Leaderless Maneuver Guidance and Event-Triggered Formation Control for Distributed Multi-Space-Robot Systems

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Leaderless Maneuver Guidance and Event-Triggered Formation Control for Distributed Multi-Space–Robot Systems

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Abstract

In this paper, the distributed displacement-based formation and leaderless maneuver guidance control problems of multi-space–robot systems are investigated by introducing event-triggered control update mechanisms. A distributed formation and leaderless maneuver guidance control framework is first constructed, which includes two parallel controllers, namely, an improved linear quadratic regulator and a distributed consensus-based formation controller. By applying this control framework, the desired formation configuration of multi-space–robot systems can be achieved and the center of leaderless formation can converge to the target point globally. Second, a pull-based event triggering mechanism is introduced to the distributed formation controller, which makes the control update independent of the events of neighboring robots, avoids unnecessary control updates, and saves the extremely limited energy of space robots. Furthermore, the potential Zeno behaviors have been excluded by proving a positive lower bound for the inter-event times. Finally, numerical simulation verifies the effectiveness of the theoretical results.

Keywords: multi-space–robot systems, formation stabilization, maneuver guidance, distributed control, event-triggered mechanisms.

1 Introduction

With the development of space robotics, the cooperative control of multi-space–robot systems (MSRSs) has a wide application prospect in various space missions, such as the construction of large space structures [1, 2], space debris removal [3, 4], space transportation [5], and the detection of gravitational waves in space [6, 7]. As the basis

for conducting above complex operations and various tasks, the maneuver guidance and formation control of MSRSs are often needed. Therefore, this topic has attracted significant attention from researchers in various scientific communities.

As stated in Ref. [8], on MSRSs, the purpose of the maneuver guidance and formation control is to drive a group of space robots with autonomous decision-making ability to achieve the

required formation geometry from any bounded initial states, while maintaining the configuration to maneuver to the target position on the adjacent orbit together. To achieve this goal, the distributed control is a promising approach for the MSRSs along with the rapid development of consensus theory these years. One of the main theoretical challenges mainly comes from how to control MSRSs only based on local information. A large number of results on distributed formation control exist that can be divided into the following categories: bearing-based formation control [9, 10], position-based control [11, 12], displacement-based control [13, 14], and distance-based control [15, 16]. Basic principles and execution differ between these methods. Specially, the displacement-based formation-control method, each robot only needs to measure the displacement between itself and its neighbors based on the specified communication topology, and use the displacement information to design the controller to achieve the desired formation configuration.

As is well known, the communication, computing, and energy resources of MSRSs are extremely limited. To reduce the resource occupation and energy consumption of robotic communication and control updates, the sampled-data method is used [17–19]. This scheme updates the controller periodically, which reduces the energy consumption to a certain extent. However, it will still cause unnecessary waste. For example, when the state of the robot tends to be stable, the controller will still communicate and update the controller according to a pre-determined period. Therefore, a technique called an event-triggered control scheme [20] has been rapidly developed recently. A triggering condition will be introduced in the event-triggered control scheme. Communication and controller updates will be carried out only when the triggering condition is met. For example in [20–24], when the state error of the system exceeds a certain threshold, the control update takes place. The results show that compared with sample-based control scheme, event-triggered control can effectively reduce the update times of controller and save computing, communication and energy resources. Later, Dimarogonas [25] extended the event-triggered control scheme to multi-agent systems. Because of its strong engineering application value, this scheme has been

widely studied in multi-robot control communities [26–28]. Although the event-triggered control scheme also has many applications in space missions, the previous works mainly focus on the attitude cooperative control of MSRSs [29–31]. To the best of our knowledge, there is little research on an event-triggered control scheme being applied to the formation configuration control of distributed MSRSs. In addition, a pull-based event-triggered protocol is presented, where each robot merely updates the controller at its own triggering instants [32, 33]. Therefore, it can further reduce the communication congestion of the system and save computing resources. It is worth noting that under the event-triggered control scheme, an adverse phenomenon is easily caused, namely, that the system may be triggered countless times in a limited time. We call this triggering behavior Zeno behavior [34, 35], and it is not allowed in hardware execution. Therefore, to avoid Zeno behavior, we must ensure that the positive minimum inter-event time (IET) exists.

Based on the above discussion, this paper studies the distributed displacement-based formation and maneuver guidance control problems of MSRS by introducing event-triggered control update mechanisms. The main contributions can be summarized as follows:

First, a distributed formation and maneuver guidance control framework are skillfully constructed, which includes two types of parallel controllers, namely, an improved linear quadratic regulator (LQR) and a distributed consensus-based formation controller. By applying this control framework, the desired formation configuration of MSRS can be achieved and the center of formation can globally converge to the target point in a leaderless manner.

Second, a pull-based event-triggered control update mechanism is introduced to the aforementioned distributed consensus-based formation controller. This type of mechanism makes the control updates independent from the events occurring of neighboring robots. It has the obvious advantages of avoiding unnecessary control updates and samplings, which means that its limited energy can be further saved.

Third, by deriving the analytic expression of the minimum inter-event time, it is proved that there exists a positive lower bound for the inter-event times, and the potential Zeno behaviors

of the proposed event-triggered mechanism are excluded theoretically.

The rest of this paper is organized as follows. Several notations, basic theories of graphs, an orbit coordinate system, and a formal system statement regarding this study are introduced in Section 2. In Section 3, a distributed event-triggered control scheme for MSRSs is proposed with its feasibility analyzed. A simulation example is given to show the efficacy of our results in Section 4. Finally, conclusions are presented in Section 5.

2 Preliminaries

In this section, several mathematical notations and preliminary results that will be used throughout this paper are provided.

2.1 Notations

Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote the real n -dimensional Euclidean vector space and $m \times n$ matrix space, respectively. Let I_n be an n -dimensional identity matrix. $\|\cdot\|$ expresses the Euclidean norm of a vector or the spectral norm of a matrix. M^T is the transpose of matrix M . Let $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the minimum and maximum eigenvalues, respectively. $A \otimes B$ as the Kronecker product of matrices A and B .

2.2 Basic Theory of Graph

In $\mathbb{R}^n$, the communication network topology among N robots can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = 1, \dots, N$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ are the vertex set and edges set, respectively. $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the associated adjacency matrix defined by $a_{ij} = 1$ if and only if $(i, j) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The degree matrix is $D = \text{diag}\{d_1, \dots, d_N\}$, which is a degree matrix with $d_i = \sum_{j=1, j \neq i}^N a_{ij}$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $\mathcal{L} = D - A$. A diagonal matrix $G = \text{diag}\{g_1, \dots, g_N\}$, where $g_i = 1$ if robot i can obtain the formation-center information; otherwise, $g_i = 0$. The positive-definite matrix H is defined as $H = L + G$.

For more details, the reader is referred to Ref. [36].

2.3 Orbit Coordinate System

Define a reference coordinate system where the x-axis represents the radial direction, the y-axis represents the in-track direction, and the z-axis represents the cross-track direction to facilitate the description of the robot's motion, as shown in Fig. 1. Radial refers to the direction of the geocentric radius vector, cross-track refers to the direction normal to the orbit plane (along the orbital angular momentum vector), and in-track refers to the direction perpendicular to both the radial track and cross-track.

Fig. 1 Orbit coordinate system.

2.4 System statement

Consider a MSRS with N robots. As illustrated in Fig. 2, our aim is the guidance of all robots to establish a specific formation configuration and maintain it until it reaches the target location of the adjacent orbit:

Fig. 2 Maneuver of MSRS.

$$\begin{aligned} \ddot{\mathbf{r}}_i = & -2\boldsymbol{\Omega} \times \dot{\mathbf{r}}_i - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_i) - \dot{\boldsymbol{\Omega}} \times \mathbf{r}_i \\ & + \mu \left(\frac{\mathbf{R}_0}{R_0^3} - \frac{\mathbf{R}_i}{R_i^3} \right) + \mathbf{u}_i, \end{aligned} \quad (1)$$

where $\mathbf{r}_i = [x_i, y_i, z_i]^T$ is the position vector of robot i in the orbital frame; $\boldsymbol{\Omega}$ is the orbital angular velocity of the target point; μ is the geocentric gravitational constant; $\mathbf{R}_0$ is the distance from the target point to the center of the Earth, and $\mathbf{R}_i$ is the distance from robot i to the center of the Earth; $\mathbf{R}_i = \mathbf{R}_0 + \mathbf{r}_i$; R_0 , and R_i are the corresponding moduli.

The dynamics (1) can be linearized and rewritten as a state-space equation as follows [13]:

$$\begin{aligned}\dot{X}_i(t) &= AX_i(t) + BU_i(t); \\ U_i(t) &= U_{Gi}(t) + U_{Fi}(t),\end{aligned}\quad (2)$$

where $X_i = [x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i]^T$ is the state vector of robot i ; U_i is control input, which includes two parts, i.e., the maneuver guidance law U_{Gi} and formation control law U_{Fi} ; and A and B are coefficient matrices. If the target points are located on adjacent circular orbits, or if the eccentricity of the orbit is particularly small, A and B , the system matrices, are constants and can be written as

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 0 & 2\omega & 0 \\ 0 & 0 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where ω is the mean orbital angular velocity of the target.

For maneuvering guidance, a improved LQR method is adopted, and an optimal control law is designed as follows [13]:

$$U_{Gi} = -R_G^{-1}B^T P_G X_i(t) = -K_G X_i(t), \quad (3)$$

where P_G can be obtained by solving the continuous-time algebraic Riccati equation (ARE):

$$\begin{aligned}(A + \alpha_1 I)^T P_G + P_G (A + \alpha_1 I) + Q_G \\ - P_G B R_G^{-1} B^T P_G = 0.\end{aligned}\quad (4)$$

The $\alpha_1 > 0$; $Q_G \in \mathbb{R}^{n \times n}$ and $R_G \in \mathbb{R}^{m \times m}$ are positive-definite matrices such that

$$J_{Gi} = \int_0^\infty e^{2\alpha_1 t} \cdot (X_i^T Q_G X_i + U_{Gi}^T R_G U_{Gi}) dt \quad (5)$$

has a minimum value.

We can define the formation center of a MSRS as $X_0 = \frac{1}{N} \sum_{i=1}^N X_i$, where N is the number of robots in the team, and we define the state vector from the formation center to robot i as $\eta_i = X_i - X_0$, $i = 1, 2, \dots, N$, so we have $\sum_{i=1}^N \eta_i = 0$. Then, $\eta = (\eta_1^T, \eta_2^T, \dots, \eta_N^T)^T$ is the desired formation configuration.

For formation control, a control law is proposed as follows [13]:

$$U_{Fi}(t) = -c_F K_F \Theta_i(t), \quad (6)$$

where

$$\begin{aligned}\Theta_i(t) &= \sum_{j=1}^N a_{ij} [(X_i(t) - \eta_i) - (X_j(t) - \eta_j)] \\ &\quad + g_i [(X_i(t) - \eta_i) - X_0(t)]\end{aligned}\quad (7)$$

is the neighbor tracking error of robot i in formation control; $K_F = R_F^{-1} B^T P_F$. Similarly, P_F can be obtained by solving the continuous-time ARE:

$$\begin{aligned}(A - BK_G + \alpha_2 I_n)^T P_F + P_F (A - BK_G + \alpha_2 I_n) \\ + Q_F - P_F B R_F^{-1} B^T P_F = 0,\end{aligned}\quad (8)$$

where $Q_F \in \mathbb{R}^{n \times n}$ and $R_F \in \mathbb{R}^{m \times m}$ are positive-definite matrices, similar to those in cost function (5).

Assumption 1 The pair (A, B) is stabilizable.

Assumption 2 The undirected communication topology $\mathcal{G}$ is connected, has a spanning tree, and is balanced.

Lemma 1 (Young's Inequality) For any real numbers a and b and any $\epsilon > 0$, we have

$$2ab \leq \frac{a^2}{\epsilon} + \epsilon b^2. \quad (9)$$

Lemma 2 [37] Under Assumption 1, given any $Q > 0$, there exists a unique $P > 0$ satisfying

$$A^T P + PA + Q = 0. \quad (10)$$

Lemma 3 [38] Under Assumption 1, for any $\kappa > 0$, there exists a positive-definite solution $P > 0$ to the following Riccati inequality:

$$A^T P + PA - 2\kappa P B B^T P + \kappa I < 0. \quad (11)$$

Remark 1 Under Assumptions 1 and 2, although the maneuver guidance and formation control of MSRSs can be realized, the controller needs continuous update, which will result in an unnecessary waste of resources. To solve this problem, we introduce the event-triggered control scheme, which updates the controller only when a specific event is triggered, effectively reducing, in turn, the update frequency of the controller.

3 Main results

In this section, an event-triggered control scheme for the formation control problem is presented to reduce the frequency of controller update.

The event-triggered control scheme is designed as follows:

$$\tilde{U}_{Fi}(t) = -c_F K_F \Theta_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \quad (12)$$

where the increment sequence $\{t_k^i\}_{i \in N, k=0,1,2,\dots}$ are the times when the controller performs calculations and updates, and

$$\begin{aligned} \Theta_i(t_k^i) = & \sum_{j=1}^N a_{ij} [(X_i(t_k^i) - \eta_i) - (X_j(t_k^i) - \eta_j)] \\ & + g_i [(X_i(t_k^i) - \eta_i) - X_0(t_k^i)]. \end{aligned} \quad (13)$$

The state measurement error of robot i is defined as

$$e_i(t) = X_i(t_k^i) - X_i(t), \quad t \in [t_k^i, t_{k+1}^i), \quad (14)$$

and we define $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$.

The event-triggered time sequence $t_0^i, t_1^i, t_2^i, \dots$ of robot i will be determined by the following triggering condition:

$$\|K e_i(t)\|^2$$

$$\begin{aligned} > \frac{\rho}{\lambda_N^2} \left\| K \left\{ \sum_{j=1}^N a_{ij} [(X_i(t) - \eta_i) - (X_j(t) - \eta_j)] \right. \right. \\ & \left. \left. + g_i [(X_i(t) - \eta_i) - X_0(t)] \right\} \right\|^2, \end{aligned} \quad (15)$$

where $t \in [t_k^i, t_{k+1}^i)$, $K = B^T P$, $\rho \in (0, 1)$ is a designed parameter that can be arbitrarily chosen, and $\lambda_N = \lambda_{\max}(H)$. Obviously, the triggering condition is also distributed, which only must judge whether to update the controller with the information from neighbors.

Theorem 1 Under assumptions 1 and 2, the maneuver guidance and formation control of MSRSs (2) can be achieved using the maneuver guidance law (3) and distributed event-triggered formation control scheme (12) with the event triggering condition (15).

Proof Letting $\xi_i(t) = X_i(t) - \eta_i - X_0(t)$, $\bar{A} = A - B K_G$, we can also define $\xi(t) = (\xi_1^T(t), \xi_2^T(t), \dots, \xi_N^T(t))^T$, and then we have

$$\begin{aligned} \dot{\xi}_i(t) = & \bar{A} \xi_i(t) + \bar{\eta}_i \\ & - c_F B K_F \left\{ \sum_{j=0}^N a_{ij} [\xi_i(t_k^i) - \xi_j(t_k^i)] + g_i \xi_i(t_k^i) \right\}. \end{aligned} \quad (16)$$

A MSRS is said to achieve the maneuver guidance and the desired formation η if, for any given bounded initial states,

$$\lim_{t \rightarrow \infty} \|\xi_i(t) - \xi_j(t)\| = 0, \quad (i = 1, 2, \dots, N).$$

Consider the following Lyapunov function candidate:

$$V(t) = \xi^T(t) (H \otimes P_F) \xi(t), \quad (17)$$

where P_F is a positive definite matrix and satisfies Eq. (8). Letting $e_{ij}(t) = \xi_i(t) - \xi_j(t)$, one has

$$V(t) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N a_{ij} e_{ij}^T P_F e_{ij} + \sum_{i=1}^N g_{ii} \xi_i^T P_F \xi_i > 0. \quad (18)$$

Denoting $\tilde{A} = \bar{A}^T P_F + P_F \bar{A}$, $\tilde{B} = c_F P_F B K_F$, then

$$\begin{aligned} \dot{V}(t) = & 2\xi^T(t) (H \otimes P_F) \dot{\xi}(t) \\ = & \xi^T(t) (H \otimes \tilde{A} - H^2 \otimes 2\tilde{B}) \xi(t) \\ & - \xi^T(t) (H^2 \otimes 2\tilde{B}) e(t) \end{aligned}$$

$$\begin{aligned}
& + \xi^T(t) (H \otimes P_F \bar{A}) \eta \\
& + \eta^T (H \otimes \bar{A}^T P_F) \xi(t). \quad (19)
\end{aligned}$$

By using Young's inequality, we can obtain

$$\begin{aligned}
& - \xi^T(t) (H^2 \otimes 2\tilde{B}) e(t) \\
& \leq \xi^T(t) (H^2 \otimes \tilde{B}) \xi(t) + e^T(t) (H^2 \otimes \tilde{B}) e(t) \quad (20)
\end{aligned}$$

and

$$\begin{aligned}
& \xi^T(t) (H \otimes P_F \bar{A}) \eta + \eta^T (H \otimes \bar{A}^T P_F) \xi(t) \\
& \leq \frac{1}{2} \xi^T(t) (H \otimes P_F \bar{A}) \xi(t) + \frac{1}{2} \eta^T (H \otimes P_F \bar{A}) \eta \\
& \quad + \frac{1}{2} \xi^T(t) (H \otimes \bar{A}^T P_F) \xi(t) \\
& \quad + \frac{1}{2} \eta^T (H \otimes \bar{A}^T P_F) \eta \\
& = \frac{1}{2} \xi^T(t) (H \otimes \bar{A}) \xi(t) + \frac{1}{2} \eta^T (H \otimes \bar{A}) \eta. \quad (21)
\end{aligned}$$

By using the Lemma 2, we can obtain

$$-\frac{1}{2} \xi^T(t) (H \otimes Z) \xi(t) - \frac{1}{2} \eta^T (H \otimes Z) \eta < 0. \quad (22)$$

Letting $W = -\frac{1}{2} \xi^T(t) (H \otimes Z) \xi(t) - \frac{1}{2} \eta^T (H \otimes Z) \eta$, we then have

$$\begin{aligned}
\dot{V}(t) & < \xi^T(t) (H \otimes \bar{A} - H^2 \otimes \tilde{B}) \xi(t) \\
& \quad + e^T(t) (H^2 \otimes \tilde{B}) e(t) + W. \quad (23)
\end{aligned}$$

Furthermore, if we can guarantee that

$$e^T(t) (H^2 \otimes \tilde{B}) e(t) \leq \rho \xi^T(t) (H^2 \otimes \tilde{B}) \xi(t) \quad (24)$$

with $0 < \rho < 1$, then

$$\dot{V}(t) < \xi^T(t) (H \otimes \bar{A} - H^2 \otimes (1 - \rho)\tilde{B}) \xi(t) + W. \quad (25)$$

Let $\lambda_1, \lambda_2, \dots, \lambda_N$ be the eigenvalues of matrix H , satisfying $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$. Since the matrix H is symmetric, then there exists an orthogonal matrix M such that

$$M^{-1} H M = M^T H M = J = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N). \quad (26)$$

It is clear to see that $M^T M = I_N$ and $H = M J M^T$. Defining $\hat{\xi}(t) = (M^T \otimes I_n) \xi(t)$, the following inequality then holds:

$$\begin{aligned}
\dot{V}(t) & \leq \hat{\xi}^T(t) (J \otimes \bar{A} - J^2 \otimes (1 - \rho)\tilde{B}) \hat{\xi}(t) + W \\
& = \sum_{i=1}^N \hat{\xi}_i^T(t) (\lambda_i \bar{A} - \lambda_i^2 (1 - \rho)\tilde{B}) \hat{\xi}_i(t) + W. \quad (27)
\end{aligned}$$

Considering inequality (11) with $2\kappa = \lambda_i (1 - \rho) c_F > 0$, then

$$\dot{V}(t) \leq -\kappa \lambda_i \sum_{i=1}^N \hat{\xi}_i^T \hat{\xi}_i + W < 0. \quad (28)$$

Obviously, we must only satisfy that inequality (24) is valid. A sufficient condition for (24) is that

$$\begin{aligned}
& \sum_{i=1}^N \lambda_N^2 \|K e_i(t)\|^2 \leq \rho \|(H \otimes K) \xi(t)\|^2 \\
& = \rho \sum_{i=1}^N \left\| K \left[\sum_{j=1}^N a_{ij} (\xi_i(t) - \xi_j(t)) + g_i \xi_i(t) \right] \right\|^2. \quad (29)
\end{aligned}$$

Therefore, for each robot in a MSRS, a sufficient condition is

$$\begin{aligned}
& \|K e_i(t)\|^2 \\
& \leq \frac{\rho}{\lambda_N^2} \left\| K \left[\sum_{j=1}^N a_{ij} (\xi_i(t) - \xi_j(t)) + g_i \xi_i(t) \right] \right\|^2. \quad (30)
\end{aligned}$$

Thus, $\dot{V}(t) < 0$ if inequality (30) is satisfied. Furthermore, ξ_i will asymptotically converge to zero, which means that, under the maneuver guidance law (3) and the distributed event-triggered control scheme (12) with the triggering condition (15), the maneuver guidance and formation control of MSRS (2) can be achieved. The proof is completed. $\square$

Lemma 4 [20] The Lipschitz continuity on compact sets of $f(x, u)$ and $k(x)$ implies that $f(x, k(x + e))$ is also Lipschitz continuous, and we can thus obtain $\|f(x, k(x, e))\| \leq L \|x\| + L \|e\|$.

The Zeno behavior is excluded according to the following theorem.

Theorem 2 Consider the MRSR (2) with a distributed event-triggered control scheme (12) and distributed event triggering condition (15). No robot will exhibit Zeno behavior.

Proof If inequality (30) is satisfied, the following inequality is also satisfied:

$$\|(H \otimes K) e(t)\|^2 \leq \rho \|(H \otimes K) \xi(t)\|^2 \quad (31)$$

with $0 < \rho < 1$. We can now determine the inter-event times by observing the dynamics of $\|(H \otimes K) e(t)\| / \|(H \otimes K) \xi(t)\|$. Then,

$$\begin{aligned}
& \frac{d}{dt} \left(\frac{\|(H \otimes K) e(t)\|}{\|(H \otimes K) \xi(t)\|} \right) \\
& = \frac{e^T(t) (H^T H \otimes K^T K) \dot{e}(t)}{\|(H \otimes K) e(t)\| \|(H \otimes K) \xi(t)\|}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\xi^T(t) (H^T H \otimes K^T K) \dot{\xi}(t)}{\|(H \otimes K) \xi(t)\| \|(H \otimes K) \xi(t)\|} \frac{\|(H \otimes K) e(t)\|}{\|(H \otimes K) \xi(t)\|} \\
& \leq \frac{\|(H \otimes K) \dot{\xi}(t)\|}{\|(H \otimes K) \xi(t)\|} + \frac{\|(H \otimes K) \dot{\xi}(t)\|}{\|(H \otimes K) \xi(t)\|} \frac{\|(H \otimes K) e(t)\|}{\|(H \otimes K) \xi(t)\|} \\
& = \frac{\|(H \otimes K) \dot{\xi}(t)\|}{\|(H \otimes K) \xi(t)\|} \left(1 + \frac{\|(H \otimes K) e(t)\|}{\|(H \otimes K) \xi(t)\|} \right) \\
& \leq L \left(\frac{\|(H \otimes K) e(t)\|}{\|(H \otimes K) \xi(t)\|} \right)^2. \tag{32}
\end{aligned}$$

Letting $Y = \|(H \otimes K)e(t)\| / \|(H \otimes K)\xi(t)\|$, we have the estimate $\dot{Y} \leq L(1 + Y)^2$ and conclude that $Y(t) \leq \Phi(t, \Phi_0)$, where $\Phi(t, \Phi_0)$ is the solution of $\dot{\Phi} = L(1 + \Phi^2)$ satisfying $\Phi(t, \Phi_0) = \Phi_0$. Then, the inter-execution times are bounded by the time it takes for $\Phi(t)$ to evolve from 0 to $\sqrt{\rho}$, which implies $\Phi(\tau, 0) = \sqrt{\rho}$ and $t_{k+1}^i - t_k^i > \tau = \frac{\sqrt{\rho}}{L + L\sqrt{\rho}}$. Therefore, for all robots, Zeno behavior can be excluded. The proof is completed. $\square$

Theorem 3 *Considering a group of space robots without leadership, the dynamics is shown in (2). To ensure a distributed formation control mechanism, the dynamics of the center of the formation is expressed as*

$$\dot{X}_0 = (A - BK_G) X_0 = \bar{A} X_0, \tag{33}$$

and the communication topology is required to be balanced.

Proof Calculating the time derivative of the formation-center states $X_0 = \frac{1}{N} \sum_{i=1}^N X_i$, we obtain

$$\begin{aligned}
\dot{X}_0(t) &= \frac{1}{N} \sum_{i=1}^N \dot{X}_i(t) = \bar{A} X_0(t) - c_F B P_F \frac{1}{N} \\
& \cdot \left\{ \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[(X_i(t_k^i) - \eta_i) - (X_j(t_k^j) - \eta_j) \right] \right. \\
& \left. + g_i \left[(X_i(t_k^i) - \eta_i) - X_0(t_k^i) \right] \right\}. \tag{34}
\end{aligned}$$

It can be seen from (34) that the calculation of X_0 must obtain global information. To ensure that the formation is a distributed structure, and that the constraint $\eta = (\eta_1^T, \eta_2^T, \dots, \eta_N^T)^T$, the second term in (34) can be expressed as

$$\begin{aligned}
& \sum_{i=1}^N \sum_{j=1}^N a_{ij} \left[(X_i(t_k^i) - \eta_i) - (X_j(t_k^j) - \eta_j) \right] \\
& + g_i \left[(X_i(t_k^i) - \eta_i) - X_0(t_k^i) \right]
\end{aligned}$$

$$\begin{aligned}
& = \sum_{i=1}^N \sum_{j=1}^N a_{ij} (X_i(t_k^i) - \eta_i) - \sum_{i=1}^N \sum_{j=1}^N a_{ij} (X_j(t_k^j) - \eta_j) \\
& = \sum_{i=1}^N \sum_{j=1}^N a_{ij} (X_i(t_k^i) - \eta_i) - \sum_{i=1}^N \sum_{j=1}^N a_{ji} (X_j(t_k^j) - \eta_j) \\
& = \sum_{i=1}^N \sum_{j=1}^N (a_{ij} - a_{ji}) (X_i(t_k^i) - \eta_i). \tag{35}
\end{aligned}$$

The formation is distributed if and only if (35) equals 0, and the dynamics of the formation center satisfies $\dot{X}_0 = (A - BK_G) X_0 = \bar{A} X_0$. This means $a_{ij} = a_{ji}$; that is, the communication topology must be balanced. The proof is completed. $\square$

4 Numerical simulation

To intuitively verify the effectiveness of the theoretical results reported in this paper, we validated the proposed maneuver guidance and distributed-event-triggered formation-control algorithms.

Considering a MSRS with four robots, the communication topology used is given as shown in Fig.3 with the corresponding H matrix (36):

Fig. 3 Communication topology of four robots and one virtual center.

$$H = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}. \tag{36}$$

It is supposed that the initial states of the four space robots in orbital frame are

$$\begin{aligned}
X_1 &= (180, 1070, 1000, 0, 0, 0)^T; \\
X_2 &= (250, 950, 1000, 0, 0, 0)^T; \\
X_3 &= (80, 970, 1000, 0, 0, 0)^T; \\
X_4 &= (50, 1050, 1000, 0, 0, 0)^T,
\end{aligned}$$

and the desired geometry constraints of the robots in the orbital frame are defined as follows:

$$\begin{aligned}\eta_1 &= (50, 50, 0, 0, 0, 0)^T; \\ \eta_2 &= (50, -50, 0, 0, 0, 0)^T; \\ \eta_3 &= (-50, -50, 0, 0, 0, 0)^T; \\ \eta_4 &= (-50, 50, 0, 0, 0, 0)^T.\end{aligned}$$

The weight matrices of (4) and (8) are chosen as

$$Q_G = \begin{bmatrix} \omega^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega^2 \end{bmatrix}, R_G = \begin{bmatrix} \frac{1}{\omega^2} & 0 & 0 \\ 0 & \frac{1}{\omega^2} & 0 \\ 0 & 0 & \frac{1}{\omega^2} \end{bmatrix},$$

and $Q_F = Q_G, R_F = R_G$. The values of the parameters in Eqs. (12) and (15) are $c_F = 1$ and $\rho = 0.1$.

Under the event-triggered control scheme (12), all robots can asymptotically converge to the target point in a neighboring orbit with a semi-major axis of 6,840,200 m, as shown in Fig. 4. Fig. 5 shows the initial and final configurations of the formation. One can clearly see that we have finally reached the desired formation configuration. Fig. 6 shows the state trajectories of four space robots. Moreover, Fig. 7 shows the event-triggered time instants and release intervals for each robot. Fig. 8 shows the evolutionary trajectories of U_{Fi} ($i = 1, 2, 3, 4$) from 0 to 100 seconds, from which we can see that U_{Fi} remains unchanged during the interval between two triggerings. It can be seen from Table 1 that in the total time of 5000s, the trigger period of the periodic triggering scheme of each robot is 0.001s, while the average interval of the event triggering of the four robots are 0.0905, 0.1107, 0.0908 and 0.1111 respectively. Through our method, the update times of the controller are obviously reduced. From Fig. 9, we can observe that each space robot starts from the initial position, and the distance from the robot to the formation center will not change in approximately 400s. After this, the MSRS has formed and maintained the desired formation configuration.

Table 1 Time triggering periods and event triggering average periods of four space robot.

Robot	Total time (s)	Time triggering period(s)	Event triggering average period(s)
1	5,000	0.001	0.0905
2	5,000	0.001	0.1107
3	5,000	0.001	0.0908
4	5,000	0.001	0.1111

Fig. 4 Trajectories of MSRSs.

(a) Initial formation configuration

(b) Desired formation configuration

Fig. 5 (a) Initial and (b) Desired formation configurations.

Fig. 6 State trajectories of four robots.

(a) Robot 1

(b) Robot 2

(c) Robot 3

(d) Robot 4

Fig. 7 Event triggering time instants of four space robots.

(a) Robot 1

(b) Robot 2

(c) Robot 3

(d) Robot 4

Fig. 8 $U_{Fi}, i = 1, 2, 3, 4$.

Fig. 9 Robots' deviation from formation center.

5 Conclusions

In this study, the maneuver guidance and formation maintenance problem of a leaderless MSRS under a time-invariant communication topology is studied. The time instant of the controller update is determined by a distributed trigger condition, so as to save the limited resources of the entire system. The asymptotic convergence of MSRSs, which means that the convergence time is unlimited, is investigated. However, time is required for specific space tasks. As a result, our future work will add fixed-time convergence characteristics to the maneuver guidance and formation control of multi-space-robot systems.

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