Three-dimensional omnidirectional acoustic illusion

Chuanjie Hu

Xiamen university

Yuhang Yin

Xiamen University

Huanyang Chen (✉ kenyon@xmu.edu.cn)

Xiamen University

Letter

Keywords:

Posted Date: December 14th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-1167047/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Three-dimensional omnidirectional acoustic illusion

Chuanjie Hu ^{1, 2}, Yuhang Yin ^{1, 2}, and Huanyang Chen ^{1, *}

¹ Department of Physics, Xiamen University, Xiamen 361005, China

² Institute of Electromagnetics and Acoustics & Pen-Tung Sah

Institute of Micro-Nano Science and Technology, Xiamen University,

Xiamen 361005, China

*kenyon@xmu.edu.cn

Abstract:

Acoustic waves are ubiquitous in human everyday experience, therefore, precise control over the deformation of acoustic waves is always extremely desirable, which can be used, for example, to transform or hide objects from incident waves. Acoustic illusion devices are generally implemented by transformation acoustics, which can deceive ears or sonar systems. Challenges remain, the complexed and extreme material parameters prescribed by coordinate transformation theory make the implementations particularly difficult, even with the help of acoustic metamaterials. Here, a novel method based on Fabry-Perot resonances offers a feasible solution for achieve three-dimensional (3D) omnidirectional passive acoustic illusion. We theoretically demonstrated perfect 3D acoustic illusion via Mie theory, reduced version is further designed numerically and implemented experimentally. In the future, our work opens new possibilities for the implementation of modern acoustic

illusion devices, such as camouflage for anti-sonar detection.

Introduction:

Stories of desperate travelers being lured by a mirage of the miraculous pool among the dunes in a never-ending desert is familiar. The thirsty traveler runs forward madly, trying to throw himself into the blue pool. Soon the shady water would vanish into thin air according to the regular version of the story, leaving him in the yellow dust under the burning sun again. Mirage here is the deceptive appearance of distant objects caused by the bending of light (refraction) in the air layers of varying density, i.e., optics illusion. They are tricks for the eyes, which make us think that there is something with nothing in reality. It's just like their optical cousins that made people go enamored, just with sound. The key of how to artificially realize the illusion effect lies in an object giving the detector some deceptive signals to hide itself or resemble another object, which was deemed impossible before the proposal of the coordinate-transformation-based technique.

The coordinate transformation method has been introduced in the field of electromagnetics (EM) based on the form-invariant of Maxwell's equations, linking spatial geometry with material parameters to calculate the electromagnetic parameters of devices with predefined functionalities [1,2], which is also especially applicable to acoustics. For instance, two-

dimensional acoustic cloak is proposed by equivalence demonstration between the acoustic wave equation and the two-dimensional Maxwell's equations[3]. Simultaneously, three-dimensional acoustic cloak is established by the isomorphism of the acoustic equation and conductivity equation [4], which is verified by Mie theory [5]. The question is that the material parameters derived from coordinate transformation are often inhomogeneous, anisotropic, or of extreme values that do not exist in nature [6,7], which makes the applications of invisibility cloaks and more generic illusion devices very challenging even with the help of metamaterial approaches. Despite numerous schemes of acoustic cloaks have been proposed with physically feasible parameters [8-12], only a few have been experimentally verified so far [13-19]. Different from the cloak in free space, the cloak for hiding objects near reflecting boundaries[13-17] based on the quasi-conformal map was proposed, which doesn't require extreme material parameters for realization. However, there is a flip side to the so-called ground cloak, the huge size of cloak shell compared to the hidden object makes its application limited. In addition to the coordinate transformation, there are also other mechanisms for passive acoustic illusion and invisibility cloak, such as scattering cancellation [18] and metasurfaces [19]. Although some results are fascinating, the difficulty of the design and the complexity of the structures limit the practicality as well as the implementation of the current methods. Besides, it's noted that

accompanied by various approaches for scattering manipulation proposed are the deficiencies of their respective functionalities, such as unidirectional or ground (near reflecting boundaries). Could there be a much simpler way of designing practical 3D cloak for acoustic illusion? The Fabry-Pérot resonance[20-21] is a good candidate as it can maintain the phase constant while keeping perfect transmission.

In this paper, we develop a theoretical framework and experimental realization of acoustic omnidirectional illusion devices using holey structures along the radial directions in free space. Our approach requires neither the bulk metamaterials with negative indices nor ground, yet allows us to demonstrate a series of omnidirectional acoustic illusion effects. Figure 1(a) shows a schematic diagram illustrating the design strategy, i.e. the desired illusion relies on structuring the surfaces with the help of FP cavities. The original object (indicated by the green sphere) covered with holey cloak makes its scattering field sound like the target objects (indicated by the white sphere) living in the virtual space. More specifically, inspired by the common golf ball, we designed a novel illusion cloak based on FP resonance in r -direction. Dimples all over the golf ball can reduce aerodynamic drag and wind resistance to make the ball fly farther. In fact, there is no regulation on how many dimples a golf ball can have, or what shapes or sizes ball they are. Therefore, how to optimize the ball with different numbers, patterns and shapes of dimples is a difficult task for

manufacturers, which contains a wealth of mechanisms about aerodynamics and surface geometry. Referring to the surface division on classic golf ball, a rigid shell with 336 holes has been employed as the illusion cloak.

Let us first theoretically prove that an original sphere coated by the FP cloak have an identical scattering as another sphere. As shown in Fig. 2(a), a spherical object (indicated by region I) with bulk modulus K_1 and mass density ρ_1 is coated with a FP acoustic cloak (indicated by region II) with

bulk modulus $K_2 = \frac{r}{b} K_0$ and anisotropy density

$\rho_2 = (\rho_r, \rho_\theta, \rho_\phi) = (\frac{b}{r} \rho_0, \infty, \infty)$, where K_0 and ρ_0 are the bulk modulus and density of the background [20]. Here, the inner and outer radii of the cloak are a and b , respectively. For region I, the acoustic pressure distribution can be written as

$$P_1(r, \theta, \varphi) = \sum_{l=0}^{\max} \sum_{m=-l}^l A_{lm}^1 J_l(n_1 k_0 r) Y_{lm}(\theta, \varphi) \quad (1)$$

For the cloak shell(region II), the pressure field can be written as[20]

$$P_2(r, \theta, \varphi) = \sum_{l=0}^{\max} \sum_{m=-l}^l [B_{lm}^1 e^{ik_0 b \ln r} + B_{lm}^2 e^{-ik_0 b \ln r}] Y_{lm}(\theta, \varphi) \quad (2)$$

While that the total pressure field in the background is contributed by the incident wave and the scattered wave,

$$P_4(r, \theta, \varphi) = \sum_{l=0}^{\max} \sum_{m=-l}^l [P_{in} J_l(k_0 r) + D_{lm} H_l(k_0 r)] Y_{lm}(\theta, \varphi) \quad (3)$$

where $J_l(x)$ and $H_l(x)$

$Y_{lm}(\theta, \varphi)$ are the spherical

harmonics. By matching the continual boundary conditions of P and $(1/\rho_r)[(\partial P)/(\partial r)]$ at $r=a$ and $r=b$, P_{in}

A_{lm}^1 , B_{lm}^1 , B_{lm}^2 and D_{lm} can be determined.

The detailed form of continual boundary condition is shown in Supplemental Material Section 1[24].

In Fig. 2(b), a single spherical object of radius c (denoted by region III) with bulk modulus K_3 and density ρ_3 is placed in the background

(denoted by region IV) with K_0 and ρ_0 . The pressure distribution inside the sphere can be written as,

$$P_3(r, \theta, \varphi) = \sum_{l=0}^{\max} \sum_{m=-l}^l C_{lm} J_l(n_3 k_0 r) Y_{lm}(\theta, \varphi) \quad (4)$$

Similarly, the pressure field can be written as

$$P_4(r, \theta, \varphi) = \sum_{l=0}^{\max} \sum_{m=-l}^l [P_{in} J_l(k_0 r) + D_{lm}^2 H_l(k_0 r)] Y_{lm}(\theta, \varphi) \quad (5)$$

By matching the continual boundary conditions of P and $(1/\rho_r)[(\partial P)/(\partial r)]$ at $r=c$, the coefficients C_{lm} and D_{lm}^2 can be determined, detailed conditions see Supplemental Material Section 1[24].

By carefully comparing Eqs. (S1-S3) with Eqs. (S4-S5), if $b=c$, and

$\rho_3 = \frac{a}{b} \rho_1$, $K_3 = \frac{b}{a} K_1$ and $n_3 = \frac{a}{b} n_1$ at the FP resonance conditions, the

total pressure field of object A given by Eq. (3) is identical to that of object

B given by Eq. (5). The so-called FP condition can be written as

$e^{ik_0 b \ln b/a} = e^{im\pi}$, i.e., $k_0 b \ln \frac{b}{a} = \int_a^b k_0 n(r) dr = m\pi$ [20]. As a special case, the

invisibility cloak hides an object by simply eliminating its acoustic

scattering field, which can also be achieved when the relative index of the

object B is equal to 1. In other words, there is nothing that can be detected

in the background medium, which is theoretically illustrated in

Supplemental Material Section 2[20, 24]. In fact, it is more challenging

and intriguing to realize a general illusion cloak that detect the scattering

pattern of a particular object A appears like some other object B of

predetermined choice. For example, the object A with refractive index of 3 covered by the illusion cloak could sound like the object B with refractive index of 2.

To demonstrate the perfect illusion effect more vividly, we analytically plot the acoustic pressure distribution of two objects. The dark green object represents the original sphere with mass density $\rho_1 = \rho_0$, bulk modulus $K_1 = K_0$, and a radius of $r = a = 0.03m$, covered by a 3D acoustic FP shell

with an outer radius of $r = b = 0.06m$. The gray object represents the desired mimic sphere with $\rho_3 = 0.5\rho_0$, $K_1 = 2K_0$, and a radius of $r = c = 0.06m$. Supposing the same incident plane wave given by $P_{in} = P_0 e^{i(k_0 r \cos \theta)}$ impinges from the left on two objects and results in the total acoustic pressure distributions shown in Figs. 2(c) and 2(d), which indicating that the original object has been transformed to another object in terms of scattering pattern at the operating frequency $f = \frac{m c_{air}}{2b \ln(b/a)} = 8247.4 \text{ Hz}(m=2)$. The background material here we employed is air (with the relative acoustic refractive index of $n_0 = \sqrt{\rho_0 / K_0}$). It's noted that the illusion device and an equivalent bare sphere have the identical scattering coefficients for the case of each order at a series of FP resonance frequencies. Detailed calculation and assumption can be found in Supplemental Material Section 3[24]. So far, the perfect illusion device has been theoretically proved, and it works efficiently in full 4π solid angle. However, the gradient material parameters of the acoustic cloak along r direction make the preparation difficult, which is expected to further overcome in the future with the development of fabrication technology.

Now we come to implement a reduced version of the FP acoustic cloak via homogeneous material. The simplified way is to ignore the gradient parameter along r -axis between $a \leq r \leq b$, i.e., assuming ρ_r as a constant value. To achieve the required strong anisotropy, the constituent material

must have sufficiently acoustic rigidity compared with the background medium (air). The schematic design of the illusion cloak (shown in Fig. 1) is inspired by golf ball. Specifically, this is achieved by piercing 336 holes in the rigid shell, each of which has a circular cross section and linearly shrinks radially. It's worth mentioning that other surface division approaches would also be applicable, as long as the divided holes are compact and small. The rigid walls between adjacent holes almost completely prohibit transverse propagation, which contributes extremely large component along the polar and azimuthal directions, that is, ρ_θ and ρ_ϕ equal infinity. In the current simulation and experiment, the background medium is air with $\rho_{air}=1.21\text{kg/m}^3$ and $c_{air}=343\text{m/s}$. The FP shell is made of acrylic resin with 1190kg/m^3 and 2260m/s , which can be considered acoustically rigid due to the huge acoustic impedance mismatch between the polymer and air. To sum up, it can be approximately that the sound velocity c_r of the FP shell along r direction equal to that of the air c_{air} , while there is negligible acoustic wave propagates along the polar and azimuthal directions due to extreme impedance mismatch. The FP resonance condition in r direction is found as:

$$\int_a^b k_0 n(r) dr = m\pi (m = 1, 2, 3 \dots), \text{ since } n_r \approx n_{air}, \text{ the working frequency can be calculated as } f = \frac{mc_{air}}{2(b-a)}.$$

Finally, we fabricate a real 3D FP cloak sample to demonstrate the illusion effect. The FP cloak is composed of two symmetrical hemisphere-shaped hole structures with an outer radius 6 cm,

and the inside hidden space has a radius of 3 cm. The experimental setup is shown in Fig. 1(c), see also Supplemental Material Section 9 for detail description [24]. A square array of speakers is placed head-on to the sample to generate normal incident gaussian beam, then acoustic pressure field distribution inside the measured region, shown as a black dashed rectangle in Fig. 3(a), can be measured by a moving microphone mounted on a 3-axis linear stages. The measured area here is 60 cm by 20 cm with a step of 8 mm, and located directly behind the sample. The whole experimental system is surrounded by sound-absorbing foams to reduce the noise and reflection.

Fig.3. Numerical and experiment results of the 3D illusion device in physical place and the equivalent object in virtual space with Gaussian beam incident at 5716.7Hz. (a) The simulated and experiment results of normalized pressure distribution for the original sphere covered by the illusion cloak, and (b) the corresponding near-field pattern. (c) The simulated results of pressure distribution for the equivalent sphere, while that of the illusion device as the reference, (d) the corresponding near-field pattern.

Next, a series of three-dimensional simulations and experiments around the prefixed operational frequency are carried out to demonstrated the effect of the designed illusion device. Gaussian beams are incident on the illusion device and a bare sphere along x direction. The original sphere in Fig. 3(a) is of radius $a = 0.03m$

$$\rho_1 = \rho_0 \quad \text{and} \quad K_1 = K_0$$

$$b = 0.06m$$

$$c = 0.06m \quad \text{with}$$

$\rho_1 = 0.5\rho_0$ and $K_1 = 2K_0$. The operating frequency is $f = \frac{mc_{air}}{2(b-a)} = 5716.7 \text{ Hz}$ ($m=1$). In Fig. 3(a), we plot the pressure

distribution for the original sphere in physical place covered by the cloak, and the inset displays the zoom-in view of the measured pressure distribution by experiment. The simulated results of pressure distribution for the equivalent bare sphere as shown in Fig.3(c), and that of the illusion device in the inset map as the reference. The observe plane here is located at XY plane with $z = 0 \text{ cm}$ (central plane of the device). In order to quantify the illusion effect, the corresponding near-field pattern of the Fig. 3(a) and 3(c) have been probed along half circle of radius 2.5λ with the object as the center and depicted in Fig.3(b) and 3(d), respectively. By multi-dimensional comparison, it can be observed in the numerical and experiment results that at the resonance frequency, one object covered by the cloak with reduced parameters mimics acoustically another object quite well.

Fig 4. Numerical and experiment results of the 3D illusion device in physical place and the equivalent object in virtual space with Gaussian beam incident at 5716.7Hz. (a) The simulated and experiment results of normalized pressure distribution for the original sphere covered by the illusion cloak, and (b) the corresponding near-field pattern. (c) The simulated and experiment results of pressure distribution for the equivalent sphere, (d) the corresponding near-field pattern.

From this general method, we also take another example. The parameter setting here keeps consistent with the case in Fig.3, except that the inner sphere coated by the FP shell is replaced with an acrylic resin sphere. As mentioned above, due to the extreme impedance mismatch, the resin sphere can be regarded as rigid scattering object. Figure 4(a) and 4(c) shows the simulated field distribution of the illusion cloak and a single equivalent sphere, rectangular dotted regions depict location in which the

experimental pressure distribution was mapped in the inset map. Experimental pressure fields show similar scattering waves as in the numerical simulations. The corresponding near-field pattern of the Fig. 4(a) and 4(c) have also been depicted in Fig. 4(b) and 4(d), respectively. It's worth mentioning that the material of a single equivalent sphere in virtual space could still be approximated as rigid scatterer respect to the background medium. For simplicity, a solid resin sphere of radius c is used to replace the equivalent bare sphere in the experiment. We can see that the achieved illusion effects are quite good: the original sphere in Fig. 4(a) with the help of FP cloak has the same pressure distributions as the equivalent sphere in Fig. 4(c). So far, the different test results of simulation and experiment firmly validate our designs that the FP cloak is a simple and effective approach for transforming one object to another one. In general, the key of this method is: for an acoustic detector at a series specified frequencies and arbitrary incident angles, the detector will get the desired acoustic signals while we don't need to significantly change the original object. It's worth emphasizing that the direction and type of the incident waves are usually unpredictable in practical scenes, such an illusion effect is always valid (see Supplemental Material Section 4 to 8 [24]). Thus, the omnidirectional functionality of our proposed illusion device for any incoming waves would have unparalleled significance in practical applications. In addition, for different kinds of scatterer shapes,

the related matrix of scattering coefficient and incoming coefficient could be determined by matching the boundary condition based on the so-called T matrix method and the above acoustic illusion effect is still valid [22-23].

In summary, we have developed a method to design 3D acoustic omnidirectional illusion cloak based on FP resonances. As a practical implementation of our strategy, we propose a reduced-version cloak design with simplified anisotropic parameters, which can be realized by densely perforating FP cavities with linearly shrinking cross section along the radial direction on a solid spherical shell. Good agreement is observed between the theoretical prediction and experiment measurement. Moreover, we further verified its generality for different source and objects of arbitrary shapes. Compared with transformation acoustics, this omnidirectional cloak greatly simplifies the material parameters, which would significantly facilitate practical applications of the designed structures. In the further, this unique but simpler method may find many applications in the design of modern acoustic illusion devices, such as camouflage for anti-sonar detection.

- [1] J. B. Pendry, D. Schurig, and D. R. Smith, Controlling electromagnetic fields, *Science* 312, 1780 (2006).
- [2] U. Leonhardt, Optical conformal mapping, *Science* 312, 1777 (2006).
- [3] S. A. Cummer and D. Schurig, One path to acoustic cloaking, *New J.*

Phys. 9, 45 (2007).

- [4] H. Chen and C. T. Chan, Acoustic cloaking in three dimensions using acoustic metamaterials, *Appl. Phys. Lett.* 91, 183518 (2007)
- [5] S. A. Cummer, B.-I. Popa, D. Schuring, D. R. Smith, J. Pendry, M. Rahm, and A. Starr, Scattering Theory Derivation of a 3D Acoustic Cloaking Shell, *Phys. Rev. Lett.* 100, 024301 (2008).
- [6] Y. Lai, H. Y. Chen, Z.-Q. Zhang, and C. T. Chan, Complementary Media Invisibility Cloak that Cloaks Objects at a Distance Outside the Cloaking Shell, *Phys. Rev. Lett.* 102, 093901 (2009).
- [7] Y. Lai, J. Ng, H. Chen, D. Han, J. Xiao, Z. Q. Zhang, and C. T. Chan, Illusion Optics: The Optical Transformation of an Object into Another Object, *Phys. Rev. Lett.* 102, 253902 (2009).
- [8] D. Torrent, J. Sánchez-Dehesa, Acoustic cloaking in two dimensions: a feasible approach, *New J. Phys.* 10, 063015 (2008).
- [9] Y. Cheng, F. Yang, J. Y. Xu, and X. J. Liu, A multilayer structured acoustic cloak with homogeneous isotropic materials, *Appl. Phys. Lett.* 92, 151913 (2008).
- [10] M. D. Guild, M. R. Haberman, and A. Alù, Cancellation of the acoustic field scattered from an elastic sphere using periodic isotropic elastic layers, *J. Acoust. Soc. Am.* 128, 2374 (2010).
- [11] M. Guild, M. Haberman, and A. Alù, Plasmonic-type acoustic cloak made of a bilaminate shell, *Phys. Rev. B* 86, 104302 (2012).

- [12] Y. Liu and S. He, Acoustic Illusion Using Materials with Isotropic and Positive Parameters, *Phys. Rev. Appl.* 10, 064036 (2018).
- [13] B.-I. Popa, L. Zigoneanu, and S. A. Cummer, Experimental Acoustic Ground Cloak in Air, *Phys. Rev. Lett.* 106, 253901 (2011).
- [14] W. Kan, B. Liang, X. Zhu, R. Li, X. Zou, H. Wu, J. Yang, and J. Cheng, Acoustic illusion near boundaries of arbitrary curved geometry, *Sci. Rep.* 3, 1427 (2013).
- [15] Zigoneanu, L. Popa, B. & Cummer, S. Three-dimensional broadband omnidirectional acoustic ground cloak, *Nat. Mat.* 109, 054906 (2014).
- [16] W. Kan, V. M. García-Chocano, F. Cervera, B. Liang, X. Zou, L. Yin, J. Cheng, and J. Sánchez-Dehesa, Broadband acoustic cloaking within an arbitrary hard cavity, *Phys. Rev. Appl.* 3, 064019 (2015).
- [17] W. Kan, B. Liang, R. Li, X. Jiang, X. Zou, L. Y, and J. Cheng. Three-dimensional broadband acoustic illusion cloak for sound-hard boundaries of curved geometry, *Sci. Rep.* 6, 36936 (2016).
- [18] L. Sanchis, V. M. García-Chocano, R. Llopis-Pontiveros, A. Climente, J. Martínez-Pastor, F. Cervera, and J. Sánchez-Dehesa, Three-Dimensional Axisymmetric Cloak Based on the Cancellation of Acoustic Scattering from a Sphere, *Phys. Rev. Lett.* 110, 124301 (2013).
- [19] S. Fan, S. D. Zhao, L. Cao, Y. Zhu, A. L. Chen, Y. F. Wang, K. Donda, Y. S. Wang, and B. Assouar, Reconfigurable curved metasurface for acoustic cloaking and illusion, *Phys. Rev. B* 101, 024104 (2020).

- [20] H. Chen, Y. Zhou, M. Zhou, L. Xu, and Q. H. Liu, Perfect Undetectable Acoustic Device from Fabry-Perot Resonances, *Phys. Rev. Appl.* 9, 024014 (2018).
- [21] Y. Zeng, Y. Zhou, M. Zhou, and H. Y. Chen, Illusion Elasticity in a Fluid Background, *Phys. Rev. Appl.* 11, 014015 (2019).
- [22] R. Sainidou, N. Stefanou, I. E. Psarobas, and A. Modinos, A layer-multiple-scattering method for phononic crystals and heterostructures of such, *Comput. Phys. Commun.* 166, 197 (2005)
- [23] E. Boyko, V. Bacheva, M. Eigenbrod, F. Paratore, A. D. Gat, S. Hardt, and M. Bercovici, Microscale hydrodynamic cloaking and shielding via electro-osmosis, *Phys. Rev. Lett.* 126, 184502 (2021).
- [24] See Supplemental Material Section 1 to 9.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [supplementarymaterialforillusion.docx](#)