Empirical Models to Formulate the Nonlinear Response of Rocking Shallow Foundations

Sara Hamidpour

Tarbiat Modares University

Hamzeh Shakib (✉ shakib@modares.ac.ir)

Tarbiat Modares University <https://orcid.org/0000-0002-8714-8651>

Roberto Paolucci

Politecnico di Milano

António Correia

National Laboratory of Civil Engineering: Laboratorio Nacional de Engenharia Civil

Masoud Soltani

Tarbiat Modares University

Research Article

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Posted Date: December 13th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1158880/v1>

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Empirical Models to Formulate the Nonlinear Response of Rocking Shallow Foundations

Sara Hamidpour¹ · Hamzeh Shakib¹ · Roberto Paolucci² · António Correia³ · Masoud Soltani¹

Abstract

This paper aims to introduce a simplified moment-rotation backbone model for exploring the nonlinear behavior of shallow foundations subjected to rocking. The model is developed based on parametric numerical investigations of rectangular footings on dense dry sand, taking advantage of a nonlinear macro-element model verified based on a set of experimental results. Empirical expressions are proposed for rocking stiffness degradation due to gravity loads and foundation rotation as a function of the factor of safety against vertical loads and aspect ratio of foundations. Similar to previous researches, the uplift reference rotation was introduced to explore a new closed-form expression appropriate for normalizing the foundation response in a non-dimensional form. The proposed approach for stiffness degradation and nonlinear backbone model of rocking foundations aims to be simple, to minimize the dependence on the variable parameters, and to provide physically sound selections for engineering applications.

Keywords Foundation rocking · Stiffness degradation · Moment-rotation backbone · Nonlinear macro-element model

1 Introduction

It has long been known that the dynamic interaction of the foundation with the superstructure and the underlying soil can affect and modify the response of structures under seismic loads. Such interaction has been the subject of numerous experimental and analytical research works with the aim to correlate the theory with the physical phenomenon. In this way, researchers have developed several numerical tools including rigorous and simplified techniques with the aim of promoting the most important features of efficiency, accuracy, and simplicity. Since the rocking of foundations is recognized to be the most relevant vibration mode for such interaction, this research will mainly address the rocking mode response of foundations.

For the last couple of decades, there has been a growing interest in simplified simulation methods of nonlinear soil structure interaction (SSI). Gazetas et al. (2013) and Adamidis et al. (2014) proposed an iterative equivalent-linear method for the nonlinear rocking foundations on undrained clay using simple charts for stiffness degradation based on foundation rotation and factor of safety against vertical loads (FS_v). Paolucci et al. (2013) developed an iterative linear-equivalent procedure for the nonlinear SSI in displacement-based design (DBD) method. They proposed an empirical expression for the rocking stiffness degradation due to foundation rotation as a function of FS_v and sand relative density (D_r). In the DBD framework, the iterations were used to modify the rocking stiffness of foundation regarding the imposed rotation until the convergence was acquired on the foundation rotation. In the same context, Anastasopoulos and Kontoroupi (2014) developed a simplified approximate model for the inelastic rocking response of square foundations on undrained clay. The model followed a simple nonlinear moment-rotation relation as a unique non-dimensional piecewise curve for different FS_v . Unlike the methods using equivalent-linear models, the model proposed by Anastasopoulos and Kontoroupi (2014) aimed to be straightforward not requiring iterations for the nonlinear response considerations. Deng (2012) and Deng et al. (2014) introduced a trilinear moment-rotation model for the nonlinear foundations consisted of an elastic and a plastic element in series so that the elastic element stiffness was the initial stiffness and the equivalent-linear stiffness of the plastic element was equal to the secant stiffness regarding maximum rotation and moment capacity of foundation. Taeseri et al. (2019) emphasized the shortcomings of the existing impedance formulations,

Hamzeh Shakib
shakib@modares.ac.ir

¹ Faculty of Civil and Environmental Engineering, Tarbiat Modares University, Jalal Al Ahmad Avenue 7, Tehran, Iran

² Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, Milan, Italy

³ National Laboratory for Civil Engineering, Av. do Brasil 101, Lisbon, Portugal

e.g., Gazetas (1991) in considering soil inhomogeneity and addressed its effects on the small strain rocking stiffness of embedded foundations. In line with the simplified methods, Sieber et al. (2020) investigated a nonlinear and a bilinear rocking stiffness model for simulation of bridge piers with square foundations on stiff clay and observed such simplified models were appropriate for the maximum rotation prediction while not for the rotation time history and M - θ loops.

Rocking behavior of foundations has attracted the attention of researchers and practitioners because of the beneficial characteristics like energy dissipation and self-centering capability. Deng et al. (2014) focused on bridge piers and showed their improved seismic performance when allowing rocking at foundation level. Studies of Hakhamaneshi (2014) and Hakhamaneshi and Kutter (2016) aimed to evaluate the performance of rocking foundations with different shapes and embedment depths and revealed the larger settlements in narrower rectangular and I-shaped footings. Liu et al. (2013) and Liu (2014) investigated the performance of low-rise frames and observed the beneficial features of balancing the foundation rocking and structural inelasticity. Aiming to support the concept of controlled share of ductility demand between superstructure and foundation, Pecker et al. (2014) provided an overview of the experimental activities for seismic evaluation of nonlinear foundations and theoretical achievements in using foundation macro-element models in DBD framework. Figini and Paolucci (2017) studied the coupled nonlinear response of rocking foundation and structure assuming three fixed, elastic and nonlinear base conditions and observed the beneficial effects of the nonlinear foundation and its substantial contribution to overall energy dissipation. Gajan and Kayser (2019) quantified the effects of soil uncertainties on rocking foundations performance and also Sandararajan and Gajan (2020) and Gajan et al. (2021) analyzed and summarized the results of nine experimental series in order to investigate the performance of rocking foundations and to explore the possible correlations between performance with capacity and seismic demand parameters.

Inspired by the increasing development of simplified models for foundation rocking behavior, the present research explores an approximate moment-rotation backbone model for nonlinear rocking response of surface rectangular foundations on dense dry sand. Primarily, the existing data from centrifuge and shake table experiments, which include different foundation geometries and vertical and lateral loadings were examined and used to validate the nonlinear macro-element model used for the parametric numerical simulations. The numerical analyses were performed with the finite element software SeismoStruct (2021) using a macro-element model for soil-foundation simulation. Based on the parametric study results, empirical closed-form expressions were developed for the rocking stiffness degradation and the uplift reference rotation of foundation. The concept of uplift reference rotation was then used to simplify the nonlinear response of foundations in the form of non-dimensional values. Finally, a simple trilinear moment-rotation backbone curve for nonlinear foundations was proposed with a formulation to consider the footing aspect ratio (B/L) effects. The rocking foundation hysteretic damping ratio was also investigated. The accuracy of the proposed approach was validated against available experimental data. This study goes one step further than similar studies discussed before with simplifying of two main responses of foundation namely rocking stiffness degradation and moment-rotation backbone in non-dimensional form and their prediction based on simplified empirical expressions.

2 Problem Definition

Primarily, it is necessary to explain some concepts regarding the foundation nonlinear behavior on which this research will mainly focus as the reference values for the foundation rocking stiffness.

The elastic foundation impedances represent the stiffness under small levels of applied loads or displacements and neglect yielding and gapping, while the highly nonlinear nature of soil response shows itself even from the initial stages of static gravity loading. The foundation elastic stiffness in rocking mode can be determined using the formulation of Gazetas (1991) - also adopted by ASCE-7 (2016), ASCE-41 (2017) and FEMA 356 (2000) - as a function of soil shear modules G at small strains, Poisson's ratio ν , and footing dimensions ($L \times B$) as bellow:

$$K_{R(Elastic)} = 3 \frac{G}{1 - \nu} I^{0.75} \left(\frac{L}{B}\right)^{0.15} \quad (1)$$

To consider the foundation nonlinear behavior properly, an elastic stiffness degradation model is required, which includes the effect of two main parameters including gravity loads and the foundation rotation. These two influential parameters contribute to the soil material yielding and to the soil-footing contact area reduction due to gapping, which results in the foundation rocking stiffness degradation. As it will be discussed later, the higher the amplitude of gravity loads, the greater the soil yielding and nonlinearity and consequently the greater the elastic stiffness degradation. The reduced initial effective stiffness ($K_{R(N,\theta \rightarrow 0)}$) is simply defined as the stiffness after the application of gravity loads and before rotation is applied to the foundation in this research and can be determined as a fraction of $K_{R(Elastic)}$. According to Eq. (2), $K_{R(N,\theta \rightarrow 0)}$ is a function of FS_v and footing aspect ratio (B/L) and will be empirically determined in Eq. (6).

$$K_{R(N,\theta \rightarrow 0)} = K_{R(Elastic)} \cdot f(FS_v, \frac{B}{L}) \quad (2)$$

When the foundation undergoes rocking, its stiffness decreases with the increase of the rotation angle because of the reduction of contact area and the greater nonlinearity in the soil response. The foundation rocking stiffness ($K_{R(N,\theta)}$) can be expressed as a function of the rotation angle as follows:

$$K_{R(N,\theta)} = K_{R(N,\theta \rightarrow 0)} \cdot g(\theta) \quad (3)$$

With the simultaneous consideration of the reduced initial effective stiffness and the dependency on the foundation rotation, the two previous equations can be combined to define the dynamic rocking stiffness of foundation as a function of gravity loads, footing aspect ratio and rotation angle as Eq. (4). For simplicity, the term $K_{R(N,\theta)}$ will be presented as K_R hereafter.

$$K_R = K_{R(N,\theta)} = K_{R(Elastic)} \cdot h(FS_v, \frac{B}{L}, \theta) \quad (4)$$

In this research, K_R is defined as the secant rocking stiffness of foundation, which can be calculated as the ratio of the moment to rotation ($K_R = M/\theta$) regarding the moment-rotation backbone curve of foundation as displayed in Figure 1.

The soil-foundation system is highly nonlinear with wide hysteresis loops that indicate the importance of the damping of hysteretic type. The hysteretic damping ratio can be computed for every hysteretic loop using Eq. (5):

$$\xi_R = \frac{\Delta E / E}{4\pi} \quad (5)$$

where, ΔE is the enclosed area of the M - θ loop and represents the energy dissipated during each cycle and E is the elastic energy as displayed in Figure 1. A schematic view of the studied system under rocking and a typical moment-rotation loop are illustrated in Figure 1.

Fig. 1 Schematic view of SDOF system based on rocking foundation and illustration of secant stiffness and area of hysteretic loop

3 Experimental Rocking Foundations Studies

In recent years, several experimental projects have been conducted on the rocking response of shallow foundations at different research facilities worldwide (Pecker et al. 2014). Some of the pioneering experimental projects can be summarized as: the works of Gajan and Kutter (2008), Deng et al. (2012), Hakhamaneshi (2014) and Liu (2014) in the centrifuge testing equipment at the Center for Geotechnical Modeling (CGM) at University of California, Davis; large-scale cyclic experiment of Faccioli et al. (2001) and Negro et al. (2000) at the European Laboratory for Structural Assessment (ELSA) in Italy; reduced-scale shake table experiment of Anastasopoulos et al. (2013) at the National Technical University of Athens (NTUA) in Greece; and the large-scale shake table test of Shirato et al. (2008) and Paolucci et al. (2008) and cyclic tests of Shirato et al. (2008) at the Public Works Research Institute (PWRI) in Japan. Taking advantage from the mentioned experimental programs, the results of the large-scale cyclic experiments of TRISEE (Faccioli et al. 2001; Negro et al. 2000) and PWRI (Shirato et al. 2008) as well as the results of the centrifuge experiment of Gajan and Kutter (2008) were used in this study. The experiments were selected based on ease of the data accessibility and to meet the requirements of this research in response evaluation of rocking foundation systems under slow-cyclic loads.

3.1 Employed Experimental Case Studies

TRISEE project is a large-scale cyclic experimental program conducted at the ELSA laboratory in Ispra (Italy). The aim of this program was to investigate the nonlinear interaction between shallow foundation and soil under cyclic loads. The experimental set-up consisted of a rigid caisson filled with high relative density (HD, $D_r = 85\%$) and low relative density (LD, $D_r = 45\%$) Ticino river sand. The $1\text{ m} \times 1\text{ m}$ steel foundation had a concrete interface with the underlying soil in order to ensure a high friction contact. The vertical load was kept constant during the tests and the specimens were subjected to cyclic displacement loads until the collapse of the foundation. For detailed information about TRISEE experiment project, one may refer to Faccioli et al. (2001) and Negro et al. (2000).

The PWRI large cyclic tests conducted in Japan focused on the performance of shallow foundations under realistic seismic loads. The set-up contained a laminar box filled with dry Toyoura sand ($D_r = 60\%$ & 80%) and a pier supported on a $0.5\text{ m} \times 0.5\text{ m}$ footing. Two types of reversed displacement cyclic lateral loads with different frequencies and amplitudes were applied to the specimens. Because of the realistic excitations of various amplitude levels and careful instrumentation, this test results were considered very helpful for a better understanding of the nonlinear SSI as well as the calibration of numerical approaches. More information about the test is accessible in Shirato et al. (2008).

Gajan and Kutter (2008) performed several series of centrifuge experiments (20g) for shear walls based on shallow foundations, hereafter referred to as SSG, as a part of PEER research program at UC Davis. This program aimed at better understanding of the nonlinear SSI interaction. The $2.8m \times 0.65m$ footing was founded on dense dry Nevada sand ($D_r = 60$ & 80%) and subjected to slow lateral cyclic and dynamic excitations. In order to cover a wide variety range of parameters, the testing program included specimens that had different mass, loading amplitude and frequency and the lateral loading point height. Model configurations and experimental results can be found in Gajan (2006), Gajan and Kutter (2008) and Gajan et al. (2008). Summary information of the employed experimental case studies in this research is presented in Table 1.

It is to be noted that detailed information and data about several recent experimental projects on rocking foundations can be found in two database reports compiled by Hakhamaneshi et al. (2020) and Gavras et al. (2020) in the framework of FoRCy and FoRDy database for cyclic and dynamic loading experiments, respectively, which were used in this research a well.

Table 1 Characteristic parameters of the employed experimental case studies*

TEST	L (m)	B (m)	D/B	Mass (ton)	FS_v	h (m)	D_r (%)
TRISEE-HD	1	1	0	30	5	0.9	85
PWRI-5	0.5	0.5	0	0.89	6.7	1.3	80
PWRI-7	0.5	0.5	0	0.89	6.7	0.9	80
PWRI-8	0.5	0.5	0	0.89	6.7	0.9	80
PWRI-10	0.5	0.5	0	0.89	6.9	0.9	60
PWRI-11	0.5	0.5	0	0.89	6.9	0.9	60
SSG02-03	2.8	0.65	0	28	5.2	4.9	80
SSG02-05	2.8	0.65	0	58	2.5	4.8	80
SSG02-07	2.8	0.65	0	58	2.6	4.9	80
SSG04-04	2.8	0.65	0	38	4	3.5	80
SSG04-06	2.8	0.65	0	68	2.3	3.1	80

* L : foundation length, B : foundation width, D : foundation embedment depth, Mass: mass of structure-footing system, FS_v : factor of safety against vertical loads, h : SDOF system height (also lateral loading point height), D_r : soil relative density.

3.2 Evaluation of the Results of Experimental Case Studies

According to Table 1, a set of eleven experimental case studies was selected. As mentioned before, the experimental models had been subjected to slow lateral cyclic displacement loadings with different amplitudes and frequencies. Using the results of the experimental models in terms of the moment-rotation cyclic loops, three responses were investigated including the moment-rotation ($M-\theta$) backbone curves, the foundation rocking stiffness degradation and the hysteretic damping. The equivalent backbone envelope of the cyclic $M-\theta$ loops for the selected experimental cases are displayed in Figure 2. The backbone curves were also used to explore the rocking stiffness degradation due to rotation.

Fig. 2 Foundation $M-\theta$ backbone curves of experimental case studies: TRISEE_HD (left), PWRI (middle) and SSG (right)

As explained in Sect. 2, rocking stiffness degradation which is calculated as the ratio of the secant stiffness ($K_R = M/\theta$) to the initial effective stiffness of foundation ($K_{R(N,\theta \rightarrow 0)}$), provides a non-dimensional curve as displayed for experimental cases studies in Figure 3 (left). The degradation is due to the rotation and the consequent uplift, gapping and soil material yielding. Examination of the results showed that generally there were noticeable differences between the trends of stiffness degradation curves for each experimental series so that

degradation curve had a steeper slope in SSG specimens than in TRISEE or PWRI specimens. In addition, by looking at a defined value of degradation ratio in the graph, 0.6 for instance, it is observed that the rotation value is larger for PWRI (6 mrad), TRISEE (2 mrad) and SSG (0.5 mrad), respectively. This observation could be attributed to the foundation dimension effects because, as it will be discussed in more detail in the next sections, the rotation at which the foundation uplift initiates is a smaller value for longer and strip-like footings.

Figure 3 (right) displays the damping ratio calculated according to Eq. (5) for the experimental cases. As shown, the damping ratio was mostly in the range of 10% to 30%, with smaller values for PWRI and larger values for SSG specimens. In addition, it was observed that PWRI-10 and 11 had larger damping ratios compared to the other PWRI specimens as they were based on less dense sands ($D_r = 60\%$). The increase of damping ratio as a function of rotation was observed in most cases, although it has not occurred in all experimental specimens.

Fig. 3 Foundation rocking stiffness degradation (left) and hysteretic damping ratio (right) obtained from experimental case studies

4 Parametric Numerical Analyses

According to the preliminary outcomes investigated in the Sect. 3.2, it was clear that experimental results depended on different parameters including foundation geometry (size and shape), gravity loads amplitude, lateral loading protocol, and test conducting method. However, the attention of this numerical study is limited to the two parameters as the subject of parametric evaluation i.e. FS_v and the footing aspect ratio that were found to have the largest impact on the results. It should be mentioned that, sensitivity analyses were performed to identify the importance degree of FS_v and loading amplitude and frequency parameters for different sizes of foundations - results not provided for brevity purposes - among which the FS_v was recognized as the most effective one.

4.1 Numerical Models Extended based on Experimental Case Studies

The numerical models were developed based on the TRISEE-HD large-scale test and SSG04-06 and SSG02-07 centrifuge experiments using SeismoStruct software. The results of the SSG02-07 model are not presented in this paper for the sake of brevity. In addition, we also considered a numerical model similar to SSG04-06 but with half-length ($1.3m \times 0.65m$) named SSG-01 hereafter and generated to explore the effect of foundation aspect ratio on the results.

In the numerical analyses, the superstructure was simulated by a SDOF system with mass, m , and height, h , supported by a rigid footing on dense sands. An elastic column frame element was used for simulation of the superstructure, while a macro-element model was used for the simulation of the soil-foundation system. The lateral displacement loads were applied at column height, h , similar to the experiments. It is worth noting that the numerical models were the same as the corresponding experimental specimens in every way except the FS_v , which was considered as a variable parameter for the parametric evaluation purpose. For the TRISEE-HD model $FS_v = 1.5, 3, 5, 7, 9$, for SSG04-06 model $FS_v = 1.6, 2.3, 3, 4, 6$ and for the SSG-01 model $FS_v = 1.6, 3, 4, 6$ were investigated.

4.2 Macro-Element Model for Soil-Footing system

The soil structure interaction can be modeled using different methods. The direct finite element approach with cubic solid elements for soil has been used by numerous researcher (Anastasopoulos 2009; Torabi and Rayhani 2014; Xiong et al. 2018; Qaftan et al. 2020) and is considered to provide the most accurate and realistic results, however it is not cost-effective and faces numerical complexity in dealing with boundary conditions, interface elements and strongly nonlinear soil response. The Winkler foundation models can also be used whereby the soil medium under footing is idealized by uniformly distributed independent spring elements (Raychowdhury and Hutchinson 2009; El Canainy and El Naggar 2009). Although Winkler method is simple and easy to be implemented, its main limitation is not accounting for the coupling among different degrees of freedom (DOFs) in the SSI system (Pender 2007). In recent decades, macro-element models were introduced to the hybrid simulation methods of SSI problems with the aim to make the computations simple. In this model, the footing is assumed rigid and a single macro-element with a suitable yield surface and plastic potential function substitutes the soil-foundation system at the center point of the superstructure base. The advantageous feature of the macro-element models is to account for the coupling among all DOF of the system (Chatzigogos et al. 2011; Figini et al. 2012).

The macro-element model used in this research is characterized as a zero-length link element with six DOF in three-dimensional cases in SeismoStruct software. This macro-element introduces uplift based on a nonlinear elastic-uplift response, which takes into account the contact degradation through irrecoverable changes in contact interface geometry. A bounding surface plasticity model compatible with the soil type is also adopted to account for the simultaneous elastic-uplift and plastic-nonlinear response. For simulation of the soil-foundation system in SeismoStruct software, in addition to the linear visco-elastic foundation impedances and the bearing capacity, five nonlinear macro-element parameters should be defined among which only three need to be calibrated (d_{mg} , h_0 , χ_g). The foundation stiffness and dashpot coefficients can be determined typically based on standard formulations as reported in Gazetas (1991) while standard formulas such as Meyerhof (1951) may also be used for the bearing capacity. The nonlinear macro-element parameters used for numerical simulations are defined in Table 2 with a typical range of variability and suggested optimum values that allow obtaining a reasonable agreement with experiments. The suggested values were determined based on calibrations carried out on a set of experimental results introduced in Sect. 3. Figure 4 shows two examples of such calibration results for TRISEE-HD and SSG04-06 case studies. According to the results, the numerical models were reasonably accurate in predicting the most relevant response measures such as hysteretic $M-\theta$ curves, ultimate rocking moment and stiffness degradation curves. For more details about the employed macro-element model and the input parameters, one may refer to SeismoStruct (2021) and Pianese (2018).

Table 2 Characteristic parameters of the employed macro-element model

Parameter	Range	Suggested value
Uplift initiation parameter, α	3	3
Contact degradation parameter, d_{mg}	0.1-10	4
Reference plastic modulus, h_0	0.2-0.4	0.2
Exponent for loading history in unloading/reloading, n_{ur}	1	1
Plastic potential parameter, χ_g	0.5-2	1.5

Fig. 4 Validation of numerical models using macro-element against experimental results: TRISEE-HD (left) and SSG04-06 (right)

5 Rocking Stiffness Degradation of Foundations

5.1 Evaluation of the Results of Parametric Numerical Analyses

After the validation of numerical model results introduced in Sect. 4, foundation rocking stiffness degradation is evaluated in this section based on a series of parametric numerical analyses aimed at providing suitable expressions for the functions in Eq. (2) and Eq. (4). A sample of such parametric studies is shown in Figure 5, where the graphs on the top display the rocking stiffness decrease relative to the elastic stiffness ($K_R/K_{R(elastic)}$), while the graphs on the bottom display the rocking stiffness decrease relative to the reduced initial effective stiffness ($K_R/K_{R(N, \theta \rightarrow 0)}$). Interesting results were observed comparing the two groups of curves for each foundation model. The $K_R/K_{R(elastic)}$ curves present the effect of gravity loads better, while $K_R/K_{R(N, \theta \rightarrow 0)}$ curves present the effect of uplift better. In Figure 5 (top), the stiffness ratio at the starting point of each curve represents the ratio of the initial effective stiffness to the elastic stiffness ($K_{R(N, \theta \rightarrow 0)}/K_{R(elastic)}$). As an expected consequence of the greater imposed vertical stress and the consequent yielding of the soil, this ratio was a smaller value for heavily loaded foundations (smaller FS_v), which is more explored in Figure 6. It is important to note that in Figure 5 (top), by the increase of rotation angle, at some point, the degradation curves trend reversed and greater slope of stiffness degradation was observed for lightly loaded foundations. This is the consequence of the greater foundation uplift in lightly loaded foundations and the better presentation of that can be seen in Figure 5 (bottom) where the $K_R/K_{R(N, \theta \rightarrow 0)}$ curves are provided. According to these curves, the rocking stiffness of the lightly loaded foundations (larger FS_v) started to decrease at smaller angles of rotation rather than the heavily loaded ones, which emphasizes the bold role of the uplift and contact interface reduction in lightly loaded foundations. Further investigation of the results confirmed the fact that foundations with different physical properties start to detach from the underneath soil at different angles of rotation.

Fig. 5 Rocking stiffness (K_R) degradation relative to elastic stiffness ($K_{R(elastic)}$) (top) and relative to initial effective stiffness ($K_{R(N, \theta \rightarrow 0)}$) (bottom) obtained from numerical models: TRISEE-HD (left), SSG-01 (middle), SSG04-06 (right)

The findings from numerical analyses were used to derive an empirical closed-form expression for the rocking stiffness degradation due to gravity loads ($K_{R(N, \theta \rightarrow 0)}/K_{R(elastic)}$). According to Figure 6, the reduction of the elastic stiffness due to gravity loads was more pronounced for the HD cases rather than the SSG cases. For the HD case with $FS_v = 1.5$, for instance, the initial effective stiffness was almost 35% of the elastic stiffness value and in the case of $FS_v = 9$, it was almost 80%; however, for the SSG04-06, the initial effective stiffness ratio was in the range of 65% to 80% for heavily to lightly loaded foundations. The wider range of stiffness reduction in the HD models compared to the SSG models, as clarified in Figure 6, could be an indicator of the footing size and aspect ratio effects; so that the stiffness of the foundations with larger size and smaller aspect ratio showed less sensitivity to the gravity loads. The approximate closed-form expression in Eq. (6) was developed as a function of FS_v and footing aspect ratio as previously discussed in Eq. (2) by fitting the best estimate to the numerical results:

$$\frac{K_{R(N, \theta \rightarrow 0)}}{K_{R(Elastic)}} = 1 - \left[\left(\frac{1}{1.5 FS_v} \right)^{0.5} \left(\frac{B}{L} \right)^{0.35} \right] \quad (6)$$

Fig. 6 Initial effective rocking stiffness to elastic stiffness ratio ($K_{R(N, \theta \rightarrow 0)}/K_{R(elastic)}$) obtained from numerical results (dots) and Eq. (6) (lines)

5.2 Normalization of the Stiffness Degradation Curves

According to the previous section, the rotation at which the foundation starts to detach from the soil was found to be an informative criterion for the rocking response evaluation. The concept of the uplift initiation rotation was first introduced and used for normalization of the stiffness degradation and the moment-rotation curves by Gazetas et al. (2013) and Anastasopoulos et al. (2014). Based on the results of the parametric numerical study, an updated closed-form expression is proposed in this section for calculation of the uplift initiation rotation for rectangular foundations on dense dry sands, introduced here as the uplift reference rotation and denoted as θ_r . The uplift reference rotation will later be used for normalization of the stiffness degradation and the moment-rotation curves in this study.

Since cohesionless dense sands were considered in this study, the stress distribution under the rigid footing due to vertical loads is expected to be maximum at the middle and decrease toward the edges of the foundation. The minimum moment to trigger the rotation in rigid footing on elastic half-space is $M = NL/4$, where N is the vertical load and L is the footing length in the rocking plane. In this way, the triggered elastic rotation is $\theta = NL/4K$. It is important to note that since foundation rocking occurs in the presence of gravity loads, therefore the rocking stiffness, K , coincides with $K_{R(N,\theta \rightarrow 0)}$. Combining the experimental and the numerical parametric investigations results, the following expression was obtained for the uplift reference rotation:

$$\theta_r = \frac{NL}{4K_{R(N,\theta \rightarrow 0)}} \exp\left(\frac{0.5}{FS_v} - \frac{2}{FS_v}\left(\frac{B}{L}\right)\right) \quad (7)$$

It should be noticed that while in Gazetas et al. (2013), the normalization helped to remove the foundation shape effect on the stiffness degradation curves and in Anastasopoulos et al. (2014), it was suitable to provide a unique M - θ curve irrespective to the FS_v for square foundations, in this research, the normalization was employed to provide unique stiffness degradation and M - θ curves for square and rectangular foundations irrespective to the FS_v and based on the footing aspect ratio.

To achieve this, the horizontal axis of the $K_R/K_{R(N,\theta \rightarrow 0)}$ curves in Figure 5 (bottom), were normalized based on the uplift reference rotation, θ_r , determined from Eq. (7). The resultant curves are displayed by dashed lines corresponding to different FS_v values in Figure 7, which showed almost unique response regardless of the FS_v value and could be well replaced with a single curve. The fitted single curves displayed by continuous lines in Figure 7 for each study case were used to derive an empirical expression for the stiffness degradation of foundations, which will be explained in more details in the following section.

Fig. 7 Normalized stiffness degradation curves obtained from numerical models (dashed lines) and the fitted curves (continuous lines): TRISEE-HD (left), SSG-01 (middle), SSG04-06 (right)

5.3 A Simplified Formula to Predict the Rocking Stiffness Degradation of Foundations

Paolucci et al. (2013) proposed an empirical expression for foundation stiffness degradation due to foundation rotation presented in Eq. (8) in which a and m are non-dimensional parameters and a function of soil density and foundation FS_v , the proposed values of which are given in Paolucci et al. (2013).

$$\frac{K_R}{K_{R,0}} = \frac{1}{1 + a\theta^m} \quad (8)$$

By taking advantage of the normalization introduced in the previous section, a more general expression than Eq. (8) was introduced by using the normalized rotation, θ/θ_r , and the aspect ratio, B/L , as follows:

$$\frac{K_R}{K_{R(N,\theta \rightarrow 0)}} = \frac{1}{1 + 0.3\left(\frac{\theta}{\theta_r}\right)^{1.15}\left(\frac{B}{L}\right)^{-0.6}} \quad (9)$$

The above expression can be used for square and rectangular foundations with different aspect ratios where the FS_v value came into play indirectly in the Eq. (6) for $K_{R(N,\theta \rightarrow 0)}$ and Eq. (7) for θ_r .

In order to validate the proposed Eq. (7) and Eq. (9), we refer to the experimental models introduced in Sect. 3 (see Table 1 for details). For this purpose, the results of the foundation rocking stiffness degradation presented in Figure 3 were normalized in horizontal axis based on the θ_r value (Eq. 7) and displayed with dots in Figure 8. Besides, the stiffness degradation curves obtained from Eq. (9) for aspect ratios of 1.0 (corresponding to TRISEE-HD and PWRI) and 0.25 (corresponding to SSG), are displayed with a bold line in the same graph. According to the validation results shown in Figure 8, it can be concluded that normalization of the results using θ_r helped to unify the stiffness degradation curves for foundations with specific aspect ratio to almost similar response and the proposed Eq. (9) predicted the stiffness degradation of the experimental models with acceptable accuracy.

Fig. 8 Normalized stiffness degradation curves obtained from experimental case studies (dots) and the proposed Eq. (9) (bold line): TRISEE-HD and PWRI foundations with $B/L=1.0$ (left) and SSG foundations with $B/L=0.25$ (right)

6 Moment-Rotation Backbone Curve Model

As a further application of the normalization procedure based on θ_r , a set of empirical M - θ backbone curves were also acquired for the previously described numerical models and normalized by θ_r on the horizontal axis and by the ultimate moment capacity, M_u , on the vertical axis. According to the results shown in Figure 9 (dashed lines), the normalization using θ_r made the M - θ curves -like the stiffness degradation curves in Figures 7- almost unique regardless of the FS_v value so that they could be represented by a single curve for each foundation with a specific aspect ratio (continuous lines). By summarizing the findings, it can be concluded that normalization of the foundation responses using the uplift reference rotation, θ_r , provides general curves for the M - θ or the stiffness degradation with regard to the foundation aspect ratio, while the effect of FS_v is considered indirectly in θ_r formulation.

Fig. 9 Normalized moment-rotation backbone curves obtained from numerical models (dashed lines) and the fitted curves (continuous lines): TRISEE-HD (left), SSG-01 (middle), SSG04-06 (right)

In line with the previous results explanations, a general trilinear backbone model displayed in Figure 10, was proposed for the square foundations ($B/L = 1.0$) which is modifiable for rectangular foundations with different aspect ratios through an accompanying formulation (Eq. 10). The proposed model is a piecewise non-dimensional

moment-rotation backbone consisting of sections for quasi-elastic, nonlinear and perfectly plastic responses. In the proposed model, with a simplifying approximation the quasi-elastic response is considered prior to the uplift initiation in the foundation ($\theta/\theta_r < 1$). Initiation of the nonlinear response phase ($1 < \theta/\theta_r < 6$) and the plastic response phase ($\theta/\theta_r > 6$) are defined in terms of the non-dimensional rotation, θ/θ_r , which depends on the aspect ratio of foundation. In the backbone, M_u represents the ultimate moment capacity of foundation and the uplift moment, corresponding to the uplift reference rotation ($\theta/\theta_r = 1$) is equal to $M_{up} = 0.45M_u$.

Fig. 10 Proposed non-dimensional trilinear M - θ backbone curve for foundation with $B/L = 1.0$

While Figure 10 refers to square foundations, the extension of the proposed trilinear model to rectangular foundations ($B/L < 1.0$), can be expressed by the following expression, which is used for modifying the θ/θ_r values on the horizontal axis of Figure 10:

$$\left(\frac{\theta}{\theta_r}\right) = \left(\frac{\theta}{\theta_r}\right)_{B/L=1} * \left(\frac{B}{L}\right)^{0.2} \quad (10)$$

In order to validate the proposed backbone model in Figure 10 with the accompanying Eq. (10), we refer again to the experimental case studies introduced in Sect. 3. For this purpose, the M - θ curves of the experimental specimens displayed in Figure 2, were normalized in horizontal axis based on the θ_r value (Eq. 7) and on the vertical axis based on the corresponding ultimate moment, M_u , of each specimen. Figure 11 displays the results of such normalization along with the backbone curve obtained from proposed trilinear model in Figure 10 and Eq. (10). It was observed that normalization based on θ_r helped to unify the M - θ curves irrespective to FS_v variability and the proposed backbone model can predict the M - θ backbone of the foundations with acceptable accuracy. Regarding the PWRI, the experimental specimens seemed to have experienced strength deterioration after the ultimate moment reached during the test that is not very common in foundations. However, the non-deteriorating strength assumption, which is more common in rocking foundations, was considered to predict the backbone of this test series as well.

Fig. 11 Normalized moment-rotation curves obtained from experimental case studies (dashed lines) and the proposed backbone curve (bold line): PWRI foundations with $B/L=1.0$ (left) and SSG foundations with $B/L=0.25$ (right)

The backbone models, like the trilinear model proposed in this research, can be used in monotonic or pushover loadings, while in cyclic or dynamic loadings they require to be coupled with a second part of a model that

provides the energy dissipation in many cases provided by finite element models – such as link elements - or dashpot elements. In order to evaluate the application of the proposed trilinear backbone model coupled with such link element, some experimental SSI systems were numerically simulated by taking the advantage of the 3D link element available in SeismoStruct software, which is suitable for foundation flexibility simulations. The considered link element connects two initially coincident structural nodes and requires the definition of the moment-rotation (or force-displacement) response curve for the desired DoF. The validation was performed for TRISEE-HD and SSG04-06 models, introduced in Sect. 3, and a shear wall model from CAMUS 4 test, referenced with more details in Combescure and Chaudat (2000) and The project CAMUS 3 and 4 (2000). CAMUS is a series of seismic tests of 1:3 scale reinforced concrete (RC) shear walls on shake table denoted by CAMUS 1 to 4. While CAMUS 1-3 were fixed-base, CAMUS 4 was tested resting on a sand layer ($D_R = 70\%$) with the aim to investigate the effect of uplift on the overall response of RC wall structure. It represented a 5-story shear wall with 5.135 m total height, based on a $2.1m \times 0.8m$ foundation and was subjected to different base excitations including the artificial Nice excitations with PGA values of 0.33g and 0.52g which were considered in this study. It is to be noted that for the TRISEE-HD it was necessary to consider a post yield stiffness ratio larger than zero (2%) to obtain the best results while for the other tests, perfectly plastic response and the post-yield stiffness ratio of zero was appropriate. The results from such numerical simulations were compared to the experimental results and displayed in Figure 12 in terms of the rotation and the moment time series, as well as the moment-rotation hysteretic curves. It was observed that the numerical simulations by employing the proposed trilinear backbone model coupled with a link element that substituted the soil-footing system showed good accordance with both the cyclic (TRISEE-HD and SSG04-06) and the dynamic (CAMUS 4) experimental results.

Fig. 12 comparison between numerical simulations using proposed trilinear $M-\theta$ backbone model (gray lines) and experimental results (dashed lines) for TRISEE-HD, SSG04-06 and CAMUS 4 under 0.33g and 0.52g excitations: rotation time series (left), moment time series (middle) and moment-rotation hysteretic curve (right)

7 Hysteretic Damping Ratio of Foundation

The numerical analyses results were also used to determine the hysteretic damping ratio of foundation under cyclic loads. The results displayed in Figure 13 show that, in general, the hysteretic damping ratio was less than 10% for rotations smaller than 0.001 rad and increased at maximum to almost 40% with the increasing of the rotation amplitude to 0.05 rad. On average, the damping ratio had a greater amplitude for SSG04-06, SSG-01 and TRISEE-HD models, respectively. Moreover, according to the results, it seems that the role of the FS_v on the hysteretic damping ratio became less important as the foundation size increased and aspect ratio decreased.

Fig. 13 Rocking hysteretic damping ratio obtained from numerical models: TRISEE-HD (left), SSG-01 (middle), SSG04-06 (right)

8 Conclusions

This study aimed to investigate the nonlinear rocking response of foundations on dense dry sands considering the soil material and geometrical nonlinearities by taking the advantage of a nonlinear macro-element model for the soil-foundation system. The main objective has been to develop a simple approach according to which, by employing non-dimensional expressions based on few characteristic parameters, the foundation rocking moment-rotation behavior can be predicted. It was observed that the elastic stiffness of foundation was affected and reduced by two parameters namely gravity loads and foundation rotation both contributing in soil material yielding and contact surface reduction. The rotation at which the foundation edge starts to detach from the underneath soil was found to be a very useful parameter for normalizing the rotation response of foundations to acquire non-dimensional and almost unique stiffness degradation and moment-rotation curves. By evaluation of several experimental and numerical models, the authors introduced empirical expressions for the rocking stiffness degradation due to both gravity loads and foundation rotation as well as a formulation for determining the uplift reference rotation. The approximate closed-form expressions required as the input parameters, the FS_v and the aspect ratio, B/L , simple to be quantified. Finally, a non-dimensional trilinear $M-\theta$ backbone curve was developed as a function of the footing aspect ratio while the effect of FS_v came into play indirectly in normalization using uplift reference rotation. The effectiveness of the proposed approach was evaluated against experimental results. Although the approximate model has been developed based on several simplifying assumptions, it was found to produce reasonable responses compared to experimental results. The hysteretic damping ratio of foundation as the result of soil high nonlinearity under rocking cycles was also investigated which was found to have larger amplitude and to be less sensitive to FS_v in the case of narrow foundations (small aspect ratio). With the simplicity

of the proposed model combined with its non-dimensional form it is expected to be easily implemented in structures design or assessment applications to account for the nonlinear SSI e.g., works explored in Sotiriadis et al. (2017) and Paolucci et al. (2013).

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Statements and Declarations

Funding

The authors declare that no funds was received during the preparation of this manuscript.

Competing Interests

The authors declare that they have no relevant conflict of interests.

Author Contributions

All authors contributed to the study in conception, data collection and analysis.

Data Availability

The data generated during the current study are available on reasonable request from the corresponding author.