Minimizing uncertainty in complete automation of Douglas-Peucker Algorithm for geospatial mapping

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1 **Minimizing uncertainty in complete automation of Douglas-Peucker**

2 **Algorithm for geospatial mapping**

3 **Abstract**

4 It is qualitatively evident that the greater the map scale change, the greater the optimal
5 distance threshold of the Douglas-Peucker Algorithm, which is used in polyline
6 simplification. However, no specific quantitative relationships between them are known
7 by far, causing uncertainties in complete automation of the algorithm. To fill this gap,
8 the current paper constructs quantitative relationships based on the spatial similarity
9 theories of polylines. A quantitative spatial similarity relationship model was proposed
10 and evaluated by setting two groups of control experiments and taking $\langle C, T \rangle$ as
11 coordinates. In order to realize the automatic generalization of the polyline, we verified
12 whether these quantitative relationships could be fitted using the same function with
13 the same coefficients. The experiments revealed that the unary quadratic function is the
14 best, whether the polylines were derived from different or the same geographical feature
15 area(s). The results also show that using the same optimal distance threshold is
16 unreasonable to simplify all polylines from different geographical feature areas. On the
17 other hand, the same geographical feature area polylines could be simplified using the
18 same optimal distance threshold. The uncertainties were assessed by evaluating the
19 automated generalization results for position and geometric accuracy perspectives using
20 polylines from the same geographic feature areas. It is demonstrated that in addition to
21 maintaining the geographical features, the proposed model maintains the shape

characteristics of polylines. Limiting the uncertainties would support the realization of completely automatic generalization of polylines and the construction of vector map geodatabases.

Keywords: *Optimal distance threshold; Map scale change; Automatic map generalization; Geometric accuracy; Spatial similarity relationships*

1. Introduction

In realistic cartography, different cartographers always select different maps as the generalization results, and in most situations, cartographers often need to input parameter(s) in the execution of an algorithm. This humans' constant interference consequently leads to the non-automation of generalized algorithms, which decreases the efficiency of map generalization and increases the uncertainty of the resulting maps (Yan 2014; Makris et al., 2021). Linear data such as contour, road network, and rivers are the main spatial data types widely used in different fields. In the process of downsizing small-scale maps or constructing vector map databases, it is indispensable to generalize polylines from one larger scale (e.g., 1:5000) to corresponding generalized counterparts at the other smaller scale (e.g., 1:50000). Therefore, it is extremely essential to realize the completely automatic generalization of polylines.

Numerous scholars have been making tremendous efforts to investigate the realization of automatic polyline generalization issues (Ai et al. 2016; Shen et al. 2018; Kronenfeld et al. 2020). Meanwhile, various methods have been presented (Mcmaster 1987), such

42 as the Douglas-Peucker algorithm (DP) (Douglas and Peucker 1973), Li-Openshaw (Li
43 and Openshaw 1992), Bend Group algorithm (Qian HZ 2017). However, the problems
44 are only partially solved. Among these approaches, the DP remains the most well-
45 known and effective polyline simplification algorithm in map generalization (Ramer
46 1972; Hersherberger and Snoeyink 1992; Saalfeld 1999; Yan 2014; Sandu et al., 2015).
47 The evaluation and minimization of uncertainties in polyline simplification methods
48 might validate these algorithms' acceptability, consistency, and advanced application
49 for automatization.

50 Spatial similarity relationships are critically important measures for the accuracy
51 assessment of map generalization results (Yan 2015, 2016; Makris et al., 2021), which
52 have attracted the increasing attention of cartographers in recent years. However, few
53 studies have been published on the automatic generalization of polylines based on
54 spatial similarity relationships. The reason mainly focuses on three aspects: Firstly, no
55 in-depth investigations on integrating spatial similarity degree into map automatic
56 generalization are available. Only a few studies have been tested (Yan 2014), i.e., it is
57 ambiguous about how to automatically calculate the transition conditions threshold of
58 DP using the spatial similarity degree. As a result, it leads to constant human
59 intervention in the execution of DP. Secondly, very little is known about the specific
60 quantitative functional relationship between optimal distance threshold and map scale
61 change. Is it linear or nonlinear? Is it a linear equation with one variable or a quadratic
62 equation, or is it a logarithmic function? These questions are still being sought. Thirdly,

63 when referring to the effect of regional geographical features on selecting distance
64 threshold, some cartographers believe that it should be considered, while others argue
65 that there is no need (Li and Sui 2000). Besides, they assume that the reason why the
66 map is generalized is only the change of map scale, while other surrounding factors
67 remain unchanged. For example, regardless of the chosen threshold, the degree of
68 terrain fragmentation remains unchanged before and after the generalization. Hence,
69 the conflicting opinions adversely influence the determination of the optimal distance
70 threshold and cause uncertainty. A computational model based on quantitative spatial
71 similarity relationships might overcome the challenge of realizing an entirely automatic
72 generalization of the polylines.

73 Constructing the quantitative functional relationships between the optimal distance
74 threshold (T) and map scale change (C) would realize the completely automatic
75 generalization of polylines based on the DP without manual parameterization.
76 Meanwhile, the optimal distance threshold mainly depends on the magnitude of map
77 scale change. Technically, when the spatial similarity degree between the intermediate
78 polyline (L_i) simplified by DP and the standard target scale polyline (LM2) reaches the
79 maximum, the corresponding threshold is obviously the optimal distance threshold. The
80 main goal of the current research is to minimize the uncertainties in determining the
81 optimal distance threshold for improving the complete automation of the DP algorithm.

2. Methodology

In order to automatically calculate the optimal distance threshold and realize the completely automatic generalization of the individual polyline, this paper firstly constructs and validates the spatial similarity evaluation model by quantitatively selecting the evaluation index and determining its weight, and then calculates the optimal distance threshold based on the spatial similarity relations. Afterward, the optimal distance threshold and corresponding map scale change are recorded when the similarity degree between the intermediate result and the standard target scale polyline reaches the maximum. Then taking $\langle C, T \rangle$ as the coordinates, the best quantitative empirical formula is fitted using the regression analysis and least square model. The generalization results are finally validated from the position and geometric accuracy perspectives to confirm the acceptability of the introduced methods.

2.1. Principle of parameter optimization of DP algorithm

Douglas-Peucker algorithm (DP) is a well-known and effective simplification algorithm of polylines (Douglas and Peucker 1973). DP has been widely used in map generalization to reduce the number of points on a curve (Yan 2014; Gu et al. 2016). According to its simplification fundamentals, we can clearly know that the degree that polyline is simplified mainly depends on the threshold of each simplification process, i.e., the initial threshold and step size. Moreover, in which points are deleted, the order is the opposite of that when points are reserved. However, it is consistent with the

process of DP. The optimal distance threshold determines the number of points to be deleted and the degree to that a polyline is generalized. Hence, the current research is aimed to find an appropriate approach for determining the quantitative relationship between T and C. To this end, the order that points to be deleted is firstly determined by reversing the order of points to be selected. As a result, the similarity between the intermediate result and the standard target scale polyline reaches the maximum; the corresponding threshold λ_k is the optimal distance threshold that the polyline is generalized from scale M_1 to scale M_2 . Finally, the point pairs (C, T) used to fit the quantitative relations between them are recorded after multiple circulative iterations (Figure 1).

Figure 1. Flow chart of the methodology applied in this study. λ_0 is the initial distance threshold. $\Delta\lambda$ is a gradually changing value, and the value of $\Delta\lambda$ is inversely correlated with the similarity degree between L_{M2} and L_i . For example, suppose the polyline is generalized from 1:50000 to 1:250000, then if the similarity degree (S) between L_{M1} and L_i is less than 0.6, $\Delta\lambda$ will be 0.02 km, while if S is more than 0.6 and less than 0.8, $\Delta\lambda$ will be 0.01 km; and if S is more than 0.8, $\Delta\lambda$ will be 0.005 km.

2.2. Selection of evaluation index and determination its weights

The judgment of spatial similarity degree is the essence of map generalization. However, people often ambiguity about the similarity between two objects when judging the spatial similarity relationship between multi-scale maps because they remain uncertain about what properties should be considered in similarity assessments (Yan 2010). Therefore, among many polyline properties, including length, distance, complexity, sinuosity, etc., it is necessary to extract major properties that influence people's judgment at first and then determine its corresponding weights to construct a spatial similarity evaluation model.

According to previous research, 12 properties can be used to evaluate the spatial similarity between multi-scale polylines, including position (Olteanu-Raimond et al. 2015), length (Zhang et al. 2014), distance (Deng 2007, Tong et al. 2014), size, sinuosity (Danel et al. 2014), complexity, orientation (Samal 2004, Olteanu-Raimond et al. 2008), area, shape, buffer-overlapped area, degree of node and attribute. These can be further divided into geometric properties and thematic attribute properties (Li et al. 2006, Zhang 2009). Due to the variety of the geometric properties, it is not clear to choose what combination of geometric properties to produce the best solution. In addition, if all properties have been considered, correlated and redundant information is likely imported, which may lead to low computational efficiency and the accuracy of

138 the result. This phenomenon can be explained by the curse of dimensionality or
 139 Hughes's effect (Hughes 1968). Using as few indicators as possible to express as much
 140 polyline information as possible should be solved first. Therefore, absolute distance
 141 matrix D between factors is employed to overcome the above problem and measure the
 142 closeness of the indicators based on the results of genetic algorithm.

$$143 \quad D_{(12 \times 12)} = \begin{bmatrix} 0 & & & & & & & & & & & \\ 3.22 & 0 & & & & & & & & & & \\ 1.94 & 3.49 & 0 & & & & & & & & & \\ 4.22 & 2.46 & 3.95 & 0 & & & & & & & & \\ 5.14 & 4.41 & 4.13 & 3.56 & 0 & & & & & & & \\ 1.23 & 3.39 & 1.99 & 4.51 & 5.24 & 0 & & & & & & \\ 1.94 & 3.22 & 1.60 & 3.71 & 4.39 & 1.99 & 0 & & & & & \\ 1.45 & 3.18 & 2.39 & 4.46 & 5.37 & 0.99 & 2.72 & 0 & & & & \\ 5.04 & 3.25 & 4.46 & 2.32 & 2.73 & 5.27 & 5.49 & 5.49 & 0 & & & \\ 3.16 & 2.98 & 3.73 & 4.59 & 4.60 & 3.79 & 4.79 & 3.19 & 4.79 & 0 & & \\ 5.28 & 4.53 & 4.71 & 3.95 & 3.86 & 6.02 & 4.12 & 5.72 & 4.04 & 4.12 & 0 & \\ 4.21 & 4.06 & 4.02 & 4.04 & 2.62 & 4.88 & 3.42 & 4.89 & 3.45 & 3.42 & 3.42 & 0 \end{bmatrix}$$

144 where, R is a symmetric matrix, and $d_{ij} = \sum_{k=1}^n |w_{ik} - w_{jk}|$ ($i, j = 1, 2, \dots, 12, k =$
 145 $1, 2, \dots, 9$), w_{ik} represents the k th factor that affects the i th data group in Table 5 of
 146 Chehreghani's research.

147 And then, the direct cluster genealogy diagram (Figure 2) is derived based on the matrix
 148 R.

149

151 According to the F statistical analysis of cluster analysis, $F > F_{\alpha}(f_1, f_2) = 2.494$ ($\alpha =$
152 $0.05, f_1 = r - 1 = 5, f_2 = n - r = 3$), which indicates that the classes in Figure 2 are
153 significantly different, the classification strategy is reasonable. It can be determined that
154 Z belongs to (2.46, 2.62) because the attribute as the thematic property can be extracted
155 as an independent class, meanwhile, the clustering degree of other geometric properties
156 reaches maximum. Finally, the impact indices are clustered into six categories:
157 {Attribute}, {Sinuosity}, {Degree of node}, {Position, Complexity, Area, Distance,
158 Orientation}, {Size, Shape, Length}, {Buffer-overlapped area}. In this paper, only
159 geometric properties were considered because, in most situations, cartographers pay
160 more attention to the geometric attributes of polylines and ignore their thematic attribute
161 information (Deng 2007). It should be noted that {degree of node}, {Position,
162 Complexity, Area} and {Size, Length} are excluded from the selected impact factors

since they impact only a few part groups of the dataset. Finally, the chosen geometric factors are {Sinuosity}, {Distance, Orientation}, {Shape} and {Buffer-overlapped area}. Except for distance and orientation, the correlation coefficient $|R_{xy}|$ between the selected geometric factors is less or equal to 0.3, which is consistent with the basis for the exclusivity of index selection. Although distance and orientation were clustered in one class, and their correlation coefficient $|R_{xy}|$ is 0.76, they impact all groups of datasets in stability test, which indicates that both factors are very stable. Thus, Sinuosity, Distance, Orientation, Shape, and Buffer-overlapped area are chosen. It is generally not allowed to move polylines on maps in the process of map generalization, i.e., orientation between polylines is not changed after map generalization. Therefore, Sinuosity, Distance, Shape, and Buffer-overlapped area are finally chosen as evaluation indices.

Distance is a fundamental concept in geospatial science. According to the previous research, it can be known that compared with Euclidian distance (Peuquet 1992), Bottleneck distance (Efrat et al. 2001), and Fréchet distance (Nayyeri et al. 2015), Hausdorff distance is one of the most used distances for spatial objects in GIS, but it is sensitive to the shape of the objects, especially to the outliers. Besides, it does not satisfy the change law of similarity (Li et al., 2018). Therefore, Mean- Hausdorff distance (MHD) is selected as the distance similarity metric to obtain more stable and accurate result (Deng et al. 2007). On the other hand, the shape is viewed as the most crucial geometric factor that describes planar curves. The essence of shape similarity is to judge

the coincidence degree between polylines. Therefore, Li (2018) comparatively assessed the advantages and disadvantages of different measurement indices through experiments, e.g., Vertex Reserved, Length Ratio, Turning Function, and Dual-side Bend Forest Shape Similarity. The results indicated that the cumulative turning function (Fan et al. 2014) (Equation (1)) could eliminate the interference of detail difference, which is more suitable for the comparison between multi-scale polylines of the same target.

$$\widehat{Sim}_{L_1, L_2}^{shp} = (\int f(\theta_{L_1} - \theta_{L_2}) ds)^{\frac{1}{2}} = (\int_0^1 T_A(L_1) ds - \int_0^1 T_A(L_2) ds)^{\frac{1}{2}} \quad (1)$$

where, $f(\theta_{L_1} - \theta_{L_2}) = \begin{cases} |\theta_{L_1} - \theta_{L_2}| & (|\theta_{L_1} - \theta_{L_2}| \leq 180^\circ) \\ 360^\circ - |\theta_{L_1} - \theta_{L_2}| & (|\theta_{L_1} - \theta_{L_2}| > 180^\circ) \end{cases}$. The $\widehat{Sim}_{L_1, L_2}^{shp}$ is

inversely correlated with the similarity degree between L_1 and L_2 . If the angle is clockwise, α_i is positive, and if the angle is counterclockwise, α'_i is negative. The relation between α_i and α'_i is that $\alpha_i = 360^\circ - |\alpha'_i|$. For instance, if $\alpha'_i = -18^\circ$, then $\alpha_i = 360^\circ - 18^\circ = 342^\circ$.

Figure 3. Cumulative angle function of the multi-scale polylines L_1 and L_2 .

Buffer-overlapped area can reflect the horizontal distance and vertical distance of two polyline entities. However, it is worth noting that the threshold of buffer radius is the key to solving various research problems. According to the visual perception degree of position deviation on the figure, buffer radius of the buffer-overlapped area is generally set as 1~2 times of the minimum distance (0.2mm) between two points on the target scale map, e.g., if the target scale is 1:250000, then the buffer radius is 50m.

However, suppose we want to study the mechanism of spatial similarity varying with map scale change. In that case, map scale change must be the unique independent variable, and the same set of weights for different groups of datasets should be used to eliminate the influence of weights variation, which may adversely influence the results. Therefore, nine groups of weights of four stability factors are normalized to the same group of weights (Table 1) by using the Sum-Product method.

Table 1. Normalized Weights of stability factors.

Impact factor	Distance	Sinuosity	Shape	Buffer-overlapped area
Weight	0.32	0.23	0.11	0.34

The weights of sinuosity, distance, shape, buffer-overlapped area are 0.23, 0.32, 0.11, and 0.34, the critical ranking of which is consistent with the weights obtained from the opinions of experts based on spatial cognition experiment or the well-adopted criteria (Chehreghani et al. 2016). The large weight value indicates that the corresponding factor strongly influences the judgment of spatial similarity relations and vice versa. Thus,

216 according to the value of the adopted weights, the buffer-overlapped area has the
 217 greatest impact, followed by distance.

218 2.3. Construction of spatial similarity evaluation model

219 According to the definition and description of each index in Section 2.2, it is known to
 220 all that some factors are positive indicators, such as distance, shape, sinuosity, while
 221 others are negative, i.e., the larger $\widehat{Sim}_{L_1, L_2}^{Dis}$, indicates the less similarity between polylines,
 222 while the $\widehat{Sim}_{L_1, L_2}^{Buf}$ is an opposite factor. Moreover, the units and dimensions of factors are
 223 different, and their value varies greatly, which also has a great impact on the results.
 224 Therefore, to eliminate the effects of negative factors and different units on results, the
 225 similarity degree of these factors should be normalized to [0, 1] before determining the
 226 spatial similarity evaluation model, which is achieved by using the most applicable
 227 technique range standardization as follows:

$$228 \quad Sim_{L_i, L_j}^{P_k} = \begin{cases} \frac{\widehat{Sim}_{L_i, L_j}^{P_k} - \min_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\}}{\max_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\} - \min_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\}} & \text{when } \widehat{Sim}_{L_i, L_j}^{P_k} \text{ is a positive index.} \\ \frac{\max_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\} - \widehat{Sim}_{L_i, L_j}^{P_k}}{\max_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\} - \min_{1 \leq i \leq N} \{\widehat{Sim}_{L_i, L_j}^{P_k}\}} & \text{when } \widehat{Sim}_{L_i, L_j}^{P_k} \text{ is a negative index.} \end{cases} \quad (2)$$

229 $\widehat{Sim}_{L_i, L_j}^{P_k}$ is the similarity degree of the factor P_k after normalized, where the large
 230 $\widehat{Sim}_{L_i, L_j}^{P_k}$ indicates the higher spatial similarity. N is the number of groups of datasets.

231 Finally, Equation (3) (Yan 2014) can be employed to calculate the spatial similarity
 232 degree between polylines based on four factors.

$$233 \quad Sim(L_i, L_j) = \sum_{k=1}^n W_{P_k} \widehat{Sim}_{L_i, L_j}^{P_k} \quad (3)$$

where, $\text{Sim}(L_1, L_2) \in (0,1]$, $W_{pk} \in [0,1]$, n is the total number of impact factors, which is 4.

2.4. Geometric accuracy assessment

In the generalization of an individual polyline, the influence of simplification on the geometric accuracy is mainly reflected in the change of geometric features and the position of points on the polyline (Wu F et al. 2008, Chen et al. 2016). The variations of the former are mainly represented by the changes in curve length and sinuosity. The variations of the latter are mainly manifested as the global or local displacement before and after the generalization of the polyline. Such variations are evaluated using the standardized deviation of displacement (SMD), position error (PE), vector displacement, area displacement, and buffer tolerance. Among these indices, the SMD mainly evaluates the local maximum, and it is the most suitable index for comparing the different simplification results of the same linear features (Wu et al. 2008). Furthermore, PE is often used to evaluate the overall displacement (White 1985; McMaster 1987). In this research work, the geometric accuracy of the simplified polyline is evaluated based on the length ratio (LR) (Chehreghani and Abbaspour 2018), sinuosity degree (SD) (Daneshmandi and Yang 2014), SMD (Chen 2016), and PE.

By comparing the length ratio of the polyline, it can be seen how much the simplification algorithm compresses the points on the polyline, and the ratio is inversely correlated with the amount of compression (Wu 2008). The degree of curvature can be obtained by sum the included angles of adjacent straight lines on the polyline. If the

length of polylines changes more and the sinuosity degree changes less, this indicates that the geometric features of the simplified polyline are better maintained. The smaller value of SMD represents a smaller displacement of a feature on the map. This implies a lesser likeliness of intersection, which highlights that the spatial relationship is maintained very well. Since the SMD can only be used to obtain the maximum value of the polyline's local displacement, it cannot properly describe the overall displacement. So, the PE is used to evaluate the positional accuracy of a polyline. The smaller the PE value is, the better the accuracy of the point position on the curve is maintained. Furthermore, the simplification result can effectively maintain the overall shape characteristics of polylines.

3. Experiments and Results

According to different geographical features, the plain of China is divided into the Northeast China Plain, the North China Plain, and the Yangtze Plain. Thirty-eight groups of multi-scale datasets of rivers and roads, which include 1:10k, 1:25k, 1:50k, 1:100k, 1:250k, and 1:500k, were selected to calculate the quantitative relationships between S and C. these data are divided into two major group categories: control and experimental. The experimental group is composed of the multi-scale polylines datasets from the same geographical feature area, each group of the dataset includes river and road from the same sample area, and the former is composed of the multi-scale polylines datasets from the different geographical feature areas, each group of the

dataset includes river or road from the different sample areas. These multi-scale datasets include 12 groups of river samples from the Huaibei Plain (Sample area I), which belong to a part of the North China Plain, seven groups of river samples from the Northeast China Plain (Sample area II). There are 19 groups of samples from the lower Yangtze Plain (Sample area III), including 12 groups of the river and 7 groups of roads, which are separately sampled from the Jianghuai Plain and the Yangtze River Delta Plain, because both of which are part of the lower Yangtze Plain, we assume they share the same geographic features. These multi-scale vector data, as mentioned above, were derived from the data vectorization results of GF-2 images under different scales using ArcGIS10.6. The GF-2 image is obtained by the fusion of Pan and multi-spectral image, whose spatial resolution is 1m and 4m, respectively. Multi-scale datasets of rivers and roads, which include 1:5w, 1:10w, 1:25w, 1:50w, and 1:100w, were derived from the current results of the National Geomatics Center of China (NGCC).

Figure 4. Sampling area of experimental data and sampling data of multi-scale river and road in the plain area

3.1 Validation of the proposed spatial similarity evaluation model

It is qualitatively evident that the greater the map scale change, the smaller the spatial similarity degree between multi-scale polylines. However, no specific quantitative relationships between them are known by far, which hampers the complete automation of the algorithm. There are five potential candidate functions to map this gap that can describe such changing trends, including linear function, polynomials, power functions,

297 logarithmic functions, and exponential function. The three or more order polynomials
 298 have n-2 inflection point(s). Hence the curve has more than one monotonicity
 299 decreasing interval. Ultimately, only second-order polynomials should be considered.

$$300 \begin{cases} y = a_2x^2 + a_1x + a_0 & (a_2 > 0, x \in [1, -\frac{a_1}{2a_2}]) \\ y = a \ln(x) + a_0 & (a < 0, x \geq 1) \\ y = ax^b & (a > 0, -1 < b < 0, x \geq 1) \\ y = ae^{bx} & (a > 0, b < 0, x \geq 1) \\ y = ax + b & (a < 0, x \geq 1) \end{cases} \quad (4)$$

301 Taking $\langle C_{S_i, S_j}, \text{Sim}(L_i, L_j) \rangle$ as coordinates, the unknown coefficients of the Equation can
 302 be determined using the curve fitting approach. Here, for simplification, $\text{Sim}(L_i, L_j) =$
 303 $f(C_{S_i, S_j})$ is replaced with $S=f(C)$. Statistical analysis is conducted based on Microsoft
 304 Excel (V10.0) and the least square method R^2 ($R^2 \in [0,1]$) is employed to choose the
 305 best-fitted function (Yang et al. 2015). Considering that the five groups of datasets have
 306 similar variation tendencies, multi-scale polylines from the Huaibei plain are taken as
 307 an example (Figure 5).

309 **Figure 5. Fitting results of five functions and its R^2 of multi-scale polylines from the Huaibei plain.**

310 R^2 , as a good indicator, often is used to compare the candidate functions. The larger
 311 value of R^2 always indicates the better fitting curve. As can be clearly seen from Figure
 312 5, the R^2 of the power function is closest to 1 ($R^2=0.8152$), which achieves the best

fitting relationship of the curve among all candidates. Therefore, Equation (5) is chosen as a quantitative model for describing the relationships between spatial similarity degree and map scale change of individual polylines. This conclusion is consistent with Yan's conclusion (Yan 2014).

$$\text{Sim}(L_i, L_j) = \begin{cases} 1, & \text{if } L_1 \text{ and } L_2 \text{ are straight line, otherwise} \\ aC_{(L_i, L_j)}^b \end{cases} \quad (5)$$

where, $a > 0$, $-1 < b < 0$; $\text{Sim}(L_i, L_j) \in (0, 1]$, $C_{(L_i, L_j)} \in [1, +\infty)$.

To validate the reliability of the proposed model, this paper selected five groups of vector sampling multi-scale polylines from previous research of Yan (2015) and Chehreghan (2016). Figure 6 shows the fitting results of the proposed model, and Table 2 shows the accuracy comparison result with the existing evaluation model (Yan 2014).

where, (A), (C), and (E) corresponds to Figure 2, Figure 3, and Figure 4 of Yan's research (2015), respectively; (B) and (D) correspond to Figure 11 and Figure 13 of Chehreghan's research (2016), respectively.

Figure 6. Quantitative relationships fitting results of validation data between S and C.

Table 2. Accuracy comparison result with the existing evaluation model.

Group	Previous evaluation	Proposed evaluation	Accuracy improvement
(A)	$S=1.0164C^{-0.3427}$,	$S=0.903C^{-0.085}$, $R^2=0.9135$	4.16%
(C)		$S=0.6716C^{-0.213}$, $R^2=0.9470$	7.98%
(E)	$R^2=0.8770$	$S=0.7165C^{-0.28}$, $R^2=0.9781$	11.52%

326 Compared with the previous model, the fitting accuracy of the evaluation model
327 proposed in this paper is improved by 4.16%~11.52%. However, compared with
328 Chehreghan's conclusion, there is a nonlinear power function relationship between
329 spatial similarity and map scale change (Figure 6 (B), (D)). Although the trend of three
330 groups of the multi-scale polyline (Figure 6(A),(C),(E)) is the same, they cannot be
331 described using the same power function whether they are the same ground objects of
332 different types or the different ground objects of same type, which is some different
333 with Yan's and Chehreghan's conclusion.

334 In order to further verify whether these quantitative relationships can be fitted using the
335 same function with the same coefficients, this paper selects 38 groups of the dataset of
336 different plains, and the curve fitting results are shown in Figure 7.

338 Figure 7. Fitting results between S and C of the multi-scale polylines.

339 Figure 7(A) shows the fitting results between S and C of multi-scale individual
 340 polylines from the same geographical feature area, and Figure 7(B) shows the fitting
 341 result of multi-scale individual polylines from the different geographical feature areas.

342 Through the comparative analysis of the above fitting results in Figure 7, it can be seen
343 clearly that the relationship between spatial similarity degree and map scale change of
344 different groups of the dataset from the same geographical feature plain can be
345 described using the same power function curve with the same parameters, e.g., in Figure
346 7(A1), 12 groups of multi-scale polylines of the river are all from the Huaibei Plain, the
347 quantitative relationship between S and C can be fitted using the same power function
348 $S=0.9245C^{-0.224}$, and the fitting accuracy R^2 is up to 0.8152. Moreover, 19 groups of
349 multi-scale polylines of river and road are all from the Lower Yangtze Plain in Figure
350 7(A4), although they are the different types of the polyline, and some groups of
351 polylines of which are sampled from the Jianghuai Plain, while others are sampled from
352 the Yangtze River Delta Plain, the fitting accuracy is also up to 0.7966. However,
353 Figure 7(B) shows that no matter whether the polylines are the same or different type(s)
354 of features, the coefficients of their power function are entirely different for polylines
355 from the different geographical feature areas. For example, as shown in Figure 7(B1),
356 12 groups of multi-scale polylines are from the Huaibei Plain, and the other 12 groups
357 of multi-scale polylines are from the Lower Yangtze Plain, both of them are the same
358 type of ground objects, but the fitting accuracy of power function is only 0.5005. In
359 Figure 7(B3), 12 groups of multi-scale polyline of the river are from the Huaibei Plain,
360 and seven groups of multi-scale polyline of the road are from the Lower Yangtze Plain;
361 both are the different types of ground objects, the fitting accuracy is also only 0.4155.

3.2. Determination of distance threshold

The essence of the DP algorithm is to compare the straight-line distance connecting the start point and endpoint with the optimal distance threshold. Therefore, this part aims to find the optimal distance threshold, which determines the degree of the polyline to be simplified, corresponding to map scale change to realize the completely automatic generalization of polylines.

It is qualitatively evident that the greater the map scale change, the greater the optimal distance threshold of DP. Hence, three potential candidate functions can be employed to fit the changing trend between λ and C: polynomial, linear Equation, and logarithmic function. Since the other polynomials have $n-1$ inflection point(s), e.g., cubic polynomial, which indicates that the curve is not monotonic. Hence, only the second-order polynomial ($x \in [0, -b/2a]$) satisfies the variation tendency that the dependent variable increases with the independent variable. Ultimately, only quadratic polynomial, linear Equation, and logarithmic function are considered (Equation (6)).

$$\left. \begin{array}{ll} y = ax^2 + bx + d & (x \in [1, -\frac{b}{2a}]) \\ y = a\ln(x) + b & (x \geq 1) \\ y = ax + b & (x \geq 1) \end{array} \right\} \quad (6)$$

According to the principle described in section 2.1, the point pairs (C, T) of the optimal distance threshold and map scale change are continuously recorded with the gradual generalization of the polyline. The fitting results between them are shown in Figure 8.

Figure 8. Optimization result of candidate functions between T and C.

As can be clearly seen from Figure 8, the R^2 of the unary quadratic function is closest to 1 ($R^2=0.9585$), which achieves the best fitting relationship of the curve among all the candidates. Therefore, the unary quadratic function (Equation (7)) is chosen as the quantitative model for describing the relationships between T and C.

$$\begin{cases} T = 0 & (C = 1) \\ T = aC^2 + bC + d & (a < 0, C \in (1, \max\{C_i\})) \end{cases} \quad (7)$$

In order to further verify whether the quantitative relationships between T and C can be fitted using the same function with the same coefficients, the experimental datasets of this part are divided into two parts: control and experimental. The former consists of multi-scale datasets from the different geographical feature areas, and the quadratic functions are shown in Figure 9(a). The latter consists of multi-scale datasets from the different geographical feature areas, and the quadratic functions are shown in Figure 9(b).

Figure 9. Quantitative relationship fitting results between T and C of multi-scale polylines.

It can be clearly seen from Figure 9(a) that the R^2 of each dataset in the control group is no less than 0.9585, and the maximum is up to 0.9917, indicating a very high fitting accuracy. However, Figure 9(a1) shows that no matter whether the polylines are the same or different type(s) of features (e.g., B and C both are roads; D and E are the different types of features of the real world), the coefficients of their unary quadratic function are entirely different for polylines from the different geographical feature area. Consequently, these results convincingly demonstrate that it is unreasonable to describe all five groups of datasets using the same single quadratic Equation with the same coefficients; besides, R^2 is only 0.4636. Therefore, it is impossible to simultaneously realize the completely automatic generalization of all polylines using the same optimal distance threshold.

Based on the results presented in Figure 9(b), the first group of experimental data which is consisted of four groups of multi-scale polyline from Shanxi Mountain (Figure 9(b1)), the other consists of 19 groups of multi-scale polylines from the Lower Yangtze River Plain (Figure 9(b2)). The R^2 values of the two groups of datasets are not less than 0.8521, with the best in the first groups ($R^2 = 0.9012$). Hence, the fitting accuracy is satisfactory. i.e., it is affirmative to realize the completely automatic generalization of all two groups of sampling datasets, respectively. Therefore, these results convincingly demonstrate that it is reasonable to realize the complete automation of the DP generalization algorithm for the polylines from the same geographical feature area. Therefore, polylines from the same geographical feature area are the objects of the following research.

3.3. Automated generalization of a polyline

Taking the multi-scale polylines from the Shanxi Mountain as an example, the theoretical optimal distance threshold is determined if the original and target map scales are given (Table 3). For example, suppose the original and target map scales are 1:50000, 1:250000, respectively, and then the map scale change C is 5. Hence, the optimal distance threshold T will be 0.0095 km. Afterward, six groups of experimental datasets are automatically generalized using DP based on the corresponding theoretical optimal distance threshold.

Table 3. Parameter T optimally results of multi-scale polylines.

T=-0.0001C ² +0.0098C-0.037 (C∈(1,50])						T=-0.00003C ² +0.0026C-0.0008 (C∈(1,50])						
1	C	5	10	25	50	2	C	2.5	5	10	25	50
	T/km	0.0095	0.0510	0.1455	0.2030		T/km	0.0056	0.0114	0.0222	0.0455	0.0542

Figure 10 shows the results of the experimental datasets based on the proposed method.

Figure 10. Experiment results of the different datasets from the same geographical feature area.

3.4. Accuracy evaluation

The proposed model is validated for its acceptability and consistency. In this regard, length ratio (LR), Sinuosity ratio (SR), PE, and SMD are employed to evaluate the geometric and position accuracy of generalization results (Wu et al. 2008).

Table 4. Geometric accuracy comparison between simplification results and available data.

Features	Target scale reference Data (L_{M2})				Simplification results of proposed method (L)			
	LR	SR	PE/km	SMD (%)	LR'	SR'	PE'/km	SMD'
Road1	0.9901	0.9145	0.0211	0.4661	0.9915	0.9004	0.0578	0.4565

Road2	0.9540	0.9227	0.0321	0.5334	0.9668	0.9154	0.0772	0.5102
Road3	0.9759	0.9032	0.0415	0.6029	0.9869	0.9056	0.0295	0.5621
River1	0.9983	0.7708	0.0478	0.6046	0.9983	0.7658	0.0448	0.5803
River2	0.9832	0.8827	0.0021	0.5662	0.9874	0.8789	0.0398	0.5663
River3	0.9966	0.8678	0.0101	0.6211	0.9975	0.8339	0.0641	0.6128

Table 4 also reveals that comparing with LM2, ΔLR of samples is less than zero, and ΔSR is almost greater than zero, the error of the compression amount (ΔLR), but the sinuosity degree change rate (ΔSR) of the road and river are the smallest. It highlights that the compression amount (LR) and the sinuosity degree change rates (SR) of L are consistent with LM2, and the proposed approaches can better maintain the geometric features of road and river; the data compression effect is superior.

Figure 11. Error rate comparison of geometric and position accuracy of simplification results

The road and river simplification results are comparable to the standard target scale polyline in terms of maintaining the position accuracy (Figure 11). Compared with the standard target scale polylines, the position errors of road and river are significantly superior to the error-index values of LM2 except for two groups of samplings, but the maximum position errors are only 0.012. It highlights that the simplification results of the proposed method could better maintain the global position accuracy of polylines.

As shown in Table 4, the SMD is almost greater than zero, but the maximum of SMD of L only is up to 0.0408, i.e., the index SMD also indicates that the simplification results of the proposed method could better maintain the local position accuracy of polylines. In summary, the proposed method effectively maintains the local and global shape characteristics of roads and rivers.

4. Discussions

Compared with special subjective parameter determination algorithms (Douglas and Peucker 1973; Li and Openshaw 1992; Ai et al. 2016; Qian 2017), which require humans' constant interference, this paper constructed the quantitative relationships between the optimal distance threshold and map scale change using the spatial similarity relations of polylines based on two groups of control experiments, which consider polylines from the same or different geographical feature areas. Finally, based on the theoretical optimal distance, the polylines from the same geographical feature areas are completely automatically generalized using the Douglas-Peucker algorithm.

First, compared with the linear Equation and logarithmic function, the R^2 of the unary quadratic function is no less than 0.8521. Therefore, the unary quadratic functions are the best as the quantitative model for describing the relationships between the optimal distance threshold and map scale change.

Second, no matter whether the polylines are from the same or different geographical feature area(s), the unary quadratic function can be employed to describe the

quantitative relationship between the optimal distance threshold and map scale change.

However, coefficients a , b , and c are the same for the polylines from the same geographical feature area, whereas the coefficients are different for the polylines from the different geographical feature areas. Similar results were also found between the similarity degree and map scale change. Thus, there are two main objective factors affecting map generalization: scale and regional geographic characteristics. This conclusion illustrates those regional geographic characteristics influence the determination of the quantitative relationship between the optimal distance threshold and map scale change. Therefore, it is reasonable to generalize the polylines from the same geographical area using the same single optimal distance threshold.

Third, according to the quantitative functional relation of the polylines from the same geographical feature area, if the map scale change is given, the theoretical optimal distance threshold can be calculated, and vice versa. Compared with semi-automatic or non-automatic polyline simplification algorithm (Gu 2016; Qian HZ 2017; Li CM 2017), this not only can facilitate the automation of map generalization software but avoid human intervention in the process of map generalization, which will promote the efficiency of the algorithm and the accuracy of map generalized results.

Last, this paper validated the acceptability and consistency of the geometric and position accuracy by comparing them with the standard target scale polyline. The experiment results indicate that the SMD of the road and the river simplified by the proposed method is better than the standard target scale polyline, i.e., geometric

features retention of the proposed method is more superior. This conclusion is consistent with the conclusions drawn by Wu (2008), who compared the Douglas-Peucker, Li-Openshaw, Circle, and asymptotic algorithms. Therefore, it can be concluded that the proposed method can better maintain the geometric features and shape characteristics of polylines on the whole in terms of geometric and position accuracy. In this paper, multi-scale individual polylines are taken as research objects. However, the generalization of polyline group objects is not only dependent on the features of ground features; it is closely related to the surrounding ground features. Therefore, due to the defects of the DP algorithm itself, the simplification results may intersect or intersect with other adjacent features that are close in distance, e.g., two adjacent contours with a smaller distance between them simplified by the DP algorithm may intersect. It is recommended that future work should conduct extensive experiments over polyline group objects, e.g., contour cluster, road network, reticulate drainage.

5. Conclusions

This paper proposed a model, which describes the quantitative relationships between the optimal distance threshold and map scale change, to minimize the uncertainties in the complete automation of the Douglas-Peucker algorithm. It can calculate the theoretical optimal distance threshold of the polylines from the same geographical feature area taking map scale change as the only independent variable and vice versa.

It is indicated that realizing the complete automation of the Douglas-Peucker algorithm is affirmative for the polylines from the same geographical feature area. The findings of the current study not merely provide an idea and method for realizing the complete automation of map generalization by looking for the quantitative relationship between the parameter of algorithm and map scale change but also facilitate the automation of map generalization algorithms and system of polyline and improve the accuracy of map generalized results. The proposed model optimizes algorithm threshold settings, eventually reducing the inevitable differences between automated-simplified and cartographers' ratified simplification results. Reduced uncertainties in realizing the Douglas-Peucker Algorithm automation would support improved spatial data matching and establish a vector map database.

References

- Ai, T.H., Shu, K., Yang, M., et al., 2016. Envelope generation and simplification of polylines using Delaunay triangulation. *International Journal of Geographical Information Science*, 31(2), 297-319.
- Chen, J. N., Qian, H. Z., Wang, X., et al. 2016. Geometric accuracy assessment of linear features' simplification algorithm. *Geomatics and Information Science of Wuhan University*, 45(4):486-493. [in Chinese]
- Chehreghan, A., Abbaspour, R.A., 2018. A geometric-based approach for road matching on multi-scale datasets using a genetic algorithm. *Cartography and Geographic Information Science*, 45(3), 155-269.

531 Chehreghan, A., Abbaspour, R.A., 2016. An assessment of spatial similarity degree
532 between polylines on multi-scale, multi-source maps. *Geocarto International*,
533 32(5), 471-487.

534 Danell, P.A., Yang, P., 2014. Quantitative methods for comparing different polyline
535 models. *Journal of Geographic Information System*, 6, 88-98.

536 Deng, M., Li, Z.L., Chen, X.Y., 2007. Extended Hausdorff distance for spatial objects
537 in GIS. *International Journal of Geographical Information Science*, 21(4):459–475.

538 Douglas, D.H, Peucker, T. K., 1973. Algorithms for the reduction of the number of
539 points required to represent a digitized line or its caricature. *The Canadian*
540 *Cartographer*, 2(10):112-122.

541 Efrat, A. et al. Geometry helps in bottleneck matching and related problems.
542 *Algorithmica*, 2001, 31 (1):1–28.

543 Gu, T., Chen, X. Y., Liu, C.Q., 2016. A modified line simplification method combined
544 Douglas-Peucker with Li-Openshaw. *Journal of East China University of*
545 *Technology*, 39(4):396-400. [in Chinese]

546 Hershberger, J., Snoeyink, J., 1992. Speeding up the Douglas-Peucker line-simplification
547 algorithm. In: *Proceedings of the fifth symposium on data handling UBC*
548 *Technical report TR-92-07*, pp:134-143.

549 Hughes, G.F., 1968. On the mean accuracy of statistical pattern recognizers. *IEEE Trans.*
550 *Inf. Theory*, 14, 55-63.

551 Kronenfeld, B. J., Stanislawski, L.V., Battenfield, B.P., et al, .2020. Simplification of

polylines by segment collapse:minimizing areal displacement while preserving
area. *International Journal of Cartography*, 6(1):22-46.

Li, Z. L., Openshaw, S., 1992. Algorithms for automated line generalization based on a
natural principle of objective generalization. *International Journal of Geographic
Information Systems*, 16(5):373-389.

Li, C. M., Guo, P.P., Yin, Y., et al., 2017. A line simplification algorithm considering
spatial relations between two lines. *Acta Geodaetica et Cartographica Sinica*,
46(4):498-506. [in Chinese]

Li, Z. L., Sui, H.G., 2000. An integrated technique for automated generalization of
contour maps. *The Cartographic Journal*. 37(1):29-37.

Li, ZX, Zhai, J.S., Wu, F., 2018. Shape similarity assessment method for coastline
generalization. *ISPRS International Journal of GeoInformation*, 7(7):283.

Li, B., Fonseca, F.T., 2006. TDD: A comprehensive model for qualitative spatial
similarity assessment. *Spatial Cognition and Computation*, 6(1):31-61.

Makris, A., Silva, C.L.d., Bogorny, V. et al. Evaluating the effect of compressing
algorithms for trajectory similarity and classification problems. *Geoinformatica* 25,
679–711 (2021). <https://doi.org/10.1007/s10707-021-00434-1>.

McMaster, R. B., 1986. A statistical analysis of mathematical measures for linear
simplification. *The American Cartographer*, 13(2):103-116.

McMaster, R. B., 1987. Automated line generalization. *Cartographica*, 24(2):74-111.

Nayyeri, A. et al., Computing the Fréchet Distance between polygons with holes. In M.

573 M. Halldórsson, K. Iwama, N. Kobayashi & B. Speckmann, eds. Automata,
574 Languages, and Programming: 42nd International Colloquium, ICALP 2015, 6–
575 10 July 2015, Kyoto, Japan, Proceedings, Part I. Heidelberg: Springer Berlin
576 Heidelberg, 997–1009.

577 Olteanu-Raimond, A., Mustière, S., Ruas, A., 2015. Knowledge formalisation for vector
578 data matching using belief theory. *Journal of Spatial Information Science*, 10:21–
579 46.

580 Olteanu-Raimond, A., et al., 2008. Data matching - a matter of belief. In: Ruas A, Gold
581 C, editors. *Headway in spatial data handling. Lecture notes in geoinformation and*
582 *cartography*. Berlin: Springer, p. 501-519.

583 Peuquet, D.J., 1992. An algorithm for calculating minimum euclidean distance between
584 two geographic features. *Computers & Geosciences*, 18 (8), 989–1001.

585 Qian, H. Z., He, H. W., Wang, X., et al. Line feature simplification method based on
586 Bend group division[J]. *Geomatics and Information Science of Wuhan University*,
587 2017, 42(8):1096-1103. [in Chinese]

588 Ramer, U., 1972. An iterative procedure for the polygonal approximation of plane
589 curves. *Computer Graph Image Process*, 1(3):244-256.

590 Saalfeld, A. (1999). Topologically consistent line simplification with the Douglas-
591 Peucker algorithm. *Cartography and Geographic Information Science*, 26(1), 7-18.

592 Sama,l A., Seth, S., Cueto, K., 2004. A feature-based approach to conflation of
593 geospatial sources. *International Journal of Geographical Information Science*,

594 18(5), 459–489.

595 Sandu Popa, I., Zeitouni, K., Oria, V. et al. Spatio-temporal compression of trajectories
 596 in road networks. *Geoinformatica* 19, 117–145 (2015).
 597 <https://doi.org/10.1007/s10707-014-0208-4>.

598 Tong, X.H., Liang, D., Jin, Y.M., 2014. A linear road object matching method for
 599 conflation based on optimization and logistic regression. *International Journal of*
 600 *Geographical Information Science*, 28(4):824-846.

601 Wu, F., Zhu, K. P.. 2008. Geometric accuracy assessment of linear features'
 602 simplification algorithm. *Geomatics and Information Science of Wuhan University*,
 603 33(6):600-603. [in Chinese]

604 White, E. R., 1985. Assessment of line-generalization algorithms using characteristic
 605 points. *The American Cartographer*, 12(1):17-28.

606 Yan, H. W., 2010. Fundamental Theories of Spatial Similarity Relations in Multi-scale
 607 Map Spaces. *Chinese Geographical Science*, 20(1):18–22.

608 Yan, H. W., Li, J., 2014. Spatial similarity relations in multi-scale map spaces [M].
 609 Springer International Publishing Switzerland.

610 Yan, H. W., Shen, Y. Z., Li, J., 2016. Approach to calculating spatial similarity degrees
 611 of the same river basin networks on multi-scale maps. *Geocarto International*,
 612 31(7):765-782.

613 Yan, H. W., 2015. Quantitative relations between spatial similarity degree and map scale
 614 change of individual linear objects in multi-scale map spaces. *Geocarto*

615 International, 30(4):472-482.

616 Yang, W. F., Yan, H. W., Jonathan, L., 2015. Formula for calculating spatial similarity
617 degrees between poing clouds on multi-scale maps taking map scale change as the
618 only independent variable. *Geodesy and Geodynamics*, 2(6):113-125.

619 Shen, Y. L., Ai, T. H., Wang, L., et al. 2018. A new approach to simplifying polygonal
620 and linear features using superpixel segmentation. *International Journal of*
621 *Geographical Information Science*, 32(10): 2023-2054.

622 Zhang, M., Yao, W., Meng, L., 2014. Enrichment of topographic road database for the
623 purpose of routing and navigation. *International Journal of Digital Earth*, 7(5):
624 411–431.

625 Zhang, M., 2009. *Methods and implementations of road-network matching*. Munich:
626 Technical University of Munich