A Novel Decentralized Fixed-Time Tracking Control for Modular Robot Manipulators: Theoretical and Experimental Verification

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A novel decentralized fixed-time tracking control for modular robot manipulators: theoretical and experimental verification

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Abstract

This paper proposes a novel decentralized fixed-time tracking control approach, which realizes the advantages of modular robot manipulators (MRMs) with fixed-time convergence, strong robustness, and high tracking performance. First, to estimate the total uncertainty of MRMs, the fixed-time disturbance observer based on the extended state is developed. Then, combined with the disturbance observer, a novel decentralized control method based on fixed-time control strategy was developed to achieve the global fixed-time convergence of MRMs. And, stability analysis based on Lyapunov is used to obtain the fixed-time stability and convergence time of MRMs. Finally, numerical analysis and experiment respectively verify the excellent tracking ability of the proposed decentralized fixed-time tracking control algorithm.

Keywords: Fixed-time convergence, Modular robot manipulators, Disturbance observer, Lyapunov stability

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1. Introduction

Modular robot manipulators (MRMs) [1, 2], have attracted widespread attention in the robotics community because of their structural flexibility and

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adaptability better than traditional robots. So far, MRMs have been widely
 5 used in many extreme situations, such as medical assistance, disaster investigation,
 disaster relief, and space exploration. Correspondingly, it is still challenging
 to design an appropriate control system to ensure that MRMs can operate
 with high precision, safety, and reliability. These challenges include model un-
 certainty, uncertain disturbances, and highly nonlinear systems.

10 The performance of a mechanical system can be judged by various technical
 indicators, including transient response, convergence speed, steady-state
 accuracy, and robustness. At present, there are many technologies to deal with
 uncertainty and disturbance, including sliding mode control (SMC), neural net-
 work, adaptive fuzzy control, etc. Among them, SMC is widely used as an
 15 excellent robust control method due to its mature performance in system un-
 certainty and disturbance rejection [3–6]. In [7], the hierarchical SMC based
 on backstepping is presented to realize the balance and movement of the ball-
 bot with model uncertainty and disturbance. In [8], SMC based on a cascaded
 extended state observer is proposed for flexible joint robots with uncertainties
 20 and disturbances. Unfortunately, the conventional SMC usually uses a linear
 sliding mode surface to make the system state asymptotically converge to the
 equilibrium point, but its convergence time tends to be infinite. In order to
 improve efficiency in actual production, the operation time of the robot is only
 a short moment. This urgently requires the robot to converge quickly to ensure
 25 that MRMs stably reaches the target position before performing the operation
 task, so as to improve the operation accuracy. Therefore, the terminal sliding
 mode controller (TSMC) has been developed to allow the control system to
 converge within finite time [9–13]. Different from the traditional SMC, the non-
 linear sliding hyperplane is adopted in TSMC to ensure that the system state
 30 achieves rapid finite time convergence in the sliding phase [14–17]. In literature
 [18], the accurate tracking of redundant manipulators with model uncertainty
 and external disturbance is achieved by combining fuzzy adaptive and TSMC.
 In [19], the design of a non-singular TSMC was introduced to solve the problem
 of finite time trajectory tracking of the manipulator robot with full-state con-

35 straints. As mentioned in [14–19], TSMC not only has the characteristics of a
convergent point within finite time, but also has other superior characteristics,
including higher tracking accuracy, faster convergence, and so on. However, the
finite time convergence speed depends on the initial value, which usually may
not be predictable in advance. At the same time, this means that the larger the
40 initial value, the longer the convergence time will be.

To circumvent the drawbacks of convergence within finite time and at the
same time as a development of finite time control, fixed-time SMC has been
proposed recently [20]. At present, fixed-time controllers have been successfully
used to aircraft [21, 22], spacecraft [23] and motor systems [24], etc., and there
45 are also some results in robot control systems [25–27]. In [28], a robust controller
was presented to solve the problem of uncertain robot manipulators tracking
and ensure the global approximate fixed-time convergence of the robot system.
Considering a robot manipulator system with input constraints, an adaptive
fixed-time fault-tolerant control is presented in [29]. According to the above-
50 mentioned documents, the advantage of fixed-time SMC is that it can ensure
that the system converges and stabilizes within a fixed time, and the settling
time does not depend on the initial state value.

Moreover, in most practical systems, uncertainty and external interference
are inevitable [30, 31]. To boost the performance of the control system, observer-
55 based control algorithms are widely used through real-time accurate estimation
and feedback [32–36]. Among them, due to the simplicity of the disturbance
observer, it is very suitable for application in the actual system and will not
increase the calculation problem of the system. Therefore, based on the idea of
the extended state observer, all the unknown uncertainties affecting the system
60 are extended to a new state to enhance the estimation speed. Unfortunately,
the disturbance observer based on the extended state has not yet obtained or
guaranteed the stability of the fixed-time. Therefore, this paper presented a
fixed-time disturbance observer based on an extended state to estimate the
uncertainties of MRMs.

65 Inspired by the above-mentioned literature, this paper combines the ex-

tended state-based disturbance observer with fixed-time control to form a novel type of decentralized fixed-time SMC strategy based on the fixed-time disturbance observer. First, based on joint torque feedback technology (JTF), the dynamic model of MRMs is presented as a collection of dynamic subsystems
70 related to all coupled modules. Then, the uncertainty of the system is integrated into the total disturbance, which is estimated by a fixed-time disturbance observer based on an extended state. Secondly, a novel decentralized control method based on a fixed-time SMC strategy is designed in conjunction with the disturbance observer to accomplish global fixed-time convergence of
75 MRMs. In summary, the main achievements and novelties can be highlighted as follows:

1. To our knowledge, this is the first time that a fixed-time SMC approach has been introduced to solve the decentralized control problem of MRMs with model uncertainties and uncertain disturbances based on joint torque feedback
80 technology.
2. Different from the existing methods that do not consider dynamic decomposition, model uncertainty, and external disturbance compensation control, this paper presents a decentralized tracking control scheme based on a fixed-time disturbance observer, including uncertainty decomposition-based robust
85 control and observer-based fixed-time SMC. The proposed observer can realize fast and accurate lumped disturbance estimation in a fixed time, avoiding the delay in transmitting uncertain information to the system. And the established experimental platform verifies the advantages of the proposed method.

The follow-up structure of this paper is as follows: Dynamic model and
90 problem statement in Sect. 2. In Sect. 3, the decentralized controller is proposed and its stability is proved. Numerical simulations and experiments are performed to verify the effectiveness of the proposed strategy in Sect. 4. The conclusion is given in Sect. 5.

2. Dynamic model and problem statement

95 2.1. Dynamic model formulation

As in Fig.1, the MRMs system under consideration consists of n modules. Each module has a harmonic reducer and a torque sensor, which are integrated into the rotary joint. Similar to the literature [37], the following assumptions are made to apply to each joint module of MRMs:

100 Assumption 1: The flexibility of the joint is negligible.

Assumption 2: The rotor is symmetric concerning the rotation axis.

Assumption 3: The inertia between the torque sensor and the reducer can be ignored, and the torque transmission will not fail at the reducer.

The following notations are used for the i^{th} module:

105 I_{mi} : the moment of inertia of the joint motor;

γ_i : the reduction ratio ($\gamma_i \geq 1$);

q_i : the angular position of the joint module;

$f_i(q_i, \dot{q}_i)$: joint friction;

τ_{fi} : Coupling torque information obtained by joint torque sensor;

110 τ_i : DC motor output torque;

z_i : unit vector along the rotary axis of joint i ; and

z_{mi} : unit vector along the rotary axis of the i^{th} rotor.

The dynamic equations derived from [37], which represents the MRMs system with n rotating joints and joint torque sensors.

First, $i = 1$, the basic module is described as:

$$I_{m1}\gamma_1\ddot{q}_1 + f_1(q_1, \dot{q}_1) + \frac{\tau_{f1}}{\gamma_1} = \tau_1 \quad (1)$$

115

Next, $i = 2$, adjacent module is described as:

$$I_{m2}\gamma_2\ddot{q}_2 + f_2(q_2, \dot{q}_2) + I_{m2}z_{m2}^T z_1 \ddot{q}_1 + \frac{\tau_{f2}}{\gamma_2} = \tau_2 \quad (2)$$

Figure 1: Structure diagram of the joint module.

For $i \geq 3$,

$$\begin{aligned}
 I_{mi}\gamma_i\ddot{q}_i + f_i(q_i, \dot{q}_i) + I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j + \frac{\tau_{fi}}{\gamma_i} \\
 + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T (z_k \times z_j) \dot{q}_k \dot{q}_j = \tau_i
 \end{aligned} \tag{3}$$

where $f_i(q_i, \dot{q}_i)$ indicates the set of friction, which mainly includes the friction of each joint motor and reducer. Inspired by [38, 39], the expression for the friction term is as follows:

$$f_i(q_i, \dot{q}_i) = B_i \dot{q}_i + F_{qi}(q_i, \dot{q}_i) + (F_{ci} + F_{si} \exp(-F_{\tau i} \dot{q}_i^2)) \text{sign}(\dot{q}_i) \tag{4}$$

where B_i , F_{ci} , F_{si} , $F_{\tau i}$ represents the coefficient of viscous friction, the related parameter of Coulomb friction, static friction related parameter, and the Stribeck effect parameter, respectively. $F_{qi}(q_i, \dot{q}_i)$ represents the non-parametric friction term that reflects the positional correlation of friction with other friction modeling errors. $\text{sign}(\dot{q}_i)$ function is as follows:

$$\text{sign}(\dot{q}_i) = \begin{cases} 1 & \text{for } \dot{q}_i > 0 \\ 0 & \text{for } \dot{q}_i = 0 \\ -1 & \text{for } \dot{q}_i < 0 \end{cases} \tag{5}$$

2.2. Model uncertainty analysis

On the basis of Eq.(3), the dynamic model of MRMs with model uncertainty and uncertain disturbances can be written as:

$$I_{mi}\gamma_i\ddot{q}_i + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T(z_k \times z_j) \dot{q}_k \dot{q}_j + I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j + f_i(q_i, \dot{q}_i) + \frac{\tau_{fi}}{\gamma_i} + \tau_d = \tau_i \quad (6)$$

where τ_d is the uncertain disturbance. The second term $I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T(z_k \times z_j) \dot{q}_k \dot{q}_j$ and the third term $I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j$ of Eq.(6) are generated by the reconfiguration of MRMs. In order to make the equation more concise and compact, the following equation is given:

$$\delta_i = \begin{cases} 0 & i = 1 \\ I_{m2} z_{m2}^T z_1 \ddot{q}_1 & i = 2 \\ I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T(z_k \times z_j) \dot{q}_k \dot{q}_j & i \geq 3 \end{cases} \quad (7)$$

Joint friction is one of the important factors leading to the uncertainty of the model. By linearizing the friction (4) and combining it with Eq.(7), Eq. (6) can be rewritten as:

$$I_{mi}\gamma_i\ddot{q}_i + \hat{B}_i\dot{q}_i + \left(\hat{F}_{ci} + \hat{F}_{si} \exp\left(-\hat{F}_{\tau i} \dot{q}_i^2\right) \right) \text{sign}(\dot{q}_i) - Y(\dot{q}_i) \tilde{\theta}_i + F_{qi}(q_i, \dot{q}_i) + \delta_i + \frac{\tau_{fi}}{\gamma_i} + \tau_d = \tau_i \quad (8)$$

where $Y(\dot{q}_i) = \begin{bmatrix} \dot{q}_i & \text{sign}(\dot{q}_i) & \exp\left(-\hat{F}_{\tau i} \dot{q}_i^2\right) \text{sign}(\dot{q}_i) & -\hat{F}_{si} \dot{q}_i^2 \exp\left(-\hat{F}_{\tau i} \dot{q}_i^2\right) \text{sign}(\dot{q}_i) \end{bmatrix}$, and the parametric model uncertainty $\tilde{\theta}_i$ is defined as:

$$\tilde{\theta}_i = \begin{bmatrix} \hat{B}_i - B_i & \hat{F}_{ci} - F_{ci} & \hat{F}_{si} - F_{si} & \hat{F}_{\tau i} - F_{\tau i} \end{bmatrix}. \quad \hat{F}_{\tau i}, \hat{B}_i, \hat{F}_{si}, \text{ and } \hat{F}_{ci} \text{ are}$$

the estimated parameters of the friction model, respectively.

Furthermore, MRMs have the following properties as they are used in the development of the proposed decentralized fixed-time tracking control.

Property 1: The vector products between z_{mi} and z_i are bounded as:

$$|z_{mi}^T z_j| \leq 1, \quad |z_{mi}^T(z_k \times z_j)| \leq 1 \quad (9)$$

Property 2: The non-parametric friction term $F_{qi}(q_i, \dot{q}_i)$ is bounded as:

$$|F_{qi}(q_i, \dot{q}_i)| < \rho_i \quad (10)$$

Property 3: The subsystem uncertain disturbance is bounded as:

$$|\tau_d| < \rho_{fi} \quad (11)$$

3. A novel decentralized fixed-time tracking controller design

3.1. Extended state based fixed-time disturbance observer

Bsased on the dynamic model (8), the extended state based fixed-time disturbance observer can be designed as:

$$\begin{cases} \dot{\hat{x}}_{i2} = M_i \tau_i - M_i \left(\hat{B}_i \dot{q}_i + \left(\hat{F}_{ci} + \hat{F}_{si} \exp \left(-\hat{F}_{\tau i} \dot{q}_i^2 \right) \right) \text{sign}(\dot{q}_i) \right) + F_Q(q_i, \dot{q}_i) \\ \quad - M_i \frac{\tau_{fi}}{\gamma_i} + M_i Y(\dot{q}_i) \tilde{\theta}_i + \hat{v}_i + \alpha_i \|e_i\|^{\mu_i} + \beta_i \|e_i\|^{\nu_i} \\ \dot{\hat{v}}_i = \varsigma_i \|\varpi_i\|^{\varphi_i} + \xi_i \|\varpi_i\|^{\psi_i} + L_i \text{sign}(\varpi_i) \end{cases} \quad (12)$$

where $M_i = (I_{mi} \gamma_i)^{-1}$, $F_Q(q_i, \dot{q}_i) = -M_i F_{qi}(q_i, \dot{q}_i)$, $\hat{v} = -M_i \delta_i(q, \dot{q}, \ddot{q}) - M_i \tau_d$, $e_i = x_{i2} - \hat{x}_{i2}$, ς_i and ξ_i stand for the positive constant, $\varphi_i > 1, 0 < \psi_i < 1$, and L_i is a control gain.

$$\varpi_i = \dot{e}_i + \alpha_i \|e_i\|^{\mu_i} + \beta_i \|e_i\|^{\nu_i} \quad (13)$$

$$\dot{e}_i = \dot{x}_{i2} - \dot{\hat{x}}_{i2} = v_i - \hat{v}_i - \alpha_i \|e_i\|^{\mu_i} - \beta_i \|e_i\|^{\nu_i} \quad (14)$$

125 Theorem 1: Considering the MRMs system (8), the extended state based fixed-time disturbance observer designed in (12) is employed to estimate the uncertainty term with $|\dot{v}_i| \leq L_i$. Then, the estimation error of the observer converges to zero within fixed-time.

The Lyapunov function $V_{i1} = 0.5 \sum_{i=1}^n \varpi_i^2$ is adopted, and its time derivative

is calculated as:

$$\begin{aligned}
\dot{V}_{i1} &= \sum_{i=1}^n \varpi_i \dot{\varpi}_i \\
&= \sum_{i=1}^n \varpi_i \left(\dot{v}_i - \dot{v}_i \right) \\
&= \sum_{i=1}^n \varpi_i \left(\dot{v}_i - \varsigma_i \llbracket \varpi_i \rrbracket^{\varphi_i} + \xi \llbracket \varpi_i \rrbracket^{\psi_i} + L_i \text{sign}(\varpi_i) \right) \\
&\leq -\varsigma_i \sum_{i=1}^n |\varpi_i|^{\varphi_i+1} - \xi_i \sum_{i=1}^n |\varpi_i|^{\psi_i+1} \\
&\leq -2^{\frac{\varphi_i+1}{2}} n^{\frac{1-\varphi_i}{2}} \varsigma_i \left(V_{i1}^{\frac{\varphi_i+1}{2}} \right) - 2^{\frac{\psi_i+1}{2}} \xi_i \left(V_{i1}^{\frac{\psi_i+1}{2}} \right)
\end{aligned} \tag{15}$$

From (21),

$$\begin{aligned}
\dot{V}_{i1} &\leq -2^{\frac{\varphi_i+1}{2}} n^{\frac{1-\varphi_i}{2}} \varsigma_i \left(V_{i1}^{\frac{\varphi_i+1}{2}} \right) - 2^{\frac{\psi_i+1}{2}} \xi_i \left(V_{i1}^{\frac{\psi_i+1}{2}} \right) \\
\dot{V}_{i1} + 2^{\frac{\varphi_i+1}{2}} n^{\frac{1-\varphi_i}{2}} \varsigma_i \left(V_{i1}^{\frac{\varphi_i+1}{2}} \right) + 2^{\frac{\psi_i+1}{2}} \xi_i \left(V_{i1}^{\frac{\psi_i+1}{2}} \right) &\leq 0
\end{aligned} \tag{16}$$

Under Lemma 1, the settling time T_0 is bounded as the following:

$$T_0 \leq \frac{1}{2^{\frac{\varphi_i+1}{2}} n^{\frac{1-\varphi_i}{2}} \varsigma_i \left(\frac{\varphi_i+1}{2} - 1 \right)} + \frac{1}{2^{\frac{\psi_i+1}{2}} \xi_i \left(1 - \frac{\psi_i+1}{2} \right)} \tag{17}$$

Therefore, the disturbance observer can be estimated the uncertainties in

130 fixed-time.

Remark 1: In Eq.(12), the extended state gain L_i is assumed with $|\dot{v}_i| \leq L_i$. To satisfy $|\dot{v}_i| \leq L_i$, L_i is obtained through the two-layer adaptive rule given in [41],

$$\begin{cases} \dot{L}_i(t) = -(\rho_{i0} + \rho_i) \text{sign}(\delta_i(t)), \rho_{i0} > 0 \\ \dot{\rho}_i = \beta_{i1} |\delta_i(t)|, \beta_{i1} > 0 \end{cases} \tag{18}$$

where

$$\begin{aligned}
\delta_i &= L_i(t) - \frac{|\kappa_i|}{\ell_{i0}} - \ell_{i1}, \ell_{i0}, \ell_{i1} > 0 \\
\dot{\kappa}_i &= \lambda \text{fal}(-L(t) \text{sign}(\varpi_i) - \kappa_i, \eta_i, \vartheta_{i0}) \\
\text{fal}(X, \eta, \vartheta_0) &= \begin{cases} |X|^\eta \text{sign}(X), |X| > \vartheta_0 \\ \frac{X}{\vartheta_0^{1-\eta}}, |X| \leq \vartheta_0 \end{cases}
\end{aligned} \tag{19}$$

where $\rho_{i0}, \lambda, \beta_{i1} > 0$, $0 < \ell_{i0}, \ell_{i1}, \eta_i, \vartheta_{i0} < 1$ are control parameters. The approximation value of $L_i \text{sign}(\varpi_i)$ is obtained by taking a $\text{fal}(\cdot)$ function filter.

3.2. Fixed-time sliding mode controller

Based on Eq.(8), the state space expression of the dynamic model of MRMs is:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = M_i \tau_i - M_i \left(\hat{B}_i \dot{q}_i - \left(\hat{F}_{ci} + \hat{F}_{si} \exp \left(-\hat{F}_{\tau i} \dot{q}_i^2 \right) \right) \text{sign}(\dot{q}_i) \right) \\ \quad + F_Q(q_i, \dot{q}_i) + M_i Y(\dot{q}_i) \tilde{\theta}_i - M_i \frac{\tau_{fi}}{\gamma_i} + \hat{v}_i + \varpi_i \end{cases} \quad (20)$$

where $[x_{i1}, x_{i2}] = [q_i, \dot{q}_i]^T$, and ϖ_i is the disturbance estimation error and ϖ_i is bounded by $\varpi_i \leq \Gamma_i$, Γ_i is a positive constant.

Let $e_i = x_{i1} - x_{id}$, $r_i = \dot{e}_i + \lambda_i e_i$, x_{id} is the desired trajectory, and λ_i is a positive constant. Then, to achieve a fixed-time control, fixed-time sliding mode surface [19] is designed as

$$s_i = \dot{e}_i + \alpha_i \llbracket e_i \rrbracket^{\mu_i} + \beta_i \llbracket e_i \rrbracket^{\nu_i} \quad (21)$$

where $0 < \nu_i < 1$ and $\mu_i > 1$. The gain α_i, β_i are positive constants.

Differentiating (21) results in

$$\dot{s}_i = \ddot{e}_i + \alpha_i \mu_i |e_i|^{\mu_i-1} \dot{e}_i + \beta_i \nu_i |e_i|^{\nu_i-1} \dot{e}_i \quad (22)$$

For (22), If $e_i = 0$ and $\dot{e}_i \neq 0$, there will be singularity problems because of $\alpha_i - 1 < 0$. Inspired by the above sliding mode hyperplane, a novel type of fixed-time SMC hyperplane is designed, the expression is as follows:

$$s_i = e_i + c_i^{-1/m_i} \llbracket \dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i} \rrbracket^{\frac{1}{m_i}} \quad (23)$$

where $c_i > 0, b_i > 0$, $\frac{1}{2} < m_{i1} < 1$ and $n_i > 1$.

Remark 2: when $s_i = 0$, (23) implies $\dot{e}_i = -b_i \llbracket e_i \rrbracket^{n_i} - c_i \llbracket e_i \rrbracket^{m_i}$, which can be verified based on Lemma 1 that the settling time T_s is bounded by $T_s < T_{\max} \triangleq$

$$\frac{1}{b_i(n_i-1)} + \frac{1}{c_i(1-m_i)}$$

Figure 2: Schematic diagram of the presented control method.

The derivative of Eq. (23) with respect to time is given:

$$\dot{s}_i = \dot{e}_i + c_i^{-\frac{1}{m_i}} \frac{1}{m_i} |\dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i}|^{\frac{1}{m_i}-1} \left(\ddot{e}_i + b_i n_i |e_i|^{n_i-1} \dot{e}_i \right) \quad (24)$$

Inserting (20) into (24) yields

$$\begin{aligned} \dot{s}_i &= \dot{e}_i + c_i^{-\frac{1}{m_i}} \frac{1}{m_i} |\dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i}|^{\frac{1}{m_i}-1} \left(\ddot{x}_{i2} - \ddot{x}_{id} + b_i n_i |e_i|^{n_i-1} \dot{e}_i \right) \\ &= \dot{e}_i + c_i^{-\frac{1}{m_i}} \frac{1}{m_i} |\dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i}|^{\frac{1}{m_i}-1} \left(\begin{aligned} &M_i \tau_i + F_Q(q_i, \dot{q}_i) - M_i \frac{\tau_{fi}}{\gamma_i} + M_i Y(\dot{q}_i) \tilde{\theta}_i + \hat{v}_i \\ &- M_i \left(\hat{B}_i \dot{q}_i + \left(\hat{F}_{ci} + \hat{F}_{si} \exp(-\hat{F}_{\tau i} \dot{q}_i^2) \right) \text{sign}(\dot{q}_i) \right) \\ &+ \varpi_i - \ddot{x}_{id} + b_i n_i |e_i|^{n_i-1} \dot{e}_i \end{aligned} \right) \end{aligned} \quad (25)$$

The decentralized control law is designed as:

$$\tau_i = \tau_f + \tau_{eq} \quad (26)$$

Design compensator τ_f to compensate non-parametric friction terms $F_Q(q_i, \dot{q}_i)$, the specific expression form is as follows:

$$\tau_f = \begin{cases} -\rho_i \frac{r_i}{|r_i|} & |r_i| > \varepsilon_i \\ -\rho_i \frac{r_i}{\varepsilon_i} & |r_i| \leq \varepsilon_i \end{cases} \quad (27)$$

where ε_i is a tunable positive constant.

If the parameter uncertainty can be taken into account as an unknown constant, it can be compensated by an integral compensator. However, in practical engineering applications, the parameter uncertainty of the model varies with factors such as temperature, and $\tilde{\theta}_i$ is decomposed as:

$$\tilde{\theta}_i = \tilde{\theta}_c + \tilde{\theta}_v \quad (28)$$

where, $\tilde{\theta}_c$ is a constant unknown vector, and $\tilde{\theta}_v$ is a variable with an upper limit, i.e., $|\tilde{\theta}_v^i| < \rho_n^i, n = 1, 2, 3, 4, 5, 6$. In order to compensate $\tilde{\theta}_c$ and $\tilde{\theta}_v$, the following compensation controller is designed:

$$u_{pc}^i = -k_1 \int_0^t Y(\dot{q}_i)^T r_i d\tau \quad (29)$$

$$u_{pvn}^i = \begin{cases} -\rho_n^i \frac{\varsigma_n^i}{|\varsigma_n^i|}, & |\varsigma_n^i| > \varepsilon_{pn}^i \\ -\rho_n^i \frac{\varsigma_n^i}{\varepsilon_{pn}^i}, & |\varsigma_n^i| \leq \varepsilon_{pn}^i \end{cases} \quad n = 1, 2, 3, 4, 5, 6 \quad (30)$$

where $\varsigma_n^i = Y(\dot{q}_i)^T r_i$, and ε_{pn}^i is the positive control parameter.

$$\begin{aligned} \tau_{eq} = & M_i^{-1} \left(\ddot{x}_{id} - \hat{v}_i - b_i n_i |e_i|^{n_i-1} \dot{e}_i - c_i^{\frac{1}{m_i}} m_i |\dot{e}_i + b_i [e_i]^{n_i}|^{2-\frac{1}{m_i}} \dot{e}_i \right) \\ & - (\Gamma_i + a_i) \text{sign}(s_i) - \lambda_{i1} [s_i]^{m_{i1}} - \lambda_{i2} [s_i]^{m_{i2}} \\ & + \hat{B}_i \dot{q}_i + \left(\hat{F}_{ci} + \hat{F}_{si} \exp(-\hat{F}_{\tau i} \dot{q}_i^2) \right) \text{sign}(\dot{q}_i) + Y(\dot{q}_i)(u_{pc}^i + u_{pv}^i) + \frac{\tau_{fi}}{\gamma_i} \end{aligned} \quad (31)$$

Theorem 2. Considering the MRMs system (20), the decentralized fixed-time tracking controller is designed as (26). The system error of MRMs will converge to zero within fixed-time, and under Lemma 1, the settling time is bounded by:

$$T < T_{max} \triangleq T_0 + T_1 + T_s + \varepsilon(\tau) \quad (32)$$

where T_0 , T_1 and T_s are defined in (17), Remark 2, and (36). $\varepsilon(\tau)$ is a small time margin related to the boundary width.

The following Lyapunov candidate and its time derivative are computed as:

$$\begin{aligned}
V_{i2} &= 0.5 \sum_{i=1}^n s_i^2 \Rightarrow \dot{V}_{i2} = \sum_{i=1}^n s_i \dot{s}_i \\
&= \sum_{i=1}^n s_i \left(\dot{e}_i + \Delta_i \left(M_i \tau_i - M_i \left(\hat{B}_i \dot{q}_i - \left(\hat{F}_{ci} + \hat{F}_{si} \exp \left(-\hat{F}_{\tau i} \dot{q}_i^2 \right) \right) \text{sign}(\dot{q}_i) \right) \right. \right. \\
&\quad \left. \left. + F_Q(q_i, \dot{q}_i) - M_i \frac{\tau_{fi}}{\gamma_i} + \hat{v}_i + \varpi_i - \ddot{x}_{id} + b_i n_i |e_i|^{n_i-1} \dot{e}_i \right) \right) \\
&= \sum_{i=1}^n s_i \left(\dot{e}_i + \Delta_i \left(\ddot{x}_{id} - \hat{v}_i - b_i n_i |e_i|^{n_i-1} \dot{e}_i - c_i^{\frac{1}{m_i}} m_i |\dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i}|^{2-\frac{1}{m_i}} \dot{e}_i \right. \right. \\
&\quad \left. \left. - (\Gamma_i + a_i) \text{sign}(s_i) - \lambda_{i1} \llbracket s_i \rrbracket^{m_{i1}} - \lambda_{i2} \llbracket s_i \rrbracket^{m_{i2}} + M_i \frac{\tau_{fi}}{\gamma_i} \right. \right. \\
&\quad \left. \left. + M_i \left(\hat{B}_i \dot{q}_i + \left(\hat{F}_{ci} + \hat{F}_{si} \exp \left(-\hat{F}_{\tau i} \dot{q}_i^2 \right) \right) \text{sign}(\dot{q}_i) \right) \right. \right. \\
&\quad \left. \left. + M_i \tau_f - M_i \left(\hat{B}_i \dot{q}_i - \left(\hat{F}_{ci} + \hat{F}_{si} \exp \left(-\hat{F}_{\tau i} \dot{q}_i^2 \right) \right) \text{sign}(\dot{q}_i) \right) \right. \right. \\
&\quad \left. \left. - F_Q(q_i, \dot{q}_i) - M_i \frac{\tau_{fi}}{\gamma_i} + \hat{v}_i + \varpi_i - \ddot{x}_{id} + b_i n_i |e_i|^{n_i-1} \dot{e}_i \right) \right) \quad (33)
\end{aligned}$$

For i th joint, and $n=1,2,3,4,5,6$, if $|\varsigma_n^i| > \varepsilon_{pn}^i$, exist $\varsigma_n^i(\tilde{\theta}_v + u_{pvn}^i) < 0$, if $|\varsigma_n^i| \leq \varepsilon_{pn}^i$, $\varsigma_n^i(\tilde{\theta}_v + u_{pvn}^i) \leq \varsigma_n^i \sum_{n=1}^6 (\rho_n^i \frac{\varsigma_n^i}{|\varsigma_n^i|} - \rho_n^i \frac{\varsigma_n^i}{\varepsilon_{pn}^i})$, and reaches its maximum at the $|\varsigma_n^i| \leq \frac{\varepsilon_{pn}^i}{2}$.

$$\begin{aligned}
\dot{V}_{i2} &\leq \Delta_i \left(-\lambda_{i1} \sum_{i=1}^n |s_i|^{m_{i1}+1} - \lambda_{i2} \sum_{i=1}^n |s_i|^{m_{i2}+1} + \sum_{n=1}^6 \frac{\rho_n^i \varepsilon_{pn}^i}{4} \right) \\
&\leq \Delta_i \left(-2^{\frac{m_{i1}+1}{2}} 2^{\frac{m_{i1}}{2}} \lambda_{i1} \left(V_{i2}^{\frac{m_{i1}+1}{2}} \right) - 2^{\frac{m_{i2}+1}{2}} \lambda_{i2} \left(V_{i2}^{\frac{m_{i2}+1}{2}} \right) + \ell \right) \quad (34)
\end{aligned}$$

where $\Delta_i = c_i^{-\frac{1}{m_i}} \frac{1}{m_i} |\partial_i|^{\frac{1}{m_i}-1}$, $\partial_i = \dot{e}_i + b_i \llbracket e_i \rrbracket^{n_i}$, $\ell = \sum_{n=1}^6 \frac{\rho_n^i \varepsilon_{pn}^i}{4}$. In order to facilitate the analysis, the state $(e_i, \dot{e}_i)$ is divided into two regions $\Omega_{i1} = \{(e_i, \dot{e}_i) | \Delta_i \geq 1\}$ and $\Omega_{i2} = \{(e_i, \dot{e}_i) | \Delta_i < 1\}$.

i. If the system state is in area Ω_{i1} , formula (37) can be obtained:

$$\dot{V}_{i2} \leq -2^{\frac{m_{i1}+1}{2}} 2^{\frac{m_{i1}}{2}} \lambda_{i1} \left(V_{i2}^{\frac{m_{i1}+1}{2}} \right) - 2^{\frac{m_{i2}+1}{2}} \lambda_{i2} \left(V_{i2}^{\frac{m_{i2}+1}{2}} \right) + \ell \quad (35)$$

It is worth noting that $V_i = 0$ can be understood as $s_i = 0$. This means that the system is stable at a fixed-time and has a bounded settling time T_s , and is

$$T_s \leq \frac{1}{2^{\frac{m_{i1}+1}{2}} n^{\frac{1-m_{i1}}{2}} \lambda_{i1} \left(\frac{m_{i1}+1}{2} - 1 \right) \ell} + \frac{1}{2^{\frac{m_{i2}+1}{2}} \lambda_{i2} \left(1 - \frac{m_{i2}+1}{2} \right) \ell} \quad (36)$$

ii. If the system state is in area Ω_{i2} , $0 < \Delta_i < 1$, when $\partial_i \neq 0$. The terminal sliding mode hyperplane $s_i = 0$ can still prove to be an attractor. What remains is to prove that $\partial_i=0$ is not attractive except for the origin[40]. Therefore, it
150 can be concluded that the sliding surface $s = 0$ can be reached from anywhere in the phase plane within fixed time $t_r < T_r + \varepsilon(\tau)$ [40]. Once the sliding surface $s = 0$ is reached, it follows from (21), the system states will reach the origin within the fixed time T_s . Hence, the total settling time T is bounded by (32). This completes the proof.

155 Remark 3: Although fixed-time control has attracted widespread attention, the global fixed-time stability of the robot system remains to be studied. This paper proposes a novel decentralized fixed-time control, which achieves fixed-time convergence, strong robustness, and high-precision tracking control effects. The main contributions of this paper are: 1) The extended state based fixed time
160 observer is designed to compensate for the influence of uncertain terms in the controller, and the adaptive value is obtained through the principle of the two-layer adaptive. The adaptive value adapts according to the change of the total uncertainty, avoiding the need for the upper bound of uncertainty. 2) Compared with TSMC, a novel decentralized fixed-time sliding mode control proposed in
165 this paper has higher accuracy, faster convergence speed, and effectively reduces chattering.

4. Theoretical and experimental analysis

4.1. Simulations

Considering the tracking problem for MRMs with model uncertainties and
170 uncertain disturbances, the validity of fixed-time controller is verified in this subsection through numerical simulations. The parameter settings of MRMs are designed in Table 1, including the nominal parameters of the friction model, numerical simulation parameters, and experimental parameters.

Table 1: Parameters setting

Parameter type	Name	Value	Name	Value	Name	Value
Model parameters	$\hat{B}_i$	0.0123 <i>Nms/rad</i>	$\hat{F}_{\tau i}$	20 <i>s²/rad²</i>	I_{mi}	118 <i>gcm²</i>
	$\hat{F}_{ci}$	0.0015 <i>Nm</i>	$\hat{F}_{si}$	0.02557 <i>Nm</i>	γ_i	101
Numerical simulation	$\alpha_i \ \beta_i$	5 8	$\varphi_i \ \psi_i$	1.5 0.6	$\varsigma_i \ \xi_i$	7 3
control parameters	$\mu_i \ \nu_i$	1.25 0.5	$m_i \ n_i$	0.75 5/3	$c_i \ b_i$	10 20
	$\Gamma_i \ a_i$	45 1.5	$\lambda_{i1} \ \lambda_{i2}$	15 15	$m_{i1} \ m_{i2}$	1.75 0.75
	$\varepsilon_i \ \rho_i$	0.1 0.07	$\lambda_i \ \vartheta_0$	100 0.6	$\rho_{i0} \ \beta_{i1}$	2 3
	$\ell_{i1} \ \ell_{i0}$	1.2 0.75	$\lambda \ \eta$	4 0.6		
Experimental control parameters	$\lambda_{i1} \ \lambda_{i2}$	17 13	$\varepsilon_i \ \rho_i$	0.1 0.07	$\lambda_i \ \vartheta_0$	120 2
	$\alpha_i \ \beta_i$	10 12	$\varphi_i \ \psi_i$	1.75 0.75	$\varsigma_i \ \xi_i$	12 5
	$\mu_i \ \nu_i$	1.75 0.75	$m_i \ n_i$	0.75 9/7	$c_i \ b_i$	13 25
	$\Gamma_i \ a_i$	50 1.5	$m_{i1} \ m_{i2}$	1.75 0.55	$\rho_{i0} \ \beta_{i1}$	3 5
	$\ell_{i1} \ \ell_{i0}$	2 0.75	$\lambda \ \eta$	5 0.6		

The tracking trajectory of the two joints is as follows

$$\begin{aligned} q_{1d} &= 0.5\sin(2.2t) - 0.3\sin(1.5t), & 0 \leq t \leq 20 \\ q_{2d} &= 0.5\sin(0.6t) - 0.3(\cos(2t) - 1), & 0 \leq t \leq 20 \end{aligned} \quad (37)$$

Case 1: The initial positions are $[q_1(0), q_2(0)]^T = [0.04, 0.02]^T$. The position tracking error comparison of the numerical simulation is shown in Fig.3 and Fig.4. The above figures show that the system produces a large error fluctuation during the initial operation and quickly returns to the equilibrium point. After stabilization, the residuals near the equilibrium point are small, which proves that the system error reaches the equilibrium point in a fixed time. From the simulation results that the proposed control approach achieves a good tracking process and has higher control accuracy under model uncertainty and uncertain disturbance. Fig.5 and Fig.6 show the control torque of two different control methods under joint 1 and joint 2. As shown in the figures, since the proposed control strategy estimates the total uncertainty through the observer, the chat-

185 tering phenomenon has been significantly weakened. In addition, the proposed
fixed-time control law also helps to reduce chattering.

Figure 3: Joint 1 tracking error curves

Figure 4: Joint 2 tracking error curves

Figure 5: Joint 1 control torque curves

Figure 6: Joint 2 control torque curves

Figure 7: Joint 1 tracking error curves

Figure 8: Joint 2 tracking error curves

Figure 9: Joint 1 control torque curves

Figure 10: Joint 2 control torque curves

Case 2: The initial conditions are zero. The position tracking error comparison of the numerical simulation is shown in Fig.7 and Fig.8. Fig.9 and Fig.10 show the control torque of two different control methods under joint 1 and joint 2. The two control methods compared provide high tracking accuracy and fast convergence. Therefore, they can be utilized for trajectory tracking control of uncertain MRM. Note that the numerical simulation parameters are the same in both cases, only the initial conditions are different.

4.2. Experimental analysis

The proposed decentralized fixed-time tracking control approach is further verified on a two-link MRMs experimental platform, as shown in Fig.11. The two-link MRMs consist of two sets of joint modules and links. Each joint module contains a DC brushed motor produced by Maxon, with a maximum output torque of 190mNm. A harmonic drive device is connected to the output of the motor, and an incremental encoder produced by Maxon is installed on the motor end with an accuracy of 500 count/rev to measure the displacement of the motor end. An absolute encoder produced by Netzer is installed on the link end, which has a 19-bit precision and is used to collect the absolute position data of the link end. Adopt the advanced power amplifier (PLA) produced by Quanser Company to drive the motor of the joint module and the QPIDE data acquisition card to collect the data of each sensor. The joint torque sensor is installed between the link module and the joint module to measure the joint

coupling torque.

Figure 11: Experimental platform.

An experiment of two-link MRMs with tracking control is built to validate the effectiveness of the proposed control algorithm. Numerical simulations and experiments show that the uncertain disturbance conditions are the same, $\tau_{1d} = 0.02q_{1d}^2$ and $\tau_{2d} = 0.02\sin q_{2d}$ for joint 1 and joint 2, respectively. The parameters of TSMC are $\beta_i = 4$, $q_i = 5$, and $p_i = 3$ [17]. All the initial conditions are zero. The desired positions are set as:

$$\begin{aligned} q_{1d} &= 0.2\sin(3\pi t/10), & 0 \leq t \leq 20 \\ q_{2d} &= 0.2\sin(\pi t/5), & 0 \leq t \leq 20 \end{aligned} \quad (38)$$

Figs.12 and 13 show the tracking curves of MRMs under the TSMC and the proposed decentralized fixed-time control strategy, respectively. From these figures, both methods ensure good tracking performance of the MRMs system, which signifies that the trajectory tracking task is successfully achieved under the two control methods. The trajectory error of the system is shown in Figs. 14, 15, and the two control methods keep the tracking error of each joint within an acceptable range. From the error comparison curves, MRMs based on the

TSMC method has tracking error less than 6×10^{-3} rad and 1.5×10^{-3} , respectively. The errors of the proposed decentralized fixed-time control method are less than 4×10^{-3} rad and 0.5×10^{-3} rad, respectively. From the control torque curves in Figs. 16, 17, it can be seen that the proposed control method has a certain effect on reducing chattering. As we can see, the two control methods ensure good tracking performance. Similar to the numerical simulation, the proposed decentralized fixed-time control approach is better than the performance of TSMC.

Figure 12: Joint 1 position curves.

Figure 13: Joint 2 position curves.

Figure 14: Joint 1 error curves.

Figure 15: Joint 2 error curves.

Figure 16: Joint 1 control torque curves.

Figure 17: Joint 2 control torque curves.

5. Conclusion

This paper uses a fixed-time control algorithm based on the extended state
225 disturbance observer to solve model uncertainties and uncertain disturbances of
MRMs and the decentralized tracking control problem of uncertain disturbance.
With the help of JTF technology, a dynamic model of MRMs subsystem is es-
tablished. The saturation-based robust controller is used to avoid the influence
of non-parametric friction terms. A novel fixed-time SMC method based on the
230 extended state fixed-time disturbance observer is designed, which is used for the
global decentralized fixed-time tracking control of MRMs with model uncertain-
ties and uncertain disturbances. Through the Lyapunov stability theorem, it is
demonstrated that the tracking error of MRMs converges in a fixed time under
the proposed control method, and the settling time can be set arbitrarily with
235 the design parameters, independent of the initial state of the system. Numerical
simulations and experimental results demonstrate that the proposed approach
improves the control performance of the system. Future efforts will focus on
global fixed-time stable fault-tolerant control for MRMs subject to uncertain
environmental disturbances and actuator constraints.

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Declarations

Conflict of interest

The authors declare that they have no conflict of interest.

250 Data Availability Statements

All data generated or analyzed during this study are included in this published article. The model and parameter data are obtained in the table of "parameter setting" in the paper.

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