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A Memristor Based Chaotic Oscillator for Weak Signal Detection and Its Circuitry Realization

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Abstract A new type of weak signal detection system that combines the memristor and Van der pol-Duffing chaotic system has been proposed in this paper, and the dynamic characteristics of the system are studied. It is observed that the system can change from a chaotic state to a periodic state under different driving force amplitudes. Moreover, as compared with several classical chaotic oscillators, the numerical simulation results show that the system has stronger anti-noise performance with the detectable signal-to-noise ratio reaches -163dB, and has a wider detection range. Its detection accuracy is up to 1×10^{-9} . More importantly, this paper provides the circuit implementation scheme of the system, and the weak signal can be detected with our designed circuit. This may offer a possible way for weak signal detection system with good performance in anti-noise performance, detection range and accuracy.

Keywords Van der pol-Duffing oscillator · Memristor · Signal detection · Circuit design

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1 Introduction

The weak signals detection with chaotic systems have become increasingly popular due to chaotic systems' characteristics of sensitivity to parameters and immunity to noise. The chaotic system's state can change from chaotic to periodic under the influence of periodic signals. Using this characteristic, the weak signal can be detected through the transition of chaotic system's state. Some valuable studies have come to the fore, and they use the Duffing chaotic system success to detect the weak signals in a very low SNR condition, and make it possible to apply this theory to deal with engineering problems [1-3]. However, Duffing chaotic system has good detection performance only when the system's driving force frequency $\omega = 1\text{rad/s}$, which greatly restricts its application. Gradually, scholars turned their attention to model fusion, such as couple Van der pol with Duffing. It has been proved that the dynamic behavior of the Duffing system is richer, and more robust in weak signal detection after being coupled with the Van der pol system [4,5]. Even so, there are still some problems to be solved urgently, such as enhancing detection accuracy, improving anti-noise performance, and enlarging detection range.

In order to solve the issues above, researchers have been motivated to investigate memristor-based oscillator system due to the memristor's non-volatile memory characteristic [6-10]. They found that compared with the conventional method, the Duffing chaotic system based on memristor has richer nonlinear dynamic characteristics and is robust to weak signals [11]. However, the problem of limited detection frequency is still unsolved, which needs further improvement. Considered the Van der pol-Duffing system has a better robust performance for different frequency signals than Duffing system, the Van der pol-Duffing coupled with the memristor may bring a better performance in weak signal detection. Therefore, it is desirable to systematically investigate the characteristics of the memristor-based Van der pol-Duffing system, such as detection accuracy, anti-noise performance and detection range.

Motivated by the above discussions, we present the mathematical model of a memristor-based Van der pol-Duffing chaotic oscillator. The proposed mathematical model of the memristor-based Van der pol-Duffing is investigated by the numerical as well as by real-time hardware experimental analysis.

2 Construction and Characteristics of Memristor-Based Chaotic Oscillator

2.1 Construction of chaotic oscillator

The memristor is a new type of passive two-terminal element, which is defined by the relationship between the magnetic flux (φ) and the charge (q) flowing through the memristor. The memristor used in this paper is charge-controlled, and its constitutive relationship can be expressed as:

$$\varphi(q) = aq + bq^3 \quad (1)$$

the memristance value is expressed as:

$$M(q) = d\varphi/dq = a + 3bq^2 \quad (2)$$

The mathematical models of the Duffing chaotic oscillator and the Van der pol oscillator are:

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = \gamma \cos(\omega t) \quad (3)$$

$$\ddot{x}(t) + \alpha(1 - x^2(t))\dot{x}(t) + x(t) = 0 \quad (4)$$

where x is the dynamical variable, k and α are damping ratio, γ and ω are the amplitude and angular frequency of the driving force $\gamma \cos(\omega t)$, respectively. According to the mathematical model of two chaotic oscillators, the coupled Van der pol-Duffing chaotic oscillator is expressed as:

$$\ddot{x}(t) - \alpha(1 - x^2(t))\dot{x}(t) - x(t) + x^3(t) = \gamma \cos(\omega t) \quad (5)$$

In the design of the memristor-based chaotic oscillator model, we used the relationship between the Van der pol-Duffing oscillator and the memristance value to perform equation coupling, and replaced the cubic nonlinearity with the charged-controlled memristor characteristic nonlinear. The state equation for the memristor-based chaotic oscillator is expressed as:

$$\ddot{x}(t) - \alpha(1 - x^2(t))\dot{x}(t) - x(t) + x^3(t) = \gamma \cos(\omega t) \quad (6)$$

Thus, by replacing the cubic nonlinearity in the classical Van der pol-Duffing, we design a new type of memristor-based chaotic oscillator. For ease of description, the above oscillator equations are expressed as the following differential oscillator equation:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = \alpha(1 - x_2^2)x_3 - M(q)x_2 + \gamma \cos(x_4), \\ \dot{x}_4 = \omega. \end{cases} \quad (7)$$

where $M(q) = a + 3bx_1^2$ is the memristance function.

Fig. 1: Lyapunov exponent graph of the new system

2.2 Oscillator Characteristic Analysis

In the numerical simulation study, the oscillator shows complex dynamics with the change of the driving force amplitude. When fixed system parameters as $\alpha = 0.5$, $a = -1$, $b = 1/3$, the Jacobian matrix of the calculation system is shown in Eq.8.

$$J_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2x_1x_2 - x_2x_3 - x_1^2 + 1 & 0.5 - 0.5x_2^2 - \gamma \sin(\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

The Lyapunov exponents can be calculated according to the Jacobian, and they are always used to characterize the characteristics of the system motion. For chaotic oscillators with attractors, there must be a positive Lyapunov exponent. Fixed driving force angular frequency $\omega = 1 \text{ rad/s}$, initial value is set to $(0, 0.01, 0, 0)$. Changing driving force amplitude γ from 0 to 5, setting the step size to 0.001, and using the Runge-Kutta method for numerical calculation, the obtained system Lyapunov exponent diagram is shown in Fig. 1. It can be found that in the range of the driving force amplitude (γ) less than 1, and the system's Lyapunov exponents change significantly. When γ increases to more than 1, the system's positive Lyapunov exponents change to negative, and the exponents present $(0, 0, -, -)$, the oscillator is no longer in chaos.

We focus on the driving force amplitude γ in the range of 0~1. Fig. 2 is the $x_2 - x_2$ phase diagram corresponding to different amplitude γ when the angular frequency $\omega = 1 \text{ rad/s}$ of the system driving force. It can be observed that when γ is small, the oscillator is in a quasi-periodic state. With the increase of γ , the oscillator will enter the chaotic state. If the γ increases further, the oscillator will exit the chaotic state and enter the large-scale periodic state.

Fig. 2: System $x_2 - x_3$ phase diagram under different driving force amplitudes

After repeated simulation experiments, it can be determined that the amplitude threshold of the proposed oscillator from chaotic motion to periodic motion is $\gamma = 0.791504$ when $\omega = 1 \text{ rad/s}$.

3 Memristor-based oscillator for weak signal detection

The weak signal embedded in the strong background noise can be represented by the following model:

$$r(t) = s(t) + n(t) \quad (9)$$

where $s(t)$ is the weak cosine signal $\gamma \cos(\omega t + \theta)$, and $n(t)$ is the white noise signal $N(0, \sigma^2)$, where σ^2 is noise variance. In this paper, the proposed system is compared with chaotic oscillator including Duffing, Van der pol-Duffing and Duffing combined with memristor.

According to the detection principle, the prerequisite is to find the threshold of chaos-to-period conversion, which can be judged by the Lyapunov exponent. Simultaneously, the detection range of the system can also be observed from its Lyapunov exponent spectrum. In order to compare the detection range of these four systems, the Lyapunov exponents of them are calculated, which are shown in Fig. 3, the abscissa is the system's driving force amplitude γ .

Fig. 3: Lyapunov exponent spectrum: (a) Duffing; (b) Van der pol-Duffing; (c) Duffing with memristor; (d) Proposed model, the inset is the maximum LE

It can be observed in Fig. 3(a), Fig. 3(b) and Fig. 3(c) that these systems all have the limitation of narrow-domain detection. With the increase of parameter γ , the system's motion state will return from the periodic state to the chaotic state, which may lead to misjudgment of detection. However, as showed in Fig. 3(d), with the increasing of γ , the proposed system can keep periodic motion after completing the transition from chaos to period. The inset is the maximum Lyapunov exponent of the proposed model. Therefore, it can be said that the proposed detection model has a wide detection range.

The signal-to-noise ratios of four systems mentioned above were investigated. After adding weak cosine signals with Gaussian noise, the phase trajectories of the above four systems are shown in Fig. 4, and the amplitude of signals is uniformly set to $1 \times 10^{-9}V$. It can be found that under the same parameter settings, the anti-noise ability of the chaotic system is greatly improved after adding the memristor. The phase trajectory diagrams of the new detection system shown in Fig. 4(d-ii) and Fig. 4(d-iii) are more compact, which indicates that the proposed system is more stable than other three. Moreover, the proposed system can detect effectively when the noise variance is 0.01, while other systems require noise variance of 0.001 or less to detect effectively. The minimum SNR is calculated to be -163.0103dB, which is much smaller than the other three systems and some groups' reports.

$$SNR = 10 \log \left(\frac{(1 \times 10^{-9})^2}{2 \times 0.01} \right) \quad (10)$$

Table 1 shows the comparison between the proposed method and the detection methods mentioned in the references. It mainly compares three aspects: amplitude detection accuracy, signal-to-noise ratio and detection range. According to the data in the table, the proposed method has these advantages.

Fig. 4: Phase trajectories of four systems after adding weak cosine signals with Gaussian noise with different variances (σ^2). (a) Duffing; (b) Van der pol-Duffing; (c) Duffing combined memristor; (d) Proposed model. (i) $\sigma^2=0.1$; (ii) $\sigma^2=0.01$; (iii) $\sigma^2=0.001$

Compared with the traditional Duffing (Method I) and Van der pol-Duffing (Method II) detection oscillators, the oscillator proposed in this paper not only greatly improves the detection accuracy and signal-to-noise ratio, but also can realize wide area detection. Compared with the novel detection oscillator in Method III, under the same detection accuracy, the proposed oscillator has a lower detectable SNR. In addition, in Method IV, it also adds a memristor, different from this paper is that the basic model it uses is the Chua's circuit. It can be seen from the comparison of the data in Table 1 that although the detection accuracy of the two oscillators is relatively close, the SNR of Method IV is still much higher than the proposed oscillator. Furthermore, the Lyapunov exponent shows that the proposed oscillator in this paper can implement a wide range detection because this system will never go back to chaotic system when the amplitude of driving force is greater than the state transition threshold value.

From the above comparison, it can be concluded that the proposed chaotic oscillator has a better noise immunity, higher detection accuracy and wide detection range, which indicates that it has an excellent detection performance.

Table 1: Comparison of different methods

Method	Chaotic model	Amplitude detection accuracy	SNR(dB)
Method I in [1]	Duffing	4×10^{-7}	-91
Method II in [12]	Van der pol-Duffing	0.01	-45
Method III in [13]	Novel oscillator	1×10^{-9}	-93
Method IV in [10]	Chua's circuit with memristor	4×10^{-8}	-81
Proposed method	Van der pol-Duffing with memristor	1×10^{-9}	-163

4 System Construction and Experiment

4.1 Circuit design of the chaotic system

The circuit implementation of a chaotic system is mainly classified into two types: analog implementation and digital implementation. Since most of the operational amplifiers, multipliers and other basic components can easily realize the calculation functions of addition, subtraction, multiplication, division, integration and differentiation, and easy access to external circuits without changing the electrical properties of the circuit itself. Therefore, in this paper, we give an analog implementation circuit of the proposed chaotic model by using the above components.

Considering the difficulty of physical realization of the memristor element, this paper uses the equivalent circuit model to replace the memristor. The memristor model circuit is shown in Fig.5(a), and the characteristic curve(I-V) of the memristor is shown in Fig. 5(b).

The circuit design of the proposed chaotic oscillator is mainly realized by the integrator, multiplier, inverter, and adder modules in the circuit. The circuit schematic of Eq. 7 is illustrated in Fig. 6. The circuit includes three inverting integrators, four multipliers, one inverting adder, two inverters and a cosine signal source. Following consideration of the difficulty and cost of circuit implementation, the operational amplifiers used in the integrator, multiplier and inverter are all dual-channel operational amplifiers TL082, and the multiplier is the four-quadrant analog multiplier AD633. The dynamic equations corresponding to Fig. 7 are:

$$\begin{cases} \frac{dV_1}{dt} = \frac{-V_2}{R_{13}C_3}, \\ \frac{dV_2}{dt} = \frac{-V_3}{R_{10}C_2}, \\ \frac{dV_3}{dt} = -\left(-\frac{R_6}{R_1}V_2^2V_3 + \frac{R_6}{R_2}V_3 + \frac{R_6}{R_3}V_2 + \frac{R_6}{R_4}V_2V_1^2 + \frac{R_6}{R_5}\gamma \cos(\omega t)\right). \end{cases} \quad (11)$$

Among them, the negative signs in the first and second equations in the equation are due to the reverse characteristic of the integrator, and the integral time constant is $\tau = R_7C_1 = R_{10}C_2 = R_{13}C_3$. In order to make the integrator

Fig. 5: (a) Schematic of memristor equivalent circuit and (b) characteristic curve

gain is 0dB when the driving signal $\omega = 1\text{rad/s}$, set $R_7 = R_{10} = R_{13} = 100k\Omega$, $C_1 = C_2 = C_3 = 10\mu F$, therefore $\tau = 1$, other circuit parameters are set as: $R_1 = R_2 = 20k\Omega$, $R_3 = R_4 = R_5 = R_6 = R_8 = R_9 = R_{11} = R_{12} = 10k\Omega$, $R_{18} = R_{20} = R_{22} = R_{24} = 9k\Omega$, $R_{17} = R_{19} = R_{21} = R_{23} = 1k\Omega$, $R_{14} = R_{15} = R_{16} = R_{32} = R_{33} = R_{34} = 100k\Omega$. The supply voltage is $\pm 15V$, the relevant expressions of the parameters a and b in the formula are: $a = -R_6/R_3$, $b = R_6/R_4$.

In the circuit, the output voltages of the amplifiers U3B, U2B, and U1B are denoted as V_1 , V_2 , and V_3 , which represent the state variables x_1 , x_2 , and x_3 respectively. Fig. 6 Fig. 8 show the circuit schematic, various motion states of the chaotic circuit, and trajectory diagram respectively. Note that the external driving force signal in the experiment uses a signal generator to generate a $f = 0.159155\text{Hz}$ ($\omega = 1\text{rad/s}$) cosine signal, and the various motion states presented by the circuit are also the result of changing the driving force amplitude.

It can be seen from the experimental data of the circuit that the state transition of the oscillator conforms to the theoretical change trend, indicating that the designed circuit is feasible. However, the experimental data indicates that because the angular frequency used is too small, the circuit test effect is not as clear as the simulation experiment, so we changed the driving force frequency of the system to $f = 1\text{kHz}$ ($\omega = 2\pi \times 10^3\text{rad/s}$), and carried out the circuit test again.

Change the value of the capacitor C_1 , C_2 , C_3 to 1.59nF , so that the gain of the integrator is 0 dB, and the frequency of the driving force is changed to $f = 1\text{kHz}$. Fig. 9 shows the circuit experiment of the system, and the experimental results obtained are shown in Fig. 10. The experimental results shown in Fig.10 correspond to the driving force's amplitudes of 1.5V, 2V, and 3.5V.

It can be clearly seen from the above experimental results that when the driving force's frequency is changed to $f = 1\text{kHz}$, the system switches between multi-period, chaotic, and single-period more obviously. And after changing

Fig. 6: Schematic of the proposed model

Fig. 7: Chaotic system at the driving forces frequency $f = 0.159155Hz$

Fig. 8: Waveforms of V2-V3 under different driving force amplitude

the driving force's frequency, the amplitude corresponding to each state has also changed.

5 Weak signal detection experiment

To further verify the feasibility of the above system in weak signal detection, a practical detection system was constructed. After repeated experiments, it is found that when the system is in a chaotic state, continuing to increase the amplitude of the driving force will make it enter a multi-period state and finally enter a signal-period. A critical point for the transition from chaotic state to multi-period is at 2.069V of the system's driving force, and the dynamic behavior is shown as Fig. 11. The signal to be tested $r(t)$ is a cosine signal with a frequency of $1kHz$ and an amplitude of $0.001V$. And $r(t)$ is embedded in strong background noise as shown in Fig. 12.

First of all, the system driving force is set at the critical voltage of 2.069V, and the system operation trajectory is shown in Fig.13(a). Input a noise signal to the detect system, the system's trajectory is shown as Fig.13(b), and the

Fig. 9: Chaotic system at the driving forces frequency $f = 1kHz$

Fig. 10: Waveforms of V2-V3 under the driving force frequency $f = 1kHz$

Fig. 11: Critical state of the system

Fig. 12: Test signal $r(t)$ embedded in the strong noise

Fig. 13: Trajectory of the detection system under different input signals

experiment indicates that the noise signal can't influence the state of the detect system. Then, the test signal $r(t)$ with a strong background noise is input to the system. At the influence of the test signal $r(t)$, the detect system enters the multi-period state as shown in Fig.13(c). Next, adjust the amplitude of the driving force. When γ decreases to 2.068V, the system goes back to the chaotic state, and the trajectory of the system is shown as Fig.13(d). In the process of detection, the amplitude change of the system's driving force is 0.001V, and the detect system is only sensitive to the periodic signal. Therefore, the amplitude of the test signal $r(t)$ is equal to the change of the system's driving force 0.001. This result shows that the parameters estimated are consistent with the actual values, and the system is feasible in weak signal detection.

6 Conclusion

In this paper, we construct a memristor-based Van der pol-Duffing chaotic oscillator, and the numerical simulation indicates that the proposed oscillator has a better performance in anti-noise and detection range. The detected signal-to-noise ratio of the oscillator can reach -163dB, indicating that the oscillator is sensitive to periodic signals. The hardware circuit has been designed and an actual signal detection case is given in this paper. The experimental results show that using this memristor-based chaotic oscillator can detect the signal embedded in strong noise, and estimate its amplitude parameters effectively.

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