Modelling and Analysis of Coupling Dynamics of a Lower Limb Exoskeleton

Yao Yan

University of Electronic Science and Technology of China

Zhenlei Chen

University of Electronic Science and Technology of China

Cheng Huang

University of Electronic Science and Technology of China

Qing Guo (✉ guoqinguestc@uestc.edu.cn)

University of Electronic Science and Technology of China <https://orcid.org/0000-0003-0522-1243>

Research Article

Keywords: Human-exoskeleton coupling dynamics, interactive model, misalignment, light-weight structure, coupling points and tightness

Posted Date: January 31st, 2022

DOI: <https://doi.org/10.21203/rs.3.rs-1165874/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.
[Read Full License](#)

Modelling and analysis of coupling dynamics of a lower limb exoskeleton

Yao Yan^a, Zhenlei Chen^a, Cheng Huang^a, Qing Guo^{a,*}

^a*School of Aeronautics and Astronautics, University of Electronic Science and Technology of China, Chengdu, 611731, China*

Abstract

Exoskeleton is a promising technology to enhance the mobility of aged and disable people. With comfortable human-exoskeleton interaction, mechanical and control designers aim at accurately tracking a desired trajectory, while saving the wearer's energy expenditure is the preference from the viewpoint of biomechanics. Given all of these effects, we propose a new model, consisting of the swing dynamics of both human and robot's lower extremities coupled by damped springs representing elastic and viscous properties of band and human tissue. With the coupling coefficients identified with an experimental platform, the analytical model yields consistent results compared with an experimental exoskeleton, especially in the prediction of the interactive forces. Further analyses are then performed based on this validated model, revealing the influences of desired trajectory, mass ratio, misalignment, coupling points, health condition and band tightness on the human-exoskeleton coupling dynamics. It is found that introducing gravity compensation and tuning the feedback gain improve the tracking accuracy, but hardly change the interactive force. The most comfortable interaction requires a healthy wearer coupled with a light-weight exoskeleton without any misalignment, but properly changing the trajectory, coupling points and tightness can partly reduce the interactive forces if the ideal condition is unachievable.

Keywords: Human-exoskeleton coupling dynamics, interactive model, misalignment, light-weight structure, coupling points and tightness

Nomenclature

$\dot{x}$	Derivative of x with respect to time t	
$\text{sgn}(x)$	Sign function of x	
$\text{sat}(x, y)$	Saturation function of x between the values $[-y, y]$	
θ_d	Vector of desired joint angles	[rad]
θ_{i_1}	Vector of joint angles: human ($i_1 = h$) or robot ($i_1 = r$)	[rad]
τ_h^a	Vector of human active joint torques	[N m]
τ_h^{ida}	Vector of human active joint torques obtained by inverse dynamics analysis	[N m]

*Corresponding author. e-mail: guoqinguestc@uestc.edu.cn

Email addresses: y.yan@uestc.edu.cn (Yao Yan), zhenlei_chen@std.uestc.edu.cn (Zhenlei Chen), h.cheng.alesso@foxmail.com (Cheng Huang)

τ_{i_1}	Vector of joint torques: human ($i_1 = h$) or robot ($i_1 = r$)	[N m]
$\tau_{i_1}^i$	Vector of coupling joint torques: human ($i_1 = h$) or robot ($i_1 = r$)	[N m]
τ_r^c	Vector of robot's controlling joint torques	[N m]
τ_r^f	Vector of robot's frictional joint torques	[N m]
$\mathbf{C}_{i_1}(\theta_{i_1}, \dot{\theta}_{i_1})$	Coriolis and centripetal matrix: human ($i_1 = h$) or robot ($i_1 = r$)	[N m s rad ⁻¹]
$\mathbf{e}_{i_1}$	Vector of tracking errors: human ($i_1 = h$) or robot ($i_1 = r$)	[rad]
$\mathbf{F}_{i_2}$	Vector of coupling forces: thigh ($i_2 = t$) or shank ($i_2 = s$)	[N]
$\mathbf{G}_{i_1}(\theta_{i_1})$	Gravational vector: human ($i_1 = h$) or robot ($i_1 = r$)	[N m]
$\mathbf{J}_{i_1, i_2}$	Jacobian matrix: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
$\mathbf{M}_{i_1}(\theta_{i_1})$	Inertial matrix: human ($i_1 = h$) or robot ($i_1 = r$)	[N m s ² rad ⁻¹]
$\mathbf{s}_{i_1, i_2}$	Coordinate of coupling point: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
$\Delta \mathbf{s}_{i_2}$	Vector of human-exoskeleton relative displacement: thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
α	Angle between human and robot's thighs due to hip misalignment	[rad]
β	Angle of hip misalignment	[rad]
γ	Angle between human and robot's shanks due to hip misalignment	[rad]
θ_{i_1, i_3}	Joint angle: human ($i_1 = h$) or robot ($i_1 = r$); hip ($i_3 = h$) or knee ($i_3 = k$)	[rad]
τ_{h, i_3}^a	Human active joint torque: hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
$\tau_{h, i_3}^{\text{ida}}$	Human active joint torque based on inverse dynamics analysis: hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
$\tau_{h, i_3}^{\text{max}}$	Maximum active joint torque of healthy people: hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
$\tau_{r, i_3}^{\text{sf}}$	Static frictional torque in robot's joint: hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
τ_{i_1, i_3}	Joint torque: human ($i_1 = h$) or robot ($i_1 = r$); hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
τ_{i_1, i_3}^i	Coupling joint torque: human ($i_1 = h$) or robot ($i_1 = r$); hip ($i_3 = h$) or knee ($i_3 = k$)	[N m]
ω	Swing frequency	[rad s ⁻¹]
a	Magnitude of hip misalignment	[m]
b_{i_2}	Coupling damping: thigh ($i_2 = t$) or shank ($i_2 = s$)	[N s m ⁻¹]
c_{i_3}	Coefficient of viscous friction in robot joint: hip ($i_3 = h$) or knee ($i_3 = k$)	[N m s rad ⁻¹]
c_{i_1, i_2}	Center of mass: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
d_a	Vertical distance from the coupling point to the laser sensor above it	[m]

d_b	Vertical distance from the coupling point to the laser sensor below it	[m]
D	Horizontal displacement at the coupling point	[m]
D_a	Horizontal displacement measured by the laser sensor above the coupling point	[m]
D_b	Horizontal displacement measured by the laser sensor below the coupling point	[m]
e_{i_1, i_3}	Tracking error in each joint: human ($i_1 = h$) or robot ($i_1 = r$); hip ($i_3 = h$) or knee ($i_3 = k$)	[rad]
F_0	Force constant in the coupling coefficient identification	[N]
F_x	Horizontal force measured by the force sensor	[N]
g	Gravitational acceleration	[m s ⁻²]
H	Wearer's height	[m]
h_c	Health condition	
i_1	Index of human ($i_1 = h$) and robot ($i_1 = r$)	
i_2	Index of thigh ($i_2 = t$) and shank ($i_2 = s$)	
i_3	Index of hip ($i_3 = h$) and knee ($i_3 = k$)	
I_{i_1, i_2}	Moment of inertia: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[kg m ²]
k_d	Derivative feedback gain in robot control	[N m s rad ⁻¹]
k_{i_2}	Coupling stiffness: thigh ($i_2 = t$) or shank ($i_2 = s$)	[N m ⁻¹]
k_p	Proportional feedback gain in robot control	[N m rad ⁻¹]
l_{i_1, i_2}	Length: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
m_{i_1, i_2}	Mass: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[kg]
M	Wearer's body mass	[kg]
$\overline{P}_{h, i_3}^a$	Time-averaged human active joint power: hip ($i_3 = h$) or knee ($i_3 = k$)	[W]
$\overline{P}_{r, i_3}^c$	Time-averaged robot's controlling joint power: hip ($i_3 = h$) or knee ($i_3 = k$)	[W]
r_{i_2}	Coupling tightness: thigh ($i_2 = t$) or shank ($i_2 = s$)	
r_m	Mass ratio	
s_{i_1, i_2}	Distance from segment proximal end to coupling point: human ($i_1 = h$) or robot ($i_1 = r$); thigh ($i_2 = t$) or shank ($i_2 = s$)	[m]
t	Time	[s]
T	Swing period	[s]

1. Introduction

The world is ageing rapidly. The number of people aged 65 and older was estimated to grow from 617.1 million in 2015 to 1.6 billion by 2050 [1]. Besides, 720 million people aged 15 years and older currently have functional disability [2]. Both of aged and disable people suffer from mobility disability, which limits their accessibility to better public and health services, regular social contacts, and civic life [3]. To strengthen human mobility and assist disable patients, the concept of exoskeleton has been proposed in 1960s by US Armed Force and General Electric [4], which attaches a wearable robot to a wearer to augment his mobility. Due to the underdevelopment of sensing, material and control technologies, exoskeleton did not attract too many attentions before 2000, but the latest two decades witnessed a vigorous growth in the studies of exoskeleton [5]. Depending on the assistive strategy, exoskeletons may actuate single or multiple joints, including specific joints, upper and lower extremities, or even a full body [6]. More specifically for assisting lower limb locomotion, many famous exoskeletons, such as BLEEX [7] and HULC [8] for strength augmentation, LOPES [9] and C-ALEX [10] for patient rehabilitation, and ReWalk [11] and HAL [12] for locomotion assistance, have been proposed.

Various lower limb exoskeletons can be generally classified into two groups, the active and passive. An active exoskeleton regards its wearer as an external disturbance to the robot and actively drives its joints to accurately track a desired trajectory [13]. The desired trajectory can be a pre-defined one, such as sinusoidal or measured from a healthy person [14], or timely adjusted given the wearer's intention and effort [15]. For a given trajectory, many control algorithms, such as proportional-integral-derivative (PID) [16, 17], adaptive neural network [18], sliding-mode [19], etc., can be employed to achieve accurate tracking. By contrast, passive exoskeletons have no active joints, which attach exquisite structures to a healthy wearer to reduce his metabolic energy consumption [20, 21] or muscle contraction [22] during normal movements. However, many studies found only few drops in these metrics for better human mobility [23].

Besides the tracking accuracy from the view point of robot control and energy expenditure from the view point of human mobility, another issue is the human-exoskeleton coupling force which is critical for interaction comfort [24]. To understand the comfort, various new pressure sensors have been proposed to monitor the interactive forces on legs [25] and back [26]. Influences of the coupling force on the robot control have been realized by the control research community, and they use various methods to compensate the human-robot impedance for better tracking performance [27, 28]. Meanwhile, machine learning is also employed in adjusting the parameters of impedance model to minimize the interactive force for more comfortable human-exoskeleton coupling [29]. However, instead of revealing the physical relationship between the interactive movements and forces, impedance model uses artificial inertia, damping and stiffness coefficients, and it cannot be regarded as a constitutive equation but a filter [30]. Therefore, the impedance model does not yield accurate prediction of the interactive force and cannot be used to study the human-exoskeleton coupling dynamics.

Up to now, most investigations regarded the human-exoskeleton system as a whole and their motions are synchronized [31–33], while very rare studies tried to accurately model the human-exoskeleton coupling effect to the best of our knowledge. For example, Shi et al. [34] proposed a coupling model for both of human and exoskeleton dynamics, but the coupling torques were only compensated by adaptive scheme, without predicting the interactive force for assessing human comfort. Despite the effectiveness of these methods in robot control, it was found that the simulation results hardly match experimental tests from

the viewpoint of biomechanics if the realistic contact formulation is absent [35]. Therefore, some scholars started to investigate the coupling model. For example, in the study of assisting subjects with muscle deficiency during sit-to-stand motion, Serrancoli et al. [36] solved optimal control problems with a digital model in OpenSim to calibrate the subject-exoskeleton contact mode, so that the simulation can predict of the contact forces.

Due to the lack of analytical methods for the analysis of human-exoskeleton coupling dynamics, this paper proposes a new 4 degrees of freedom (DoF) model of the lower limb swing motion in Section 2, where the coupling is represented by damped springs to represent the elastic and viscous properties of the band and human tissue. After coefficients of the coupling forces identified with an experimental platform, the new model is then validated in Section 3, which yields consistent prediction of the interactive forces compared with an exoskeleton apparatus. Based on this validated model, Section 4 studies the influence of various factors, such as desired trajectories, mass ratio, misalignment, coupling points, health condition and band tightness, on the human-exoskeleton coupling dynamics. Finally, conclusions are drawn in Section 5.

2. Modelling of the human-exoskeleton coupling system

To simplify the variables and parameters used in the modelling and analysis of human-exoskeleton coupling dynamics, we use 3 indices as follows.

- $i_1 \in \{h, r\}$ in subscript represents human (h) and robot (r),
- $i_2 \in \{t, s\}$ in subscript represents thigh (t) and shank (s),
- $i_3 \in \{h, k\}$ in subscript represents hip (h) and knee (k).

2.1. System description

As displayed in Fig. 1, the human-exoskeleton lower-limb swing dynamics consists of 4 degrees of freedom (DoF) for the rotation angles, θ_{i_1, i_3} , of human ($i_1 = h$) and robot's ($i_1 = r$) hips ($i_3 = h$) and knees ($i_3 = k$). This model also considers the misalignment between human and robot's hips, which has a magnitude of a [m] and an angle of β [rad]. For human ($i_1 = h$) and robot's ($i_1 = r$) thigh ($i_2 = t$) and shank ($i_2 = s$), the mass, moment of inertia, length, centre of mass, and coupling point are denoted as m_{i_1, i_2} [kg], I_{i_1, i_2} , [kg m²], l_{i_1, i_2} [m], c_{i_1, i_2} [m], and s_{i_1, i_2} [m], respectively.

With the vectors of human ($i_1 = h$) and robot's ($i_1 = r$) joint angles denoted as $\theta_{i_1} = (\theta_{i_1, h}, \theta_{i_1, k})^T$, the human-exoskeleton swing dynamics displayed in Fig. 1 can be modelled by using Euler-Lagrange equations with external forces, yielding

$$\begin{cases} \mathbf{M}_h(\theta_h) \ddot{\theta}_h + \mathbf{C}_h(\theta_h, \dot{\theta}_h) \dot{\theta}_h + \mathbf{G}_h(\theta_h) = \tau_h, \\ \mathbf{M}_r(\theta_r) \ddot{\theta}_r + \mathbf{C}_r(\theta_r, \dot{\theta}_r) \dot{\theta}_r + \mathbf{G}_r(\theta_r) = \tau_r, \end{cases} \quad (1)$$

where the symmetric positive-symmetric inertia matrices, Coriolis and centripetal matrices, and gravita-

Figure 1: Human-exoskeleton coupling dynamics in the swing of lower limbs, where their hips are misaligned.

tional vectors for the wearer ($i_1 = h$) and robot ($i_1 = r$) are given as follows

$$\begin{aligned}
 \mathbf{M}_{i_1}(\boldsymbol{\theta}_{i_1}) &= \begin{pmatrix} I_{i_1,t} + m_{i_1,t}c_{i_1,t}^2 + m_{i_1,s}l_{i_1,t}^2 & 0 \\ 0 & 0 \end{pmatrix} + m_{i_1,s}l_{i_1,t}c_{i_1,s}\cos(\theta_{i_1,k}) \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} + (I_{i_1,s} + m_{i_1,s}c_{i_1,s}^2) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \\
 \mathbf{C}_{i_1}(\boldsymbol{\theta}_{i_1}, \dot{\boldsymbol{\theta}}_{i_1}) &= m_{i_1,s}l_{i_1,t}c_{i_1,s}\sin(\theta_{i_1,k}) \begin{pmatrix} -\dot{\theta}_{i_1,k} & -\dot{\theta}_{i_1,k} - \dot{\theta}_{i_1,h} \\ \dot{\theta}_{i_1,h} & 0 \end{pmatrix}, \\
 \mathbf{G}_{i_1}(\boldsymbol{\theta}_{i_1}) &= g \begin{pmatrix} m_{i_1,t}c_{i_1,t}\sin(\theta_{i_1,h}) + m_{i_1,s}l_{i_1,t}\sin(\theta_{i_1,h}) + m_{i_1,s}c_{i_1,s}\sin(\theta_{i_1,h} + \theta_{i_1,k}) \\ m_{i_1,s}c_{i_1,s}\sin(\theta_{i_1,h} + \theta_{i_1,k}) \end{pmatrix}.
 \end{aligned} \tag{2}$$

Here, $g = 9.81 \text{ [m s}^{-2}\text{]}$ is the gravitational acceleration. The torques in human and robot joints, $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_r$ in Eq. (1), can be decomposed into

$$\begin{aligned}
 \boldsymbol{\tau}_h &= \boldsymbol{\tau}_h^i + \boldsymbol{\tau}_h^a, \\
 \boldsymbol{\tau}_r &= \boldsymbol{\tau}_r^i + \boldsymbol{\tau}_r^f + \boldsymbol{\tau}_r^c,
 \end{aligned} \tag{3}$$

where $\boldsymbol{\tau}_h^i$ and $\boldsymbol{\tau}_h^a$ are the vectors of interactive and active torques in human joints, and $\boldsymbol{\tau}_r^i$, $\boldsymbol{\tau}_r^f$ and $\boldsymbol{\tau}_r^c$ are the vectors of interactive, frictional and controlling torques in robot's joints, respectively.

2.2. Joint toques and power supplies

The interactive torques, $\boldsymbol{\tau}_h^i$ and $\boldsymbol{\tau}_r^i$, are induced by the human-exoskeleton coupling. Given the misalignment and geometric constraint displayed in the top of Fig. 1, the coupling points at human thigh and shank can be obtained by the following equations once the coupling points at robot's thigh and shank

are known.

$$\begin{aligned} s_{h,t} &= s_{r,t} \cos(\alpha) + a \cos(\beta), \\ s_{h,s} &= (l_{r,t} - s_{r,t}) \cos(\alpha) + s_{r,s} \cos(\gamma) - (l_{h,t} - s_{h,t}), \end{aligned} \quad (4)$$

where α (γ) is the angle between human and robot's thighs (shanks) owing to the hip misalignment. The angles can be determined by sine rule as follows

$$\begin{aligned} \alpha &= \sin^{-1} \left(a \frac{\sin(\beta)}{s_{r,t}} \right), \\ \gamma &= \sin^{-1} \left((l_{r,t} - s_{r,t}) \frac{\sin(\alpha)}{s_{r,s}} \right). \end{aligned} \quad (5)$$

In the xoy frame shown in Fig. 1, the coupling points at human and robot's thighs, and human and robot's shanks are located at

$$\mathbf{s}_{h,t} = \begin{pmatrix} s_{h,t} \sin(\theta_{h,h}) + a \cos(\beta) \\ -s_{h,t} \cos(\theta_{h,h}) + a \sin(\beta) \end{pmatrix}, \quad \mathbf{s}_{r,t} = \begin{pmatrix} s_{r,t} \sin(\theta_{r,h}) \\ -s_{r,t} \cos(\theta_{r,h}) \end{pmatrix}, \quad (6)$$

and

$$\begin{aligned} \mathbf{s}_{h,s} &= \begin{pmatrix} l_{h,t} \sin(\theta_{h,h}) + s_{h,s} \sin(\theta_{h,h} + \theta_{h,k}) + a \cos(\beta) \\ -l_{h,t} \cos(\theta_{h,h}) - s_{h,s} \cos(\theta_{h,h} + \theta_{h,k}) + a \sin(\beta) \end{pmatrix}, \\ \mathbf{s}_{r,s} &= \begin{pmatrix} l_{r,t} \sin(\theta_{r,h}) + s_{r,s} \sin(\theta_{r,h} + \theta_{r,k}) \\ -l_{r,t} \cos(\theta_{r,h}) - s_{r,s} \cos(\theta_{r,h} + \theta_{r,k}) \end{pmatrix}. \end{aligned} \quad (7)$$

At the coupling points at the thigh ($i_2 = t$) and shank ($i_2 = s$), relative human-exoskeleton displacement can be obtained by

$$\Delta \mathbf{s}_{i_2} = \mathbf{s}_{h,i_2} - \mathbf{s}_{r,i_2}. \quad (8)$$

Derivative of Eq. (8) with respect to time, t , yields the corresponding relative velocities as follows

$$\begin{aligned} \Delta \dot{\mathbf{s}}_{i_2} &= \dot{\mathbf{s}}_{h,i_2} - \dot{\mathbf{s}}_{r,i_2} \\ &= \mathbf{J}_{h,i_2} \dot{\boldsymbol{\theta}}_h - \mathbf{J}_{r,i_2} \dot{\boldsymbol{\theta}}_r, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \mathbf{J}_{i_1,t} &= \begin{pmatrix} s_{i_1,t} \cos(\theta_{i_1,h}), 0 \\ s_{i_1,t} \sin(\theta_{i_1,h}), 0 \end{pmatrix}, \\ \mathbf{J}_{i_1,s} &= \begin{pmatrix} l_{i_1,t} \cos(\theta_{i_1,h}) + s_{i_1,s} \cos(\theta_{i_1,h} + \theta_{i_1,k}) & s_{i_1,s} \cos(\theta_{i_1,h} + \theta_{i_1,k}) \\ l_{i_1,t} \sin(\theta_{i_1,h}) + s_{i_1,s} \sin(\theta_{i_1,h} + \theta_{i_1,k}) & s_{i_1,s} \sin(\theta_{i_1,h} + \theta_{i_1,k}) \end{pmatrix}, \end{aligned} \quad (10)$$

are Jacobians for human ($i_1 = h$) and robot's ($i_1 = r$) velocities at the coupling points, respectively. Due to the elastic and viscous properties of the non-rigid human-exoskeleton coupling, as seen in Fig. 2, the relative motion yields interactive reactions exerted on both of human and robot's lower extremities. By using a simple linear model for the coupling, interactive forces at the thigh ($i_2 = t$) and shank ($i_2 = s$)

Figure 2: Relative motions and interactive forces between the wearer and robot at thigh and shank

can be represented as follows

$$\mathbf{F}_{i_2} = -r_{i_2}(k_{i_2}\Delta\mathbf{s}_{i_2} + b_{i_2}\Delta\dot{\mathbf{s}}_{i_2}), \quad (11)$$

where k_{i_2} and b_{i_2} are the coupling stiffness and damping at the thigh ($i_2 = t$) and shank ($i_2 = s$) when the binding is tight, and $r_{i_2} \in [0\%, 100\%]$ represents the coupling tightness. When r_{i_2} decreases from 100% towards 0%, the binding is gradually loosened, yielding no interactive force for $r_{i_2} = 0\%$. Next, by using the Jacobians listed in Eq. (10), the coupling forces are mapped into the joint space for the interactive torques exerted on human and robot's joints as follows [37]

$$\begin{aligned} \boldsymbol{\tau}_h^i &= \mathbf{J}_{h,t}^T \mathbf{F}_t + \mathbf{J}_{h,s}^T \mathbf{F}_s, \\ \boldsymbol{\tau}_r^i &= -\mathbf{J}_{r,t}^T \mathbf{F}_t - \mathbf{J}_{r,s}^T \mathbf{F}_s. \end{aligned} \quad (12)$$

Human active torque, $\boldsymbol{\tau}_h^a$, is from muscle contraction to track a desired periodic trajectory, $\boldsymbol{\theta}_d$, which can be sinusoid or measured from a healthy person [14]. More specifically in this study, a simple sinusoid trajectory as follows is used

$$\boldsymbol{\theta}_d = \begin{pmatrix} 40^\circ + A \sin(\omega t) \\ -40^\circ + A \sin(\omega t + 90^\circ) \end{pmatrix} \quad (13)$$

where A [deg] and ω [rad s⁻¹] are the amplitude and frequency of the swing motion. Correspondingly, the swing period is $T = 2\pi/\omega$ [s]. For a healthy person, the active joint torques required to track the desired trajectory can be estimated by inverse dynamics analysis (IDA) [38] as follows

$$\boldsymbol{\tau}_h^a = \boldsymbol{\tau}_h^{\text{ida}} = \begin{pmatrix} \tau_{h,h}^{\text{ida}} \\ \tau_{h,k}^{\text{ida}} \end{pmatrix} = \mathbf{M}_h(\boldsymbol{\theta}_d)\ddot{\boldsymbol{\theta}}_d + \mathbf{C}_h(\boldsymbol{\theta}_d, \dot{\boldsymbol{\theta}}_d)\dot{\boldsymbol{\theta}}_d + \mathbf{G}_h(\boldsymbol{\theta}_d), \quad (14)$$

where $\tau_{h,i_3}^{\text{ida}}$ is the active torque at human hip ($i_3 = h$) and knee ($i_3 = k$) obtained by inverse dynamics analysis. This analysis assumes sufficient human power, but a paralysed person cannot provide any joint torques, i.e., $\boldsymbol{\tau}_h^a = \mathbf{0}$ [N m]. Given human health condition, we employ a saturation function to represent insufficient human effort as follows

$$\boldsymbol{\tau}_h^a = \begin{pmatrix} \tau_{h,h}^a \\ \tau_{h,k}^a \end{pmatrix} = \begin{pmatrix} \text{sat}(\tau_{h,h}^{\text{ida}}, h_c \tau_{h,h}^{\text{max}}) \\ \text{sat}(\tau_{h,k}^{\text{ida}}, h_c \tau_{h,k}^{\text{max}}) \end{pmatrix} \quad (15)$$

where τ_{h,i_3}^a is the active torque in human hip ($i_3 = h$) and knee ($i_3 = k$), $h_c \in [0\%, 100\%]$ is the health condition, $\tau_{h,h}^{\max} = 110$ [N m] ($\tau_{h,k}^{\max} = 120$ [N m]) is the maximum torque in the hip (knee) of a healthy person [39, 40], and

$$\text{sat}(x, y) = \begin{cases} -y, & \text{if } x < -y, \\ x, & \text{if } -y \leq x \leq y, \\ y, & \text{if } x > y, \end{cases} \quad (16)$$

is the saturation function.

The exoskeleton assists its wearer to track the desired trajectory with a minimized tracking error, which can be realized by a simple proportional-derivative (PD) feedback as follows

$$\boldsymbol{\tau}_r^c = \begin{pmatrix} \tau_{r,h}^c \\ \tau_{r,k}^c \end{pmatrix} = k_p \mathbf{e}_r + k_d \dot{\mathbf{e}}_r. \quad (17)$$

Alternatively, compensation of the robot's gravity as follows can be included in the PD feedback as well

$$\boldsymbol{\tau}_r^c = \begin{pmatrix} \tau_{r,h}^c \\ \tau_{r,k}^c \end{pmatrix} = k_p \mathbf{e}_r + k_d \dot{\mathbf{e}}_r + \mathbf{G}_r(\boldsymbol{\theta}_r). \quad (18)$$

Here, τ_{r,i_3}^c is the control torques in robot's hip ($i_3 = h$) and knee ($i_3 = k$), k_p and k_d are the proportional and derivative gains, and $\mathbf{e}_r = \boldsymbol{\theta}_d - \boldsymbol{\theta}_r = (e_{r,h}, e_{r,k})$ is the vector of tracking errors in robot's hip and knee. Besides, friction in the robot's joints introduces frictional joint torque, $\boldsymbol{\tau}_r^f$, which can be modelled by Coulomb dry friction with viscous effect as follows

$$\boldsymbol{\tau}_r^f = \begin{pmatrix} -\text{sgn}(\dot{\theta}_{r,h})\tau_{r,h}^{\text{sf}} - c_h \dot{\theta}_{r,h} \\ -\text{sgn}(\dot{\theta}_{r,k})\tau_{r,k}^{\text{sf}} - c_k \dot{\theta}_{r,k} \end{pmatrix}, \quad (19)$$

where τ_{r,i_3}^{sf} and c_{i_3} represent the static friction and coefficient of viscosity in the robot's hip ($i_3 = h$) and knee ($i_3 = k$), and $\text{sgn}(\cdot)$ is the sign function of $\cdot$.

Human muscle contraction and robot control supply energy for the swing motion, where the active and controlling joint torques, $\boldsymbol{\tau}_{h,a}$ and $\boldsymbol{\tau}_{r,c}$, yield time-averaged power as follows

$$\begin{aligned} \bar{P}_{h,i_3}^a &= \frac{1}{nT} \int_{mT}^{(m+n)T} \tau_{h,i_3}^a \dot{\theta}_{h,i_3} dt, \\ \bar{P}_{r,i_3}^c &= \frac{1}{nT} \int_{mT}^{(m+n)T} \tau_{r,i_3}^c \dot{\theta}_{r,i_3} dt, \end{aligned} \quad (20)$$

where $\bar{P}_{h,i_3}^a$ is the time-averaged active power in human hip ($i_3 = h$) and knee ($i_3 = k$), and $\bar{P}_{r,i_3}^c$ is the time-averaged controlling power in robot's hip ($i_3 = h$) and knee ($i_3 = k$), and m and n are selected positive integers.

3. Model validation

3.1. Experiment setups

To validate the model proposed above, as displayed in Fig. 3(a), we built an experimental lower-limb exoskeleton, which has 2 DoF with its hip fixed on a rigid frame. The main structure was made of steel,

with its hip and knee actuated by two servo motors (GDM1-100N2/120N2). While the human-exoskeleton interactive forces measured by four 3D force sensors (JNSH-2-10kg-BSQ-12), the robot's joint angles are measured by two absolute encoders, INC-4-150 and INC-3-125.

Parameter values and the joint friction of the exoskeleton, Eqs (2) and (19), were identified by neighbourhood field optimization [41, 42], yielding the robot's inertia matrix, Coriolis and centripetal matrix, gravitational vector, and frictional vector as follows

$$\begin{aligned}
\mathbf{M}_r(\boldsymbol{\theta}_r) &= \begin{pmatrix} 25.68 + 2.65 \cos(\theta_{r,k}) & 6.91 + 1.32 \cos(\theta_{r,k}) \\ 6.91 + 1.32 \cos(\theta_{r,k}) & 6.91 \end{pmatrix}, \\
\mathbf{C}_r(\boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r) &= 1.32 \sin(\theta_{r,k}) \begin{pmatrix} -\dot{\theta}_{r,k} & -\dot{\theta}_{r,k} - \dot{\theta}_{r,h} \\ -\dot{\theta}_{r,h} & 0 \end{pmatrix}, \\
\mathbf{G}_r(\boldsymbol{\theta}_r) &= g \begin{pmatrix} 78.39 \sin(\theta_{r,h}) + 33.32 \sin(\theta_{r,h} + \theta_{r,k}) \\ 33.32 \sin(\theta_{r,h} + \theta_{r,k}) \end{pmatrix}, \\
\boldsymbol{\tau}_{r,f} &= \begin{pmatrix} -31.38 \text{sgn}(\dot{\theta}_{r,h}) - 15.55 \dot{\theta}_{r,h} \\ -43.67 \text{sgn}(\dot{\theta}_{r,k}) - 27.51 \dot{\theta}_{r,k} \end{pmatrix}.
\end{aligned} \tag{21}$$

Besides the robot, the wearer has a mass of $M = 66$ [kg] and a height of $H = 1.78$ [m], which are used to estimate the wearer's average anthropometric data by using the following relationships [43]

$$\begin{aligned}
l_{h,t} &= 0.245H, & c_{h,t} &= 0.433l_{h,t}, & m_{h,t} &= 0.1M, & I_{h,t} &= 6.23 \times 10^{-4}MH^2, \\
l_{h,s} &= 0.285H, & c_{h,s} &= 0.606l_{h,s}, & m_{h,s} &= 0.061M, & I_{h,s} &= 8.57 \times 10^{-4}MH^2.
\end{aligned} \tag{22}$$

To identify the human-exoskeleton coupling stiffness and damping, we designed a platform shown in Fig. 3(b), where two laser displacement sensors (ILD1420-200) and one 3D force sensor (LH-SZ-02-100N) are used to measure human motion relative to the rigid frame and the corresponding interactive force when human lower limb is tied to the platform. The laser sensors are kept a distance to the band so that deformation in the coupling area does not affect the measurement, and the relative displacement at the coupling points is indirectly estimated by

$$D = \frac{d_a D_b + d_b D_a}{d_a + d_b}, \tag{23}$$

where D_a and d_a (D_b and d_b) are the horizontal displacement measured by the sensor above (below) the coupling point and its vertical distance to the coupling point. If zero interactive force, $F_x = 0$, shows up for $D = D_0$ and $\dot{D} = 0$, then the force can be represented as

$$F_x = F_0 + k_{i_2} D + b_{i_2} \dot{D}, \tag{24}$$

where $F_0 = -k_{i_2} D_0$, and k_{i_2} and b_{i_2} are the identified stiffness and damping at the thigh ($i_2 = t$) and shank ($i_2 = s$), respectively. With 10 groups of identification performed with this platform, the medians of identified stiffness and damping displayed in Fig. 3(c) yield the interactive forces consistent with the

Figure 3: (a) Experimental apparatus of the low-limb exoskeleton, with the robot's hip joints actuated by two servo motors and measured by two absolute encoders, and the interactive forces measured by four force sensors. (b) Platform for the identification of human-exoskeleton coupling parameters, with relative displacement measured by two laser sensor, and interactive force measured by a force sensor. (c) Coupling parameters identified by the platform, and comparison between measured and fitted forces.

measured data. Thus, the following medians corresponding to Fig. 3(c) will be used for further analysis

$$\begin{aligned} k_t &= 3.283 \times 10^3 \text{ [N m}^{-1}\text{]}, & b_t &= 0.367 \times 10^3 \text{ [N s m}^{-1}\text{]}, \\ k_s &= 5.645 \times 10^3 \text{ [N m}^{-1}\text{]}, & b_s &= 0.418 \times 10^3 \text{ [N s m}^{-1}\text{]}. \end{aligned} \quad (25)$$

With the parameter values listed in Eqs (21), (22) and (25), Eq. (1) will be solved by forth-order Runge-Kutta method for simulation result. Meanwhile, experiments will be performed with the platform shown in Fig. 3(a), and the robot's joint angles and interactive forces measured by the encoders and force sensors will be used to validate the analytical model.

3.2. Results comparison

As an example, both of the simulation and experiment results for $a = 0$ [mm], $A = 20^\circ$, $\omega = \frac{\pi}{2}$ [rad s⁻¹], $k_p = 1000$ [N m rad⁻¹] and $k_d = 100$ [N m s rad⁻¹] are obtained and displayed in Fig. 4, showing the magnitude of interactive forces at the thigh and shank, and the robot's joint angles in 5 swing periods from 4 [s] to 24 [s]. The left (right) two columns are for the PD control without (with) gravity compensation, while the first and third (second and forth) columns are for a paralytic (healthy) wearer. In the experiment, the paralysis was imitated by the healthy wearer consciously deactivating his muscle contraction. The first two rows, Figs 4(a-h), display the interactive forces, showing that the analytical model yield consistent results compared with the experiment. It is also shown that recovery of human health condition, h_c , significantly decreases the interactive forces both at the thigh and shank. For instance, $|\mathbf{F}_t|$ in Fig. 4(a) oscillating between 28 [N] and 48 [N] declines to the fluctuation between 8 [N] and 30 [N] in Fig. 4(b). Similar phenomenon is also found in $|\mathbf{F}_s|$, no matter the gravity is compensated

Figure 4: Time series of the magnitudes of interactive forces in the (a-d) thigh and (e-h) shank, and robot's (i-l) hip and (m-p) knee angles. Corresponding with PD control without (with) gravity compensation, panels in the first and second columns in the left (right) part are for $h_c = 0\%$ and 100% , respectively.

Figure 5: Maximum magnitudes of the interactive forces in the (a-d) thigh and (e-h) shank, and maximum tracking errors of the robot's (i-l) hip and (m-p) knee, are plotted as functions of the proportional feedback gain, k_p . Corresponding with PD control without (with) gravity compensation, panels in the first and second columns in the left (right) part are for $h_c = 0\%$ and 100% , respectively.

in feedback control or not.

Although the gravity compensation does not affect the magnitude of interactive forces, it significantly improves the tracking accuracy of the robot joints, especially in the robot hip shown in Figs 4(i-l). Compared with the simulation result, as seen in Fig. 4(i), there is a large deviation between the experiment result and the desired trajectory, which could be induced by uncertainties of the platform, such as external disturbance, errors in the parameter identification or the anthropometric data. Nonetheless, as seen in Figs 4(j-l), the tracking errors are extensively shrunk by either actuating the human muscle or compensating the robot gravity. By contrast, the robot's knee in Figs 4(m-p) shows a better consistence between the simulation and experiment results, especially when the gravity compensation is involved.

Then the influence of feedback gain on the human-exoskeleton coupling dynamics is studied, with the maximum magnitude of interactive force and maximum absolute tracking error plotted as functions of k_p in Fig. 5. By comparing the first and third (second and forth) columns, one can see that the gravity compensation seldom affects the coupling forces, while it indeed decreases the tracking errors obtained in the experiment. By contrast, comparison between the first and second (third and forth) columns shows

that the recovery of human health extensively decreases the coupling forces and simultaneously improves the tracking accuracy.

From Figs 5(a-d), one can observe a slight increase in the coupling force at the thigh with respect to the enhancement of the proportional gain from 400 [N m rad⁻¹] to 2000 [N m rad⁻¹], no matter of the health condition or the gravity compensation. For a paralysed wearer, as seen in Figs 5(a) and (c), $|\mathbf{F}_t|$ varies between 45 [N] and 50 [N], which is significantly decreased to the variation between 30 [N] and 35 [N] for a healthy wearer in Figs 5(b) and (d), i.e., a paralysed wearer suffers around 15 [N] extra interactive force at the thigh compared with a healthy person. A similar trend is seen in Figs 5(e-h) for the coupling force at the shank, but it seems that $|\mathbf{F}_s|$ is insensitive to the increase of k_p . However, $|\mathbf{F}_s|$ is very sensitive to human health condition, where the drop in the force is up to around 35 [N] for a healthy wearer compared with a paralysed patient. From Figs 5(i-p), the simulation results show that the health condition and gravity compensation seldom affect the tracking errors. However, the experiment reveals that both of them help the exoskeleton to improve its tracking accuracy, especially in robot's hip. By contrast, both of the analytical and experimental results unveil that enhancement of the proportional gain can improve the tracking accuracy, but this trend becomes more and more marginal.

4. Simulation analysis

Since the mathematical model yields consistent results compared with the experiment, especially in the prediction of coupling forces which are very critical for human comfort, the following analysis will be based on the model in Eq. (1), control with gravity compensation, and the parameter values in Eqs (21), (22) and (25). Moreover, except the given parameter values, other parameters will adopt the values used in Section 3.2. Namely, the default values are $a = 0$ [mm], $\beta = 0$ [rad], $A = 20^\circ$, $\omega = \frac{\pi}{2}$ [rad s⁻¹], $k_p = 1000$ [N m rad⁻¹], $k_d = 100$ [N m s rad⁻¹], and $h_c = 100\%$.

4.1. Influence of trajectory and mass ratio

Human joints have limited rotational ranges. For example, the hip has a maximum flexion angle of 90° when the knee is extended, and a maximum extension angle of 10° when the knee is flexed [44]. Given the anthropometry, the desired trajectory should have constrained swing amplitude, $A \leq 40^\circ$, for human safety. The walk-to-run transition stride frequency was estimated as 70.8 strides per minutes [45], so that the swing frequency is chosen as 0.2π [rad s⁻¹] $\leq \omega \leq 2.36\pi$ [rad s⁻¹] to match the walking motion of average people. It would be cumbersome for a heavy robot to track a trajectory with large amplitude or high frequency, and a light-weight design is very crucial to improve its performance [23]. The main structure of the exoskeleton in Fig. 3 is made of steel, which is very heavy to operate. If it can be replaced by light-weight materials, such as aluminium alloy or carbon fibre reinforced composite, its weight can be halved or even quartered. Therefore, the mass ratio is selected for $r_m \in [25\%, 100\%]$. Correspondingly, the matrices and vector in Eqs (2) and (21), $\mathbf{M}_r(\boldsymbol{\theta}_r)$, $\mathbf{C}_r(\boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r)$ and $\mathbf{G}_r(\boldsymbol{\theta}_r)$, will be replaced by $r_m \mathbf{M}_r(\boldsymbol{\theta}_r)$, $r_m \mathbf{C}_r(\boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r)$ and $r_m \mathbf{G}_r(\boldsymbol{\theta}_r)$ for various light-weight structures.

Influence of the swing amplitude and frequency, and mass ratio on human and robot's tracking errors are illustrated in Fig. 6. In general, increase of either the swing amplitude or the mass ratio results in growth of tracking errors in all of the human and robot's joints. In the right-most column of Fig. 6, the errors undergo a growth with respect to the increase of swing frequency, but they all reach their maxima before ω reaches 2π [rad s⁻¹]. The errors in human and robot's hips are especially evident, reaching

Figure 6: Tracking errors in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the swing frequency, ω . From the left panels to the right, the mass ratio r_m increases from 25% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, and thick dashed blue lines are for $A = 10^\circ, 20^\circ, 30^\circ$ and 40° , respectively.

almost 1 [rad] for $\omega = 2\pi$ [rad s⁻¹]. From the right-most column to the left, decrease of the mass ratio postpones the appearance of maxima, and the largest error is roughly 0.25 [rad] for $r_m = 25\%$, which is a quarter of the one for $r_m = 100\%$. Namely, a light-weight structure helps the human-exoskeleton system to improve its tracking accuracy during walking.

Moreover, the light-weight design is also very critical for the human-exoskeleton interactive forces besides the tracking accuracy. As seen in Figs 7(a-b), a light-weight structure, $r_m \leq 50\%$, results in insensitivity of the interactive force at the thigh to the swing frequency, which continuously increases with respect to the swing amplitude. With respect to the increase of the mass ratio in Figs 7(a-d), the thick dashed blue line tilts up for a large frequency, indicating that simultaneously increasing the weight, swing amplitude and frequency will induce a very large interactive force at the thigh. Moreover, as seen in Figs 7(e-h), this problem is even severer in the interaction at shank. For a slow swing, $\omega < \pi$ [rad s⁻¹], a small steady interactive force, $|\mathbf{F}_s| < 30$ [N], is observed, no matter of the mass ratio or the swing amplitude. For a fast swing, $\omega > \pi$ [rad s⁻¹], the interactive force increases almost linearly with respect to the increase of the frequency, and the slope becomes steeper for heavier weight or larger amplitude. In the most severer case in Fig. 7(h), $|\mathbf{F}_s|$ can reach 400 [N], which results in extremely uncomfortable interaction.

Figure 7: Magnitude of the interactive forces in the (a-d) hip and (e-h) knee are plotted as functions of the swing frequency, ω . From the left panels to the right, the mass ratio r_m increases from 25% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, and thick dashed blue lines are for $A = 10^\circ$, 20° , 30° and 40° , respectively.

The corresponding average active joint power is displayed in Fig. 8, showing a general trend that a larger swing amplitude results in larger joint power. For a slow swing, $\omega < \pi$ [rad s⁻¹], human hip provides positive power, while negative power is found in human knee. On the contrary, human hip (knee) consumes (supplies) energy in case of fast swing, $\omega > \pi$ [rad s⁻¹]. With respect to the increase of swing frequency, power supplied by human knee and robot's hip keeps growing, while human hip consumes more and more energy. In robot's knee, as shown in Figs 8(m-n), a uniform increase of the power with respect to the increase of swing frequency is also observed for small mass ratios ($r_m \leq 50\%$), but maxima appear for large mass ratios in Figs 8(o-p). It worth noting that the average power of the robot joints can be as large as 600 [W] for swing with large amplitude and frequency, which is very demanding for commercial motors, and hampers the minimization and light-weight design of the exoskeleton. Especial for the robot knee which is at the distal end of the thigh, increasing its weight for larger power supply introduces extra burden to the swing of lower limbs.

4.2. Influence of misalignment and coupling points

Misalignment is also very crucial for the human-exoskeleton coupling dynamics, where dislocation of the joint can cause discomfort, pain or even long-term injury [46]. As seen in Fig. 9, the tracking errors start to fluctuate with respect to the change of misalignment angle, β , in case of the presence of misalignment, $a > 0$ [mm]. Except the error in robot's hip shown in Figs 9(i-l) which is not significantly influenced by the misalignment, the fluctuation amplitude is evidently enlarged by the increase of deviation between human and robot's hips. Taking human hip displayed in Fig. 9(a) as an example, a constant absolute tracking error of $|e_{h,h}| = 0.096$ [rad] is changed into the fluctuation with a maximum of 0.275 [rad] by a misalignment of $a = 20$ [mm] and $\beta = \pi/2$ [rad]. From the left-most column to the right-most in Fig. 9, the coupling point at the thigh is moving from the thigh's proximal end to the distal end, which extensively shrinks the fluctuation amplitude of the errors in human hip and knee, and robot's knee. However, the effect of moving the coupling point becomes less and less evident for $s_{r,t} > 0.15$ [m]. By contrast, as seen in Figs 10(a-d) and (i-p), moving the coupling point at shank does not have a significant

Figure 8: Average power in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the swing frequency, ω . From the left panels to the right, the mass ratio r_m increases from 25% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, and thick dashed blue lines are for $A = 10^\circ, 20^\circ, 30^\circ$ and 40° , respectively.

Figure 9: Tracking errors in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,t}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

influence on the tracking errors in human hip, and robot's hip and knee, but Figs 10(e-h) show an evident decrease in the tracking error in human knee when $s_{r,s}$ increases from 0.05 [m] to 0.15 [m]. Simply put, the misalignment has a crucial influence on the tracking accuracy, but this problem can be alleviated by properly moving the coupling points away from the proximal end of each segment.

The misalignment is even more crucial for the coupling forces. As shown in Fig. 11(a), the interactive force at the thigh is only 11 [N] in case that human and robot's hips are aligned, while it jumps to 130 [N] for $a = 20$ [mm] and $\beta = 0$ [rad]. At the shank displayed in Fig. 11(e), $|\mathbf{F}_s|$ even reaches 500 [N] for $a = 20$ [mm] and $\beta = \pi/2$ [rad]. Unlike the problem of tracking errors discussed above, the uncomfortable interaction at the thigh cannot be alleviated by moving the coupling points. As displayed in Figs 11(a-d) and 12(a-d), $|\mathbf{F}_t|$ cannot be evidently decreased by increasing either $s_{r,t}$ or $s_{r,s}$. On the contrary, Figs 11(a-d) show a continuous increase of $|\mathbf{F}_t|$ with respect to $s_{r,t}$ in case of no misalignment. By contrast, as seen in Figs 11(e-h) and 12(e-h), moving the coupling points improves the comfort at the shank, where the maximum force of 500 [N] is decreased to roughly 200 [N] when $s_{r,t}$ increases from 0.05 [m] to 0.15 [m]. However, the improvement becomes very marginal for $s_{r,t} > 0.15$ [m] ($s_{r,s} > 0.15$ [m]).

In the aspect of joint power, Figs 13 and 14 show insignificant variation with respect to the change of misalignment and coupling points. The largest fluctuation appears in Fig. 13(m) for $a = 20$ [mm],

Figure 10: Tracking errors in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,s}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

Figure 11: Magnitude of the interactive forces in the (a-d) hip and (e-h) knee are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,t}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

Figure 12: Magnitude of the interactive forces in the (a-d) hip and (e-h) knee are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,s}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

Figure 13: Average power in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,t}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

Figure 14: Average power in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the angle of alignment, β . From the left panels to the right, the coupling point in the robot's thigh, $s_{r,s}$, increases from 0.05 [m] to 0.35 [m]. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $a = 0$ [mm], 5 [mm], 10 [mm], 15 [mm] and 20 [mm], respectively.

showing the robot knee power varying between 15 [W] and 17 [W]. Variation the coupling points has a mild influence on the joint power, but the fluctuation amplitude is indeed suppressed by moving the coupling points away from the proximal end of each segment. In general, the robot's joints and human hip provide positive energy to the swing motion, while the human knee consumes the energy with small negative power.

4.3. Influence of health condition and band tightness

Health condition is an index of maximum muscle contraction the wearer can provide to actuate his joint. For $h_c = 0\%$, the wearer has lower extremity paralysis and is incapable to swing his leg independently, so that he can only be passively driven by the interaction with exoskeleton. In that case, a tight binding is normally required. For example, the tracking errors in human hip and knee shown in Figs 15(a) and (e) are extremely large for $h_c < 20\%$, but increasing r_t from 10% to 100% for a tight binding at the thigh, as seen in Figs 15(a-h), can extensively suppress the errors. By contrast, the human health condition and band tightness have insignificant influence on the robot's tracking accuracy, which is already very small. This reveals the importance of applying the feedback control in the robot's joints. Another interesting phenomenon unveiled in Fig. 15 is that the tracking errors in human joints and robot knee are always increased by slight recovery of human health for $h_c < 5\%$, which is then reduced to level off before h_c reaches 20%. When the binding at the thigh is loose, as seen in Figs 15(a-b) and (e-f), a tighter binding at the shank will result in even larger tracking errors in human joints. On the contrary, the right two columns of Fig. 15 illustrate that a tight binding at the thigh makes the tightness at the shank insignificant, and all the tracking errors minimized.

For a wearer with $h_c > 20\%$, as displayed in Figs 16(a-d), the interactive force at the thigh continuously grows with respect to the increase of r_t for a tight coupling at the thigh. For an unhealthy wearer with $h_c \approx 5\%$, the maximum $|\mathbf{F}_t|$ shows up for $r_t = 40\%$ and $r_s = 100\%$ in Fig. 16(b), which reaches about 120 [N]. For a paralysed wearer ($h_c = 0\%$), Figs 16(a-d) indicate that $|\mathbf{F}_t|$ is enlarged by either increasing r_t or decreasing r_s . The interactive force at the shank, as seen in Figs 16(e-f), also achieves its maximum for $h_c \approx 5\%$, which can be as large as 200-300[N], when the tightness is large at the shank ($r_s = 70\%$ and 100%) but small at the thigh ($r_t = 10\%$ and 40%). For a wearer with $h_c > 20\%$, the interactive force at the shank stays at a very low level ($|\mathbf{F}_s| < 18$ [N]). For a paralysed wearer ($h_c = 0\%$), $|\mathbf{F}_s|$ is strengthened by raising the tightness at the shank, while r_t has no apparent influence on $|\mathbf{F}_s|$.

The joint power displayed in Fig. 17 shows the general trends that human hip and robot's knee provide more energy, robot's hip supplies less energy, and human knee consumes more energy when the wearer gradually recovers his health. In the right two columns of Fig. 17, influence of r_s on the joint power becomes insignificant in case r_t is large (70%-100%). By contrast, the left two columns of Fig. 17 show significant fluctuations in the joint power for $r_t \leq 40\%$ and $h_c < 20\%$. Moreover, the fluctuation amplitude is further enhanced by the increase of r_s . However, the joint power is much less significant than the tracking errors or interactive forces discussed above, since the largest change observed in Fig. 17(e) is no more than 4 [W].

Figure 15: Tracking errors in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the human health condition, h_c . From the left panels to the right, the coupling strength at the thigh, r_t , increases from 10% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $r_s = 10\%$, 40% [mm], 70%, and 100% [mm], respectively.

Figure 16: Magnitude of the interactive forces in the (a-d) hip and (e-h) knee are plotted as functions of the human health condition, h_c . From the left panels to the right, the coupling strength at the thigh, r_t , increases from 10% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $r_s = 10\%$, 40% [mm], 70%, and 100% [mm], respectively.

Figure 17: Average power in (a-d) human hip, (e-h) human knee, (i-l) robot hip and (m-p) robot knee, are plotted as functions of the human health condition, h_c . From the left panels to the right, the coupling strength at the thigh, r_t , increases from 10% to 100%. In each panel, the thin solid grey, thin dashed red, thick solid green, thick dashed blue lines and the purple dots are for $r_s = 10\%$, 40% [mm], 70% , and 100% [mm], respectively.

5. Conclusions

To study the human-exoskeleton coupling dynamics during the swing motion of lower limbs, this paper proposed a 4 DoF model involving the rotational angles of human and robot's hips and knees. The exoskeleton bound to the wearer at its thigh and shank, where the interactions were modelled as a linear damped springs to represent the elastic and viscous properties of the band and human tissue. In robot's joints, the exoskeleton was driven by a PD control, while its motion was hampered by the joint friction. Based on inverse dynamics analysis and given human health condition, saturation of human active torques was employed to represent insufficient muscle contraction of an unhealthy wearer.

With the coupling coefficients identified by a platform, this analytical model was then validated by comparison with an experimental exoskeleton. The analytical method yielded consistent prediction of the human-exoskeleton coupling dynamics compared with the experimental results, no matter how the health condition, gravity compensation, or the proportional gain were varied. It was found in the experiment that gravity compensation reduced the maximum absolute tracking errors in robot's joints, especial in its hip, but it had no significant influence on the magnitude of interactive forces. For a paralysed wearer without gravity compensation in the control, the maximum tracking errors in robot's hip reduced for about 0.3 [rad] (0.1 [rad]) with respect to the enhancement of proportional feedback gain from $k_p = 400$ [N m rad⁻¹] to 2000 [N m rad⁻¹] according to experimental (analytical) results. This error was further reduced by about 0.1 [rad] or 0.2 [rad] by the introduction of human actuation or gravity compensation. On the other hand, only a small increase of the interactive forces around 50 [N] at thigh and shank was observed, no matter the robot's gravity was compensated in the control or not. However, the recovery of human health reduced the coupling forces for about 15 [N] and 35 [N] at thigh and shank for more comfortable human-exoskeleton interaction.

Then more detailed analysis was performed with this validated analytical model. It was found that a light-weight design of the structure helps to reduce the tracking errors, interactive forces and joint power, especially when the desired trajectory has a large magnitude and frequency. For example, the tracking errors in human and robot's hips, the interactive force at shank, and the controlling power in robot's hip can be as large as 1 [rad], 400 [N] and 600 [W] for a swing motion of $A = 40^\circ$ and $\omega = 2\pi$ [rad s⁻¹], while these indices can be halved or even quartered by a light-weight exoskeleton with a mass ratio of $r_m = 25\%$. We also found that misalignment between human and robot's hips causes server problems in tracking errors and interactive forces, of which the errors in all joints and force at the shank can be reduced by moving the coupling points at the thigh and shank away from the segment's proximal end. However, moving the coupling point has an insignificant influence on the interactive force at the thigh. Moreover, the interactive forces can be as large as 150 [N] and 500 [N] at the thigh and shank for the wearer to bear in case of a large misalignment, $a = 20$ [mm], which should be carefully compensated in the exoskeleton design for the wearer's comfort. Given human health condition, it was found that the coupling tightness at the thigh and shank is not important once the wearer is healthy enough ($h_c > 20\%$). However, it could be very dangerous for a wearer who slight recovered ($h_c \approx 5\%$), because insufficient muscle contraction could result in very large tracking errors and interactive forces. In that case, a tight coupling at thigh is very critical to reduce the tracking errors and coupling forces.

Acknowledgements

This research is supported by National Natural Science Foundation of China (Grants No. 12072068, 11932015, 51775089, and 11872147), and Sichuan Science and Technology Program (Grant No. 2020YFG0137). The authors thank Mr. Gan Liu and Mr. Li Chen for operating the exoskeleton and data collection.

Compliance with ethical standards.

Conflict of interest. The authors declare that they have no conflict of interest concerning the publication of this manuscript.

Data accessibility. The numerical and experimental data sets generated and analysed during the current study are available from the corresponding author on reasonable request.

References

- [1] W. He, D. Goodkind, P. Kowal, An Aging World: 2015 International Population Reports, U.S. Census Bureau, Washington, DC, 2016.
- [2] WHO, World report on disability, World Health Organization, Geneva, 2011.
- [3] W. A. Satariano, J. M. Guralnik, R. J. Jackson, R. A. Marottoli, E. A. Phelan, T. R. Prohaska, Mobility and aging: new directions for public health action, *American Journal of Public Health* 102 (8) (2012) 1508–1515. doi:10.2105/AJPH.2011.300631.
- [4] N. Aliman, R. Ramli, S. M. Haris, Design and development of lower limb exoskeletons: a survey, *Robotics and Autonomous Systems* 95 (2017) 102–116. doi:https://doi.org/10.1016/j.robot.2017.05.013.
- [5] G. Bao, L. Pan, H. Fang, X. Wu, H. Yu, S. Cai, B. Yu, Y. Wan, Academic review and perspectives on robotic exoskeletons, *IEEE Transactions on Neural Systems and Rehabilitation Engineering* 27 (11) (2019) 2294–2304. doi:10.1109/TNSRE.2019.2944655.
- [6] T. Yan, M. Cempini, C. M. Oddo, N. Vitiello, Review of assistive strategies in powered lower-limb orthoses and exoskeletons, *Robotics and Autonomous Systems* 64 (0) (2015) 120–136. doi:10.1016/j.robot.2014.09.032.
- [7] A. B. Zoss, H. Kazerooni, A. Chu, Biomechanical design of the Berkeley lower extremity exoskeleton (BLEEX), *IEEE/ASME Transactions on Mechatronics* 11 (2) (2006) 128–138. doi:10.1109/TMECH.2006.871087.
- [8] K. N. Gregorczyk, L. Hasselquist, J. M. Schiffman, C. K. Bense, J. P. Obusek, D. J. Gutekunst, Effects of a lower-body exoskeleton device on metabolic cost and gait biomechanics during load carriage, *Ergonomics* 53 (10) (2010) 1263–1275. doi:10.1080/00140139.2010.512982.
- [9] B. Koopman, E. H. F. Van Asseldonk, H. Van Der Kooij, Estimation of human hip and knee multi-joint dynamics using the LOPES gait trainer, *IEEE Transactions on Robotics* 32 (4) (2016) 920–932. doi:10.1109/TRO.2016.2572695.

- [10] R. Hidayah, L. Bishop, X. Jin, S. Chamrathy, J. Stein, S. K. Agrawal, Gait adaptation using a cable-driven active leg exoskeleton (c-alex) with post-stroke participants, *IEEE Transactions on Neural Systems and Rehabilitation Engineering* 28 (9) (2020) 1984–1993. doi:10.1109/TNSRE.2020.3009317.
- [11] P. Asselin, C. M. Cirnigliaro, S. Kornfeld, S. Knezevic, R. Lackow, M. Elliott, W. A. Bauman, A. M. Spungen, Effect of exoskeletal-assisted walking on soft tissue body composition in persons with spinal cord injury, *Archives of Physical Medicine and Rehabilitation* 102 (2) (2021) 196–202. doi:https://doi.org/10.1016/j.apmr.2020.07.018.
- [12] K. Miura, M. Koda, K. Tamaki, M. Ishida, A. Marushima, T. Funayama, H. Takahashi, H. Noguchi, K. Mataka, Y. Yasunaga, H. Kawamoto, Y. Sankai, A. Matsumura, M. Yamazaki, Exercise training using hybrid assistive limb (HAL) lumbar type for locomotive syndrome: a pilot study, *BMC Musculoskeletal Disorders* 22 (1) (2021) 533. doi:10.1186/s12891-021-04421-3.
- [13] S. Oh, E. Baek, S.-k. Song, S. Mohammed, D. Jeon, K. Kong, A generalized control framework of assistive controllers and its application to lower limb exoskeletons, *Robotics and Autonomous Systems* 73 (2015) 68–77. doi:10.1016/j.robot.2014.10.001.
- [14] J. Cao, S. Q. Xie, R. Das, MIMO sliding mode controller for gait exoskeleton driven by pneumatic muscles, *IEEE Transactions on Control Systems Technology* 26 (1) (2018) 274–281. doi:10.1109/TCST.2017.2654424.
- [15] W. Meng, Q. Liu, Z. Zhou, Q. Ai, B. Sheng, S. S. Xie, Recent development of mechanisms and control strategies for robot-assisted lower limb rehabilitation, *Mechatronics* 31 (2015) 132–145. doi:10.1016/j.mechatronics.2015.04.005.
- [16] M. S. Amiri, R. Ramli, M. F. Ibrahim, Hybrid design of PID controller for four dof lower limb exoskeleton, *Applied Mathematical Modelling* 72 (2019) 17–27. doi:10.1016/j.apm.2019.03.002.
- [17] J. Zhang, S. Zhang, J. Xu, Human-machine coupled dynamic modelling and control of lower limb exoskeleton for rehabilitation, *Journal of Dynamics and Control* 19 (04) (2021) 55–63. doi:10.6052/1672-6553-2020-091.
- [18] H. Jabbari Asl, T. Narikiyo, M. Kawanishi, Adaptive neural network-based saturated control of robotic exoskeletons, *Nonlinear Dynamics* 94 (1) (2018) 123–139. doi:10.1007/s11071-018-4348-1.
- [19] D. Wang, J. Wu, L. Deng, Z. Li, Y. Wang, A real-time optimization control method for coagulation process during drinking water treatment, *Nonlinear Dynamics* 105 (4) (2021) 3271–3283. doi:10.1007/s11071-021-06794-5.
- [20] S. H. Collins, M. B. Wiggin, G. S. Sawicki, Reducing the energy cost of human walking using an unpowered exoskeleton, *Nature* 522 (2015) 212–215.
- [21] J. Zhang, P. Fiers, K. A. Witte, R. W. Jackson, K. L. Poggensee, C. G. Atkeson, S. H. Collins, Human-in-the-loop optimization of exoskeleton assistance during walking, *Science* 356 (6344) (2017) 1280. doi:10.1126/science.aal5054.

- [22] M. M. Alemi, J. Geissinger, A. A. Simon, S. E. Chang, A. T. Asbeck, A passive exoskeleton reduces peak and mean emg during symmetric and asymmetric lifting, *Journal of Electromyography and Kinesiology* 47 (2019) 25–34. doi:<https://doi.org/10.1016/j.jelekin.2019.05.003>.
- [23] A. J. Young, D. P. Ferris, State of the art and future directions for lower limb robotic exoskeletons, *IEEE Transactions on Neural Systems and Rehabilitation Engineering* 25 (2) (2017) 171–182. doi:[10.1109/TNSRE.2016.2521160](https://doi.org/10.1109/TNSRE.2016.2521160).
- [24] M. K. Duong, H. Cheng, T. T. Huu, J. Qiu, Minimizing human-exoskeleton interaction force by using global fast sliding mode control, *International Journal of Control, Automation and Systems* 14 (4) (2016) 1064–1073. doi:[10.1007/s12555-014-0395-7](https://doi.org/10.1007/s12555-014-0395-7).
- [25] S. M. M. De Rossi, N. Vitiello, T. Lenzi, R. Ronsse, B. Koopman, A. Persichetti, F. Vecchi, A. J. Ijspeert, H. Van der Kooij, M. C. Carrozza, Sensing pressure distribution on a lower-limb exoskeleton physical human-machine interface, *Sensors* 11 (1) (2011). doi:[10.3390/s110100207](https://doi.org/10.3390/s110100207).
- [26] C. Zhang, X. Zang, Z. Leng, H. Yu, J. Zhao, Y. Zhu, Human-machine force interaction design and control for the hit load-carrying exoskeleton, *Advances in Mechanical Engineering* 8 (4) (2016) 1687814016645068. doi:[10.1177/1687814016645068](https://doi.org/10.1177/1687814016645068).
- [27] W. Huo, S. Mohammed, Y. Amirat, Impedance reduction control of a knee joint human-exoskeleton system, *IEEE Transactions on Control Systems Technology* 27 (6) (2019) 2541–2556. doi:[10.1109/TCST.2018.2865768](https://doi.org/10.1109/TCST.2018.2865768).
- [28] Z. Li, B. Huang, Z. Ye, M. Deng, C. Yang, Physical human-robot interaction of a robotic exoskeleton by admittance control, *IEEE Transactions on Industrial Electronics* 65 (12) (2018) 9614–9624. doi:[10.1109/TIE.2018.2821649](https://doi.org/10.1109/TIE.2018.2821649).
- [29] R. Huang, H. Cheng, J. Qiu, J. Zhang, Learning physical human-robot interaction with coupled cooperative primitives for a lower exoskeleton, *IEEE Transactions on Automation Science and Engineering* 16 (4) (2019) 1566–1574. doi:[10.1109/TASE.2018.2886376](https://doi.org/10.1109/TASE.2018.2886376).
- [30] M. Mokhtari, M. Taghizadeh, M. Mazare, Impedance control based on optimal adaptive high order super twisting sliding mode for a 7-DoF lower limb exoskeleton, *Meccanica* 56 (3) (2021) 535–548. doi:[10.1007/s11012-021-01308-4](https://doi.org/10.1007/s11012-021-01308-4).
- [31] Q. Xie, Q. Meng, Q. Zeng, Y. Fan, Y. Dai, H. Yu, Human-exoskeleton coupling dynamics of a multi-mode therapeutic exoskeleton for upper limb rehabilitation training, *IEEE Access* 9 (2021) 61998–62007. doi:[10.1109/ACCESS.2021.3072781](https://doi.org/10.1109/ACCESS.2021.3072781).
- [32] Z. Li, C. Deng, K. Zhao, Human-cooperative control of a wearable walking exoskeleton for enhancing climbing stair activities, *IEEE Transactions on Industrial Electronics* 67 (4) (2020) 3086–3095. doi:[10.1109/TIE.2019.2914573](https://doi.org/10.1109/TIE.2019.2914573).
- [33] Y. Zhu, J. Cui, J. Zhao, Biomimetic design and biomechanical simulation of a 15-dof lower extremity exoskeleton, in: 2013 IEEE International Conference on Robotics and Biomimetics (ROBIO), 2013 IEEE International Conference on Robotics and Biomimetics (ROBIO), 2013, pp. 1119–1124.

- [34] D. Shi, W. Zhang, W. Zhang, L. Ju, X. Ding, Human-centred adaptive control of lower limb rehabilitation robot based on human-robot interaction dynamic model, *Mechanism and Machine Theory* 162 (2021) 104340. doi:10.1016/j.mechmachtheory.2021.104340.
- [35] J. S. Stoltze, J. Pallari, B. Eskandari, A. S. C. Oliveira, C. I. Pircoveanu, J. Rasmussen, M. S. Andersen, Development and functional testing of an unloading concept for knee osteoarthritis patients: a pilot study, *Journal of Biomechanical Engineering* 144 (1) (2021). doi:10.1115/1.4051847.
- [36] G. Serrancolí, A. Falisse, C. Dembia, J. Vantilt, K. Tanghe, D. Lefeber, I. Jonkers, J. De Schutter, F. De Groote, Subject-exoskeleton contact model calibration leads to accurate interaction force predictions, *IEEE Transactions on Neural Systems and Rehabilitation Engineering* 27 (8) (2019) 1597–1605. doi:10.1109/TNSRE.2019.2924536.
- [37] K. M. Lynch, F. C. Park, *Modern Robotics: Mechanics, Planning, and Control*, Cambridge University Press, Cambridge, 2017.
- [38] R. Pámies-Vilà, O. Pătkau, A. Dória-Cerezo, J. M. Font-Llagunes, Influence of the controller design on the accuracy of a forward dynamic simulation of human gait, *Mechanism and Machine Theory* 107 (2017) 123–138. doi:10.1016/j.mechmachtheory.2016.09.002.
- [39] J. C. Dean, A. D. Kuo, N. B. Alexander, Age-related changes in maximal hip strength and movement speed, *The Journals of Gerontology: Series A* 59 (3) (2004) 286–292. doi:10.1093/gerona/59.3.M286.
- [40] S. Laforest, D. M. M. St-Pierre, J. Cyr, D. Gayton, Effects of age and regular exercise on muscle strength and endurance, *European Journal of Applied Physiology and Occupational Physiology* 60 (2) (1990) 104–111. doi:10.1007/BF00846029.
- [41] Z. Chen, H. Xiong, X. Wang, Q. Guo, Y. Shi, Y. Yan, G. Liu, D. Jiang, Model identification of 2-DoF lower limb exoskeleton with neighborhood field optimization, *IFAC-PapersOnLine* 53 (2) (2020) 8704–8709. doi:10.1016/j.ifacol.2020.12.284.
- [42] Z. Chen, Q. Guo, Y. Yan, Y. Shi, Model identification and adaptive control of lower limb exoskeleton based on neighborhood field optimization, *Mechatronics* 81 (2022) 102699. doi:10.1016/j.mechatronics.2021.102699.
- [43] D. A. Winter, *Biomechanics and Motor Control of Human Movement*, John Wiley & Sons, Inc., New Jersey, 2009.
- [44] A. I. Kapandji, *The Physiology of the Joints*, Churchill Livingstone, Edinburgh, 2011.
- [45] E. A. Hansen, L. A. R. Kristensen, A. M. Nielsen, M. Voigt, P. Madeleine, The role of stride frequency for walk-to-run transition in humans, *Scientific Reports* 7 (1) (2017) 2010. doi:10.1038/s41598-017-01972-1.
- [46] M. B. Näf, K. Junius, M. Rossini, C. Rodriguez-Guerrero, B. Vanderborght, D. Lefeber, Misalignment compensation for full human-exoskeleton kinematic compatibility: state of the art and evaluation, *Applied Mechanics Reviews* 70 (5) (2019). doi:10.1115/1.4042523.