Reliability Evaluation of CNC Machine Tools Considering Competing Failures of Fault Failure Data and Machining Accuracy Degradation Data

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Reliability evaluation of CNC machine tools considering competing failures of fault failure data and machining accuracy degradation data

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Abstract

Traditional reliability evaluation of CNC machine tools usually considers a single failure mode of fault failure or degradation failure, or considers fault failure and degradation failure to be independent of each other. However, in the actual working conditions, fault failure and degradation failure are mutually affected, and the reliability evaluation of the competing failure models of CNC machine tools by considering the two failure modes comprehensively can get more accurate evaluation results. Therefore, this paper proposes a reliability evaluation method for CNC machine tools considering fault failure data competing with machining accuracy degradation data. A fault failure model of CNC machine tools is established based on a non-homogeneous Poisson process. The fault failure model is updated according to the different effects of each maintenance result of the failure on machining accuracy. By integrating multiple geometric errors of CNC machine tools through multi-body system theory, the amount of machining accuracy degradation is extracted. A machining accuracy degradation failure model is established using the Wiener process. Considering the correlation between fault failure and degradation failure, a competing failure model based on the Coupla function is developed for evaluating the reliability of CNC machine tools. Finally, the effectiveness of the proposed method is verified by example analysis.

Keywords Machine tool, Reliability evaluation, Fault failure, Degradation failure, Competing failure

1 Introduction

CNC machine tools are the essential equipment of the machinery industry, occupying an important position in the aerospace, military, transportation and other sectors.The reliability of CNC machine tools is an essential performance indicator, directly affects the production efficiency and processing quality of products [1]. Traditional reliability evaluation methods for CNC machine tools are divided into three main directions: reliability evaluation based on fault failure data, reliability evaluation based on accuracy degradation data, and competing failure evaluation where fault failure data and accuracy degradation data are independent of each other.

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The most researched is the reliability evaluation for the fault failure data of CNC machine tools. As a typical repairable system, the fault failure data of CNC machine tools is an essential source for reliability evaluation. Fan proposed a reliability evaluation algorithm based on failure data with the spindle of the CNC grinding machine as the research object, established a mathematical model of the failure-free time using five standard distribution functions, and finally determined a distribution model of the spindle system through optimization tests [2]. Zhang proposed an improved Bootstrap sampling method for the problem of difficulty in obtaining failure data and applied this method to reliability evaluation [3]. Kool proposed a technique to combine simulation data with fault monitoring data for reliability evaluation, which alleviates the high cost of reliability evaluation [4]. Wang proposed a risk assessment method for CNC lathes based on faulty data and the combination of fuzzy set theory and gray theory, which avoids the shortcomings of traditional RPN analysis and improves the reliability of CNC lathes [5]. Lin studied the reliability assessment of complex electromechanical systems from the perspective of fault propagation and proposed a fault propagation model considering fault data for system reliability evaluation [6].

Zhang performed Bayesian reliability evaluation of a modified Bayesian method with the results of gradient reliability as a priori information and validated it on a heavy-duty CNC machine [7]. Zhang proposed a reliability evaluation method based on cascade failure analysis and failure impact degree evaluation, which considered the impact of failure propagation on reliability [8]. For some high-reliability equipment, reliability evaluation based on fault failure data requires many tests over a long period. Unlike fault failure data, the accuracy data will degrade significantly with the increase of equipment service time, so reliability evaluation of CNC machine tools based on degraded data is an effective method.

CNC machine tools as a typical complex electromechanical system, in the working process, due to vibration, wear, temperature rise and other factors will cause the CNC machine tool machining accuracy degradation, when the machine tool accuracy degradation to a certain threshold value considered that the CNC machine tool failure occurs [9]. Dai used the signal characteristics of the machining process instead of the traditional temporal data to fit the equipment degradation model for effective reliability evaluation of the tool [10]. Duan proposed a reliability evaluation method based on multiple uncorrelated and correlated degraded data for small sample data for the reliability of CNC machine tools [11]. Wu used machining performance degradation data to estimate the reliability level of the machine with only a small number of samples and conducted a machining performance degradation experiment on an OTM650 device for validation [12]. Yuan proposed a Bayesian strategy-based reliability analysis method for accelerated performance degradation, and the method was validated with a functional milling head of a CNC machine tool [13]. Guo proposed a new multi-stress constant stress accelerated degradation test optimization method for electric spindles to improve the accuracy of electric spindle reliability evaluation by shortening the accelerated degradation test cycle [14].

Fault failure and degradation failure are both primary failure forms of mechanical equipment. A more accurate evaluation can be made by fully considering both failure forms in the reliability evaluation of equipment. The model that combines the correlation of multiple failure modes is called the competitive failure model, and the competitive failure model is widely used in reliability evaluation. Fei proposed a reliability modeling approach based on degradation failure and traumatic failure s-correlated competing risks that fully consider the correlation between degradation failure and traumatic failure [15]. An proposed a new reliability model for systems with shock loads above a certain level experiencing the associated competing failure processes, which essentially also considers the competing relationship between two forms of failure, degradation and fault [16]. Hao proposed a new reliability evaluation method for evaluating the reliability of degraded electronic systems with competing risks of soft and hard related failures [17]. Tang developed a reliability model for systems affected by two dependent competing failure processes, considering the correlation between the size of the additional damage during soft failure and the magnitude of the stress of the impact load during hard failure [18]. Cha proposed a correlated competitive risk model for reliability analysis of technology units with degradation phenomena and catastrophic failures [19]. Tao developed a competitive failure model with random shocks that can be effectively applied to condition monitoring and reliability evaluation of plain bearings [20].

Competing failure models based on multiple failure modes are widely used in equipment such as electronic components and machine tool parts.However they are less used in the reliability evaluation of CNC machine tools, which are still mainly based on a single failure mode or considered fault failure and degradation failure to be independent of each other [21]. The primary reason is that CNC machine tools as a complex electromechanical system, affecting the accuracy of many error factors, can not extract an adequate amount of accuracy degradation to reflect the degradation of machining accuracy failure, in addition to the long process of accuracy degradation, and fault throughout the degradation process, the mutual influence and correlation between the two, intensifying the difficulty of comprehensive consideration of degradation and failure for reliability evaluation. CNC machine tools as a typical repairable system, the effect of

maintenance after each fault failure will have a significant impact on the initial value of accuracy at the current point in time. Therefore, considering the fault failure data of CNC machine tools with machining accuracy degradation performance competition failure reliability evaluation can better reflect the actual working conditions, for the subsequent preventive maintenance and reliability growth has some significance.

In this paper, a fault failure model of CNC machine tools is established based on the non-homogeneous Poisson process (NHPP). The fault failure model is updated according to the different effects of failure repair results on machining accuracy. At the same time, the multiple geometric errors of CNC machine tools are integrated through the theory of multi-body system, the machining accuracy degradation amount is extracted and the machining accuracy degradation failure model is established by using the Wiener process, and finally, the competitive failure reliability model of CNC machine tools is established based on the correlation between fault failure and degradation failure by considering the Coupla function comprehensively, and the validity of the model is verified by examples.

Main contribution:This paper integrates the correlation between fault failure and machining accuracy degradation failure of CNC machine tools based on Coupla function and establishes a competitive failure model of CNC machine tools for reliability evaluation; The competitive failure model takes into account the impact of fault repair results on machining accuracy and can better reflect the actual working conditions; Integration through multi-body system theory The accuracy degradation amount extracted by multiple errors can reflect the machining accuracy degradation trend simply and effectively.

The remainder of this paper is organized as follows: Section II analyzes the competitive failure system of CNC machine tools. Section III establishes the failure model of tne CNC machine tool based on NHPP and updates the failure model considering the influence of different maintenance results on accuracy. Section IV extracts the machining accuracy degradation amount through the multi-body system theory and establishes the machining accuracy degradation failure model based on the Wiener process. Section V establishes a competitive failure model of CNC machine tools based on the Coupla function. Section VI verifies the validity of the reliability evaluation model by example. Section VII concludes the whole paper.

2 Competitive Failure System Analysis

CNC machine tools as a typical complex electromechanical system, with fault failure and degradation failure of two failure forms, fault failure are mainly refers to the sudden failure caused by machine downtime that can not continue to work, degradation failure refers to the growth of processing time, CNC machine tools due to vibration, wear, temperature and other effects of the machine accuracy exceed the failure threshold accuracy failure [22]. The two forms of failure are interactive throughout the operation of the CNC machine tool, and each sudden failure of the CNC machine tool will impact the accuracy of the machine tool. The failure process when only degradation is considered in the operation of the machine tool is shown in Figure 1, and L denotes the accuracy failure threshold.

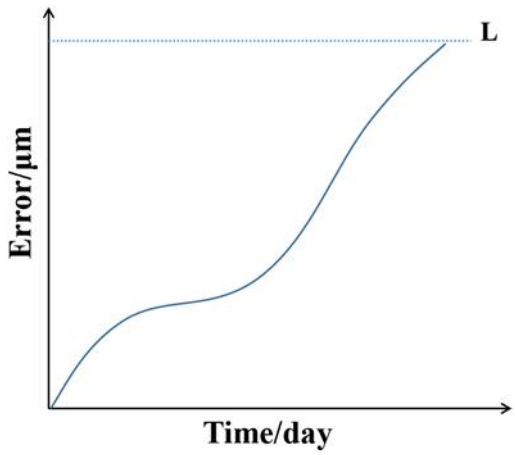

Fig.1 Only degradation failure process

When the object of study is the complete CNC machine tool, the failure of all subsystems and components inside the CNC machine tool may lead to machine downtime. The complex structure inside the entire machine also makes the failure mode more complicated. The failure process of the CNC machine tool as a typical repairable system when only fault failure is considered is shown in Figure 2, running from the moment T_0, $T_i(i=1..\ .n)$ denotes the time of each failure, without considering the

repair time, and $R_i(i=1.. .n)$ denotes the time between each failure.

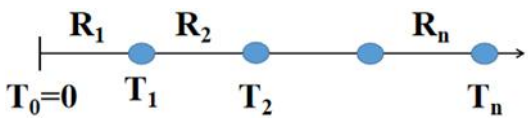

Fig.2 Only fault failure process

In the work process of CNC machine tools, two forms of failure exist at the same time. The most significant performance is that after the completion of each fault repair, the degradation of CNC machine tool accuracy start point will change, the competitive failure process of CNC machine tools as shown in Figure 3. $M_i(i=1... .n)$ denotes the repair time after each downtime failure, and $I_i(i=1.... .n)$ indicates the degree of impact on the machine accuracy after each repair is completed, from the figure can be seen, in T_1 time point failure and after M_1 time repair, the machine back to work the starting accuracy error has increased significantly, at this time can be equated to the machine accuracy of the failure threshold is reduced, the subsequent degradation of the failure time is shortened; in T_2 time point failure and after M_2 time repair, the machine back to work the starting accuracy error and the machine before downtime remains the same. Machine re-working the starting accuracy error and machine downtime before maintaining the same, indicating that the fault of the failure here on the machine precision did not have a significant impact; in T_3 time point failure and after M_3 time repair, the machine re-working the starting accuracy error reduced, this is generally more than some maintenance work disguised as a machine tool accuracy correction, such as CNC machine tool spindle repair correction, CNC system maintenance, etc.

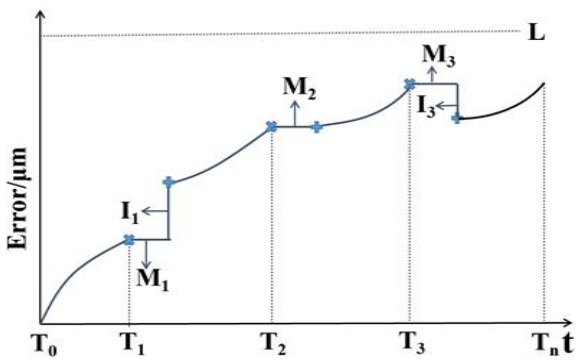

Fig.3 Competitive failure process

So CNC machine tools are different from other non-repairable systems. In a fixed period , the fault failure of CNC machine tools and degradation failure have apparent competition. Fault is discrete data, degradation is continuous data. In the CNC machine tool within the specified working time, machine accuracy degradation is always present, so the competing failure system is manifested in two forms: 1. fault failure is not generated, only degradation failure; 2. fault failure and degradation failure coincide during the working time.

3 Fault modeling

3.1 Fault modeling based on NHPP

CNC machine tools are typical repairable systems, the fault of CNC machine tools occurs randomly within a certain period, and the repair and replacement of defective parts of CNC machine tools are difficult to restore the machine tool to a "good as new" condition, so on this basis, the non-homogeneous Poisson process (NHPP) is an effective method for modeling the reliability of CNC machine tools [23].

NHPP is defined as follows [24], if the counting process $\{X(t),t \geqq 0\}$ is a homogeneous Poisson process with intensity function $\lambda(t)$, then the following conditions are satisfied.

(1) $X(0)=0$;

(2) $\{X(t),t \geqq 0\}$ is an independent incremental process;

(3) $X(t)$ satisfies the following two equations;

$P\{X(t+h)-X(t)=1\}=\lambda(t)h+o(h)$

$P\{X(t+h)-X(t) \geqq 2\}=o(h)$

where $\lambda(t)$ is called the intensity function, which can also be called the fault density, and $\lambda(t)\Delta t$ indicates the probability of occurrence of the fault in Δt time, from the intensity function. The average number of faults in the time interval (0,t] can be obtained as shown in Equation 1.

$$E\left[N(t)\right]=w(t)=\int_0^t \lambda(t)dt \qquad (1)$$

In general, the failure function of CNC machine tools has the shape of a bathtub curve, when the fault strength function $\lambda(t)$ is expressed by two superimposed NHPP, as shown in Equation(2), $\lambda_1(t)$ and $\lambda_2(t)$ indicate two different forms of strength fault function of CNC machine tools with early failure and accidental failure, as shown in

Equation (3)(4).

$$\lambda(t)=\lambda_1(t)+\lambda_2(t) \tag{2}$$

$$\lambda_1(t)=\lambda_1\beta_1 t^{\beta_1-1} \qquad \lambda_1>0,\ \beta_1>0 \tag{3}$$

$$\lambda_2(t)=\lambda_2\beta_2 t^{\beta_2-1} \qquad \lambda_2>0,\ \beta_2>0 \tag{4}$$

The cumulative fault intensity function $w(t)$ in the time interval (0,t] can be obtained by the above equation, as shown in Equation (5).

$$w(t)=\int_0^t \lambda(t)dt=\lambda_1 t^{\beta_1}+\lambda_2 t^{\beta_2} \tag{5}$$

Reliability is an essential indicator of the reliability of CNC machine tools, which indicates that CNC machine tools in the specified conditions and the specified time interval, the probability of completing the specified function. CNC machine tool in T=0 moment has been running, running to the failure time T=t. The failure interval time is recorded as T_f, at this time T_f reliability function expressed in $R_f(t)$, as shown in the formula (6).

$$R_f(t)=P(T_f>t)=e^{-w(t)}=e^{-\int_0^t \lambda(t)dt} \tag{6}$$

3.2 Fault model update considering the impact of different maintenance outcomes

In the fixed working time range of CNC machine tools, multiple fault data may occur, and each fault data after maintenance may have different effects on the machine's accuracy. On the reliability model of CNC machine tools established based on the NHPP, a fault model considering different maintenance results is founded. The probability of n fault of CNC machine tool in [0,t] time is shown below.

$$P(F(t)=n)=\frac{\left(e^{-\int_0^t \lambda(t)dt}\right)\left(\int_0^t \lambda(t)dt\right)^n}{n!} \tag{7}$$

Where $\lambda(t)$ represents the intensity function, and $F(t)$ represents the total number of faults. According to the different effects of fault repair results on machine tool accuracy, three forms of $F^+(t)$, $F^0(t)$ and $F^-(t)$ is used to represent the positive, no, and negative effects of fault repair results on machine tool accuracy. The positive impact is described as the starting point of machine accuracy error after the fault repair is significantly lower than before the fault repair; no impact is described as the starting point of machine accuracy error after the fault repair is equal to before the fault repair; the negative impact is described as the starting point of machine accuracy error after the fault repair is significantly higher than before the fault repair; formula (8) indicates the relationship between the three forms.

$$F(t)=F^+(t)+F^0(t)+F^-(t) \tag{8}$$

During a complete working time of a CNC machine, the probabilities of three different forms obey the following relations.

$$P(F^+(t))+P(F^0(t))+P(F^-(t))=1 \tag{9}$$

With i^+, i^0, i^-, respectively, the number of three forms of failure in a fixed period, the respective probability of failure is expressed by the formula, as shown in Equation (10).

$$P(F^+(t)=i^+)=\frac{\left(e^{-\int_0^t \lambda(t)dt}\right)\left(\int_0^t \lambda(t)dt\right)^{i^+}}{i^+!}$$

$$P(F^0(t)=i^0)=\frac{\left(e^{-\int_0^t \lambda(t)dt}\right)\left(\int_0^t \lambda(t)dt\right)^{i^0}}{i^0!}$$

$$P(F^-(t)=i^-)=\frac{\left(e^{-\int_0^t \lambda(t)dt}\right)\left(\int_0^t \lambda(t)dt\right)^{i^-}}{i^-!} \tag{10}$$

Three different forms of fault failure data are independent of each other, according to the probability of three different forms of fault to obtain the reliability function R_F (t) under the premise of considering the impact of different repair results, n indicates the total number of fault occurred as shown in Equation (11).

$$R_F(t)=\exp\left(-P(F^+(t))\cdot P(F^0(t))\cdot P(F^-(t))\right) \tag{11}$$

4 Accuracy degradation modeling

4.1 Extraction of machining accuracy degradation of CNC machine tools

CNC machine tools as a typical complex electromechanical system, the parts are closely related to each other, affecting each other, and there are many accuracy indicators, but the final impact is the machining accuracy of CNC machine tools. The use of the

multi-body system modeling method can integrate multiple geometric errors of CNC machine tools to obtain a unique degradation of machining accuracy. This paper introduces the modeling idea with a large number of three-axis vertical machining center as an example, and the three-dimensional model diagram is shown in Figure 4.

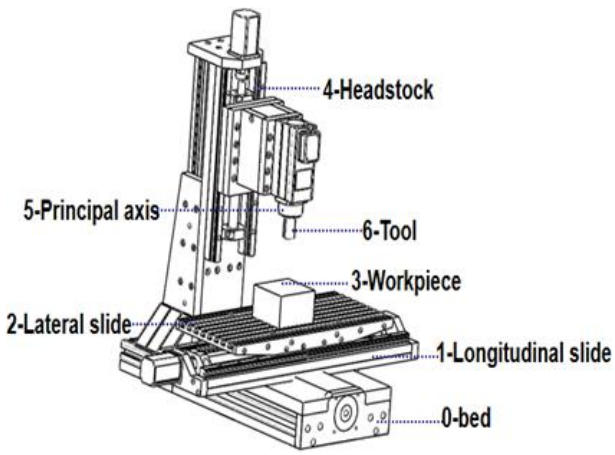

Fig.4 Structure of precision vertical machining center

The three-axis vertical machining center is abstracted into a two-branch topology, where one branch includes the bed, the longitudinal slide, the cross slide, and the workpiece. The other branch consists of the bed, spindle box, spindle, and tool. A low-order body array is used to describe the correlation of each part of the machine tool, and each part of the machine tool is treated as the corresponding "typical body", and the two branches are numbered according to the natural growth order, and the coordinate system of each unit is denoted by R [25], and the topology of the machine tool is shown in Figure 5.

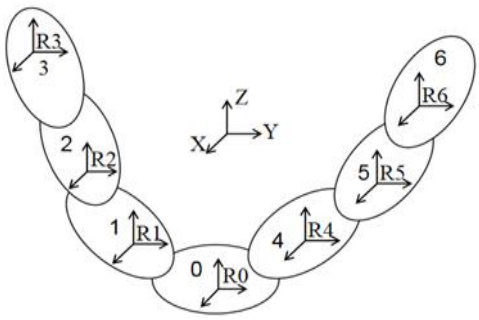

Fig.5 Topological structure of CNC machine tools

Based on the theory of multi-body system, each geometric error of the three-axis vertical machining center is transferred to the tool forming point, and the position and motion relationship between the middle of each body in the multi-body system is converted through the change of the coordinate system [26]. The geometric error sources mainly include straightness error, positioning error, angle error, and perpendicularity error. The coordinates of the tool forming point in the tool coordinate system are denoted by P_t, and the coordinates of the workpiece forming point in the workpiece coordinate system are represented by P_w. When all the errors in the coordinate system itself and between the coordinate systems are zero, the tool forming point on the workpiece coincides with the tool tip position expressed by Equation (12).

$$\left[\prod_{u=\mathrm{n},L^{\mathrm{n}}(3)=0}^{u=1} T_{L^{u}(3)L^{u-1}(3)P}\, T_{L^{u}(3)L^{u-1}(3)S}\right] P_{\mathrm{w}} = \left[\prod_{j=n,L^{n}(6)=0}^{j=1} T_{L^{j}(6)L^{j-1}(6)P}\, T_{L^{j}(6)L^{j-1}(6)S}\right] P_{\mathrm{t}} \tag{12}$$

Where $L^i(j)$ denotes the i-order low order body of body j, T_{ijp} denotes the stationary feature matrix between ij two adjacent bodies, T_{ijs} denotes the motion feature matrix between ij two adjacent bodies, and the feature matrix between adjacent bodies is represented by the associated geometric error source. Equation (13) represents the ideal forming function P_{wideal} of the tool forming point in the workpiece coordinate system.

$$P_{\mathrm{wideal}} = \left[\prod_{u=\mathrm{n},L^{\mathrm{n}}(3)=0}^{u=1} T_{L^{u}(3)L^{u-1}(3)P}\, T_{L^{u}(3)L^{u-1}(3)S}\right]^{-1} \times \left[\prod_{j=n,L^{n}(6)=0}^{j=1} T_{L^{j}(6)L^{j-1}(6)P}\, T_{L^{j}(6)L^{j-1}(6)S}\right] P_{\mathrm{t}} \tag{13}$$

In the actual machining process, the actual position of the tool forming point can deviate from the preset ideal position due to error factors, resulting in spatial position errors [27]. The combined volumetric error E of the machine tool is expressed by Equation (14).

$$E = P_{\mathrm{wideal}} - P_{\mathrm{wactual}} \tag{14}$$

The geometric errors of each CNC machine tool detected by regular tracking are substituted into the integrated volume error model. The combined volume error E in the three directions of XYZ is indicated as $E=(E_x, E_y, E_z)$, the error vector function associated with the time change is indicated as $E(t)$, the original error point of the machine tool is indicated as $E_0=(E_{x0}, E_{y0}, E_{z0})$, the error point of the machine tool at time point i is

E_i=(E_{xi}, E_{yi}, E_{zi}), and the amount of accuracy degradation at time point i is indicated as ΔE_i, which is expressed in Equation (15).

$$\Delta E_i = \sqrt{\left(E_{xi} - E_{x0}\right)^2 + \left(E_{yi} - E_{y0}\right)^2 + \left(E_{zi} - E_{z0}\right)^2} \quad (i = 1 \cdots n) \tag{15}$$

4.2 Accuracy degradation modeling based on Wiener process

The accuracy degradation of CNC machine tools is due to the joint action of random homogeneously distributed small degradation amounts of various components inside the machine tools, and these small degradation amounts are proportional to time and can be considered to obey a normal distribution, so the accuracy degradation of CNC machine tools is generally described by the Wiener process, which has good computational and analytical properties [28]. When modeling the accuracy degradation of CNC machine tools through the Wiener process, the influence of fault repair is not considered, and only the accuracy degradation data before and after the fault repair is regarded the amount of index degradation for a fixed measurement time.

Suppose the degradation of the machining accuracy of the CNC machine tool at the initial moment t_0=0 is taken as x_0=0, and the degradation of the performance index at time $t_1,\ldots,t_n$ is $x_1,\ldots,x_n$ respectively. Δx_i is the degradation of the performance of the machine tool accuracy to be measured between $t_{(i-1)}$ and t_i, and Δt_i is the interval between moments. From the properties of the Wiener process, we know that $\Delta x_i \sim N(\mu\Delta t_i,\ \sigma^2\Delta t_i)$, and the likelihood function of the parameters of the degenerate model based on the Wiener process is obtained as follows.

$$L(\mu,\ \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\sigma^2\pi\Delta t_i}} \times \exp\left(-\frac{(\Delta x_i - \mu\Delta t_i)^2}{2\sigma^2\Delta t_i}\right) \tag{16}$$

Based on this equation (16) the partial derivatives of the coefficients μ and σ are found. The equation is solved by making the partial derivatives zero. The estimates of the two coefficients can be obtained as follows.

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\frac{\Delta x_i}{\Delta t_i} \tag{17}$$

$$\hat{\sigma} = \left[\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta x_i - \hat{\mu}\Delta t_i}{\Delta t_i}\right]^{1/2} \tag{18}$$

When a one-dimensional Wiener process $\{X(t),\ t \geq 0\}$ is used to describe the machine tool machining accuracy degradation process, the accuracy failure time T of the machine tool to be tested is defined as the length of time when the amount of performance degradation first reaches the failure threshold L, the lifetime $T = \inf(t > 0 | X(t) \geq L)$, and the random variable T obeys the inverse Gaussian distribution [29]. The distribution function $F_T(t)$ and the probability density function $f_T(t)$ of the accuracy lifetime of the machine tool to be measured can be derived as follows, respectively.

$$F_T(t) = \phi\left(\frac{L - \mu t}{\sigma\left(\sqrt{t}\right)}\right) + \exp\left(\frac{2\mu L}{\sigma^2}\right)\phi\left(-\frac{L + \mu t}{\sigma\sqrt{t}}\right) \tag{19}$$

$$f_T(t) = \frac{L}{\sqrt{2\pi\sigma^2 t^3}}\exp\left[-\frac{\left(L - \mu t\right)^2}{2\sigma^2 t}\right] \tag{20}$$

Where $\Phi(.)$ denotes the standard normal function. The reliability function of the CNC machine based on the accuracy degradation process can be obtained by equation (19) as follows.

$$R_d(t) = P\left(T > t\right) = 1 - F_T\left(t\right) = 1 - \phi\left(\frac{L - \mu t}{\sigma\left(\sqrt{t}\right)}\right) - \exp\left(\frac{2\mu L}{\sigma^2}\right)\phi\left(-\frac{L + \mu t}{\sigma\sqrt{t}}\right) \tag{21}$$

5 Reliability evaluation considering competitive failure

5.1 Reliability modeling considering competitive failures

As mentioned before, the fault data and the accuracy degradation data of CNC machine tools are interrelated, so the correlation between the two types of failures must be fully considered to be able to evaluate the reliability of CNC machine tools effectively . The Coupla function is a multivariate probability distribution with a uniform marginal distribution. It is a valuable tool for generating multivariate distribution functions with different dependence structures, through which the correlation between fault failure and degradation failure can be

effectively described by this function.

The Coupla function was first proposed by Sklar, that is, Sklar's theorem, and Sklar's theorem is an existence theorem for the Coupla function, which specifically states explicity that if $H(x,y)$ is a binary joint distribution function with edge distributions $F(x)$,$G(y)$, then there exists a Coupla function $C(u,v)$ that satisfies $H(x,y) = C(F(x),G(y))$. Conversely if C is a Coupla function and F and G are two arbitrary probability distribution functions, then the H function must be a joint distribution function, and the marginal distributions are F and G [30].

The cumulative failure probabilities $F_F(t)$ and $F_d(t)$ based on equation (11) and equation(21) are obtained for fault failure and degradation failure, respectively, as shown in equation (22).

$$\begin{aligned} F_F(t) &= 1 - R_F(t) \\ F_d(t) &= 1 - R_d(t) \end{aligned} \tag{22}$$

Let $H(t_F,t_d)$ be the binary joint distribution function of fault failure time T_F and degradation failure time T_d. According to Sklar's theorem, there exists a unique Coupla function C, as shown in Equation (23), where θ is the parameter of the Coupla function.

$$\begin{aligned} &P(T_F \le t_F, T_d \le t_d) = H(t_F, t_d) \\ &= C(F_F(\mathrm{t}), F_d(t); \theta) \end{aligned} \tag{23}$$

Different types of Copula functions reflect different correlation structures between variables, and to effectively reflect the actual correlation, it is necessary to choose the appropriate Copula function, and according to the AIC principle, the Copula with the smallest AIC value is the best choice [31]. In this paper, the Frank Coupla function is chosen to describe the correlation between the two, which is expressed by Equation (24).

$$\begin{aligned} &C(F_F(\mathrm{t}), F_d(t)) = -\frac{1}{\theta}\ln \\ &\left\{1 + \frac{\left[\exp(-\theta F_F(t)) - 1\right]\cdot\left[\exp(-\theta F_d(t)) - 1\right]}{\exp(-\theta) - 1}\right\} \end{aligned} \tag{24}$$

Based on the above equation, the reliability function under the competitive failure of the CNC machine tool is obtained as shown in Equation (25).

$$\begin{aligned} R_C(t) &= P(\min(T_F, T_d) > t) \\ &= R_F(t) + R_d(t) - 1 + C(F_F(\mathrm{t}), F_d(t); \theta) \end{aligned} \tag{25}$$

5.2 Parameter estimation of the competitive failure model

To obtain reliability evaluations for the competing failure model, it is necessary to evaluate the parameters in the model, assuming that there are K machines with a fixed number of truncated fault data, the monitoring time of the i machine $[0,T_i]$, T_i is the last truncated time of the last fault data for each device, and n_i is the number of fixed number of truncated faults. $t_1,\ldots,t_l$ moments recorded the amount of degradation of performance indexes as $x_1,\ldots,x_l$. Δx_l is the amount of performance degradation of the machine accuracy to be tested between $t_{(l-1)}$ and t_l, Δt_l is the interval between moments, and n is the number of accuracy testing time points. Let the unknown parameters be $\xi=(\lambda_1,\lambda_2,\beta_1,\beta_2,\mu,\sigma,\theta)$. Then the corresponding likelihood function is obtained as shown in Equation (26).

$$\begin{aligned} L(\xi) = &\prod_{i=1}^{K}\left\{\begin{aligned} &\prod_{j=1}^{n_i}\left[\frac{\beta_1}{\lambda_1}\left(\frac{t_{ij}}{\lambda_1}\right)^{\beta_1-1} + \frac{\beta_2}{\lambda_2}\left(\frac{t_{ij}}{\lambda_2}\right)^{\beta_2-1}\right] \\ &\cdot\exp\theta\left[\left(\frac{T_i}{\lambda_1}\right)^{\beta_1} + \left(\frac{T_i}{\lambda_2}\right)^{\beta_2}\right] \end{aligned}\right\} \\ &\cdot\prod_{l=1}^{n}\frac{1}{\sqrt{2\sigma^2\pi\Delta t_l}}\cdot\exp\left(-\theta\frac{(\Delta x_l - \mu\Delta t_l)^2}{2\sigma^2\Delta t_l}\right) \end{aligned} \tag{26}$$

The corresponding logarithmic function is obtained according to the likelihood function as shown in Equation (27).

$$\begin{aligned} l(\xi) = &\sum_{i=1}^{K}\left\{\begin{aligned} &\sum_{j=1}^{n_i}\ln\left[\frac{\beta_1}{\lambda_1}\left(\frac{t_{ij}}{\lambda_1}\right)^{\beta_1-1} + \frac{\beta_2}{\lambda_2}\left(\frac{t_{ij}}{\lambda_2}\right)^{\beta_2-1}\right] \\ &+\theta\left[\left(\frac{T_i}{\lambda_1}\right)^{\beta_1} + \left(\frac{T_i}{\lambda_2}\right)^{\beta_2}\right] \end{aligned}\right\} \\ &\cdot\sum_{l=1}^{n}\ln\frac{1}{\sqrt{2\sigma^2\pi\Delta t_l}} - \theta\frac{(\Delta x_l - \mu\Delta t_l)^2}{2\sigma^2\Delta t_l} \end{aligned} \tag{27}$$

Based on Equation (27), the first-order derivatives are set to zero to obtain the maximum likelihood estimates of each parameter, and the parameters are obtained. The goodness of fit of the model is verified by root mean square error (RMSE).

6 Case Analysis

To verify the effectiveness of the proposed method, six three-axis vertical machining centers produced in the same period were used as examples, and 30 fault data of each machine tool were recorded from the factory to the end of the test. The machine accuracy degradation data were checked periodically, and some of the fault data and degradation data are shown in Table 1 and Table 2.

The fault data is truncated using a fixed number, and the last time point of each machine tool is faulted, and the degradation data is collected using a laser interferometer to collect the straightness and positioning accuracy of XYZ three axes within different time points, and the straightness and positioning accuracy are used as the geometric error source in the multi-body system, and the machining accuracy degradation amount ΔE_i is calculated for each time point through the model in section 4.1, focusing on recording the time before and after the fault data The accuracy data within the moment, and the schematic diagram of the accuracy acquisition system is shown in Figure 5.

Based on the changes in the amount of machining accuracy degradation before and after analyzing the faulty data points as described in Section 3.2, the faulty data points for each machine were classified into three forms, and the parameters of the reliability function were estimated based on the faulty and degraded data. Table 3 shows the results of the model parameter estimation and indicates the goodness of fit by TMSE.

Table 1 Partial fault data

number	**Failure time/h**									
1	49.5	420.7	752.1	761.6	799.9	1006.9	1200.9	2506.9	.....	8805.8
2	29.8	349.9	449.1	1586.2	1899.2	2538.6	3349.5	3916.8	.....	5113.8
3	179.3	457.1	958.0	1006.2	3412.6	4420.9	6058.2	6210.8	.....	7559.8
4	149.5	580.9	1020.3	1965.2	2890.0	3300.1	3749.5	4302.2	.....	8840.9
5	85.4	254.8	567.8	869.4	1700.8	2400.5	2987.6	3726.4	.....	4534.7
6	34.8	247.8	324.4	576.3	758.9	983.4	1583.9	2236.4	.....	9001.8

Table 2 Partial accuracy degradation data

number	**Machining accuracy error/μm**									**Failure reeor/ μ m**
1	5.42	9.68	6.76	7.29	8.78	15.51	8.19	9.28	.....	12
2	4.51	15.32	17.52	16.23	9.52	12.24	15.47	8.45	.....	12
3	3.21	10.72	9.24	11.56	15.75	10.45	9.78	11.89	.....	12
4	3.86	10.22	11.65	13.85	15.52	8.56	12.54	17.78	.....	12
5	2.45	7.57	12.45	10.45	17.86	8.63	6.75	10.42	.....	12
6	4.78	6.89	7.21	13.87	15.68	7.85	12.69	14.42	.....	12

Table 3 Estimations of parameters and TMSE

Parameter	λ_1	λ_2	β_1	β_2	μ	σ^2	θ	TMSE
value	0.0029	0.00038	0.53	1.72	0.0685	0.0321	9.87	0.0589

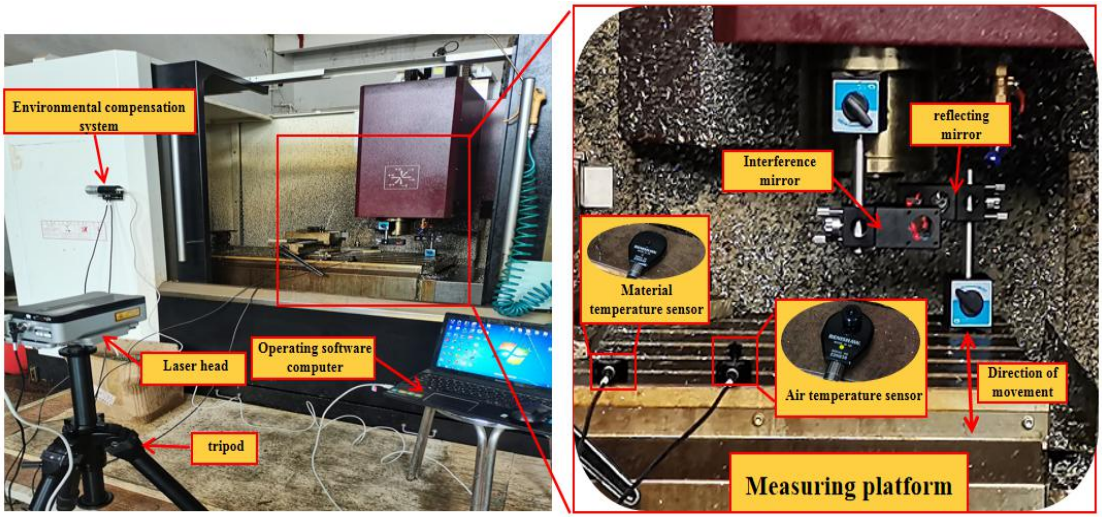

Fig. 5 Device diagram of the precision acquisition system for CNC machine tools

According to the formula (6), (21), (25), respectively, three reliability curves of fault failure, degradation failure, and competition failure are obtained as shown in Figure 6. The figure can be seen that when only degradation failure is considered, only different thresholds of machining accuracy need to be considered. According to the different thresholds required by the actual working conditions to determine the different reliability curves of degradation failure of CNC machine tools, the overall reliability of CNC machine tools under this condition is higher than that of fault failure only; The overall reliability of CNC machine tools under consideration of competing failures is lower than the other two cases. In addition, before 500h is mainly the early failure period of the machine tool, the main failure form is fault failure, reliability decreasing trend quickly, after 500h with machine parts wear, vibration and temperature rise and other effects began to appear degradation failure, the failure model and the accuracy threshold value.

Table 4 gives the mean life span (MTBF) t_a, median life $t_{0.5}$, and reliable life t_r with a reliability of 0.3 for the three different models. The results show that the competing failure model that integrates failure and degradation of the machine tool can better reflect the actual working conditions. The MTBF values obtained from the model are closer to the actual observed values in the factory.

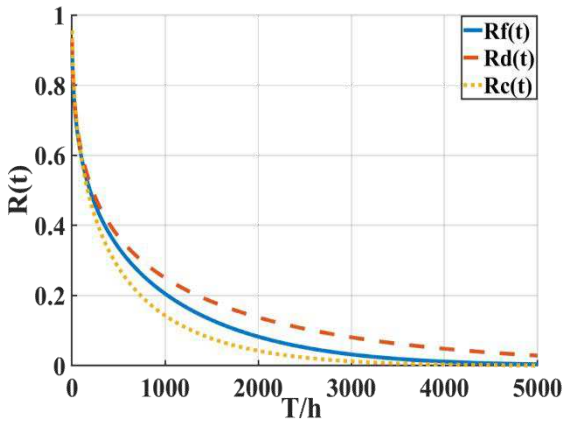

Fig.6 Reliability diagram of CNC machining center

Table 4 Comparison of three reliability models

Reliability model	t_a	$t_{0.5}$	t_r
$R_f(t)$	678h	193h	608h
$R_d(t)$	912h	220h	756h
$R_c(t)$	409h	168h	453h

7 Conclusion

In this paper, the fault failure of CNC machine tools is described by NHPP, and the failure model is updated for the different effects of fault repair results on accuracy; the amount of machining accuracy degradation is extracted by integrating the geometric errors of CNC machine tools through the multi-body system model, and the model of machining accuracy degradation failure is established by using the Wiener process. Finally, considering the correlation between the fault failure of CNC machine tools and the degradation failure of machining accuracy

comprehensively, a competing failure model of CNC machine tools was established based on the Coupla function. The model was analyzed by examples, and the following conclusions were obtained.

(1) Each downtime failure of CNC machine tools' subsequent maintenance results will impact the current initial value of machining accuracy, based on which the updated fault model is more in line with the actual working conditions.

(2) Numerous geometric errors of CNC machine tools, through the multi-body system model to integrate the many geometric errors, extract the individual machining accuracy degradation amount, and establish the degradation model through the Wiener process, can simply and accurately describe the degradation process of CNC machine tool machining accuracy.

(3) CNC machine tools in service for the early fault period, when the fault failure is mainly failure form, when the reliability of the declining trend is high-speed, after reaching the accidental failure period, due to wear, vibration and temperature rise and other factors machining accuracy error reaches the threshold of degradation failure, comprehensive consideration of the two forms of failure to establish a competitive failure model can better reflect the actual working conditions. Through the reliability curve to obtain the life value, the lifetime value obtained through the reliability curve is closer to the real observed value, which can make a more accurate assessment of the reliability of CNC machine tools.

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Conflicts of interest

Not applicable

Availability of data and material

Not applicable

Code availability

Not applicable

Authors' contributions

Cong Feng: Background research, Data curation, Software, Validation writing-original draft, Editing.
Zhaojun Yang: Methodology, Review & editing, Supervision.
Chuanhai Chen: Supervision, Project administration, Funding acquisition
Jinyan Guo: Review & editing, Supervision.
Jiangong Leng: Supervision
Jie Zhou: Assist in experiment, Data curation
All authors read and approved the final manuscript.

Ethics approval

Not applicable

Consent to participate

Not applicable

Consent for publication

Not applicable

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