New Approach Based on Fuzzy Hypergraphs in Granular Computing (An Application to the Urban Vulnerability Assessment)

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New approach based on fuzzy hypergraphs in granular computing

(an application to the urban vulnerability assessment)

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Abstract

Classifying objects based on the simultaneous impact of various parameters has always been challenging due to heterogeneity, impact conflict, and sometimes parameter uncertainty. The purpose of this study is to provide a method for classifying such data. In the proposed method, fuzzy hypergraphs were used to define the granular structures in order to apply the simultaneous effect of heterogeneous and weighted parameters in the classification. This method has been implemented and validated on Fisher intuitive research in relation to the classification of iris flowers. Evaluation and comparison of the proposed method with Fisher's experimental and results showed higher efficiency and accuracy in flower classification. The proposed method has been used to assess the seismic risk of 50,000 buildings based on 10 heterogeneous parameters. Seismic risk classification showed that more than 88% of buildings were classified, and 12% of buildings that could not be classified due to excessive scatter of parameter values were classified using a very small confidence radius. The results indicate the ability of the proposed method to classify objects with the least similarity and number of effective parameters in classification.

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1. Introduction

One of the main elements for effective response, before natural disasters, is the assessment of urban vulnerability. Urban vulnerability assessment requires access to information and advanced methods of information analysis.

Past experiences have shown that earthquakes, as a complex natural phenomenon, are not predictable. However, their effects can be measured by different mathematical patterns and techniques. Many parameters contribute to the damage caused by an earthquake.

Parameters which, in addition to uncertainties, are also conflicting in estimating the damage. The multiplicity of parameters, uncertainty and conflict and the placement of the urban zone faces many challenges in a risk class. Finally, seismic risk assessment is affected by the challenges of classification. So far, different approaches have been proposed to assess the urban vulnerability by focusing on the control of uncertainties and opposite conditions of the parameters.

In spite of the development of a vulnerability structure based on the GIS software, the obtained results are not above an average estimate of physical and life-threatening losses. This limitation implies discarding the sensitivity of infrastructure systems. The lack of attention to infrastructure systems leads to a reduction in reliability and cascading failure(1). Most of the conventional methods of determining vulnerability risk are merely a specific allocation of risk-taking (2–4).Some studies describe seismic risk using the weight scoring method and cumulative measurement (total weight) of individual events in each criterion(5,6).

A method called Diagram Index (DI) is proposed to determine the interdependence between several properties, based on graph theory. In the DI index, the performance estimation of the fundamental infrastructure of urban systems is investigated, starting with physical damage and then the effects that each node has on the performance of other nodes(1).

Previous research has shown that in principle the resonance descriptors are exacerbating the physical hazards. This increases concerns about social, economic and non-flexibility, or lack of ability to deal with physical hazards(7–9).

In some methods, by identifying the different structures, the vulnerability of buildings is defined on the basis of being in an urban area. Then the relationship between intensity and damage is obtained by means of the harm matrix(10,11). The vulnerability functions and correlation coefficients can also be used according to expected seismic input intensity(12–14).

Recently, a model of vulnerability information systems is proposed as “Common Relevant Operational Picture” (CROP). In order to minimize the risk of relief in the crop model, the possibility of coordination between regions is enabled by allocating small subset of a common database for each area(15).

Granular Computing (GrC) is a novel computational paradigm for processing complex information entities called "Information Granules", which is used to explore hidden, useful and unknown communication in the dataset(16,17).

In general, the data granules are a set of entities that are usually placed in a numeric level due to the similarity, functional closeness, consistency or similar cases, as a whole unit(18). In addition to being able to be an independent seed, granules can be a member of another granules. Then we will have a family of the granules, which are considered as a whole. The granules on one level, though they may be relatively independent, are somehow connected to each other in a certain

degree and generally exhibit a particular structure, which is the inner structure of granules. At present, granules are more than a coherent set of methods or principles and an approach for data analysis at different levels in order to detect data, is recommend.

As mentioned above, granule computing is not a process or algorithm. There is no specific method which is called “Granule Computing”. Granule Computing is used to study the data, which recognizes how different laws in the data can appear at different levels of granularity. With the development of granules, different models of granular computing are proposed and investigated. Among the models are fuzzy sets, rugged sets, and outer space as the main computational models(19).

For example, a granular computing-based model is presented based on data tables, and in this model, different methods for construction, interpretation and representation of the granules were investigated(20). Also, some other concepts of granular computing are presented based on fuzzy set(21).

Over the past half century, graph theory as a classical mathematical tool, has been applied to many applications in different fields such as economics, computer science, geometry, number theory, optimization, topology and transportation. The graph theory is currently used in granular computing and the representation of seed structures for computer problems research. For example, the adjacency matrix of a simple undirected graph (constrained) is interpreted as a Boolean information table(22).

In graph theory, however, some characteristics of the case study may have some sort of uncertainty. For example, in the earthquake and seismic vulnerability assessment, the effect and distance of buildings from blast centers (gas stations and gas pumps) are not precise. In such cases, fuzzy sets and fuzzy logic are used to deal with uncertainty management.

To solve new combinatorial problems, it was necessary to develop the concept of graph theory. In 1960, hypergraph theory was introduced as a generalization of graphs, and one of its primary objectives was to generalize some classical results of graph theory(23).

The hypergraph theory offers a more powerful tool to analyze and solve real-world problems in the modeling of complex systems. Graphs have the ability to communicate with a maximum of two nodes. It is virtually impossible to use graphs for analysis and discussion of more than two node. One of the motivations for introducing hypergraph is for solving this type of graph problems. Hence, the hypergraph can analyze the connection between members of the finite set and even the infinite nodes. Unlike ordinary graphs, each edge in the hypergraph can consist of an arbitrary number of vertices and each edge in this hypergraph is called hyperedge. In the hypergraph, the vertices can be described in a n-ary relation.

Some researchers developed a hypergraph segmentation algorithm based on multi-level patterns, in which a sequence of successive hypergraphs are constructed(24). It shows that a simple granule of the universe can be characterized by a fuzzy equivalence relation or a given partition.

For the first time, Zadeh, introduced a special type of subset X and named it a fuzzy subset, by grading the inaccurate degree of belonging of the members of a subset of the reference set X (25).

By combining the benefits of the two fuzzy concepts and hypergraph, fuzzy hypergraph is presented as a modeling structure with fuzzy relation and multiple relationships between objects(26).

Previous studies have shown that fuzzy graphs are expressed on the basis of fuzzy relations(27,28).

In the studies, some operations have been defined over fuzzy graphs(29,30). Fuzzy graph theory has many applications in various fields, including computer networks, data mining, clustering, differentiation, and recording. Analog fuzzy logic has been discussed in many of the theoretical

concepts of graph theory and in the case of fuzzy graphs is particularly interesting for researchers(31,32). There are also other definitions such as the supplement of fuzzy graph and the characteristics of fuzzy trees(33,34). Recently, the concept of M-fuzzy based graphs and some of its properties have been proposed(35).

Finally, an interesting concept called fuzzy hypergraph based on the original idea was generalized and redefined by Kaufman(36). In addition, the concept of fuzzy hypergraphs has been considered and the concept of interval values of fuzzy hypergraphs has been introduced(37,38). In recent years, the concept of intuitive fuzzy hypergraphs as well as bipolar fuzzy hypergraphs and some of their basic features have been expressed(39,40).

Classification is easily applicable when faced with a small set of attributes. For example, in a collection of white, black, red and beige cars, we can easily put them in 4 classes. But if the same set includes other features such as year of manufacture, manufacturer, engine size, price, etc., classification becomes a bit complicated. Now suppose you want to classify in a collection of thousands of records and hundreds of unordered attributes with conflicting effects. How can classes and classifying a collection of elements be defined?

Increasing the accuracy and focus on some features for classification reduces the accuracy in influencing other features in defining classes and identifying the appropriate class.

Considering the hierarchical effect of weighted and heterogeneous parameters in the classification of elements, always reduces the classification accuracy. Highlighting the role of high-level parameters and diminishing the effect of parameters in the lower layers are the main challenges of classifying elements with equal and heterogeneous properties. In this paper, to increase the accuracy in seismic vulnerability assessment and the possibility of better and more accurate decision-making at boundary points, the concepts of fuzzy and granule hypergraphs have been

used. To define urban seismic risk classes with flexible and floating boundaries and limits, a granular structure based on fuzzy hypergraph has been used. In this method, much more granules are shown. Previous research has been done to investigate how granules are made in different fields, using hypergraphs(41,42).By examining and evaluating each building in the mentioned granular structure, the risk level of the building is calculated and determined. This approach makes it possible to make better decisions at the border points of the classes. In this process, in addition to considering the simultaneous effect of all effective parameters in the classification, the degree of interaction of the parameters with each other is also considered. For verification, the proposed method was implemented in the classification of iris flowers. Fisher has intuitively classified irises(43). The conformity of the classification with Fisher's experimental and intuitive methods indicates the efficiency of the presented research method. This method is then used to assess seismic risk.

2. General

In this section, the concepts of granules, granule computing, fuzzy, hypergraph and fuzzy hypergraph are defined.

2.1. Granule, Granule Computing

Granular computing is a well-known method in computation and applied mathematics for data processing. Information is divided into granules through the granulation process(20).

Information granule is a set of entities that are obtained from the numerical analysis of the properties of the entity (data) and then it is classified at a certain level based on similarity, functional proximity, physical proximity, or indistinguishability.

Granular data is the lowest level of data that is targeted into a set. This refers to the degree of detail and the size of the class to which the data is divided. For example, dividing people's names into either a single field, consisting of both names and surnames together, or separate fields. The more segmented and specific the data, the more granularity is considered.

Putting the data at its proper level depends on the level of knowledge and awareness of the impact of data details on granulation.

Classes are described according to whether the grains are large or detailed. The structure of these levels of granulation is determined by the laws of nature or human cognition.

2.2 Fuzzy Sets

The key to understanding the difference between a fuzzy set and a classic set is the concept of fuzzy membership degree. In classical logic, the concepts of belonging or non-belonging of a member to a set, are quite definite and precise. So, an object is or is not a member of a set and hence the membership function can only have two values, 0 and 1. In 1965, Lotfi A. Zadeh, introduced a special type of subset X and called it a fuzzy subset by grading the inaccurate degree of belonging members of a subset from a reference set X (44).

Definition 2.1 A "fuzzy subset A " of reference set X or a "fuzzy set A " in reference set X , characterized with membership function $\mu_A : X \rightarrow [0,1]$ and the membership of each member is graded. The meaning of $\mu_A(x) \in [0,1]$ degree of membership $x \in X$ in "fuzzy subset A " or "fuzzy set A "(44).

The set $\text{supp}\mu_A = \{x \in X \mid \mu_A(x) > 0\}$ is called the support μ_A , and $h(\mu_A)$ is the height display μ such that $h(\mu_A) = \max\{\mu_A(x) \mid x \in X\}$.

180 If u and v are fuzzy sets, then $u \wedge v = \min\{u, v\}$ and $u \vee v = \max\{u, v\}$. We write $u \leq v$
 181 if $u(x) \leq v(x)$ for all $x \in X$ also $u < v$ if $u \leq v$ and $u(x) < v(x)$ for some $x \in X$.

182

183 2.3. Hypergraph and fuzzy hypergraph

184 In 1960, the theory of hypergraphs was introduced as a generalization of graphs. Unlike normal
 185 graphs, each edge in a hypergraph can contain an arbitrary number of vertices [23].

186 **Definition 2.2** The pair $H = (X, E)$ is called a hypergraph on a finite set X if E is a finite family of
 187 non-empty subsets X such that:

188 1) $\forall E_i \in E, E_i \neq \emptyset \quad i = 1, \dots, m$

189 2) $\bigcup_{i=1}^m E_i = X$

190 Each $E_i \in E$ is called an edge, and each edge contains a number of elements called vertices.

191 Sometimes a set of vertices is represented by $X(H)$ and the edge set by $E(H)$ (45).

192 A hypergraph is called a simple hypergraph if E does not contain duplicate edges and

193 $\forall A, B \in E, A \subset B$ then $A=B$.

194 **Example 2.1** Suppose $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $E = \{E_1, E_2, E_3, E_4\}$ such that $E_1 = \{1, 3, 8, 10\}$

195 $, E_2 = \{1, 4, 5\}, E_3 = \{6, 5, 8, 9\}, E_4 = \{2, 7, 8\}$.

196 By definition 2.2 $H = (X, E)$ is a hypergraph on the set X . Each of the subsets E_1, E_2, E_3, E_4 edges and
 197 their members are the vertices of the hypergraph.

Figure (1): Diagram of two fuzzy relations defined on the X set

Definition 2.3 Let X is a finite set and $\mathcal{E}$ a finite family of nontrivial fuzzy subsets on the set X such that $X = \bigcup \{\text{supp } \mu \mid \mu \in \mathcal{E}\}$, then the pair $H = (X, \mathcal{E})$ is a fuzzy hypergraph on the set X , and X the set of vertices and $\mathcal{E}$ is called the set of fuzzy edges (the family of edges fuzzy) of H (37).

Definition 2.4 A fuzzy hypergraph $H = (X, \mathcal{E})$ is called simple if $\mathcal{E}$ does not contain duplicate fuzzy edges and whenever $u, v \in \mathcal{E}, u \leq v$ then $u = v$ (37).

Definition 2.5 Suppose $H = (X, \mathcal{E})$ is a fuzzy hypergraph and $\mu \in \mathcal{E}$ and $0 \leq \lambda \leq 1$. λ -cut of μ is defined as follows $\mu_\lambda = \{x \in X \mid \mu(x) \geq \lambda\}$ (37).

Also, assuming that $E_\lambda = \{\mu_\lambda \mid \mu \in \mathcal{E}\}$ and $X_\lambda = \bigcup \{\mu_\lambda \mid \mu \in \mathcal{E}\}$. If $E_\lambda \neq \emptyset$ then the hypergraph

$H_\lambda = (X_\lambda, E_\lambda)$ is called a λ -level hypergraph of H .

210 **Example 2.2** By defining a fuzzy relation, we define a fuzzy hypergraph on X set.

211 • Relation μ_A a set of numbers close to 7

$$\mu_A(x) = \begin{cases} 0 & \text{where } x = 1 \\ 0 & \text{where } x = 2 \\ 0.3 & \text{where } x = 3 \\ 0.7 & \text{where } x = 4 \\ 0.8 & \text{where } x = 5 \\ 0.9 & \text{where } x = 6 \\ 1 & \text{where } x = 7 \\ 0.9 & \text{where } x = 8 \\ 0.8 & \text{where } x = 9 \\ 0.7 & \text{where } x = 10 \end{cases} \quad \mu_A : X \rightarrow [0,1]$$

$$A = \{(1,0), (2,0), (3,0.3), (4,0.7), (5,0.8), (6,0.9), (7,1), (8,0.9), (9,0.8), (10,0.7)\}$$

$$\text{supp}\mu_A = \{3, 4, 5, 6, 7, 8, 9, 10\}$$

$$\text{let } \lambda=0.6 \text{ so } \mu_{A_{0.6}} = \{4, 5, 6, 7, 8, 9, 10\}$$

212 • Relation μ_B a set of numbers that is neither too big nor too small.

$$\mu_B(x) = \begin{cases} 0 & \text{where } x = 1 \\ 0.2 & \text{where } x = 2 \\ 0.6 & \text{where } x = 3 \\ 0.8 & \text{where } x = 4 \\ 1 & \text{where } x = 5 \\ 0.8 & \text{where } x = 6 \\ 0.6 & \text{where } x = 7 \\ 0.2 & \text{where } x = 8 \\ 0 & \text{where } x = 9 \\ 0 & \text{where } x = 10 \end{cases} \quad \mu_B : X \rightarrow [0,1]$$

$$B = \{(1,0), (2,0.2), (3,0.6), (4,0.8), (5,1), (6,0.8), (7,0.6), (8,0.2), (9,0), (10,0)\}$$

$$\text{supp}\mu_B = \{2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\text{let } \lambda=0.6 \text{ so } \mu_{B_{0.6}} = \{4, 5, 6, 7\}$$

213 If $\varepsilon = \{\mu_A, \mu_B\}$ then $X \neq \text{supp}\mu_A \cup \text{supp}\mu_B = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$, by Definition 2.3 the pair

214 $H=(X, \varepsilon)$ isn't a fuzzy hypergraph on the X set. Figure (1) shows the fuzzy relations μ_A, μ_B with

215 sets $\mu_{A_{0.6}}, \mu_{B_{0.6}}$.

216 Now we define under Relation μ_c on X set. μ_c a set of numbers close to 0.

$$\mu_c(x) = \begin{cases} 0.9 & \text{where } x = 1 \\ 0.8 & \text{where } x = 2 \\ 0.7 & \text{where } x = 3 \\ 0.6 & \text{where } x = 4 \\ 0.5 & \text{where } x = 5 \\ 0.4 & \text{where } x = 6 \\ 0.3 & \text{where } x = 7 \\ 0.1 & \text{where } x = 8 \\ 0 & \text{where } x = 9 \\ 0 & \text{where } x = 10 \end{cases} \quad \mu_c : X \rightarrow [0,1]$$

$$C = \{(1,0.9), (2,0.8), (3,0.7), (4,0.6), (5,0.5), (6,0.4), (7,0.3), (8,0.1), (9,0), (10,0)\}$$

$$\text{supp}\mu_c = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

217 If $E = \{\mu_A, \mu_C\}$ then $X = \text{supp}\mu_A \cup \text{supp}\mu_c = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, by Definition 2.3 the pair

218 $H=(X, E)$ is a fuzzy hypergraph on the X set.

219

220 3. Methodology

221 In hierarchical structures, not all the properties that affect classification are considered at the same

222 time. This problem-solving method actually simplifies the problem by ignoring some features, and

223 then the problem-solving pattern is gradually modified at different levels. Finally, the main
224 problem, which is usually complex, is solved.

225 The proposed method in this paper provides the possibility of classification, taking into account
226 all the effective classification features simultaneously.

227 Defining the structure of classes is one of the main challenges in classifying elements with the
228 simultaneous impact of several properties. In this method, granular structure based on fuzzy
229 hypergraphs is defined to create the class structure. The proposed granular structure is very flexible
230 based on the classes defined in each of the properties. This flexible structure, in addition to
231 increasing the accuracy of decision-making at border points, also has the ability to classify
232 elements with high dispersion of properties and the least similarity.

233 The definition of classes in set X derives from the concept of these classes in each set of properties
234 of the elements of X set.

235 Finally, the proposed methods implemented in classifying 50,000 buildings to determine the
236 seismic risk level by considering the simultaneous impact of 10 properties.

237 **Definition 3.1** Suppose $X = \{(x_1, x_2, \dots, x_n) \mid x_i \in X_i, i=1,2,\dots,n\}$ is a finite set. So that each element of X
238 has n properties. X_i are sets of property values.

239 let $K_1, K_2, \dots, K_m$ are m title classes on set X . Note that at this stage there is no precise definition
240 for the classes on the set X .

241 Each E_{ij} class is the concept of K_j class on the X_i property for $i=1,2,\dots,n$, $j=1,2,\dots,m$. We define
242 the E matrix:

243

$$E = \begin{pmatrix} E_{11} & K & E_{1m} \\ M & O & M \\ E_{n1} & L & E_{nm} \end{pmatrix}$$

244 The elements of E matrix in each row are the classes defined on properties value set. For example,
 245 elements $E_{11}, E_{12}, \dots, E_{1m}$ are m classes defined on X_1 set.

246 **Lemma 3.2** $\bigcup_{j=1}^m E_{ij} = X_i$ where E_{ij} are classes on sets X_i , $i=1,2,\dots,n$ (sets located in a row).

247 **Proof:**

248 From Definition 3.1, $E_{ij} \subseteq X_i$ for any $i=1,2,\dots,n, j=1,2,\dots,m$, thus $\bigcup_{j=1}^m E_{ij} \subseteq X_i$. (I)

249 Conversely, since for any $i=1,2,\dots,n, j=1,2,\dots,m$, E_{ij} sets are classes defined on X_i sets so:

250
$$\forall a \in X_i \exists t \in \{1,2,\dots,m\} \text{ such that } a \in E_{it}, \text{ then } X_i \subseteq \bigcup_{j=1}^m E_{ij}. \quad (\text{II})$$

251 Combining (I), (II) we have $\bigcup_{j=1}^m E_{ij} = X_i$ for $i=1,2,\dots,n$.

252 **Lemma 3.3** The pair $H_i=(X_i, E_i)$ is hypergraph on X_i sets, when $E_i = \{E_{ij} \mid E_{ij} \neq \emptyset, j=1,2,\dots,m\}$
 253 and $i=1,2,\dots,n$.

254 **Proof:**

255 From Lemma 3.2, $\bigcup_{j=1}^m E_{ij} = X_i$ for $i=1,2,\dots,n$, by Definition 2.2 the pair $H_i=(X_i, E_i)$ is
 256 hypergraph on X_i sets.

257

258 **Definition 3.4** Corresponding to matrix E in the Definition 3.1, we define θ_x matrix for all $x \in X$,
 259 as follows:

$$260 \quad \theta_x = \begin{pmatrix} \theta_{11}(x_1) & K & \theta_{1m}(x_1) \\ M & O & M \\ \theta_{n1}(x_n) & L & \theta_{nm}(x_n) \end{pmatrix}$$

261 such that:

262

$$263 \quad \theta_{ij}(x_i) = \begin{cases} \mu_{ij}(x_i) & \text{where } x_i \in E_{ij} \\ -\mu_{ij}^c(x_i) & \text{where } x_i \notin E_{ij} \end{cases}$$

264 where:

$$265 \quad \mu_{ij} : E_{ij} \rightarrow [0,1], \mu_{ij}^c : E_{ij}^c \rightarrow [0,1], E_{ij}^c = X_i - E_{ij}$$

266

$$267 \quad \text{for } i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

268

269 For each $e \in E_{ij}$, value of the $\mu_{ij}(e)$ function is called e-membership degree in the E_{ij} set, also the
 270 value of the $\mu_{ij}^c(e)$ function is called e-membership degree in the E_{ij}^c set. The elements of the
 271 matrix are functions.

272

273 **Definition 3.5** The j^{th} score of the element $x \in X = \{(x_1, x_2, x_3, \dots, x_n) \mid x_i \in X_i, i=1, 2, \dots, n\}$ is a special
 274 number, that can be calculated from the matrix θ_x as follows:

$$\text{score}_j(x) = \sum_{i=1}^n \theta_{ij}(x) \text{ where } \theta_{ij}(x) \in \theta_x \text{ for } j=1,2,3,\dots,m.$$

For example, the sum of the elements of the first column in the matrix θ_x is $\text{score}_1(x)$.

Definition 3.6 Let $x \in X = \{(x_1, x_2, \dots, x_n) \mid x_i \in X_i, i=1,2,\dots,n\}$, from Definition 3.5 we define:

$$L_j = \{x \in X \mid \text{score}_j(x) \geq 0\} \text{ where } j=1,2,\dots,m$$

and

$$L_{m+1} = \{x \in X \mid \text{score}_j(x) < 0 \text{ for all } j=1,2,\dots,m\}.$$

Each of sets L_j are granules. By Definition 2.2 $H=(X,L)$ is a hypergraph on X . The proposed granular structure is shown in Figure (2).

The elements of $L_1, L_2, \dots, L_m$ granules have at least a $\text{score}_j(x) \geq 0$ value (see Figure (2)). Zone A contains an element of set X that has no value $\text{score}_j(x) \geq 0$ and only has $\text{score}_j(x) < 0$ values (set L_{m+1}). In other words, they are not in any granule (class). Zone B is composed of elements that have a $\text{score}_j(x) \geq 0$ in all granules (classes).

Figure (2): The proposed granular structure

Definition 3.7 Let $x \in X = \{(x_1, x_2, \dots, x_n) \mid x_i \in X_i, i=1, 2, \dots, n\}$, based on the $\text{score}_j(x)$ for $j=1, 2, \dots, m$

defined several $\text{DoM}_{\text{score}_j}$ values (degree of membership) or $\text{DoNM}_{\text{score}_j}$ values (non-membership)

as follows:

if $\text{score}_j(x) \geq 0$ then $\text{DoM}_{\text{score}_j}(x) = \mu(\text{score}_j(x))$ where $\mu: L_j \rightarrow [0, 1]$ for $j=1, 2, \dots, m$.

if $\text{score}_j(x) < 0$ then $\text{DoNM}_{\text{score}_j}(x) = \eta(\text{score}_j(x))$ where $\eta: L_{m+1} \rightarrow [0, 1]$ for $j=1, 2, \dots, m$.

Definition 3.8 Let $x \in X = \{(x_1, x_2, \dots, x_n) \mid x_i \in X_i, i=1, 2, \dots, n\}$ from Definition 3.7 we define:

$$\text{DoM}(x) = \max \left\{ \text{DoM}_{\text{score}_j}(x) \text{ for } j=1, 2, \dots, m \right\}$$

$$K_j = \left\{ x \mid \text{DoM}(x) = \text{DoM}_{\text{score}_j}(x) \right\} \text{ for } j=1, 2, \dots, m$$

299 If $\text{DoM}(x)$ doesn't exist, define $\text{DoNM}(x)$ as follows:

$$300 \quad \text{DoNM}(x) = \min \left\{ \text{DoNM}_{\text{score}_j}(x) \text{ for } j=1,2,\dots,m \right\}$$

$$301 \quad K_{m+1} = \{x \mid \exists \text{DoNM}(x)\}.$$

302

303 By Definition 3.7, a fuzzy hypergraph on the set X is defined. The elements of the set X that have
 304 the highest $\text{DoM}_{\text{score}_j}(x)$ value form the subset, which in turn defines K_j class in Definition 3.1.

305 This way of constructing subsets defines the granular structure, which is considered as a class. The
 306 other values of $\text{DoM}_{\text{score}_j}(x)$ indicate the belonging of element $x \in X$ to other classes, with
 307 different degrees of membership. The absence of a value of $\text{DoM}_{\text{score}_j}(x)$ for $x \in X$ indicates that

308 the property values are too scattered. Such elements simply have different degrees of
 309 $\text{DoNM}_{\text{score}_j}(x)$ non-belonging to classes. The set of all these elements are considered a subset of
 310 X as a class K_{m+1} (the smallest degree of non-belonging $\text{DoNM}_{\text{score}_j}(x)$).

311 **Lemma 3.9** $X = \bigcup_{j=1}^{m+1} K_j$ where K_j are classes on set X .

312 **Proof:**

$$313 \quad \text{Let } x \in \bigcup_{j=1}^{m+1} K_j \Rightarrow \exists 1 \leq t \leq m+1 \text{ such that } x \in K_t \subset X \Rightarrow \bigcup_{j=1}^{m+1} K_j \subseteq X. \quad (\text{III})$$

314 Conversely

315 Case 1:

316 For any $x \in X$ there exists $t \in \{1, 2, \dots, m\}$ such that $\text{score}_t(x) \geq 0$. From Definition 3.8 $\text{DoM}(x)$

317 value is defined and $x \in K_t$

318 then

$$319 \quad x \in \bigcup_{j=1}^m K_j \Rightarrow X \subseteq \bigcup_{j=1}^m K_j \subseteq \bigcup_{j=1}^{m+1} K_j. \quad (\text{IV})$$

320 Case 2:

321 $\text{score}_t(x) < 0$ for any $t \in \{1, 2, \dots, m\}$ by Definition 3.6 $x \in L_{m+1}$. From Definition 3.8 $\text{DoNM}(x)$

322 value is defined and $x \in K_{m+1}$

323 then

$$324 \quad x \in \bigcup_{j=1}^{m+1} K_j \Rightarrow X \subseteq \bigcup_{j=1}^{m+1} K_j. \quad (\text{V})$$

325 By (III), (IV), (V) we have $X = \bigcup_{j=1}^{m+1} K_j$.

By Def :3.1	Determine the value sets for the properties of the elements of the set X ,under the title of sets X_i .
By Def :3.1	Define the X set, base on the X_i sets, $X = \{(x_1, x_2, x_3, \dots, x_n) \mid x_i \in X_i\}$.
By Def :3.1	Expression of m-class titles, to classify the elements of set X under the heading of classes: $K_1, K_2, K_3, \dots, K_m$.
By Def :3.1	Implement the concept of classes K_j on each set of values as sets $E_{ij} \subset X_i$ and form classes on sets of values.
By Def :3.1	Formation of matrix E based on classes E_{ij} .
By Def :3.4	Definition of membership functions μ_{ij} on classes E_{ij} as $\mu_{ij}: E_{ij} \rightarrow [0,1]$.
By Def :3.4	Definition of membership functions μ_{ij}^c on sets E_{ij}^c as follows: $\mu_{ij}^c: E_{ij}^c \rightarrow [0,1]$ where $E_{ij}^c = X_i - E_{ij}$
By Def :3.4	Formation of matrix θ_x for elements of set X.
By Def :3.5	Form a matrix θ_x to score the elements of set X as $\text{score}_j(x)$.
By Def :3.6	Form $L_j \subseteq X$ sets so that $H=(X,L)$ is a hypergraph on the set X. $L = \{L_1, L_2, L_3, \dots, L_{m+1}\}$
By Def :3.7	Define functions μ, η on sets $L_1, L_2, L_3, \dots, L_{m+1}$ to determine values as follows: $\text{DoNM}_{\text{score}_j}(x), \text{DoM}_{\text{score}_j}(x)$
By Def :3.8	Determine the values of $\text{DoNM}(x), \text{DoM}(x)$ to define a fuzzy hypergraph, the structure of classes $K_1, K_2, K_3, \dots, K_{m+1}$ and the degree of membership.

Figure (3): Diagram of the proposed method

329 **Lemma 3.1** $H=(X,E)$ is fuzzy hypergraph on X where $E=\{K_j \mid K_j \neq \emptyset, j=1,2,\dots,m+1\}$.

330 **Proof:**

331 By Lemma 3.9 $X=\bigcup_{j=1}^{m+1} K_j$, from Definition 2.3 pair $H=(X,E)$ is a fuzzy hypergraph on the set X .

332 Thus, the flexural granular structure is defined based on fuzzy hypergraphs that have
 333 the ability to classify data with the least similarity. An example is given below which
 334 uses the proposed method to assess the seismic vulnerability of buildings.

335 The steps of the proposed method are shown in Figure (3). In this diagram $i=1,2,\dots,n, j=1,2,\dots,m$.

336

337 3.1. Validation

338 In 1936, Fischer, field-examined 150 irises (43). As shown in Table (1), he used his intuitive
 339 experience to classify these flowers into three categories based on four characteristics (see Table
 340 (2)). For validation, iris flowers were examined and classified by Fisher (data and results of the
 341 study are available) and then classified again using the proposed method.

Table (1): Types of irises

Row	Flower Name
1	Setosa
2	Virginica
3	Versicolor

Table (2): Characteristics of flowers

Row	Characteristics	Size (cm)
1	Sepal length	{4.3,4.4,...,7.9}
2	Sepal width	{2,2.1,...,4.4}
3	Petal length	{1,1.1,...,6.9}
4	Petal Width	{0.1,0.2,...,2.5}

342

343 The collection of flowers are defined and introduced as follows:

$$344 \quad X = \{(x_1, x_2, x_3, x_4) \mid x_j \in X_j, j=1, \dots, 4\}$$

345 such that:

$$346 \quad X_1 = \{4.3, 4.4, \dots, 7.9\}, X_2 = \{2, 2.1, \dots, 4.4\}, X_3 = \{1, 1.1, \dots, 6.9\}, X_4 = \{0.1, 0.2, \dots, 2.5\}$$

347 In the next step, we implement the concept of the classes in Table (1) for each of the properties
348 of Table (2) as sets X_i . With this implementation, classes (sets E_{ij}) are defined in each property.

349 Now, according to the definition of 3.1, we form a matrix E :

350

$$351 \quad E = \begin{pmatrix} \{4.3, 4.4, \dots, 5.8\} & \{4.9, 5.0, \dots, 7.0\} & \{4.9, 5.0, \dots, 7.9\} \\ \{2.0, 2.1, \dots, 3.4\} & \{2.2, 2.3, \dots, 3.8\} & \{2.3, 2.4, \dots, 4.4\} \\ \{1.0, 1.1, \dots, 1.9\} & \{3.0, 3.1, \dots, 5.1\} & \{4.5, 4.6, \dots, 6.9\} \\ \{0.1, 0.2, \dots, 0.6\} & \{1.0, 1.1, \dots, 1.8\} & \{1.4, 1.5, \dots, 2.5\} \end{pmatrix}$$

352 3.1.1 Description

353 The elements of the first row of the matrix, the classes defined on the set of values of the sepals,
354 are one of the four characteristics of the lily (set X_1). This classification is defined based on Fisher's
355 intuitive observations (the other lines are the same). These classes define $H_1=(X_1, E_1)$, $H_2=(X_2, E_2)$,
356 $H_3=(X_3, E_3)$, $H_4=(X_4, E_4)$, hypergraphs on four sets of Fisher values (Lemma 3.3), respectively. The
357 sepal length property is classified E_{11} , E_{12} , E_{13} such that:

$$358 \quad E_{11} = \{4.3, 4.4, \dots, 5.8\}, E_{12} = \{4.9, 5.0, \dots, 7.0\}, E_{13} = \{4.9, 5.0, \dots, 7.9\}$$

359 Clearly $E_{1j} \neq \emptyset$, for $j=1, 2, 3$ and $E_1 = \bigcup_{j=1}^3 E_{1j}$ (Lemma 3.3).

360 Each class represents a type of flower.

361 Each of the classes in Table (2) represents a type of iris. The proposed method for determining the
 362 structure and members of each type of iris defines a fuzzy hypergraph on the X set. Based on
 363 Definition 3.4, we form a matrix of θ_x for each iris flower. For this purpose, we define fuzzy
 364 membership functions $\mu_{ij}(x)$, $\mu_{ij}^c(x)$ on sets E_{ij}^c , E_{ij} .

365
$$\mu_{ij}^c(x) : E_{ij}^c \rightarrow [0,1] \text{ such that } \mu_{ij}^c(x) = 0$$

366
$$\mu_{ij}(x) : E_{ij} \rightarrow [0,1]$$

367
$$\mu_{ij}(x) = \begin{cases} \frac{(x - \text{mode}_{E_{ij}})}{(\text{mode}_{E_{ij}} - \min_{E_{ij}})} + 1 & \text{where } x \leq \text{mode}_{E_{ij}}, \text{mode}_{E_{ij}} \neq \min_{E_{ij}} \\ \frac{(x - \text{mode}_{E_{ij}})}{(\text{mode}_{E_{ij}} - \max_{E_{ij}})} + 1 & \text{where } x > \text{mode}_{E_{ij}}, \text{mode}_{E_{ij}} \neq \max_{E_{ij}} \\ 0.00001 \times (x - \text{mode}_{E_{ij}}) + 1 & \text{where } \begin{matrix} \text{mode}_{E_{ij}} = \min_{E_{ij}} \\ \text{or} \\ \text{mode}_{E_{ij}} = \max_{E_{ij}} \end{matrix} \end{cases}$$

$i=1,2,3,4 \quad j=1,2,3$

368 It is clear that $\text{mode}_{E_{ij}}$ is a member of E_{ij} with the most repetition.

369 Now, using Definition 3.5, the values of $\text{score}_j(x) \geq 0$ for the elements of set X are calculated. To
 370 better clarify the content, a sample of iris, which is shown with the symbol x , has been placed in
 371 the process of the proposed method. Consider the sample iris with specifications
 372 $x = (5.1, 3.5, 1.4, 0.2)$. We form matrix θ_x based on membership functions $\mu_{ij}(x)$ and $\mu_{ij}^c(x)$:

373

374

$$\theta_x = \begin{pmatrix} 0.875 & 0.95 & 0.967 \\ 0.643 & 0 & 0.375 \\ 0.8 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

375

376 Then, by Defining 3.5, the values of $\text{score}_j(x)$ are calculated:

377

$$\text{score}_1(x) = 3.318$$

$$\text{score}_2(x) = 0.95$$

$$\text{score}_3(x) = 1.341$$

378 Now by Defining 3.7 and using the membership function μ , the values of $\text{DoM}_{\text{score}_j}$ for $j = 1, 2, 3$
 379 are calculated.

380

$$\mu: [0, 4] \rightarrow [0, 1] \text{ such that } \mu(x) = x / 4$$

381

$$\text{DoM}_{\text{score}_1} = 0.829464286$$

$$\text{DoM}_{\text{score}_2} = 0.2375$$

$$\text{DoM}_{\text{score}_3} = 0.33512931$$

382 According to the Definition 3.8, the sample iris belongs to class K_1 (see Table (3)) with a
 383 membership degree of 0.829464286. In Table (4), the performed classification and Fisher's
 384 experimental classification are given and compared for some samples of irises.

385 In samples 1-2-3-54-56-59-60-61, the experimental results are consistent with the classification of
 386 the proposed method. This agreement is more than 90% in 150 samples. In sample 58, the
 387 experimental results do not match the classification of the proposed method. But the degree of
 388 membership to each class is very low, and this is due to the over-dispersion of the sample property

values. The experimental results do not match to the classification of the proposed method for No. 53.

Table (3): Flower Class

Row	Flower Name	Class
1	Setosa	K_1
2	Virginica	K_2
3	Versicolor	K_3

Table (4): Flower Sample

Sample No	Flower characteristics				Flower type based on experimental method	Flower type based on proposed method	Degree of belonging to the class	Class membership rate (DoM_{score_j})		
	Sepal length	Sepal width	Petal length	Petal Width				Iris-setosa	Iris-virginica	Iris-versicolor
1	5.1	3.5	1.4	0.2	Iris-setosa	Iris-setosa	0.829464286	0.829464286	0.33512931	0.2375
2	4.9	3	1.4	0.2	Iris-setosa	Iris-setosa	0.914285714	0.914285714	0.25	0.25
3	4.7	3.2	1.3	0.2	Iris-setosa	Iris-setosa	0.757142857	0.757142857	0.1875	0.125
53	6.9	3.1	4.9	1.5	Iris-versicolor	Iris-virginica	0.76329023	0.232142857	0.76329023	0.433333333
54	5.5	2.3	4	1.3	Iris-versicolor	Iris-versicolor	0.679166667	0.09375	0.238146552	0.679166667
56	5.7	2.8	4.5	1.3	Iris-versicolor	Iris-versicolor	0.8625	0.209821429	0.627155172	0.8625
58	4.9	2.4	3.3	1	Iris-versicolor	Iris-setosa	0.25	0.25	0.0625	0
59	6.6	2.9	4.6	1.3	Iris-versicolor	Iris-versicolor	0.733333333	0.214285714	0.570402299	0.733333333
60	5.2	2.7	3.9	1.4	Iris-versicolor	Iris-versicolor	0.75	0.330357143	0.389008621	0.75
61	5	2	3.5	1	Iris-versicolor	Iris-versicolor	0.333333333	0.25	0.25	0.333333333

In this method, in addition to the possibility of classifying samples with any number of characteristics and any amount of information scatter, it is also possible to determine the degree of membership to the class. There is also a lot of flexibility in this method by selecting membership functions according to the classification structure. The results of the proposed method are shown in the following diagrams.

Figure (4): Setosa membership degree chart

Figure (4) shows that 95% of the elements with a membership degree of more than 0.3 are in the Iris-setosa flower class, and this shows the accuracy of the method.

Figure (5): Versicolor membership degree chart

Figure (5) also shows that the membership of elements in the Iris-versicolor flower class is more than 0.3.

Figure (6): Virginica membership degree chart

Figure (6) shows that a number of elements with a membership of less than 0.2 are in the Iris-virginica class. This is due to the high dispersion of data, but the proposed method has the ability to classify, which is one of the strengths of this method. The comparison of the classes of Table (5) shows the differences between the proposed method and the results of the field study and Fisher's intuitive experiences.

Table: (5): Comparison of the classes

Sample No	Flower characteristics				Flower type based on experimental method	Flower type based on proposed method	Degree of belonging to the class	Class membership rate		
	Sepal length	Sepal width	Petal length	Petal Width				Iris-setosa	Iris-virginica	Iris-versicolor
51	7	3.2	4.7	1.4	Iris-versicolor	Iris-virginica	0.494252874	0.214285714	0.494252874	0.491666667
52	6.4	3.2	4.5	1.5	Iris-versicolor	Iris-virginica	0.816810345	0.214285714	0.816810345	0.6
53	6.9	3.1	4.9	1.5	Iris-versicolor	Iris-virginica	0.76329023	0.232142857	0.76329023	0.433333333
55	6.5	2.8	4.6	1.5	Iris-versicolor	Iris-virginica	0.797772989	0.178571429	0.797772989	0.620833333
57	6.3	3.3	4.7	1.6	Iris-versicolor	Iris-virginica	0.748347701	0.196428571	0.748347701	0.416666667
58	4.9	2.4	3.3	1	Iris-versicolor	Iris-setosa	0.25	0.25	0.0625	0
67	5.6	3	4.5	1.5	Iris-versicolor	Iris-virginica	0.948275862	0.3125	0.948275862	0.825
69	6.2	2.2	4.5	1.5	Iris-versicolor	Iris-virginica	0.646551724	0	0.646551724	0.55
71	5.9	3.2	4.8	1.8	Iris-versicolor	Iris-virginica	0.753663793	0.214285714	0.753663793	0.3875
73	6.3	2.5	4.9	1.5	Iris-versicolor	Iris-virginica	0.690014368	0.071428571	0.690014368	0.445833333
78	6.7	3	5	1.7	Iris-versicolor	Iris-virginica	0.751364943	0.25	0.751364943	0.379166667
79	6	2.9	4.5	1.5	Iris-versicolor	Iris-virginica	0.882543103	0.214285714	0.882543103	0.75
84	6	2.7	5.1	1.6	Iris-versicolor	Iris-virginica	0.732543103	0.142857143	0.732543103	0.4
85	5.4	3	4.5	1.5	Iris-versicolor	Iris-virginica	0.965517241	0.375	0.965517241	0.85
86	6	3.4	4.5	1.6	Iris-versicolor	Iris-virginica	0.763793103	0.178571429	0.763793103	0.475

87	6.7	3.1	4.7	1.5	Iris-versicolor	Iris-virginica	0.801364943	0.232142857	0.801364943	0.541666667
135	6.1	2.6	5.6	1.4	Iris-virginica	Iris-versicolor	0.4625	0.107142857	0.41558908	0.4625

Table (6): Title of risk

Row	Risk level	Class
1	Low-risk	K ₁
2	Moderate-risk	K ₂
3	High-risk	K ₃
4	Very High-risk	K ₄

4. Case study

In this example, the proposed method is used to assess the seismic risk of 50,000 buildings. The classification of buildings is done in four classes in Table (6), taking into account the simultaneous impact of ten building Characteristics of Table (7).

Table (7): Characteristics of building

Index	Rate	Risk level in Characteristics			
		Low	Moderate	High	Very High
Danger of earth liquefaction (Slope of the earth in percentage)	X ₁ =[0-45]	E ₁₁ =[0-3.86]	E ₁₂ =[3.86-12.11]	E ₁₃ =[12.11-23.35]	E ₁₄ =[23.35-45]
Earth movement (Magnitude of Mercalli earthquake)	X ₂ =[1-10]	E ₂₁ =[1-3]	E ₂₂ =[3-5]	E ₂₃ =[5-6]	E ₂₄ =[6-10]

Distance of buildings from explosive stations (gas and gas stations) in meters	$X_3=[0-2000]$	$E_{31}=[100-2000]$	$E_{32}=[50-100]$	$E_{33}=[100-2000]$	$E_{34}=[0-20]$
Type of materials and structures used in buildings	$X_4=\{1,2,\dots,8\}$	$E_{41}=\{1,2\}$	$E_{42}=\{3,4\}$	$E_{43}=\{5,\}$	$E_{44}=\{7,8\}$
Type of work of buildings (per person)	$X_5=[0-1000]$	$E_{51}=[1-25]$	$E_{52}=[25-75]$	$E_{53}=[70-100]$	$E_{54}=[100-1000]$
Age of buildings (by year)	$X_6=[0-100]$	$E_{61}=[0-5]$	$E_{62}=[5-15]$	$E_{63}=[15-25]$	$E_{64}=[25-100]$
Height of buildings (in meters)	$X_7=[1-200]$	$E_{71}=[1-3]$	$E_{72}=[3-5]$	$E_{73}=[5-15]$	$E_{74}=[15-200]$
Building quality	$X_8=\{1,2,3,4\}$	$E_{81}=1$	$E_{82}=2$	$E_{83}=3$	$E_{84}=4$
Density of buildings (in percentage)	$X_9=[0-100]$	$E_{91}=[0-25]$	$E_{92}=[25-50]$	$E_{93}=[50-75]$	$E_{94}=[75-100]$
Passage width (in meters)	$X_{10}=[0-100]$	$E_{10\ 1}=[20-100]$	$E_{10\ 2}=[15-20]$	$E_{10\ 3}=[12-15]$	$E_{10\ 4}=[1-12]$

438

439 We display the set of buildings as set X as follows :

440 $X=\{(x_1, x_2, x_3, \dots, x_{10}) \mid x_i \in X_i, i=1, \dots, 10\}$, X_i sets are defined in Table (7).

441

442 Table (8): Characteristics frequency table

characteristics	Min	Max	average	standard deviation
x_1	1	44.998	3.7758	12.6818
x_2	1	10	2.3182	2.8091
x_3	0	2000	19.6833	514.5335
x_4	1	8	2.1224	2.2875
x_5	1	1000	16.2825	406.8242
x_6	0	100	5.9288	31.8835

x_7	2	200	7.2686	53.8127
x_8	1	4	1.5818	1.1188
x_9	0	100	5.9418	31.8808
x_{10}	1	100	5.9477	31.7565

443

444

445 The boundaries of the classes for each of the ten properties are specified in Table (7) (risk level in
446 characteristics columns). As mentioned earlier, defining the class structure in Table (6) for
447 buildings (set X) is one of the main challenges. For this purpose, based on the proposed method,
448 we define a granular structure based on fuzzy hypergraphs.

449 In the first step, based on the Definition 3.1 elements of the matrix E, which are the classes of each
450 property, X_i is defined based on the concept of the classes in Table (6). As shown in Table (7),
451 these classes are listed in four columns: Low, Moderate, High, Very High.

452 To provide the best data, the frequency tables of properties (see Table (8)) and the percentage of
453 class properties (Table (9)) and the composition chart (see Figure (7)) of the class properties are
454 given.

455

Table (9): Percentage of characteristics belonging to classes table

Characteristics Symbol	low-risk	moderate-risk	high-risk	very high-risk
x_1	%18.996	%43.714	%12.704	%24.586
x_2	%20.434	%20.798	%10.2	%48.568
x_3	%46.37	%14.552	%8.486	%30.592
x_4	%24.846	%25.108	%25.116	%24.93
x_5	%31.882	%15.174	%7.558	%45.386
x_6	%18.034	%24.956	%6.452	%50.558

x_7	%4.31	%8.978	%26.526	%60.186
x_8	%25.08	%24.628	%25.288	%25.004
x_9	%49.276	%16.82	%16.534	%17.37
x_{10}	%53.928	%3.328	%2.112	%40.632

Figure (7): Percentage of characteristics belonging to classes chart

Now we define the fuzzy membership functions on the E_{ij} interval. In this example, Gaussian functions are used to determine the degree of membership of properties to each of the four classes defined on the characteristics set. Functions are defined as follows:

It is clear $E_{ij} \subset X_i$, $E_{ij}^c = X_i - E_{ij}$ for $i=1,2,\dots,10$, $j=1,2,3,4$.

$$\mu_{i1} : E_{i1} \rightarrow [0,1] \text{ such that } \mu_{i1}(x) = e^{\frac{\ln(0.001) \times (x - a_{i1})^2}{c_{i1}^2}} \text{ where } E_{i1} = [a_{i1}, c_{i1}] , i=1,2,3,\dots,10$$

$$464 \quad \mu_{i1}^c : E_{i1}^c \rightarrow [0,1] \text{ such that } \mu_{i1}^c(x) = e^{\frac{\ln(0.001) \times (x-b_{i1})^2}{b_{i1}^2}} \text{ where } E_{i1}^c = (c_{i1}, b_{i1}] , i=1,2,3,...,10$$

$$465 \quad \mu_{i2} : E_{i2} \rightarrow [0,1] \text{ such that } \mu_{i2}(x) = e^{\frac{\ln(0.001) \times (x - \frac{c_{i2}+d_{i2}}{2})^2}{(\frac{c_{i2}+d_{i2}}{2})^2}} \text{ where } E_{i2} = [c_{i2}, d_{i2}] , i=1,2,3,...,10$$

$$466 \quad \mu_{i2}^c : E_{i2}^c \rightarrow [0,1] \text{ such that } \mu_{i2}^c(x) = \begin{cases} e^{\frac{\ln(0.001) \times (x + (b_{i2} - \frac{(c_{i2}+d_{i2})}{2}))^2}{b_{i2}^2}} & \text{where } x \in [a_{i2}, c_{i2}) \\ e^{\frac{\ln(0.001) \times (x - (b_{i2} - \frac{(c_{i2}+d_{i2})}{2}))^2}{b_{i2}^2}} & \text{where } x \in (d_{i2}, b_{i2}] \end{cases} \quad i=1,2,3,...,10$$

$$467 \quad \mu_{i3} : E_{i3} \rightarrow [0,1] \text{ such that } \mu_{i3}(x) = e^{\frac{\ln(0.001) \times (x - \frac{d_{i3}+e_{i3}}{2})^2}{(\frac{d_{i3}+e_{i3}}{2})^2}} \text{ where } E_{i3} = [d_{i3}, e_{i3}] , i=1,2,3,...,10$$

$$468 \quad \mu_{i3}^c : E_{i3}^c \rightarrow [0,1] \text{ such that } \mu_{i3}^c(x) = \begin{cases} e^{\frac{\ln(0.001) \times (x + (b_{i3} - \frac{e_{i3}+d_{i3}}{2}))^2}{b_{i3}^2}} & \text{where } x \in [a_{i3}, d_{i3}) \\ e^{\frac{\ln(0.001) \times (x - (b_{i3} - \frac{e_{i3}+d_{i3}}{2}))^2}{b_{i3}^2}} & \text{where } x \in (e_{i3}, b_{i3}] \end{cases} \quad i=1,2,3,...,10$$

$$469 \quad \mu_{i4} : E_{i4} \rightarrow [0,1] \text{ such that } \mu_{i4}(x) = e^{\frac{\ln(0.001) \times (x-b_{i4})^2}{(b_{i4}-e_{i4})^2}} \text{ where } E_{i4} = [e_{i4}, b_{i4}] , i=1,2,3,...,10$$

$$470 \quad \mu_{i4}^c : E_{i4}^c \rightarrow [0,1] \text{ such that } \mu_{i4}^c(x) = e^{\frac{\ln(0.001) \times (x-a_{i4})^2}{e_{i4}^2}} \text{ where } E_{i4}^c = [a_{i4}, e_{i4}) , i=1,2,3,...,10$$

471 In the following, the membership of $x_{10} = (5, 4, 669, 2, 60, 43, 103, 3, 11, 49)$ element in the four risk

472 categories is examined. We form matrix θ_x (Definition 3.4) based on the above membership

473 functions. By forming matrix θ_x , the score values are calculated based on the Definition 3.5.

$$\theta_x = \begin{pmatrix} -0.0043 & 0.3809 & -0.02870 & -0.9183 \\ -0.0832 & 1 & -0.0068 & -0.537 \\ 0.0469 & -0.0329 & -0.0399 & -0.0441 \\ 1 & -0.1667 & -0.5 & -0.8333 \\ -0.0022 & 0.7586 & -0.0015 & -0.9762 \\ -0.106 & -0.045 & -0.0166 & 0.0185 \\ -0.1969 & -0.1718 & -0.1359 & 0.1497 \\ -0.6667 & -0.3333 & 1 & -0.3333 \\ 0.2625 & -0.024 & -0.1969 & -0.9198 \\ 0.1658 & -0.0391 & -0.0565 & -0.0983 \end{pmatrix}$$

From Definition 3.5:

$$\begin{aligned} \text{score}_1(x) &= 0.4159 \\ \text{score}_2(x) &= 1.3267 \\ \text{score}_3(x) &= 0.0172 \\ \text{score}_4(x) &= -4.5179 \end{aligned}$$

By Definition 3.7 and functions μ, η the values $\text{DoNM}_{\text{score}_j}, \text{DoM}_{\text{score}_j}$ are calculated for $i=1,2,3,4$.

$$\mu : [0, n] \rightarrow [0, 1] \text{ such that } \mu(x) = e^{\frac{-6.90*(x-10)^2}{100}}$$

and

$$\eta : [-n, 0] \rightarrow [0, 1] \text{ such that } \eta(x) = e^{\frac{-6.90*(x+10)^2}{100}}$$

From Definition 3.7

$$\begin{aligned}\text{DoM}_{\text{score}_1}(x) &= \mu(\text{score}_1(x)) = \mu(0.4159) = 0.0018 \\ \text{DoM}_{\text{score}_2}(x) &= \mu(\text{score}_2(x)) = \mu(1.3267) = 0.0056 \\ \text{DoM}_{\text{score}_3}(x) &= \mu(\text{score}_3(x)) = \mu(0.0172) = 0.001 \\ \text{DoNM}_{\text{score}_4}(x) &= \eta(\text{score}_4(x)) = \eta(-4.5179) = 0.1253.\end{aligned}$$

Because there exists the largest membership degree therefore, by Definition 3.8 we consider element $x_{10} = (5, 4, 669, 2, 60, 43, 103, 3, 11, 49)$ to belong to the moderate-risk class with a membership grade of 0.0056.

Due to the high dispersion of the properties of this element, the membership grade is not strong in any of the grade classes, but this method has the ability to classify with the least similarity.

This is how other buildings are classified. Table (10) shows the percentage of 50,000 buildings belonging to the four classes.

Table (10): Percentage of affiliation

Low-risk	Moderate-risk	High-risk	Very-high	No classification
%15.154	%10.882	%26.936	%35.468	%11.56

5. Conclusions

In this paper, a method for assessing the vulnerability of urban areas was presented using granular calculations and the concepts of fuzzy hypergraphs. In the proposed model, it was shown that fuzzy hypergraphs, compared to other methods, have the ability to define a flexible granular structure for different urban hazard classes. As an innovation of the proposed method, we can mention its

500 flexibility in classifying elements, considering the role of all effective parameters, both equal
501 weight and heterogeneity in classification.

502 In verifying the proposed method, it has been shown that the classification of iris flowers with the
503 proposed method is more than 90% consistent with the intuitive classification of Fisher. The low
504 degree of membership of some flowers in all three classes indicates an excessive dispersion of
505 characteristics.

506 Using the proposed method in assessing the seismic vulnerability of 50,000 buildings, it is possible
507 to simultaneously play the role of 10 heterogeneous and sometimes equal weight properties in the
508 classification .

509 Examination of the obtained results indicates the flexibility and accuracy of the proposed method
510 in classifying elements with any number of heterogeneous and equal parameters. This study shows
511 that fuzzy hypergraphs have the ability to define a flexible granular structure, taking into account
512 the simultaneous role of all effective parameters in classification.

513

514 **Declarations**

515 • Availability of data and materials

516 The datasets used and/or analyzed during the current study are available from the
517 corresponding author on reasonable request.

518 • Competing interests

519 The authors declare that they have no competing interests.

520 • Funding

521 There are no sources of funding for the research reported.

522 • Authors' contributions

Zarandi Baghini performed a new method of granular computing and developed the fuzzy hypergraphs and also analyzed and interpreted the computer proceeding data regarding the urban vulnerability and the verification of the previous experimental study. Tabatabaei Mirhosseini performed the background to the study, and was a major contributor in writing the manuscript. Babaei read and approved the final manuscript.

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