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Research Article

Keywords: finite-time control, Euler-Lagrange systems, input constraints, full-order terminal sliding mode control, adaptive control

Posted Date: January 10th, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1162027/v1>

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Full-Order Sliding Mode Bounded Control for Euler-Lagrange Systems with External Disturbances

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Abstract: This paper researches two finite-time bounded control methods for Euler-Lagrange systems exposed to external disturbances. A novel full-order terminal sliding mode surface that is convenient for solving the input constraints is designed based on the characters of the hyperbolic tangent function. By using the designed full-order terminal sliding mode surface, the finite-time controller with input constraints can deal with external disturbances with the exactly known upper bound. Further, an adaptive finite-time bounded controller is designed to deal with the external disturbances with the upper bound that cannot be accurately known. Finally, the finite-time stability of the system is proved by using Lyapunov theory and numerical simulations.

Key words: finite-time control; Euler-Lagrange systems; input constraints; full-order terminal sliding mode control; adaptive control

1. Introduction

In recent years, the control methods for Euler-Lagrange (EL) systems have absorbed the concerns of many scholars due to the potential applications in the mechanical and electrical systems, such as robot manipulators [1, 2], spacecraft systems [3, 4], and quadrotor helicopters [5,6]. In view of this, the

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sliding mode control [7], neural network control [8], backstepping control [9], and fuzzy control [10] are widely employed in EL systems. In the practical working environment, the external disturbance that shouldn't be ignored is an important factor for the control performance of EL systems. Therefore, many control strategies to suppress the influence of external disturbance have been proposed, such as adaptive control [11] and disturbance observer [12]. However, it is noteworthy that the control strategies in the above references do not restrict the magnitude of the control torque.

As actuators in the actual physical systems are physically limited, how to construct an anti-saturation controller for EL systems is a major and unavoidable mission in practical applications. Ignoring the saturation of actuators will lead to a problem that the control performance is decreased. To deal with the problem, various classical anti-saturation control strategies have been proposed in recent years, such as anti-saturation proportional derivative control and anti-saturation proportional integral derivative control [13,14], anti-saturation adaptive control [15], anti-saturation robust integral of the sign of the error control [16], anti-saturation sliding mode control [17]. In addition to the classical anti-saturation methods, the method of designing the controller and making it bounded has also been studied by many scholars [18,19]. However, some researchers have also pointed out that some theories in the classical anti-saturation methods need to be improved, such as the flaw that the minimum value of the indicator of the saturation degree of the control vector is zero, which causes the proposed controller to be invalid [20], the flaw that the auxiliary systems are unable to investigate the effect of the input saturation, which implies that the semi-global stability stated is doubtful [21], the flaw that the Lyapunov function is not guaranteed to be radially unbounded, which resulted in the proof result of the controller is not convincing [22], the flaw that the defined sets are not necessarily bounded, which causes the result of the proof to be questioned [23], the flaw that the negligence in the comparison of

the theorems, which implies that the results of the comparison may be not convincing [24]. To this end, how to design the control strategy to deal with input constraints is still a challenging task.

Since the convergence rate for EL systems is an important indicator of the control performance, some scholars have proposed finite-time control methods that have the advantage of faster convergence than asymptotic convergence methods [25] and exponential convergence methods [26]. In a general way, the finite-time controllers are usually designed by using the homogeneous method [27], adding a power integrator method [28], terminal sliding mode (TSM) control method [29], and so on. The finite-time controller that is designed by homogeneous method is suitable for EL systems without external disturbances. The finite-time controller that is designed by adding a power integrator method can guarantee that the EL systems with external disturbances converge to a region near the equilibrium point. The finite-time controller that is designed by the TSM control method for EL systems has strong robustness and can converge to the equilibrium point. Based on the TSM control method, the full-order terminal sliding mode (FOTSM) control method which has the advantage that the chattering phenomenon in the system response is greatly reduced was studied by many scholars [30-33]. In addition, the singular problem [30], bounded external disturbances [31], and mismatched uncertainty that widely exist in various practical systems [32,33] have been all solved very well by using FOTSM control method. However, the above-mentioned control methods based on FOTSM do not consider the limited input of the actuator.

Therefore, although the control methods for EL systems have achieved remarkable results, it is still a challenging problem to design a finite-time control law for EL systems with input constraints and external disturbances. Inspired by the above discussions, this paper designs a finite-time FOTSM controller to deal with input constraints and external disturbances. The main contributions of this paper

are given as follows:

- i) A novel FOTSM surface that is convenient for solving the input constraints is designed based on the hyperbolic tangent function.
- ii) By using the FOTSM surface, a chattering-free finite-time controller is designed for EL systems to deal with the input constraints and external disturbances with exactly known upper bound.
- iii) To deal with the external disturbances with the upper bound that cannot be accurately known, an adaptive finite-time bounded controller is constructed based on the hyperbolic tangent function.

The rest of this paper is organized as follows. Section II introduces the system model. The controller and stability analysis are given in section III. Finally, the simulation results and conclusions are shown in section IV and section V, respectively.

2. System model and problem statement

The system model in this paper is selected as an EL equation which is used to represent a class of nonlinear systems, such as robot manipulators, spacecraft systems, and quadrotor helicopters. The structure of the EL system is expressed as

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}} + \mathbf{G}(\mathbf{p}) = \boldsymbol{\tau} + \boldsymbol{\tau}_d \quad (1)$$

where $\mathbf{p} \in \mathbb{R}^{n \times 1}$ is the generalized vector, $\dot{\mathbf{p}} \in \mathbb{R}^{n \times 1}$ is the generalized velocity, $\ddot{\mathbf{p}} \in \mathbb{R}^{n \times 1}$ is the generalized acceleration, $\mathbf{M}(\mathbf{p}) \in \mathbb{R}^{n \times n}$ and $\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) \in \mathbb{R}^{n \times n}$ are the generalized inertia matrix and the Coriolis matrix, respectively, $\mathbf{G}(\mathbf{p}) \in \mathbb{R}^{n \times 1}$ is the force that is caused by potential energy, $\boldsymbol{\tau} \in \mathbb{R}^{n \times 1}$ and $\boldsymbol{\tau}_d \in \mathbb{R}^{n \times 1}$ denote the control force and external disturbances, respectively.

The EL system (1) has the following properties.

Property 1 [23]. $\mathbf{G}(\mathbf{p})$ satisfies $\sup_{\mathbf{p} \in \mathbb{R}^{n \times 1}} \|\mathbf{G}(\mathbf{p})\| \leq \varpi$, where ϖ is a positive constant.

Property 2 [23]. The skew-symmetric matrix $\frac{1}{2}\dot{\mathbf{M}}(\mathbf{p}) - \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})$ satisfies $\boldsymbol{\xi}^T \left[\frac{1}{2}\dot{\mathbf{M}}(\mathbf{p}) - \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) \right] \boldsymbol{\xi} = 0$,

where $\xi \in \mathbb{R}^{n \times 1}$.

To facilitate designing the bounded finite-time controller, some lemmas and assumptions are given as follows.

Lemma 1 [17]. Considering the system $\dot{\xi} = f(\xi)$, $f(0) = 0$, $\xi \in \mathbb{R}^{n \times 1}$, if there exists a continuous function $V(\xi) : U \rightarrow \mathbb{R}$ such that the following conditions hold:

(i) $V(\xi)$ is positive definite.

(ii) There exist real numbers $\rho_1 > 0$, $0 < \rho_2 < 1$ and an open neighborhood $U_0 \subset U$ of the origin such that $\dot{V}(\xi) \leq -\rho_1 V(\xi)^{\rho_2}$, $\xi \in U_0 \setminus \{0\}$.

Then the origin is a finite-time stable equilibrium of the system. If $U_0 = U = \mathbb{R}^{n \times 1}$, the origin is a globally finite-time stable equilibrium.

Assumption 1. The external disturbance τ_d and its time derivative $\dot{\tau}_d$ satisfy $\|\tau_d\| \leq \bar{\chi}_1$ and $\|\dot{\tau}_d\| \leq \bar{\chi}_2$, where $\bar{\chi}_1$ and $\bar{\chi}_2$ are positive constants.

Control objective: This paper is working on designing a robust bounded controller for the system (1) such that p and $\dot{p}$ can converge to zero in finite time, i.e., $\lim_{t \rightarrow T} p(t) = 0$, $\lim_{t \rightarrow T} \dot{p}(t) = 0$.

3. Controller design

In this section, two FOTSM controllers are proposed to realize the control objective. The first algorithm is proposed based on the fact that the upper bound of the external disturbances can be exactly known. The second algorithm can realize the control objective with the upper bound of the external disturbances cannot be accurately known.

3.1 Finite-time FOTSM controller

A FOTSM surface is designed as equation (2), where $k_1 > 0$, $k_2 > 0$, $\alpha > 0$, $0 < \varphi_1 < 1$, $\varphi_2 = \frac{2\varphi_1}{1+\varphi_1}$.

$$\sigma = \ddot{p} + M^{-1}(p)(C(p, \dot{p})\dot{p} + k_1 \tanh(\text{sig}(\alpha p)^{\varphi_1}) + k_2 \tanh(\text{sig}(\alpha \dot{p})^{\varphi_2})) \quad (2)$$

$$\text{sig}(\boldsymbol{\xi})^\varphi = [\text{sig}(\xi_1)^\varphi, \text{sig}(\xi_2)^\varphi, \dots, \text{sig}(\xi_n)^\varphi]^\text{T} \quad \forall \boldsymbol{\xi} = [\xi_1, \xi_2, \dots, \xi_n]^\text{T} \quad (3)$$

$$\text{sig}(\xi_i)^\varphi = \text{sign}(\xi_i) |\xi_i|^\varphi, \quad \forall \xi_i \in \mathbb{R}, \quad i = 1, 2, \dots, n, \quad 0 < \varphi < 1 \quad (4)$$

Remark 1. The FOTSM surface designed based on the characteristics of the hyperbolic tangent function in this paper is convenient for solving the input constraints.

By the use of FOTSM surface (2), a bounded finite-time controller is provided as equation (5), where $k_3 > \bar{\eta} = \bar{\chi}_1 + \bar{\chi}_2$ is a positive constant, $\iota = \max\{\|\mathbf{M}^{-1}(\mathbf{p})\|, \|\dot{\mathbf{M}}^{-1}(\mathbf{p})\|\}$.

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \tanh(\boldsymbol{\tau}_1) \quad (5)$$

$$\boldsymbol{\tau}_0 = -k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\varphi_1}) - k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\varphi_2}) + \mathbf{G}(\mathbf{p}) \quad (6)$$

$$\dot{\boldsymbol{\tau}}_1 = -\cosh^2(\boldsymbol{\tau}_1) \mathbf{M}(\mathbf{p}) (k_3 \iota \text{sign}(\boldsymbol{\sigma}) + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \tanh(\boldsymbol{\tau}_1)) \quad (7)$$

Remark 2. The inference of $\dot{\boldsymbol{\sigma}} = \mathbf{0}$ in [19] is unnecessary in this paper by introducing a second derivative term $\ddot{\mathbf{p}}$ in the sliding mode surface, and the value of $\ddot{\mathbf{p}}$ can be obtained by a tracking differentiator. In fact, a second-order sliding mode surface is required to achieve $\dot{\boldsymbol{\sigma}} = \mathbf{0}$. Besides, the controller (5) designed in this paper can reduce chattering due to the $\text{sign}(\boldsymbol{\sigma})$ is inside $\dot{\boldsymbol{\tau}}_1$.

Theorem 1. In view of the EL system (1), if the external disturbance $\boldsymbol{\tau}_d$ satisfies **Assumption 1**, where $\bar{\eta}$ is an exact known constant, the following conclusions can be obtained by using the designed FOTSM surface (2) and the controller (5).

- (i) $\boldsymbol{\sigma}$ can converge to zero in finite time.
- (ii) $\mathbf{p}$ and $\dot{\mathbf{p}}$ can converge to zero in finite time.
- (iii) $\boldsymbol{\tau}$ is bounded and satisfies

$$\|\boldsymbol{\tau}\| \leq \sqrt{n}(k_1 + k_2 + \varpi + 1) \quad (8)$$

Proof: To prove that $\boldsymbol{\sigma}$ can converge to zero in finite time, V_1 is designed as

$$V_1 = \frac{1}{2} \boldsymbol{\sigma}^\text{T} \boldsymbol{\sigma} \quad (9)$$

Taking the time derivative of V_1 , it can get that

$$\dot{V}_1 = \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} \quad (10)$$

Substituting the control law (5) into the FOTSM surface (2), the following relationship can be obtained as

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \mathbf{M}^{-1}(\mathbf{p})(-\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}} - \mathbf{G}(\mathbf{p}) + \boldsymbol{\tau}_0 + \tanh(\boldsymbol{\tau}_1) + \boldsymbol{\tau}_d) \\ &\quad + \mathbf{M}^{-1}(\mathbf{p})(\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}} + k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) + k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2})) \\ &= \mathbf{M}^{-1}(\mathbf{p}) \tanh(\boldsymbol{\tau}_1) + \mathbf{M}^{-1}(\mathbf{p}) \boldsymbol{\tau}_d \end{aligned} \quad (11)$$

Then, according to equation (11), one can obtain

$$\dot{V}_1 = \boldsymbol{\sigma}^T (\mathbf{M}^{-1}(\mathbf{p}) \dot{\boldsymbol{\tau}}_d + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \boldsymbol{\tau}_d + \mathbf{M}^{-1}(\mathbf{p}) \frac{1}{\cosh^2(\boldsymbol{\tau}_1)} \dot{\boldsymbol{\tau}}_1 + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \tanh(\boldsymbol{\tau}_1)) \quad (12)$$

In view of equation (7), one can obtain

$$\begin{aligned} \dot{V}_1 &= \boldsymbol{\sigma}^T (\mathbf{M}^{-1}(\mathbf{p}) \dot{\boldsymbol{\tau}}_d + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \boldsymbol{\tau}_d - k_3 t \text{sign}(\boldsymbol{\sigma})) \\ &\leq -(k_3 - \bar{\eta}) t \|\boldsymbol{\sigma}\| \end{aligned} \quad (13)$$

Equation (13) can be rewritten as

$$V_1 < -\theta_1 V_1^{\frac{1}{2}} \quad (14)$$

where

$$\theta_1 = \sqrt{2}(k_3 - \bar{\eta})t \quad (15)$$

According to **Lemma 1**, $\boldsymbol{\sigma}$ can converge to zero in finite time.

Now, (i) is proved.

$\boldsymbol{\sigma} = \mathbf{0}$ means

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{p}} + \mathbf{C}(\mathbf{p}, \dot{\mathbf{p}})\dot{\mathbf{p}} + k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) + k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2}) = \mathbf{0} \quad (16)$$

To prove that $\mathbf{p}$ and $\dot{\mathbf{p}}$ are asymptotically stable, V_2 is selected as:

$$V_2 = \sum_{i=1}^n \int_0^{p_i} k_1 \tanh(\text{sig}(\alpha \delta)^{\vartheta_1}) d\delta + \frac{1}{2} \dot{\mathbf{p}}^T \mathbf{M}(\mathbf{p}) \dot{\mathbf{p}} \quad (17)$$

The time derivative of equation (17) is

$$\dot{V}_2 = \sum_{i=1}^n k_1 \tanh(\text{sig}(\alpha p_i)^{\vartheta_1}) \dot{p}_i + \dot{\mathbf{p}}^T \mathbf{M}(\mathbf{p}) \ddot{\mathbf{p}} + \frac{1}{2} \dot{\mathbf{p}}^T \dot{\mathbf{M}}(\mathbf{p}) \dot{\mathbf{p}} \quad (18)$$

In view of equation (16), one can obtain

$$\begin{aligned} \dot{V}_2 = & \sum_{i=1}^n k_1 \tanh(\text{sig}(\alpha p_i)^{\vartheta_1}) \dot{p}_i + \dot{\mathbf{p}}^T (-\mathbf{C}(\mathbf{p}, \dot{\mathbf{p}}) \dot{\mathbf{p}} - k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) \\ & - k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2})) + \frac{1}{2} \dot{\mathbf{p}}^T \dot{\mathbf{M}}(\mathbf{p}) \dot{\mathbf{p}} \end{aligned} \quad (19)$$

According to **Property 2**, equation (19) can be expressed as

$$\dot{V}_2 = -k_2 \dot{\mathbf{p}}^T \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2}) \leq 0 \quad (20)$$

Therefore, it can get that $\mathbf{p}$ and $\dot{\mathbf{p}}$ are asymptotically stable by the use of LaSalle's principle. Then, the proof of $\mathbf{p}$ and $\dot{\mathbf{p}}$ can converge to zero in finite time is similar to that in [31], which is omitted here.

Now, (ii) is proved.

According to **Property 1** and controller (5), the equation (21) can be obtained as

$$\begin{aligned} \|\boldsymbol{\tau}\| = & \left\| -k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) - k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2}) + \mathbf{G}(\mathbf{p}) + \tanh(\boldsymbol{\tau}_1) \right\| \\ \leq & k_1 \left\| \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) \right\| + k_2 \left\| \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2}) \right\| + \|\mathbf{G}(\mathbf{p})\| + \|\tanh(\boldsymbol{\tau}_1)\| \\ \leq & \sqrt{n}(k_1 + k_2 + \varpi + 1) \end{aligned} \quad (21)$$

Now, **Theorem 1** is proved.

Remark 3. According to equation (21), it can be seen that the upper bound of the controller proposed in this paper only depends on the parameters k_1 , k_2 , and ϖ . Therefore, the upper bound of the controller can be adjusted to a required range by changing the parameters.

3.2 Adaptive finite-time FOTSM controller

Based on the FOTSM surface (2), an adaptive finite-time bounded controller is provided as equation (22), where k_3 and κ_1 are positive constants, $\hat{\eta}$ is the estimation value of $\bar{\eta}$.

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \tanh(\boldsymbol{\tau}_1) \quad (22)$$

$$\boldsymbol{\tau}_0 = -k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\vartheta_1}) - k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\vartheta_2}) + \mathbf{G}(\mathbf{p}) \quad (23)$$

$$\dot{\boldsymbol{\tau}}_1 = -\cosh^2(\boldsymbol{\tau}_1) \mathbf{M}(\mathbf{p}) (k_3 \text{sign}(\boldsymbol{\sigma}) + \kappa_1 \tanh(\hat{\eta}) \gamma \text{sign}(\boldsymbol{\sigma}) + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \tanh(\boldsymbol{\tau}_1)) \quad (24)$$

The adaptive control law is designed as

$$\dot{\hat{\eta}} = \frac{\kappa_2}{\kappa_1} \cosh^2(\hat{\eta}) \gamma \|\sigma\| \quad (25)$$

where κ_2 is a positive constant, $\gamma = \frac{\sqrt{a + \kappa_1^2 \tanh^2(\hat{\eta}(0))}}{\kappa_1 \tanh(\hat{\eta}(0))} + 1$ is a positive constant.

Theorem 2. In view of the EL system (1), if the external disturbance τ_d satisfies **Assumption 1**, where $\bar{\eta}$ is not an accurately known constant, the following conclusions can be obtained by using the designed FOTSM surface (2) and the controller (22).

- (i) σ can converge to zero in finite time.
- (ii) p and $\dot{p}$ can converge to zero in finite time.
- (iii) τ is bounded and satisfies

$$\|\tau\| \leq \sqrt{n}(k_1 + k_2 + \varpi + 1) \quad (26)$$

Proof: Defining $\tilde{\eta} = \bar{\eta} - \kappa_1 \tanh(\hat{\eta})$, a Lyapunov function V_3 is designed as

$$V_3 = \frac{1}{2} \sigma^T \sigma + \frac{1}{2\kappa_2} \tilde{\eta}^2 \quad (27)$$

Taking the time derivative of V_3 , it can get that

$$\dot{V}_3 = \sigma^T \dot{\sigma} - \frac{\kappa_1}{\kappa_2 \cosh^2(\hat{\eta})} \tilde{\eta} \dot{\hat{\eta}} \quad (28)$$

In view of the controller (22), the FOTSM surface can be rewritten as

$$\begin{aligned} \sigma &= (-M^{-1}(p)C(p, \dot{p})\dot{p} - M^{-1}(p)G(p) + M^{-1}(p)\tau_0 + M^{-1}(p)\tanh(\tau_1) + M^{-1}(p)\tau_d) \\ &\quad + M^{-1}(p)(C(p, \dot{p})\dot{p} + k_1 \tanh(\text{sig}(\alpha p)^{\rho_1}) + k_2 \tanh(\text{sig}(\alpha \dot{p})^{\rho_2})) \\ &= M^{-1}(p)\tanh(\tau_1) + M^{-1}(p)\tau_d \end{aligned} \quad (29)$$

In view of equation (29), equation (28) can be written as

$$\begin{aligned} \dot{V}_3 &= \sigma^T (M^{-1}(p)\dot{\tau}_d + \dot{M}^{-1}(p)\tau_d + M^{-1}(p)\frac{1}{\cosh^2(\tau_1)}\dot{\tau}_1 \\ &\quad + \dot{M}^{-1}(p)\tanh(\tau_1)) - \frac{\kappa_1}{\kappa_2 \cosh^2(\hat{\eta})} \tilde{\eta} \dot{\hat{\eta}} \end{aligned} \quad (30)$$

Substituting equation (24) into equation (30) can yield

$$\begin{aligned}
\dot{V}_3 &= \boldsymbol{\sigma}^T (\mathbf{M}^{-1}(\mathbf{q}) \dot{\boldsymbol{\tau}}_d + \dot{\mathbf{M}}^{-1}(\mathbf{q}) \boldsymbol{\tau}_d - k_3 \text{sign}(\boldsymbol{\sigma}) - \kappa_1 \iota \tanh(\hat{\eta}) \gamma \text{sign}(\boldsymbol{\sigma})) - \frac{\kappa_1}{\kappa_2 \cosh^2(\hat{\eta})} \tilde{\eta} \dot{\hat{\eta}} \\
&= \bar{\eta} \iota \|\boldsymbol{\sigma}\| - k_3 \|\boldsymbol{\sigma}\| - \iota \|\boldsymbol{\sigma}\| \gamma \kappa_1 \tanh(\hat{\eta}) - (\bar{\eta} - \kappa_1 \tanh(\hat{\eta})) \gamma \iota \|\boldsymbol{\sigma}\| \\
&= -k_3 \|\boldsymbol{\sigma}\| + (1 - \gamma) \bar{\eta} \iota \|\boldsymbol{\sigma}\|
\end{aligned} \tag{31}$$

Since $\gamma > 2$, where a is a positive constant, then one can obtain

$$\dot{V}_3 \leq -k_3 \|\boldsymbol{\sigma}\| \leq 0 \tag{32}$$

Thus, it can be obtained that $\boldsymbol{\sigma}$ and $\tilde{\eta}$ are all bounded. Then, another Lyapunov function is chosen as equation (33) to demonstrate $\boldsymbol{\sigma}$ is finite-time convergence.

$$V_4 = \frac{1}{2} \boldsymbol{\sigma}^T \boldsymbol{\sigma} \tag{33}$$

According to equation (29), the time derivative of V_4 is expressed as

$$\begin{aligned}
\dot{V}_4 &= \boldsymbol{\sigma}^T \dot{\boldsymbol{\sigma}} \\
&= \boldsymbol{\sigma}^T (\mathbf{M}^{-1}(\mathbf{p}) \dot{\boldsymbol{\tau}}_d + \dot{\mathbf{M}}^{-1}(\mathbf{p}) \boldsymbol{\tau}_d - k_3 \text{sign}(\boldsymbol{\sigma}) - \kappa_1 \iota \tanh(\hat{\eta}) \gamma \text{sign}(\boldsymbol{\sigma})) \\
&= -k_3 \|\boldsymbol{\sigma}\| + (\bar{\eta} - \kappa_1 \tanh(\hat{\eta}) \gamma) \iota \|\boldsymbol{\sigma}\|
\end{aligned} \tag{34}$$

Note that $\hat{\eta}(0) > 0$ is the inertial value of $\hat{\eta}$. Then, $\hat{\eta}(t) > \hat{\eta}(0) > 0$ can be obtained according to $\dot{\hat{\eta}} > 0$.

If $\hat{\eta}(0)$ is chosen to satisfy $\kappa_1 \tanh(\hat{\eta}(0)) \geq \frac{1}{2} \bar{\eta}$, such that $\kappa_1 \tanh(\hat{\eta}(0)) > |\tilde{\eta}(0)|$, it can get that

$$\begin{aligned}
\bar{\eta} - \kappa_1 \tanh(\hat{\eta}) \gamma &\leq \bar{\eta} - \kappa_1 \tanh(\hat{\eta}(0)) \left(\frac{\sqrt{a + \kappa_1^2 \tanh^2(\hat{\eta}(0))}}{\kappa_1 \tanh(\hat{\eta}(0))} + 1 \right) \\
&= \tilde{\eta}(0) - \sqrt{a + \kappa_1^2 \tanh^2(\hat{\eta}(0))} \\
&\leq \sqrt{\tilde{\eta}^2(0)} - \sqrt{a + \kappa_1^2 \tanh^2(\hat{\eta}(0))} \\
&\leq \sqrt{\kappa_1^2 \tanh^2(\hat{\eta}(0))} - \sqrt{a + \kappa_1^2 \tanh^2(\hat{\eta}(0))} \\
&\leq 0
\end{aligned} \tag{35}$$

Then, equation (34) can be rewritten as

$$\dot{V}_4 \leq -k_3 \|\boldsymbol{\sigma}\| = -\theta_2 V_4^{\frac{1}{2}} \tag{36}$$

where

$$\theta_2 = \sqrt{2} k_3 \tag{37}$$

According to **Lemma 1**, $\boldsymbol{\sigma}$ can converge to zero in finite time.

Now, (i) is proved. Then, similar to **Theorem 1**, (ii) and (iii) in **Theorem 2** can be proved.

Now, **Theorem 2** is proved.

Remark 4. Different from the proof results in some references, such as $\theta_2 = k \|\sigma\|$, the value of θ_2 is a constant in this paper, which theoretically avoids the violation of $\rho_1 > 0$ in **Lemma 1** when the state variable $\sigma = \mathbf{0}$.

Remark 5. Although the specific value of $\bar{\eta}$ is unnecessary in the second controller proposed in this paper, the approximate value of $\bar{\eta}$ is still required to be known to guarantee that the appropriate values of κ_1 and $\hat{\eta}(0)$ can be chosen to satisfy $\kappa_1 \tanh(\hat{\eta}(0)) \geq \frac{1}{2} \bar{\eta}$. Since the maximum value of $\kappa_1 \tanh(\hat{\eta}(0))$ is unconstrained, one can easily choose the parameters κ_1 and $\hat{\eta}(0)$ large enough based on the experience to guarantee the constraints proposed in this paper.

4. Simulation results

To verify the correctness of the two controllers, the simulation is presented in this section. The model of a two-link manipulator is given as [19]

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \ddot{\mathbf{p}} + \begin{bmatrix} -h\dot{p}_2 & -h(\dot{p}_2 + \dot{p}_2) \\ h\dot{p}_1 & 0 \end{bmatrix} \dot{\mathbf{p}} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \boldsymbol{\tau} + \boldsymbol{\tau}_d \quad (38)$$

where $\mathbf{p} = [p_1 \ p_2]^T$, $M_{11} = r_1 + 2r_2 \cos p_2 + 2r_4 \sin p_2$, $M_{12} = r_2 + r_3 \cos p_2 + r_4 \sin p_2$, $M_{21} = M_{12}$, $M_{22} = r_2$, $h = r_3 \sin p_2 - r_4 \cos p_2$, $G_1 = c_1 \cos p_1 + c_2 \cos(p_1 + p_2)$, $G_2 = c_2 \cos(p_1 + p_2)$ with $c_1 = m_1 g l_{c1}$, $c_2 = m_e g l_{ce}$, $r_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2$, $r_2 = I_e + m_e l_{ce}^2$, $r_3 = m_e l_1 l_{ce} \cos \delta_e$, $r_4 = m_e l_1 l_{ce} \sin \delta_e$. The parameters of the manipulator are given as $m_1 = 5\text{kg}$, $m_e = 2.5\text{kg}$, $l_1 = 1.0\text{m}$, $l_{c1} = 0.5\text{m}$, $l_{ce} = 0.5\text{m}$, $\delta_e = 0^\circ$, $I_1 = 0.36\text{kg} \cdot \text{m}^2$, $I_e = 0.24\text{kg} \cdot \text{m}^2$, $g = 9.8\text{m/s}^2$. The initial conditions for the system (38) are set as $\mathbf{p} = [0.2; 0.1]$ and $\dot{\mathbf{p}} = [0.1; -0.1]$. The external disturbance is chosen as

$$\boldsymbol{\tau}_d = \begin{cases} 0 & t \leq 100 \\ 0.01 \times [\sin(t/70), \cos(t/80)]^T & t > 100 \end{cases}$$

$G_1 \leq c_1 + c_2 \leq 36.75 \text{ N} \cdot \text{m}$, $G_2 \leq c_2 \leq 12.25 \text{ N} \cdot \text{m}$ can be obtained by calculation.

The FOTSM surface (2) can be rewritten as equation (39) according to the system (38).

$$\sigma = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \ddot{\mathbf{p}} + \begin{bmatrix} -h\dot{p}_2 & -h(\dot{p}_2 + \dot{p}_2) \\ h\dot{p}_1 & 0 \end{bmatrix} \dot{\mathbf{p}} + k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\varphi_1}) + k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\varphi_2}) \quad (39)$$

To verify the correctness of **Theorem 1**, the finite-time bounded controller for the system (40) is provided as follow:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \tanh(\boldsymbol{\tau}_1) \quad (40)$$

$$\boldsymbol{\tau}_0 = -k_1 \tanh(\text{sig}(\alpha \mathbf{p})^{\varphi_1}) - k_2 \tanh(\text{sig}(\alpha \dot{\mathbf{p}})^{\varphi_2}) + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \quad (41)$$

$$\dot{\boldsymbol{\tau}}_1 = -\cosh^2(\boldsymbol{\tau}_1) \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} (k_3 \text{sign}(\sigma) + \begin{bmatrix} \dot{M}_{11} & \dot{M}_{12} \\ \dot{M}_{21} & \dot{M}_{22} \end{bmatrix}^{-1} \tanh(\boldsymbol{\tau}_1)) \quad (42)$$

The selected parameters are given as

$$k_1 = 0.18, k_2 = 1.8, k_3 = 0.003, \alpha = 1, \varphi_1 = 0.9.$$

Figs. 1 and 2 show the curves of $\mathbf{p}$ and $\dot{\mathbf{p}}$ under controller (40). It can clearly express that both $\mathbf{p} = \mathbf{0}$ and $\dot{\mathbf{p}} = \mathbf{0}$ can be reached in 30 s, which guarantees that the control performance is acceptable for the actual system. Fig. 3 gives the curves of $\boldsymbol{\tau}$ in the controller (40), from which it can be seen that the maximum control torque is 37 N·m which satisfies the control constraints. It can also be seen that the control torque based on FOTSM surface is chattering-free.

Fig. 1 Positions of link 1 and link 2 under controller (40)

Fig. 2 Velocities of link 1 and link 2 under controller (40)

Fig. 3 Control torques under controller (40)

To verify the correctness of **Theorem 2**, the adaptive finite-time bounded controller for the system (38) is provided as follow:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_0 + \tanh(\boldsymbol{\tau}_1) \quad (43)$$

$$\boldsymbol{\tau}_0 = -k_1 \tanh(\text{sig}(\alpha \boldsymbol{p})^{\varphi_1}) - k_2 \tanh(\text{sig}(\alpha \dot{\boldsymbol{p}})^{\varphi_2}) + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \quad (44)$$

$$\dot{\boldsymbol{\tau}}_1 = -\cosh^2(\boldsymbol{\tau}_1) \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} (k_3 \text{sign}(\boldsymbol{\sigma}) + \kappa_1 \iota \tanh(\hat{\eta}) \gamma \text{sign}(\boldsymbol{\sigma}) + \begin{bmatrix} \dot{M}_{11} & \dot{M}_{12} \\ \dot{M}_{21} & \dot{M}_{22} \end{bmatrix}^{-1} \tanh(\boldsymbol{\tau}_1)) \quad (45)$$

$$\dot{\hat{\eta}} = \frac{\kappa_2}{\kappa_1} \cosh^2(\hat{\eta}) \gamma \iota \|\boldsymbol{\sigma}\| \quad (46)$$

The selected parameters are given as

$$k_1 = 0.18, k_2 = 1.8, k_3 = 0.001, \alpha = 1, \varphi_1 = 0.9, \kappa_1 = 0.003, \kappa_2 = 0.0001, \gamma = 2.001.$$

The simulation results are shown in Figs. 4-6. Fig. 4 and Fig. 5 show the curves of $\boldsymbol{p}$ and $\dot{\boldsymbol{p}}$, respectively. It can clearly show that the control performance is acceptable for the actual system from that $\boldsymbol{p} = \mathbf{0}$ and $\dot{\boldsymbol{p}} = \mathbf{0}$ can be reached in 30 s. Fig. 6 shows the curves of $\boldsymbol{\tau}$, from which it can be seen that the control torque based on FOTSM surface is chattering-free, and the maximum control torque is 37 N·m which satisfies the control constraints.

Fig. 4 Positions of link 1 and link 2 under controller (43)

Fig. 5 Velocities of link 1 and link 2 under controller (43)

Fig. 6 Control torques under controller (43)

Under the premise that the control parameters are chosen the same, it can be seen that the control torque in controller (40) and controller (43) are almost equal. In addition, it can also be seen that the control performance in controller (43) is better than controller (40) in control accuracy. Thus, the second adaptive FOTSM controller has a better performance compared with the first controller designed in this paper.

To verify the superiority of the controller proposed in this paper, some simulation results are given for comparison with the work of [19]. Equation (47) can be derived from equation (30) in [19] under the system model (38).

$$\begin{aligned} \tau = & -k_1 \text{sat}(\alpha p) - k_2 \text{sat}(\alpha \dot{p}) - k_3 \text{sign} \left(\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-T} \sigma \right) \\ & - k_4 \frac{\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-T} \sigma}{\left\| \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-T} \sigma \right\|} \tanh(\hat{\eta}) + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \end{aligned} \quad (47)$$

The adaptive law and sliding mode surface of controller (47) can be expressed as equations (48) and (49), respectively.

$$\dot{\hat{\eta}} = \frac{\nu}{k_4} \cosh^2(\hat{\eta}) \left\| \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-T} \sigma \right\| \quad (48)$$

$$\begin{aligned} \sigma = \dot{p} + \int_0^t \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} \begin{bmatrix} -h\dot{p}_2 & -h(\dot{p}_2 + \dot{p}_2) \\ h\dot{p}_1 & 0 \end{bmatrix} \dot{p} \\ + k_1 \text{sat}(\alpha p) + k_2 \text{sat}(\alpha \dot{p}) dt \end{aligned} \quad (49)$$

with

$$\text{sat}(\xi_i) = \begin{cases} \text{sig}(\xi_i)^\varphi & \text{for } |\xi_i| \leq 1 \\ \text{sign}(\xi_i) & \text{for } |\xi_i| > 1 \end{cases} \quad \forall \xi_i \in \mathbf{R}, i = 1, 2, \dots, n, 0 < \varphi < 1 \quad (50)$$

The selected parameters are the same as those in reference [19]. The simulation results are shown in

Figs. 7-9.

Fig. 7 Positions of link 1 and link 2 under controller (47)

Fig. 8 Velocities of link 1 and link 2 under controller (47)

Fig. 9 Control torques under controller (47)

It can be seen that on the premise of the same convergence time, both Fig. 7 and Fig. 8 have more obvious chattering than Fig. 4 and Fig. 5. Besides, the controller (43) proposed in this paper has the effect of reducing chattering and has better control performance, such as smaller overshoot, smoother control torque, higher control accuracy.

Conclusions

This paper designs two robust controllers based on a novel FOTSM surface which have the

advantages of finite-time convergence to deal with input constraints and external disturbances. The first bounded controller can make the state of the system converge to the equilibrium point in finite time when the system has the external disturbances with exactly known upper bound. The second one can make the state of the system converge to the equilibrium point in finite time when the system has external disturbances with the upper bound that cannot be accurately known. Finally, the finite-time stabilization of the system is proved by using Lyapunov theory and numerical simulations.

Statements and Declarations

This work was supported by the National Natural Science Foundation of China (No. 61803307, 62111530051), Aeronautical Science Foundation of China (No. 201901053004), Natural Science Basic Research Plan in Shaanxi Province of China (No. 2020JQ-209), China Postdoctoral Science Foundation (No. 2020M683565), and the Fundamental Research Funds for the Central Universities (No. 3102020ZDHKY05).

The authors have no relevant financial or non-financial interests to disclose.

All data generated or analysed during this study are included in this published article.

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