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Electron spin, nutation, and wave-particle duality

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Abstract We present a diagonal spacetime 4-manifold to model the wave-particle duality with two derivations of the energy ratio of wave to particle to be 1 to 3, by which we calculate the least time for an electron nutation as based on its quadrupole spin states in a 720-degree rotation of its associated electromagnetic wave in a model of *Zitterbewegung*.

Keywords: spin 3-D geometry; electric quadrupole; field energy

1. Introduction and motivation

The pursuit of ever faster computing speed in juxtaposition with an increasingly acute environmental concern has motivated the search for more efficient uses of electricity, such as through a better manipulation of the electron spin, as the information (0,1)-bits correspond to the magnetic dipoles, so that the higher the speed of electron nutation, the quicker the information processing (cf. [1,2]). Meanwhile, nano science with its engagement in electromagnetic waves has been more and more entwined with quantum mechanics, resulting in a mixed field of quantum nanoscience, e.g., [3,4]. Yet herein lies a geometric incongruence: whereas spin polarization as practiced by the former targets the physical 3-space, e.g., [5,6], the construct of spin as originated from Dirac's spinor of period 4π

$\square^3$

4π turn (cf. [1,2,10] for electric quadrupole): that is, relative to a fixed reference frame, its associated wave begins with a *semi-circular* rotation from $(0, -1, 0) \equiv \mathbf{S}$ to $(-1, 0, 0) \equiv \mathbf{W}$ and to $(0, 1, 0) \equiv \mathbf{N}$, entailing an initial spin axis $-\mathbf{z}$, and then at $\mathbf{N}$ the wave, altering its spin axis to $\mathbf{x}$, begins with another *semi-circular* rotation from $\mathbf{N}$ to $(0, 0, 1) \equiv \mathbf{T}$ and back to $\mathbf{S}$, or a linear transformation of $(x, y, z) \xrightarrow{-\pi/z} (-x, -y, z) \xrightarrow{\pi/x} (-x, y, -z)$

2π motion, the wave then resumes as

$(-x, y, -z) \xrightarrow{\pi_z} (x, -y, -z) \xrightarrow{\pi_{-x}} (x, y, z)$, whence returning to its initial state. As such, for every 4π – *spin* the spinning axes undergo the following succession:

$-z \rightarrow x \rightarrow z \rightarrow -x$, or ‘(down, right, up, left),’ with four distinct 90° – *nutation*’s.

This departure from the standard 2-state spin presents the possibility of manipulating electronic spins for more rapid nutation’s. We now calculate the least time for such a 90° – *nutation* from the above-described motion. Consider the relation

$$m = \frac{m_o}{\sqrt{1 - \left(\frac{|\mathbf{v}|}{c}\right)^2}}, |\mathbf{v}| < c,$$

or

$$\begin{aligned} m^2 c^4 &= m_o^2 c^4 + m^2 c^2 |\mathbf{v}|^2 \\ &= m_o^2 c^4 + m^2 c^2 |i\mathbf{v}|^2, \end{aligned}$$

where i as applied to $\mathbf{v}$ alters linear momentum into angular momentum to reflect the wave rotation of m . Then $m c^2 = m_o c^2 \mp i m c |\mathbf{v}|$

since $m c^2 \cdot m c^2 = (m_o c^2 - i m c |\mathbf{v}|)(m_o c^2 + i m c |\mathbf{v}|)$ (cf. [11] for imaginary mass); as such,

$$\begin{aligned} m c^2 &= m_o c^2 \mp i m c |\mathbf{v}| \\ &= m c^2 \sqrt{1 - \left(\frac{|\mathbf{v}|}{c}\right)^2} \mp i m c |\mathbf{v}|, \end{aligned}$$

where the first term corresponds to the particle of m and the second term, the wave of m ; assuming the energy ratio of particle to wave being 3:1 (to be derived below), then one has

$$\begin{aligned} \sqrt{1 - \left(\frac{|\mathbf{v}|}{c}\right)^2} &= \frac{3}{4}, \text{ or} \\ \left(\frac{|\mathbf{v}|}{c}\right) &= \sqrt{\frac{7}{16}} = 0.66. \end{aligned}$$

This shows that an electron by way of Breit–Wheeler pair production may be described as an electromagnetic wave of length λ spinning along a pair of perpendicular semi-circles, stopping at the two intersection points (for altering the angular momentum, cf. *Zitterbewegung* [12]), whence slowing down the otherwise speed c to $0.66c$ [9].

Therefore, at either stopping point the duration Δt is

$$\frac{\lambda/c}{\lambda/c + 2\Delta t} = 0.66, \text{ or } \Delta t = \frac{0.34}{1.32} \frac{\lambda}{c} = 0.26 \times \frac{2 \times 10^{-12}}{3 \times 10^8} = 1.7 \times 10^{-21} s$$

(λ = electron Compton wave length),

and as such the least time for an electron nutation is

$$\frac{1}{2} \left(\frac{\lambda}{c} + 2\Delta t \right) = 5 \times 10^{-21} s \text{ (in [1] the observed nutation period was about } 10^{-12} \text{ second).}$$

We now address the energy ratio of particle to wave. Consider a singular photon γ with an electromagnetic wavelength λ as contained in a ball B of radius $R = \lambda/2$ in a flat spacetime; denoting the mass/energy of B by M and performing an integration of the divergence of the gravitational field $\frac{GM/4}{r^2}$ over a compact annular volume of B (where the validity of the Newton's formulation is accounted for by the weak gravitational field as associated with GM , in details below), one arrives at an angular momentum of mass $3M/4$, with the interpretation that the spinning wave inside B containing $M/4$ *draws in* (due to the negative divergence), through the spherical boundary of B radially toward its centre, a mass of $3M/4$ that appears as a point particle γ traveling at speed c in the negative spin direction of a *separate* (yet coincidental) spacetime - - separate by assuming that the ball centre serves as a boundary point of B for the mass $3M/4$ to escape by Gauss divergence theorem). In summary, the identity

$$M/4 \text{ as wave} \equiv 3M/4 \text{ as particle}$$

dictates an energy entity M that has dual representations, hence the wave-particle duality, and either has a Euclidean ambient spacetime. In the following we shall derive the energy distribution of M .

2. Derivations for wave-particle energy ratio

We begin with an evaluation of gravitational effect of the waves in B :

$$\begin{aligned}
& \iiint_B \operatorname{div} \left(\frac{GM/4}{r^2} \right) dx dy dz \\
&= R^3 \int_{\text{setting } r \in [1/e, 1]} \left(\iint_{S_r^2} \operatorname{div} \left(\frac{GM/4}{r^2} \right) dA \right) dr \\
&= R^3 \int_{1/e}^1 \left(\iint_{S_r^2} \left(-2 \cdot \frac{GM/4}{r^3} \right) dA \right) dr \\
&= R^3 \int_{1/e}^1 \left(-2 \cdot \frac{GM/4}{r^3} \cdot 4\pi r^2 \right) dr \\
&= -R^3 \cdot 2\pi GM \int_{1/e}^1 \frac{dr}{r} \\
&= -R^3 \cdot 2\pi GM \equiv -(\pi R^3) \cdot \frac{2GM}{c^2} \cdot c^2 \\
&\equiv \# \text{ (to connect to the particle aspect below),}
\end{aligned} \tag{1}$$

where $2GM/c^2 = R = \text{Schwarzschild radius}$ for a *confinement* of the field energy $M/4$ from dissipating, inside which the metric tensor of Einstein field equations $g_{11} < 0$ engenders imaginary spacetimes (instead of the usual treatment of a signature flip [13]) and thus time t runs in a circle of $\mathbb{R}^2$ by a Wick rotation in $\mathbb{R}$ and then an embedding of $\mathbb{R} \times \mathbb{R} \subset \mathbb{R}^3$, i.e., $e^{it} = \cos t + i \sin t \leftrightarrow (\cos t, \sin t) \in \mathbb{R}^2$.

Now consider the angular momentum of $3M/4$,

$$\begin{aligned}
\frac{3}{4} McR &= \frac{3}{4} M (2GM) = \frac{3}{4} \cdot 2GM^2, \\
&\text{where } c = 1.
\end{aligned} \tag{2}$$

Multiplying # of Equation (1) by the mass density

$$\rho \equiv \frac{M}{\left(\frac{4\pi}{3} R^3 \right)},$$

one arrives at

$$\begin{aligned}
& \left(-R^3 \cdot 2\pi GM \right) \cdot \frac{M}{\left(\frac{4\pi}{3} R^3 \right)} \\
& = -2GM^2 \cdot \frac{3}{4} \\
& = - \text{angular momentum of } (3M/4)
\end{aligned}$$

of Equation (2). Thus,

$$M/4 \text{ as wave} \equiv 3M/4 \text{ as particle.}$$

The above derivation relied on one crucial assumption, namely, the integration range being $[R/e, R]$; here, we note that by a change of variable of

$r = e^{\theta/2\pi} \in [1, e]$, or $\theta = 2\pi \ln r \in [0, 2\pi]$, one has

$$\int_1^e \frac{1}{r} dr = \int_{r=1}^e e^{-\theta/2\pi} de^{\theta/2\pi} = \frac{1}{2\pi} \int_0^{2\pi} 1 \cdot d\theta = 1, \quad (3)$$

with the meaning that the gravitational energy spread out from $r = 1$ to e in a disk of radius e accounts for the entire 100% energy of $M/4$ (the integrand) and it is uniformly distributed from $\theta = 0$ to 2π (a configuration analogous to the displayed chart numbered as 6 in [3]). That is, $M/4$ resides in the annular volume of ball B in the wave universe, and $3M/4$ exists as the photon γ in the particle universe, which punctures a micro black hole b of radius 1 inside B , with $b - \{\gamma\}$ being the intersection of the two universes.

The above geometry arises from a departure from Dirac's square-root approach to the mass-shell equation by taking instead the complex conjugates as in [14]; in doing so, the abstract Clifford algebra as involved in the spinor solution underlying the Standard Model (SM) is avoided (see [15] for the comment on quantum mechanics being 'very ungeometric') and the construct of intrinsic spin is replaced with a pair of perpendicular semi-circular rotations in the Euclidean space (cf. [14]). Here, amid many re-examinations of SM, it is noteworthy that the working equations for the strong force in particular are reckoned as 'QCD inspired' rather than derived [16], which has great significance however: it means that all the experimentally verified algebraic results continue to be of engineering value, as the above introduced spacetime provides only a geometric background. In the same vein, gauge invariance specializes to frame invariance for this Euclidean model. In this connection, the proposed geometry respects the CPT invariance, where a left-handed positron is identified with a right-handed electron via a spatial

transformation, hence providing an explanation of the anti-matter asymmetry and thereof a potentially more efficient way to produce and store anti-particles, benefiting medical technology for example [9], and otherwise it furnishes the quantum vacuum a physical space of wave energies, with their energy densities identified as probability densities via their formal resemblance.

Notwithstanding the above justification of the annular domain of integration, we present another derivation of the same result through a consideration of the gravitational effect of the point mass $M_p \equiv \gamma$ and that of the wave mass $M_w \equiv \lambda$, where $M_p + M_w \equiv M$. To begin with, M_p is one singular energy point at the centre of B that draws any other energy entity (all the way) toward it, but M_w , while having its centre of mass coinciding with M_p , draws any other energy entity only up to the boundary of B for otherwise M_w would lose its identity. I.e., one is comparing the *same* amount of (gravitational) mass that is contained in the volume of B between (a) concentrated at the centre of B as *one singular point* and (b) spread out as a volume inside B . For (a) one has:

$$\left. \frac{GM_p}{r^2} \right|_q = a_{r,q} \quad \forall r \in (0, R] \quad \forall q \in S_r^2, \quad (4)$$

so that for each sphere S_r^2

$$\frac{a_{r,q}}{4\pi G} \cdot 4\pi r^2 = M_p, \quad (5)$$

implying that for every $r \in (0, R]$ the spherical area $4\pi r^2$ carries the *same* gravitational mass as $4\pi R^2$ of the ball boundary; thus, $4\pi R^2 \cdot R = 3\text{vol}(B)$ measures the gravitational effect of M_p . For (b) the calculation of the gravitational effect of M_w is of no difference from that of any matter of mass M_w occupying a volume of $\text{vol}(B)$, where one can spread out M_w uniformly over the sphere (shell) S_R^2 and recover the same gravitational mass as if it were concentrated at the centre of B . Since S_R^2 contains $\text{vol}(B)$, the gravitational effect of M_w is measured by $\text{vol}(B)$. Combining the above (a) and (b), one has

$$M = M_p + M_w = \frac{3}{4}M + \frac{1}{4}M \text{ (correspondingly)}. \quad (6)$$

To be sure, the above argument essentially reduces to a comparison between

$$4\pi R^2 \int_0^R 1 dr = 4\pi R^3 \quad (7)$$

for M_p as the volume of a (solid) cylinder of dimensions $4\pi R^2 \times R$, and

$$4\pi \int_0^R r^2 dr = \frac{4\pi}{3} R^3 \quad (8)$$

for M_w as the volume of a ball of radius R ; as such, the fraction $1/3$ originates from the anti-derivative of r^2 , and this issue had its presence in Feynman's analysis of the electromagnetic mass [17].

3. Summary Remark

In this note we presented two derivations of the energy ratio of particle to wave to be 3:1. By this relationship, we calculated the least nutation time for an electron, with a view to increasing the nutation frequency for more efficient use of electricity. We note that our model here is of a geometric nature, to serve as a supplement to SM. Yet this topic of wave-particle duality is so fundamental that it has been addressed since the early days of quantum mechanics such as [18-20]. Later treatments can be found in, e.g., [21-25]. Most of these analyses have been a re-confirmation of the principle of complementarity that no experiments are possible to measure both the wave and the particle attributes at the same time notwithstanding they have made contributions on the quantitative conditions for 'which way (welcher-weg)' the duality will assume [26]. In closing, we identify our model here with a diagonal spacetime manifold. 'Diagonal manifold,' artificial as it may resonate, nevertheless leads to a definition of the topological construct of the Euler characteristic.

Conflict of interest statement - - As the sole author of this paper, I declare that there is no conflict of interest.

Data availability statement - - The arguments contained in this paper go by mathematical derivations, hence entailing no data.

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