also those around you. In this article, we use game theory to model human spontaneous quarantine behavior. People will decide whether to change their behavior by comparing their own benefits with those of others. Assume that human behavior update strategy conforms to Fermi rule. Individual i randomly selects an individual, assuming individual j , compares the payoffs and change his/her strategy with the following probability:

$$w_{i \rightarrow j} = \frac{1}{1 + \exp[-\varepsilon(p_j - p_i)]}.$$

ε indicates the impact of payoff differences on individuals desire to change strategies. The larger the ε , the greater the impact of the payoff difference on the strategy choice.

Human behavior are divided into quarantine (q) and refuse to quarantine (r). x is the proportion of people in quarantine. Due to the severity of the epidemic and the sequelae after treatment, it is assumed that the recovered individuals will be quarantined by themselves, and the infected individuals are forcibly quarantined, so the freely moving individuals are all susceptible or latent. Before the diagnosis, the latent individuals look no different from the susceptible individuals, so we represent both susceptible and latent people as healthy people, but there is a risk of infection. Quarantine of latent people can avoid infecting neighbors, and quarantine of uninfected people can avoid being infected. The total population is divided into three categories based on whether they are hospitalization and whether they are spontaneous quarantine: infected and quarantine (Iq), health and quarantine (Hq), health and refuse to quarantine (Hr).

For the quarantine strategy, individuals will not only lose their freedom, but also no income. Suppose the relative loss of quarantine is q_1 and $0 < q_1 < 1$. Accordingly, we assume that the loss of an infected person is 1. For those who are not quarantined, there is a risk of infection, so we assume that their benefits are proportional to the number of unquarantined lurkers.

The benefits are summarized as follows:

$$p_i = \begin{cases} -1 - q_1 & \text{infected and quarantine,} \\ -q_1 & \text{health and quarantine,} \\ (-1 - q_1)(1 - x)\bar{E} & \text{health and refuse to quarantine.} \end{cases} \quad (2)$$

Since $E(t)$ represents undetected infected persons, the value of $E(t)$ is unknown, and people can only evaluate their benefits by Estimating the proportion of latent, which is denoted by $\tilde{E}$.

We use $\bar{E}(t)$ to denote the extent of the spread of infectious diseases in the network, $\bar{E}(t) = \sum_k p(k)E_k(t)$. Similarly, $\bar{S}(t) = \sum_k p(k)S_k(t)$, $\bar{I}(t) = \sum_k p(k)I_k(t)$, $\bar{R}(t) = \sum_k p(k)R_k(t)$. $(1 - x)\bar{E}(t)$ is the probability of a latent person who is not quarantined, that is, the probability of a susceptible person being infected. When a disease is prevalent, disease-related information spreads through various online platforms, so people receive the same information about the epidemic. Human behavior is based on epidemiologically related information, and we are going to model human behavior based on a homogeneous hybrid network.

The disease state of an individual will not actively change, people can only choose whether to quarantine. People diagnosed with the infection will be forced to be quarantined in hospitals, so they cannot actively choose a strategy. In this evolutionary game model, an infectious individual represent individual in E -state, that is, latent persons and asymptomatic and undiagnosed infected persons. The quarantined individual learns the strategy of the unquarantined individual and changes his strategy with a certain probability, so that x decreases:

$$x^- = x(t)(1 - x(t))(\bar{S}(t) + \bar{E}(t))^2 w_{Hq \rightarrow Hr}.$$

The unquarantined individual learns the strategy of the quarantined individual and changes his strategy with a certain probability, so that x increases:

$$x^+ = x(t)(1 - x(t))(\bar{S}(t) + \bar{E}(t))^2 w_{Hr \rightarrow Hq} + x(t)(1 - x(t))(\bar{S}(t) + \bar{E}(t))(\bar{I}(t) + \bar{R}(t)) w_{Hr \rightarrow Iq}.$$

Through the above two formulas, we can derive the evolutionary game equation of x as follows:

$$\frac{dx(t)}{dt} = \delta(x^+ - x^-).$$

Here δ represents the speed of behavior change relative to the speed of the epidemic. Spontaneous quarantine reduces epidemic infections and inhibits the spread of epidemics. Coupling the epidemic model and behavioral dynamics, the system will become:

$$\begin{cases} \dot{S}_k(t) = -\beta k S_k(t) \Theta(t), \\ \dot{E}_k(t) = \beta k S_k(t) \Theta(t) - (\alpha + \gamma_1) E_k(t), \\ \dot{I}_k(t) = \alpha E_k(t) - \gamma_2 I_k(t), \\ \dot{R}_k(t) = \gamma_1 E_k(t) + \gamma_2 I_k(t), \\ \dot{x}(t) = \delta(x^+ - x^-). \end{cases} \quad (3)$$

Here $\Theta(t) = \sum_k k p(k)(1 - x(t))E_k(t) / \langle k \rangle$.

The quarantine strategy avoids infection by controlling contact between people, so x only affects the infection rate.

The disease-free equilibrium (DFE) of the subsystem of degree k is $E_k^0 = (1, 0, 0, 0)$. The DFE of the system (1) is $E^0 = (E_1^0, E_2^0, \dots, E_N^0)$. The next generation matrix method is adopted to solve the basic regeneration number²⁴. The non-negative matrix, F , of the infection terms and the non-singular M -matrix, V , of the transition terms, are given by

$$F = \begin{pmatrix} F_{11} & 0 \\ 0 & 0 \end{pmatrix}, \quad (4)$$

$$V = \begin{pmatrix} V_{11} & 0 \\ V_{21} & V_{22} \end{pmatrix}, \quad (5)$$

$$\text{Here } F_{11} = \frac{\beta}{\langle k \rangle} \begin{pmatrix} 1 \\ 2 \\ \vdots \\ N \end{pmatrix} (1 \times p(1) \ 2 \times p(2) \cdots N \times p(N)), \quad V_{11} = -(\alpha + \gamma_1)E, \quad V_{21} = \alpha E, \quad V_{22} = \gamma_1 E, \quad E \text{ is the identity}$$

matrix.

$$R_0 = \frac{\beta \langle k^2 \rangle}{(\alpha + \gamma_1) \langle k \rangle}.$$

The dynamic system of individual behavior is divided into two parts: one is the interaction between quarantined and unquarantined individuals in the population of undetected infected persons and susceptible persons, $\frac{dx}{dt} = \delta x(1-x)(\overline{S(t)} + \overline{E(t)})(\overline{S(t)} + \overline{E(t)})(\omega_{Hr \rightarrow Hq} - \omega_{Hq \rightarrow Hr})$. and the other is the influence of isolated and hospitalized infected persons on the behavior of unquarantined individuals, $\frac{dx}{dt} = \delta x(1-x)(\overline{S(t)} + \overline{E(t)})(\overline{I(t)} + \overline{R(t)})(\omega_{Hr \rightarrow Iq})$. In the second part of the behavioral dynamics, hospitalized infected persons are forcibly isolated, so they cannot change their strategy, and $\omega_{Hr \rightarrow Iq} > 0$, so the proportion of isolated individuals increases monotonically. Next, we analyze the second part of individual behavior dynamics.

Theorem 1. When $\tilde{E} > \frac{q_1}{1+q_1}$, the system $\frac{dx}{dt} = \delta x(1-x)(\overline{S(t)} + \overline{E(t)})(\overline{S(t)} + \overline{E(t)})(\omega_{Hr \rightarrow Hq} - \omega_{Hq \rightarrow Hr})$ has a positive equilibrium point.

Proof. Let $h = \overline{S(t)} + \overline{E(t)}$, then $\overline{I(t)} + \overline{R(t)} = 1 - h$. The second part of the individual behavior dynamics will become:

$$\frac{dx}{dt} = \delta x(1-x)h^2(\omega_{Hr \rightarrow Hq} - \omega_{Hq \rightarrow Hr}).$$

After some straightforward algebra, we have

$$\frac{dx}{dt} = \delta x(1-x)h^2 \tanh\left(\frac{\varepsilon}{2}(p_{Hq} - p_{Hr})\right).$$

Obviously, 0 and 1 are the two solutions to the dynamical system.

We know that $\tanh(0) = 0$, so when $p_{Hq} = p_{Hr}$, $\frac{dx}{dt} = 0$. When $x^* = \frac{(1+q_1)\tilde{E}-q_1}{(1+q_1)\tilde{E}}$, $p_{Hq} = p_{Hr}$. It's only when $\tilde{E} > \frac{q_1}{1+q_1}$, that $0 < x^* < 1$.

From what has been discussed above, When $\tilde{E} > \frac{q_1}{1+q_1}$, the system $\frac{dx}{dt} = \delta x(1-x)(\overline{S(t)} + \overline{E(t)})(\overline{S(t)} + \overline{E(t)})(\omega_{Hr \rightarrow Hq} - \omega_{Hq \rightarrow Hr})$ has a positive equilibrium point $x^* = \frac{(1+q_1)\tilde{E}-q_1}{(1+q_1)\tilde{E}}$.

Theorem 2. When $\tilde{E} > \frac{q_1}{1+q_1}$, the positive equilibrium of the system $\frac{dx}{dt} = \delta x(1-x)(\overline{S(t)} + \overline{E(t)})(\overline{S(t)} + \overline{E(t)})(\omega_{Hr \rightarrow Hq} - \omega_{Hq \rightarrow Hr})$ x^* is stable.

Proof. From the model, $\frac{dx}{dt} = \delta x(1-x)h^2 \tanh\left(\frac{\varepsilon}{2}((1+q_1)(1-x)\tilde{E}(t) - q_1)\right)$, where $\tanh\left(\frac{\varepsilon}{2}((1+q_1)(1-x^*)\tilde{E}(t) - q_1)\right) = 0$ and $\tanh\left(\frac{\varepsilon}{2}((1+q_1)(1-x)\tilde{E}(t) - q_1)\right)$ is a monotonically decreasing function of x . When x is not equal to 0 or 1, $\delta x(1-x)h^2 > 0$. In summary, if x^* has a small disturbance, it will return to x^* , so x^* is stable.

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Figures

Figure 1

Fitting result(fit the proportion of infected persons in the model with the proportion of confirmed individuals in Wuhan). The circles indicate the proportion of the number of confirmed cases in the total population in Wuhan. The dotted line represents the fitting result of the proportion of infected persons in the model.

Figure 2

Time series of epidemics and human behavior under different β ($\beta = 0.383$; $\beta = 0.583$; $\beta = 0.783$). The figure on the left shows the time series of the proportion of infected persons. The figure on the right shows the time series of the proportion of people in quarantine.

Figure 3

Time series of epidemics and human behavior under different q_1 , δ and ε ($\delta = 40, \delta = 60, \delta = 80$; $q_1 = 0.03, q_1 = 0.05, q_1 = 0.07$; $\varepsilon = 0, \varepsilon = 0.3, \varepsilon = 0.5$). The above three figures respectively represent the time series of the proportion of infected persons under the influence of three parameters. The following three figures show the time series of the proportion of quarantined persons under the influence of three parameters.

Figure 4

Heat map of the impact of β and other parameters on epidemics and human behavior. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 5

Heat map of the impact of ϵ , q_1 and δ on epidemics and human behavior. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 6

Epidemic time series on scale-free network and homogeneous hybrid network. Other parameters are the same as before. The dotted line in Figure (a) is a time series of the proportion of infected persons on the scale-free network. The dotted line in Figure (b) is a time series of the proportion of infected persons on a homogeneous mixed network. The dotted line in Figure (c) is a time series of the proportion of infected persons on the grid network. mixed network is much smaller than that on a scale-free network. From Figure 6a and 6b, we can conclude that preventing people from running around can effectively prevent the spread of epidemics. It can be seen from Figure 6b and 6c that the results of the model solution are consistent with the simulation results.

Figure 7

Network diagram of the model in this article.

Figure 8

Schematic diagram of epidemic status transition.