A new global optimum solution model based on multi-value dynamic programming for the cut-off grade optimization in a surface mining complex

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Title page

Title of the Manuscript

“A new global optimum solution model based on multi-value dynamic programming for the cut-off grade optimization in a surface mining complex”

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Article Highlights

- A new global optimum solution model based on multi-value dynamic programming (MDP) for the cut-off grade (COG) optimization for surface mining applications.
- This discrete *COG-MDP* model has the novelty of dealing with the simulation of eight different optimization possibilities to achieve the global maximum Net Present Value (NPV).
- A sequentially advancing algorithm has been developed for the simulation of the model for the mineral grades in the field of *COG* optimization.
- The computer model is developed in Python to contend with the complexities of processing a minimum of 500 series of dynamic variables associated with the complex system of mine, mill, and smelter and refinery with an accuracy value of 0.01% in grade bins.

A new global optimum solution model based on multi-value dynamic programming for the cut-off grade optimization in a surface mining complex

Abstract

In techno-economic concern, cut-off grade (*COG*) optimization is the key for efficient mineral liquidation from the huge metalliferous surface mining sector. In this paper, a sequentially advancing algorithm based on discrete multi-value dynamic programming (MDP) has been developed to calculate the global optimum *COG* of a large-scale open-pit metalliferous deposit. The proposed *COG* optimization algorithm aims to overcome the limitations of straightforward classical techniques in determining the optimum *COG*. This discrete *COG-MDP* model is the first of its kind and has the novelty of dealing with the simulation of eight dynamic possibilities to achieve the maximal global Net Present Value (NPV). A high-level programming language (Python) has been used to develop the computer model to deal with the complexity of handling a minimum of 500 series of dynamic variables. This model can generate results in polynomial-time from the complex of mining, milling, and smelting and refining system corresponding to various limiting conditions. The prime objective considered in the model is to optimize the *COG* of a metalliferous deposit. A working open-pit copper mining complex from India has been used to validate the model. In this study, the optimum *COG* for the Malanjhand copper deposit has been found to be (0.33%, 0.23%, 0.52%, 0.26%, 0.27%, 0.22%, 0.24%) with a maximum NPV of ₹ (12204, 14653, 16948, 14609, 21454, 26717, 38821) million corresponding to various scenarios. The findings also show that the present value gradually hits zero after the project's life cycle, confirming the typical pattern of other mining firms.

Keywords: Dynamic programming; global optimum model; cut-off grade (*COG*); surface mining; NPV maximization.

1. Introduction

Mineral resources are essential for contemporary society's socioeconomic development. Even though these commodities have been widely cross-referenced as "finite," their metal output has continuously risen throughout the ages. The twenty-first century has seen exponential resource extraction, corresponding to global economic expansion; consequently, long-term sustainable development is crucial. Global metal utilization is expanding at a rate of roughly 3.2% per year, boosting trade and commerce (Krausmann et al., 2018; Van Vuuren et al., 1999). Metal mining has consistently been shown to be a significant source for meeting metal demands in infrastructure, health, information systems, and other services that can support economic growth (Jowitt et al., 2020). Thus, cut-off grade (*COG*) plays a significant role in achieving the objective of attaining techno-economic sustainability, which strives to maximize recovery utilization. The field of knowledge, exploration, findings, innovations, and technological progress required to meet a predetermined *COG* has an impact on the ultimate achievable resource capacities. The worldwide average *COG* for copper is approximately 0.49% (Henckens & Worrell, 2020; Mudd, 2012; Teseletso & Adachi, 2021).

Mineral economics is not a new field; this can be followed back 200 years to the research of classical economists like as Smith (Smith, 1776), Ricardo (Ricardo, 1821), and Jevons (Jevons, 1866), among others. Smith (Smith, 1776) noted that the demand for mineral resources appears to be unlimited due to customers' preferences and desire for manufactured products, which need a large number of fossil fuels and mineral resources to be exploited. Ricardo (Ricardo, 1821) highlighted that all mineral commodities are exposed to significant fluctuations, either temporary or permanent in nature, and of significant economic impact.

Surface mine planning is a dynamic and complicated system with a host of variables and multi-stages (Fig. 1), making it a complex problem that is hard to solve and demands a continuous financial risk assessment. Open-pit scheduling intends to find the best schedule of calendar programming across the mineral value cycle (Ali et al., 2021; Fathollahzadeh et al., 2021; Rezakhah & Newman, 2020). This is because the value of a mining project is usually affected by various underlying economic and physical variables, such as commodity prices, grades, expenses, timetables, volumes, and environmental concerns, which are unknown with confidence at the project's inception.

Fig. 1. A sequential multi-stages Surface mining operational flow diagram

2. Literature Review

Mining is a capital-intensive industry in which economic concerns dominate all activities. This is necessary for its long-term viability and growth. The *COG* of a mineral deposit is determined based on the techno-economic viability of production and profit obtained from the time a mine is opened until it is closed. *COG* is the lowest grade of ore that can be processed commercially. Therefore, this criterion creates a boundary inside a particular mineral deposit between ore and waste, either accepted for milling or rejected for dumping (Taylor, 1972, 1986).

In several contexts, mineral resources are classified as either ore or waste. The use of *COG* is a standard route to explore the ore-waste ratio and the total mineral content. All the *COG* optimization techniques can be divided broadly into traditional and modern methods. Traditional methods may further be divided into four major sub-categories: mathematical programming, integer programming, linear programming, and nonlinear programming. On the other hand, modern techniques may be divided broadly into two major categories: the evolutionary algorithm and the Monte Carlo simulation.

Out of all the *COG* techniques, the maximum number of work may be attributed to the analytical approaches, but none can yield the optimal results. Henning (Henning, 1963) was the first who brought the idea of *COG* optimization into the mining business and a year later the seminal work of Lane (Lane, 1964) was introduced. Lane determined optimal *COGs* based on limited capacities, supporting Henning's (Henning, 1963) prediction that ideal *COGs* tend

to drop with time. Henning's method keeps certain operational parameters fixed throughout the project's life cycle and does not account for the potential of capacity expansion. Following Lane's work, the amount of literature on the topic has expanded exponentially.

Through, globally the breakeven *COG* and Lane's (Lane, 1964, 1988) algorithm are being used commonly to maximize profit. However, maintaining a steady breakeven *COG* across the mine life would have serious consequences, potentially exposing the mining operation to considerably sub-optimal operating results. Lane's method also solved the breakeven model's flaws. It considered the production restrictions of various phases and the relevance of the grade-tonnage characteristics. Unfortunately, Lane's (Lane, 1964, 1988) method has been widely criticized for *COG* determination that may be more suitable in certain circumstances, making it possible to disregard the optimal estimates for *COGs* (Asad, 2005). Lane's technique has stayed empirical, necessitating the development and implementation of an analytical optimization technique (Dagdelen & Kawahata, 2008, 2007). The application of mathematical programming to open-pit mine planning traces back to the 1960s. Acknowledging the limitations of Lane's approach, a few attempts have been made to overcome them through an alternative analytical framework of optimization (Azimi et al., 2012; Dagdelen & Kawahata, 2008; Ganguli et al., 2011; Yasrebi et al., 2015). The basics of Lane's (Lane, 1964, 1988) technique have been expanded in most optimization approaches. Several studies employed time-value-of-money-based optimization approaches. The *COG* optimization algorithms that involve *NPV* maximization lead to better outcomes than those that maximize the profit function.

Lane (Lane, 1988) effectively used the deterministic dynamic programming concepts to the optimization of *COG* policies. However, he advocated to find an optimal operating strategy and the associated maximum present value in a specific condition, the sum of the maximum present value surface is irrelevant. Every point along the optimal strategy path must be optimal, and all corresponding current values must be maxima. Assuming that future prices would vary, he proposed a dynamic programming method that can assure that the best *COG* is picked at all times. However, it can only be approximated by forecasting, which is fraught with risk. Dowd (Dowd, 1976) demonstrated the technique that might be used for stochastic data with the objective function of maximizing the present value of future net profits. Several conventional and meta-heuristic techniques are being applied extensively to optimize pit scheduling. Later, Wang (Q. Wang, 1995) and Tolwinski (Tolwinski, 1998) studied dynamic programming to study block sequencing. Due to the periodic extraction of finite resources, the efficiency of dynamic programming approaches has shown to be helpful in scheduling mining operations for simultaneous optimization of several policies (B. King, 2000). Linear programming, for example, might be used to maximize cash flow locally, whereas dynamic programming could be used to improve *NPV* on a global level.

Baidowi et al. (Baidowi et al., 2021) established a mathematical model for *COG* optimization in the open-pit metalliferous deposits based on overall profit maximisation, incorporating reclamation income and expenditures. The authors proposed the consideration of environmental variables in the determination of *COGs*. Few authors emphasized the importance and integration of blending optimization throughout the mining value chain (Chanda & Dagdelen, 1995; Haonan et al., 2021; Jamshidi & Osanloo, 2018).

Dehghani and Bogdanovic (Dehghani & Bogdanovic, 2018) used a bat algorithm to estimate metal price swings, which produced better outcomes than the traditional estimation approach for forecasting copper price. This resulted in a price trend. However, it is questionable when there is a significant and rapid shift in demand and supply or a technological upgrade or technology replacement. Surface miner, for example, is a technological substitute that reduces overall equipment expenditure by around half, eliminating the requirement of drilling, blasting, and crushing. A genetic algorithm was employed by Franco-sepulveda et al. (Franco-sepulveda et al., 2019) to maximize the *NPV* of a mining operation. They looked at the price of the metal from a historical date to the present day and found that the sale value was 50% of the time lower than the projected value. Su et al. (Su et al., 2020) stated that the emergence and collapse of several price bubbles are connected to supposition, macroeconomic instability, a supply-demand imbalance, and economic turmoil. Tahar et al. (Tahar et al., 2021) agreed that commodities prices shocks might have differential effects. Most of the commodities' economies are highly volatile and difficult to anticipate. Considering an incorrect pricing forecast leads to sub-optimal *COG* selections and implications.

Russell (Russell, 1970) pioneered the concept of maximizing a project's *NPV*. The most commonly used economic metric in evaluating financial projects is *NPV* (Creemers, 2018; Peymankar et al., 2021). It is worth noting that *NPV* is acutely susceptible to revenue variations (Hustrulid et al., 2013; Oraee et al., 2011; Saleh & Marais, 2006; Wiesemann et al., 2010). Ahmadi (Ahmadi, 2018) optimized the *COG* based on Lane's theory to maximize the *NPV* using MATLAB. The optimum *COG* is the grade that maximizes the chosen objective function, usually the *NPV* (Asad & Topal, 2011; Biswas et al., 2020). Various researchers (Ahmadi & Shahabi, 2018; Bascetin & Nieto, 2007; Qingfei Wang et al., 2010) have given several algorithms to determine optimum *COG*. They concluded that evaluating the *COG* based on maximizing the *NPV* produces more accurate findings than the other methods. The *COG* that leads to the highest *NPV* for the project must be selected.

Determination of optimum *COG* is an essential and crucial aspect to exploit any metalliferous deposits efficiently and economically. Metal prices are dynamic in nature and follow an asymmetrical trend. So, one cannot fix the production schedule having a random *COG* without optimizing the objective function to reach the maximum *NPV*. The stage pits can be scheduled strategically and optimally once the ultimate pit has been set, keeping the maximum discounted and undiscounted block value in mind. The target of the strategic mine plan is to maximize *NPV* while adhering to all limitations (Jélvez et al., 2020; Samavati et al., 2018). If one doesn't optimize or check the *COG* periodically, there might be a time when the mine can face a cash crisis and needs to be shut off abruptly due to the possibility of negative *IRR* in the long run. So, the mine planners should plan considering all the price dynamics, *COGs*, grade-tonnage ratio, reserve, and market scenario. At the end of the exploration stage, the ore body model is first finalized, and then the planning of mine development and project scheduling are carried out accordingly. A planning job requires a multitude of functions. In general, the mine planning comprises the steps as shown in the sequential flowchart of the whole process given below in Fig. 2.

Fig. 2. The flowchart describes the procedural steps in mine planning.

3. Optimization model framework

The proposed multi-value dynamic programming (*MDP*) framework has been established with the knowledge that the optimum *COG* for a surface mining complex is dynamic. A high-level programming language (Python) has been utilized for the simulation of the model. The entire set of mathematical programming is based on algebraic equations, calculus of variation, and stepwise computation that meet the requirements of a sequentially advancing algorithm to determine the global optimum *COG*. Real working data from the surface mining complex of Hindustan Copper Limited (HCL) has been used to validate the algorithm's efficacy and applicability.

3.1 Functional model of multi-varying conditions

The optimum *COG* in an open-pit metalliferous deposit is primarily influenced by three controlling capacity restrictions of several activities, namely mining, milling, smelting, and refining. While estimating the global optimum *COG*, eight alternative solutions may be explored based on the varied limiting factors and production capacity of the three phases. (Table 1).

Table 1 Eight dynamic models of potential combinations based on the multi-conditional scenario.

Models	Mine capacity	Mill capacity	Refinery capacity	Restrictions or constraints (L - Limited and UL – Unlimited)
1.	UL	UL	UL	All capacities are unlimited
2.	L	UL	UL	Mine capacity is limited
3.	UL	L	UL	Mill (plant) capacity is limited
4.	UL	UL	L	Refinery (smelter) capacity is limited
5.	L	L	UL	Mine and mill capacities are limited
6.	L	UL	L	Mine and refinery capacities are limited
7.	UL	L	L	Mill and refinery capacities are limited
8.	L	L	L	All (mine, mill and refinery) capacities are limited

3.2 Model assumptions

It is presumed that the whole reserve will be mined according to the grade dissemination, but only that fraction of the mined-out material that exceeds or equals the *COG* should be sent for concentration. Furthermore, the metal price, operational expenses, capital costs, rate of interest, tax percentage, and discount rate have all been assumed to be uniform during the life of mine. However, the volatility of the commodity price indices and technical developments might reshape the entire scenario of expenses related to excavation, processing and future revenue. To make the work comprehensible, it has been assumed that, at least on average, all ore mined within any twelve months will also be processed during that period.

3.3 Model framework based on multi-value dynamic programming

To determine the optimum *COG* for an open pit metalliferous deposits, one must express the problem as a mathematical programming model. The mathematical models for dealing with multi-varying *COG* or production scheduling are nonlinear discrete optimization models. The computational complexity of such issues necessitates the development of an efficient solution technique. The creation of the mathematical models and the solution technique have been described below.

The grade-tonnage (*g-t*) variance of a mineral deposit is the input state of the framework with an individual domain (G, T), as correlated in Fig. 3. For every range of (α, β) , the *g-t* variance is distributed inside the range of $G_{\alpha, \beta}$ and $T_{\alpha, \beta}$. Therefore, the functional relation is given by $f : G \rightarrow T$. All '*G*' values shown under the grade-bins is set to ' $G_{\alpha, \beta}$ '. For any '*G*' values, there are a '*T*' values corresponding to set $(T_{\alpha, \beta})$; such that $T_{\alpha, \beta} = f(G_{\alpha, \beta})$ of Range, $R \in \{G_{\alpha, \beta}, T_{\alpha, \beta}\}$.

Fig. 3. Functional framework of *g-t* variance.

Here, the *COG* (\$) for each set of ranges from $G_{\alpha, \beta}$ must be chosen, and the stages of transition from α to β must be determined for every sequence of values. The values from individual stages are stored, and the optimization for the following state begins. *COG* is the lowest grade at which a metalliferous deposit may be extracted profitably (or processed). Material with a grade higher than *COG* is termed ore, whereas a material with a grade lower than *COG* is considered waste.

The average grade ($\bar{g}_{(\alpha,\beta)}$) is determined from the ($g-t$) variance of a mineral deposit and the concentration of metallic value present (x) in it by following the weighted average formula expressed in equation (1).

$$\bar{g}_{(\alpha,\beta)} = f(x) = \frac{\int_{\alpha}^{\beta} G_{\alpha,\beta} \times T_{\alpha,\beta}(x) dx}{\int_{\alpha}^{\beta} T_{\alpha,\beta}(x) dx} \quad (1)$$

The amount of ore (T_o) and waste (T_w) from the ($g-t$) variance will be estimated from equations 2(a), 2(b), 2(c), and (3).

$$T_{o(\alpha,\beta)} = 0, \quad \text{where } G_{\alpha} < \text{¢}, \text{ and } G_{\beta} \leq \text{¢} \quad 2(a)$$

$$T_{o(\alpha,\beta)} = \sum_{\alpha, G_{\alpha} \leq \text{¢}}^{\beta, G_{\beta} > \text{¢}} T - \sum_{\alpha < \text{¢}}^{G_{\alpha} = \text{¢}} T, \quad \text{where } G_{\alpha} \leq \text{¢}, \text{ and } G_{\beta} > \text{¢} \quad 2(b)$$

$$T_{o(\alpha,\beta)} = \sum_{\alpha, G_{\alpha} \geq \text{¢}}^{\beta, G_{\beta} > \text{¢}} T - \sum_{\alpha < \text{¢}}^{G_{\alpha} = \text{¢}} T, \quad \text{where } G_{\alpha} \geq \text{¢}, \text{ and } G_{\beta} > \text{¢} \quad 2(c)$$

$$T_{w(\alpha,\beta)} = \sum_{\alpha}^{\beta} T - T_{o(\alpha,\beta)} \quad (3)$$

Where T is the total material ($T_o + T_w$) to be mined out.

The stripping ratio ($S_{\alpha,\beta}$) is a fundamental parameter to be considered while calculating the NPV of any mining complex. Mining professionals play a crucial part in determining $S_{\alpha,\beta}$ figures related to their mine advancement, total ore-waste ratio, and whether to maintain the value constant or increase it in a trend that affects their $NPVs$. Equation (4) is used for the calculation of $S_{\alpha,\beta}$.

$$S_{\alpha,\beta} = \frac{T_{o(\alpha,\beta)}}{T_{w(\alpha,\beta)}} \quad (4)$$

Mining companies make money by mining mineral resources and refining them into marketable products. During operation, the mining complex incurred both variable and fixed expenses. The revenue is determined by the tonnes of ore treated, $\bar{g}_{(\alpha,\beta)}$, the percent recoveries over-processing, and the product's current market value. Variable expenses are further divided into those related to mining and processing the ore and those related to digging and disposing of the waste. Drilling and blasting, transportation, disposal, restoration, and other expenditures are examples of variable expenses. General and administrative expenses and any additional costs that are not dependent on output are examples of fixed costs. The NPV may be calculated by discounting capital expenses and subtracting them from the discounted cash flow. Now sequentially utilizing all of the variables mentioned above, the $NPV_{\alpha,\beta}$ of a project is determined by equation (5):

$$NPV_{\alpha,\beta} = PV_{\alpha,\beta} - CC_{\alpha,\beta} \quad (5)$$

The original ruling criteria for all stages are $Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$ for achieving the optimized returns ($NPV_{\alpha,\beta}$), these input decision variables ($Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$) are regulated by the various limiting capacity functions. Depending upon the COG (¢) and the stage transition ($\alpha \leq \psi \leq \beta$), the tonnage of outputs at distinct stages, i.e., mining, milling, and smelting and refining, are estimated consequently by following any of the logical statement devised through equation (6) to (8).

Subject to:

$$Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n \quad (6)$$

$$Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n \quad (7)$$

$$Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n \quad (8)$$

If y is the overall metallurgical recovery, then

$$Y_{\alpha,\beta} = Y_{m(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (9)$$

Where:

$PV_{\alpha,\beta}$	=	present value cash flow
$CC_{\alpha,\beta}$	=	cash flow
∂	=	discount rate
n	=	period (year) indicator
$M_{(\alpha,\beta)}$	=	maximum mining capacity (tonnes / yr)
$C_{\alpha,\beta}$	=	maximum mill capacity to process ore (tonnes / yr)
$R_{(\alpha,\beta)}$	=	maximum refinery capacity of metal production (tonnes / yr)
Q_m	=	actual mine production rate (ore and waste) (tonnes / yr)
Q_c	=	actual ore processing rate in the mill (tonnes / yr)
Q_r	=	actual metal production rate in the refinery (tonnes/ yr)
$Y_{m(\alpha,\beta)}$	=	mining recovery
$Y_{c(\alpha,\beta)}$	=	mill recovery
$Y_{r(\alpha,\beta)}$	=	refinery recovery

Generally, *COG* optimization has been observed as a nonlinear dynamic system and can be solved using Dynamic Programming (DP) to obtain a globally optimum solution. The basic concept behind DP is to break down the optimization issue into a series of sub-problems, each representing the optimization of a single control action at a particular time. The value function, which translates the current state of the system to the *NPV* of future income, connects these sub-problems. The best control action at each time can be obtained by solving a sub-problem at that moment after the value function has been computed. These sub-problems are, in general, significantly easier than the original optimization issue; each sub-global problem's optimum may be quantified effectively.

Module 1: All Unlimited Conditions

When all the three stages are unlimited, it brings the most exciting and novel part of the optimization combination. This is being the first mathematical approach, which deals with all the unlimited capacities. Here all the three components *COG*, mine production, and mine-life, have been optimized simultaneously at the same time to determine the optimal *COG*. Here, for each of the *COG* – the corresponding ore-waste tonnage, stripping ratio, average grade, and cumulative tonnage has been calculated. Each *COG* value will be optimized starting with the total tonnage as the mine production from year 1 to 100. This *COG* – mine life and mine production optimization will give the year-wise production rate for the same *COG* value for years 1 to 100. After that, the *NPV* is calculated from year 1 to 100 for the same *COG* value. This whole process will be repeated for all the *COG* values of the grade-tonnage distribution curve. After this, there will be a final set of data from all the *COG* values having their

maximum NPV s with their corresponding optimal mine life and production; among all the optimum COG values, the one with the highest NPV is the global optimum COG .

The following assumptions, as given below, have been made for the sequence of working.

1. The activity time frame is deterministic.
2. The per-period resource requirements for activities and project resource availability are set and known.
3. The cash flow associated with each activity is not susceptible to change in the course; hence, no stochastic values have been used for the determination of COG .
4. There is no interruption in the performance of the activity once it has begun (no pre-emption).
5. No cancellations are permitted. Each activity must be finished to check its optimality performance.
6. Each activity is completed following the sequentially advancing algorithm. That paradigm is associated with a predictable activity length, fixed resource requirements, and a set cash flow.
7. When an activity is completed, net non-negative cash flows will occur.
8. The stockpiling option has not been considered.

Under the prior assumptions, the COG optimization aims to maximize the project's NPV and is expressed as mathematical programming; the following problem statements are developed in the form of equations to solve the dynamic COG optimality scenario.

$$\text{Maximize} \rightarrow NPV_{(\alpha,\beta)} \text{ subject to } \sum_{COG_{(\alpha,\beta)}=0}^{COG_{(\alpha,\beta)}=n} \sum_{t_{(\alpha,\beta)}=1}^{t_{(\alpha,\beta)}=n} \sum_{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=1}^{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=n} \quad (10)$$

$$\text{Maximize: } \sum_{a=0}^N \sum_{COG_{(\alpha,\beta)}=0}^{COG_{(\alpha,\beta)}=n} \sum_{t_{(\alpha,\beta)}=1}^{t_{(\alpha,\beta)}=n} \sum_{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=1}^{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=n} \cdot NPV_{(\alpha,\beta)} \cdot j_{at} \cdot \partial_t^a \quad (11)$$

Subject to:

$$COG_{(\alpha,\beta)} = \begin{cases} 0 \\ \cdot \\ \cdot \\ N \end{cases} \text{ for } \sum_{COG_{(\alpha,\beta)}=0}^{COG_{(\alpha,\beta)}=N} = NPV_{(\alpha,\beta)}(\max), R \in \{ G_{\alpha,\beta}, T_{\alpha,\beta} \}, \forall 0, 0.01, \dots N$$

$$Q_{m(\alpha,\beta)} = \begin{cases} \frac{(T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)})}{t_{(\alpha,\beta)}} \\ \cdot \\ \cdot \\ \frac{(T_{o(\alpha,\beta)n} + T_{w(\alpha,\beta)n})}{t_{(\alpha,\beta)n}} \end{cases} \text{ for } \sum_{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=1}^{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=n} = NPV_{(\alpha,\beta)}(\max), \forall 1, 2, \dots n; Q_{m(\alpha,\beta)} \in \int_{t=1}^{t=n} Q_{m_{final}(\alpha,\beta)}$$

Besides the optimization technique mentioned above for the determination of $Q_{m(\alpha,\beta)}$, the empirical-formulae based *mining production rates* have also been determined for the comparison and efficacy of the model utilizing the existing thumb rule given by Taylor (Taylor, 1986) and the modified Taylor's rule by Long (Long, 2009).

$$\text{Mine} - \text{life}_{(\alpha,\beta)Taylor} \cong 6.5 \times (\text{Ore tonnage in millions})^{\frac{1}{4}} \quad (12)$$

$$Q_{m(\alpha,\beta)Taylor} = \left\{ \frac{(T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)})}{\text{Mine-life}_{(\alpha,\beta)Taylor}} \right\} \quad (13)$$

$$Q_{m(\alpha,\beta) Long} = 0.123 * \text{tonnage}^{0.649} * 350 \quad (14)$$

$$t_{(\alpha,\beta)} = \begin{cases} 1 \\ 2 \\ \vdots \\ n \end{cases} \text{ for } \sum_{t_{(\alpha,\beta)}=1}^{t_{(\alpha,\beta)}=n} = NPV_{(\alpha,\beta)}(\text{max}), t_{(\alpha,\beta)} \in \{1, n\}, \forall 1, 2, \dots, n$$

Where, $j_{at} = \begin{cases} 1 & \text{if action 'a' is finished in time 'n', a multi value – sequential stages} \\ 0 & \text{otherwise, though no interruption in the completion of an action} \end{cases}$

∂ = The discount factor over n periods

The target function is to maximize the total of the present values of the activities' contributions discounted back to the project's commencement date. The total of the future values of the cash flows connected with the various phases (periods) of that activity, discounted to the end of the action, less the opportunity cost of the recovered investment capital required by the action, is the contribution of that activity. This figure is derived from:

$$NPV_{(\alpha,\beta)} = \text{Max} \int_{\text{cog}(\alpha,\beta)=0}^{\text{cog}(\alpha,\beta)=n} \int_{t_{(\alpha,\beta)}=1}^{t_{(\alpha,\beta)}=n} \int_{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=1}^{Q_{m(\alpha,\beta)} \in t_{(\alpha,\beta)}=n} [\{PV_{(\alpha,\beta)} - CC_{(\alpha,\beta)}\} e^{-\partial t} dt] \quad (15)$$

$$\alpha \leq \mathbb{C} \leq \beta$$

According to the mine-life, we calculate the production capacities and the following stage capacities, and then it follows the sequentially advancing optimization approach to determine the optimal COG value. Ultimately, the grade at which it contributes the Global maximal NPV value decides the global optimal COG value. The overview of the multi-value driven nested DP algorithmic approach is shown in Fig. 4.

Fig. 4. An overview of the multi-value driven nested dynamic programming algorithmic approach.

Module II: When mining capacity is limited and the rest two stages are unlimited, then the maximum value of *NPV* from the array is considered, neglecting that production rate is less than production capacity, taking the corresponding *COG*, and deciding the final capacities of the rest two stages.

When the capacity of mining is the controlling factor, then the decision state is, $Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$ are calculated using equations (16) through (18).

$$Q_{m(\alpha,\beta)} = T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)} \quad (16)$$

$$Q_{c(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{\{1 + S_{(\alpha,\beta)}\}} \times Y_{m(\alpha,\beta)} \quad (17)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (18)$$

Module III: When mining capacity and smelter & refinery capacities are unlimited, and mill capacity is limited, then the maximum value of *NPV* from the array is considered where the mill production is less than or equal to the mill capacity and take the corresponding *COG*, and hence deciding the final output from the rest two unlimited stages.

When the capacity of milling is the controlling factor, then the decision state is, $Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$ are calculated using equations (19) through (20).

$$Q_{m(\alpha,\beta)} = \left(\frac{T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)}}{T_{o(\alpha,\beta)}} \right) \times Q_{c(\alpha,\beta)} \times \left(\frac{1}{Y_{m(\alpha,\beta)}} \right) \quad (19)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (20)$$

Module IV: When mining capacity and mill capacity are unlimited, and smelter and refinery capacity is limited, then the maximum value of *NPV* from the array is considered, which satisfies the condition that corresponding smelter and refinery production is less than or equal to the smelter and refinery capacity and take the corresponding *COG*, and hence deciding the final output from the rest two unlimited stages.

When the capacity of smelting & refining is the regulating factor, then the decision state is, $Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$ are calculated using equations (21) through (22).

$$Q_{c(\alpha,\beta)} = \left(\frac{Q_{r(\alpha,\beta)}}{\bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)}} \right) \quad (21)$$

$$Q_{m(\alpha,\beta)} = \left(\frac{T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)}}{T_{o(\alpha,\beta)}} \right) \times Q_{c(\alpha,\beta)} \times \left(\frac{1}{Y_{m(\alpha,\beta)}} \right) \quad (22)$$

Module V: When mining capacity and mill capacity are limited, and smelter & refinery capacity is unlimited, then the maximum value of *NPV* from the array is considered that satisfies the condition that corresponding mine production and mill production is less than or equal to their capacity and take the corresponding *COG*, and hence deciding the final smelter and refinery capacity.

When the capacity of mining and milling are the controlling factors, then the decision state is, $Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \forall n$ and, $Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}$, $Q_{c(\alpha,\beta)}$, $Q_{r(\alpha,\beta)}$ are calculated using equations (23) through (24).

$$Q_{c(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{\{1 + S_{(\alpha,\beta)}\}} \times Y_{m(\alpha,\beta)} \quad (23)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (24)$$

Module VI: When mining capacity and smelter capacity are limited, and mill capacity is unlimited, then the maximum value of NPV is considered from the array that satisfies the condition that corresponding mine production and smelter production is less than or equal to their capacity and take the corresponding COG , and hence deciding the mill capacity.

When the capacity of mining and refining are the controlling factors, then the decision state is, $Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \forall n$ and, $Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}$, $Q_{c(\alpha,\beta)}$, $Q_{r(\alpha,\beta)}$ are calculated using equations (25) through (26).

$$Q_{c(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{\{1 + S_{(\alpha,\beta)}\}} \times Y_{m(\alpha,\beta)} \quad (25)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (26)$$

Module VII: When milling capacity and smelter and refinery capacity are limited, and mine production capacity is unlimited, then the maximum value of NPV is considered from the array that satisfies the condition that corresponding mill, smelter and refinery production is less than or equal to their capacities and take the corresponding COG , and hence deciding the final mine capacity.

When the capacity of the milling and refining are the controlling factors, then the decision state is, $Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \forall n$ and, $Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}$, $Q_{c(\alpha,\beta)}$, $Q_{r(\alpha,\beta)}$ are calculated using equations (27) through (28).

$$Q_{m(\alpha,\beta)} = \left(\frac{T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)}}{T_{o(\alpha,\beta)}} \right) \times Q_{c(\alpha,\beta)} \times \left(\frac{1}{Y_{m(\alpha,\beta)}} \right) \quad (27)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (28)$$

Module VIII: When all the three stages are limited, the maximum value of NPV from the array that satisfies the condition corresponding to all three-production stages will be considered. Among the three stages, the minimum capacity stage is the determining factor to decide the final output from the rest two stages, and thus the corresponding calculation will take place.

When all the capacities of mining, milling and smelting and refining are the controlling factors, then the decision state is, $Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \forall n$, $Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \forall n$ and, $Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}$, $Q_{c(\alpha,\beta)}$, $Q_{r(\alpha,\beta)}$ are calculated using equations (29) through (30).

$$Q_{c(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{\{1 + S_{(\alpha,\beta)}\}} \times Y_{m(\alpha,\beta)} \quad (29)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (30)$$

Ultimately the grade at which it contributes the maximal *NPV* value decides the global optimal *COG* value.

The given stage-wise logical conditions are implied to all the eight modules for obtaining the final $Q_{mf(\alpha,\beta)}, Q_{cf(\alpha,\beta)}, Q_{rf(\alpha,\beta)}$ values according to the various limiting conditions from the set of mining, milling, smelting and refining.

Stage – I:

When mining capacity is the ruling unit, then the decision state is, $Q_m \leq M_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n$;

Accordingly, the $Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$ are calculated using equations (31) through (33).

$$Q_{m(\alpha,\beta)} = T_{o(\alpha,\beta)} + T_{w(\alpha,\beta)} \quad (31)$$

$$Q_{c(\alpha,\beta)} = \frac{Q_{m(\alpha,\beta)}}{\{1 + S_{(\alpha,\beta)}\}} \times Y_{m(\alpha,\beta)} \quad (32)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (33)$$

Where $\bar{g}_{(\alpha,\beta)}$ is the average mill-head grade.

Stage – II:

When milling and refining capacities are the ruling units, then the decision state is $Q_{c(\alpha,\beta)} > C_{\alpha,\beta}$ or $Q_{r(\alpha,\beta)} >$

$R_{\alpha,\beta}$ accordingly,

$Q_{m(\alpha,\beta)}$ and $Q_{r(\alpha,\beta)}$, which are calculated using equations (34) and (35);

$$Q_c \leq C_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n \quad ;$$

$$Q_{m(\alpha,\beta)} = \frac{Q_{c(\alpha,\beta)} \times \{1 + S_{(\alpha,\beta)}\}}{Y_{m(\alpha,\beta)}} \quad (34)$$

$$Q_{r(\alpha,\beta)} = Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} \quad (35)$$

Stage – III:

When refining and mining capacities are the ruling units, then the decision state is accordingly either.

$$Q_{r(\alpha,\beta)} > R_{\alpha,\beta} \quad \text{or} \quad Q_{m(\alpha,\beta)} > M_{\alpha,\beta}$$

$$Q_r \leq R_{(\alpha,\beta)}(\alpha, \psi) \quad \forall n \quad ;$$

The value of $Q_{c(\alpha,\beta)}$ and $Q_{r(\alpha,\beta)}$ are calculated using equations (36) and (37).

$$Q_{c(\alpha,\beta)} = \frac{Q_{r(\alpha,\beta)}}{\bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)}} \quad (36)$$

$$Q_{r(\alpha,\beta)} = \frac{Q_{c(\alpha,\beta)}}{Y_{m(\alpha,\beta)} \times \{1 + S_{(\alpha,\beta)}\}} \quad (37)$$

Following the stage transition, the decision variables ($Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$) are stored as the final or ultimate decision variables ($Q_{mf(\alpha,\beta)}, Q_{cf(\alpha,\beta)}, Q_{rf(\alpha,\beta)}$) and will now be the optimum solutions for each (α, β) state of variables. These decision variables will now be utilized to calculate the project's *concentrates (tonnes/yr)*,

tailings (tonnes/yr), metal content (tonnes/yr) and (tonnes/yr), as shown through the equations (38) to (41). Following each stage of the computation, one must optimize the final value.

$$\text{Concentrate}_{(\alpha,\beta)} = \frac{Q_{c(\alpha,\beta)} * \bar{g}_{(\alpha,\beta)} * Y_{c(\alpha,\beta)}}{C_g (\%)} \quad (38)$$

$$\text{Tailings}_{(\alpha,\beta)} = Q_{m(\alpha,\beta)} - \text{Concentrate}_{(\alpha,\beta)} \quad (39)$$

$$\text{Metal}_{(\alpha,\beta)} = Q_{rf(\alpha,\beta)} \quad (40)$$

$$\text{Slag}_{(\alpha,\beta)} = \text{Concentrate}_{(\alpha,\beta)} - \text{Metal}_{(\alpha,\beta)} \quad (41)$$

The selection of the optimal COG is followed by a multi-staged sequential decision problem (Li & Yang, 2012; Biswas et al., 2020) with the output from each state feeding as an input for the following stages. Each stage has multiple different states correlated with it, as illustrated in Fig. 5.

Fig. 5. The multi-value dynamic programming framework for the multi-stage decision sequence for 'n' stages.

Bellman's principle of Optimality (Bellman, 1972) governs the optimization of the output throughout all stages. State variables ($S_{n(\alpha,\beta)}$) and Decision Variables ($Q_{m(\alpha,\beta)}, Q_{c(\alpha,\beta)}, Q_{r(\alpha,\beta)}$) have discrete values. The optimal decision at stage 'n', for the state ' S_n ' is determined by a recursive relation. It provides the decision for every state at stage ($n - 1$). In this case, $f_n(S_n)$ is the optimized objective function value at stage 'n' and ' X_n ' is the decision that maximizes over $\{X_n\}$.

$$f_n(S_n) = \text{Max}_{\{X_n\}} [R_n(X_n) + f_{n-1}\{T(S_n, X_n)\}] \quad (42)$$

R_n : Return for the decision, ' X_n ' in the present state.

$T(S_n, X_n)$: Transfer function to obtain the state ' S_{n-1} ' related to ' S_n ' and ' X_n '.

There are returns after every stage, and the return as cash flow $CF_{(\alpha,\beta)}$ of the 'n' stages has been calculated as follows. The selling price = $SP_{(\alpha,\beta)}$, operating expenses of mining, milling, and smelting and refining = $\Omega_o(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)})$, fixed costs of mining, milling, and smelting and refining = $\psi_f(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)})$, any

other costs = $AC_{(\alpha,\beta)}$, tax rate = $TR_{(\alpha,\beta)}$ and depreciation = $D_{(\alpha,\beta)}$ of the mineral – all the elements are time – indexed (t) dependent variables, and the $CF_{(\alpha,\beta)}$ has been computed for the total life of mine (n) as devised below in equation (43).

$$CF_{(\alpha,\beta)} = \{ SP_{(\alpha,\beta)} \times Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} - \sum \Omega_o(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)}) - \sum \psi_f(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)}) - AC_{(\alpha,\beta)} - D_{(\alpha,\beta)} \} (1 - TR_{(\alpha,\beta)}) + D_{(\alpha,\beta)} \quad (43)$$

The objective function is to maximize the NPV for the total mineral deposit throughout the entire life of mine (n).

It is the cumulative summation of all the cash flows ($CF_0, CF_1, CF_2 \dots CF_n$), where CF_0 is the capital invested having a fixed discount rate of ' ∂ ' up to n^{th} year given by equation (44).

$$NPV_{(\alpha,\beta)} (Max) = \frac{CF_1}{(1+\partial)} + \frac{CF_2}{(1+\partial)^2} + \dots + \frac{CF_n}{(1+\partial)^n} - CF_0 = \sum_{t=0}^n \frac{CF_t}{(1+\partial)^t} - CF_0 \quad (44)$$

Therefore, to get the maximum $NPV_{(\alpha,\beta)}$, sequential formulae for all the intermediate stages up to ' n ' are derived using the generalized equation (45).

$$NPV_{(\alpha,\beta)} = Max \int_t^n [\{ SP_{(\alpha,\beta)} \times Q_{c(\alpha,\beta)} \times \bar{g}_{(\alpha,\beta)} \times Y_{c(\alpha,\beta)} \times Y_{r(\alpha,\beta)} - \sum \Phi(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)}) - \sum \xi(M_{(\alpha,\beta)}, C_{(\alpha,\beta)}, R_{(\alpha,\beta)}) - AC_{(\alpha,\beta)} - D_{(\alpha,\beta)} \} (1 - TR_{(\alpha,\beta)}) + D_{(\alpha,\beta)} \} e^{-\partial t} dt] \quad (45)$$

Using the equation (23), if one starts from stage one, all the intermediate states are evaluated stage by stage, until all the possible values of $NPV_{(\alpha,\beta)}$ are evaluated. Then, the maximum of $NPV_{(\alpha,\beta)}$ of all stages is chosen, so that the global maximum $NPV_{(\alpha,\beta)}$ correlating to a specific grade is the mill's global optimum COG . Here, the sequentially advancing algorithm has employed the memorization approach to identify the global COG in this case.

3.4 Application and development of the computer-aided model

A computer-aided COG -MDP model has been established for determining the optimal COG using the formulae mentioned—the proposed computational framework is derived from the multi-value dynamic programming approach for estimating the global optimum COG . The programming language used to develop the computer package is *Python 3.7*, the ideal multi-paradigm programming language for developing scientific and business applications. It imparts a great extent of calculation speed and data accuracy. *Python* have a logical set of dynamic binding rules with automatic memory guidance. It is compatible with various computing paradigms, such as structured, object-oriented, and functional and procedural programming. A *PyQt* framework, version-5, is used for this model. The COG -MDP model is divided into three sections: input, output, and graphical components. The Graphical User interface (*GUI*) has been developed using *Python 3.7* and *PyQt* designer that comes with *PyQt5 Library*. In particular, there are some of the core packages like *pandas*, *SciPy*, *NumPy* and *Matplotlib* that has been used exclusively for this model. The automated model offers more operational capability and applicability.

The ultimate optimum *COG* of the mineral grade and the optimized *NPV* of the entire process is provided by the result module. The outputs of the result segment are separated into four segments: mine, mill, smelter and refinery, and earnings. The Result-graphical component, an essential element of the computer tool and *GUI* program, supports the logical conclusion. After the simulation, the computational framework displays the result as graphical data in various graphs between the provided variables. The user is presented with the following graphical displays by the tool: (1) *Avg. Grade (%)* versus *Tonnage* (2) *COG (%)* versus *Cumulative Tonnage*, *Average Grade (%)* and (3) *COG (%)* versus *NPV*.

4. Corroboration of outputs using a working surface mining complex

The work requires an actual case paper that describes a study of domain knowledge of an open-pit copper ore mine with all or at least three phases of ore production, namely extraction, concentration, and smelter and refinery, to evaluate the models. The model has been corroborated utilizing data from India's most extensive open-pit copper resources (surface mining complex).

4.1 A brief description of the mining complex

Malanjkhand Copper Project (MCP) is a subsidiary of Hindustan Copper Limited, which was commissioned on 12th November 1982. The mining lease was approved in favor of HCL in 1973. The latitude and longitude of this mine are 22°00'59" to 22°02'24" N and 80°41'51" to 80°42'38" E and forms a part of Survey of India Toposheet No. 64 B/12, located in the Balaghat District of Madhya Pradesh in India. The strike length of the deposit is 2600 m. MCP, situated at a distance of 20 km away from the Kanha National Park, contributes about 80 percent of HCL's overall copper supply. This is the largest copper mine in India (Fig. 7), with an average orebody width of 75 m. It has a total lease area of 479.9 ha.

Fig. 7. Site location Map of Malanjkhand Copper Mine, HCL.

Malanjkhand Copper Project is a massive metalliferous deposit amounting to 200 *Million tonnes (Mt)* of copper ore reserves. MCP has rich mineable copper reserves of 143 *Mt*, which is more than 70% of the country's available resources. India's largest copper mine, Malanjkhand deposit extends vertically from the surface at 580 mean reduced level (*mRL*) to a depth of (-)300 *mRL*. Open-pit mining was planned initially to a depth of 204 *m* (376 *mRL*), which is now extended to 240 *m* (340 *mRL*). Below the open-pit floor, there exist 160 *Mt* reserves of 1.34% copper grade containing over two million tonnes of copper metal. At present, the central portion of the deposit is being mined out by the open cast mining method, which has reached the Ultimate Pit Limit (*UPL*).

The general area in the vicinity of the mine is a hilly terrain consisting of narrow basins and hills. Malanjkhand rises in isolated majesty above the undulating terrain and forms a part of the Baihar Plateau. Six peaks with intervening saddles are observed that create an arcuate chain of about 2.6 *km* long, having an eastward convexity. This highest peak is about 652 *m* above mean sea level (*mSL*), while the ground level of the surrounding area is around 575 above *mSL*. This describes the general elevation of the Baihar Plateau, which rises about 270 *m* above the Balaghat Plains. The digital surface contour terrain of the above area within lease boundary and the surface plan of the project is shown is Fig. 8 (a) and (b).

Fig. 8. (a) Initial Surface Contour Terrain (*source: HCL*) **(b)** Surface Plan of Malanjkhand Copper Mine (*source: HCL*)

King ([W. King, 1886](#)) initially noted the presence of copper in Malanjkhand in 1886, and Dunn reported it in 1942 ([Dunna, 1942](#)). Later, the porphyry character of the copper deposit was discovered by Sikka in 1971, opening up new possibilities in the search for the substantial tonnage of Precambrian polymetallic porphyry copper deposits in Madhya Pradesh, India ([Sikka & Nehru, 1997](#)). The Geological Survey of India (GSI) carried out systematic geological mapping at this site in 1969. Malanjkhand Copper belt consists of a large body of copper ore in granite rocks, ranging in composition from diorite to granite. The rocks of Malanjkhand are from the Proterozoic era and can be divided into two groups, each with unique attributes, distinguished by an unconformity. The oldest group, composed of basement rocks, was formed first, followed by the younger, less metamorphosed rocks of the Chilpi Ghat beds. Three distinct forms of mineralization are clearly visible. Firstly, the mineralization is primarily

correlated with quartz, which is generally limited to the quartz reef as a fracture fill form and can be known as quartz ore. Secondly, the mineralization of the stringer pattern has quartz and calcite joints. This stringer ore is found primarily in granites. Lastly, the disseminated ore inside granites or micro granites is present within the interstices of inherent grains of the related rocks. The most predominant ores are chalcopryrite, chalcocite, and malachite in order of abundance. The secondary and oxidized ore minerals are confined to the upper part of the deposit. Due to hydrothermal vein formation, the Malanjkhanda copper ore deposit geometry varies in the strike and dip directions. The host rock is quartz reef and granite.

Surface mining (open-pit) will be continued for exploiting the deposit. As per the revised mine plan schedule, mining is contemplated to expand the pit to a depth of 340 mRL. A view of the working pit and the modelled Ultimate Pit design of the Malanjkhanda Copper Project is shown in Fig. 9 (a) and (b).

Fig. 9 (a) Working Pit of Malanjkhanda Copper Project **(b)** Ultimate Pit design based on 3-D mine planning software (source: HCL)

The computations are made using the grade-tonnage variance and operating variables (mine, mill, and refinery capacities). Economic values have been assessed in accordance with the Indian context (in ₹). Table 2 shows the mineral content inside the grade bins.

Table 2 Grade-tonnage variance of the copper deposit.

Grade from (% Cu)	Grade to (% Cu)	Tonnage (in tonnes)
0.20	0.45	25909429
0.45	0.50	3292995
0.50	0.60	6222261
0.60	0.70	6917390
0.70	0.80	6538756
0.80	0.90	5674345
0.90	1.00	3501976
1.00	1.10	3331731
1.10	1.20	3283519
1.20	1.30	2360291
1.30	1.40	1816110
1.40	1.50	1638172
1.50	5.60	9154230

4.2 Model-inputs

Geo-mining details and techno-economic variables are the vital input parameters for optimizing the objective function for maximization of *NPV*. The essential techno-economical parameters used for the *COG-MDP* model is shown in Table 3. The following data has been used as input data to determine global optimum *COG%*. This data will be imported into the *COG-NPV* optimizer tool for the calculation and software input.

Table 3 Techno-economical parameters used for the surface mining complex in Indian currency (rupees-₹).

Mining	
Particulars	Values
Maximum mining capacity (tonnes / yr)	2540000
Percentage mining recovery (%)	100
Specific investment cost for mining (₹ / tonne / yr)	160
Unit cost of excavation (₹ / tonne ore & waste)	816.91
Fixed mining cost (₹ / tonne)	28026.48
Average ore transportation cost (₹ / tonne / km)	43.30
Average waste transportation cost (₹ / tonne / km)	43.30
Average ore transportation haul distance (km)	6
Average waste transportation haul distance (km)	6
Mill	
Maximum mill capacity (tonnes / yr)	2340000
Mill concentrate grade (%)	26
Percentage mill Recovery (%)	92.5
Specific investment cost for mill (₹ / tonne / yr)	150
Unit cost of ore processing (₹ / tonne)	464.42
Fixed mill cost (₹ / tonne)	29314.02
Average concentrate transportation cost (₹ / tonne / km)	1.95
Average mill tailing transportation cost (₹ / tonne / km)	3
Average concentrate transportation haul distance (km)	180
Average mill tailing transportation haul distance (km)	4.6
Refinery	
Maximum refinery capacity (tonnes / yr)	18500
Percentage refinery Recovery (%)	99.66
Specific investment cost for refinery (₹ / tonne / yr)	500
Unit cost of refining (₹ / tonne of metal produced)	57600
Fixed refinery cost (₹ / tonne)	47800
Average metal transportation cost (₹ / tonne / km)	3
Average slag transportation cost (₹ / tonne / km)	3
Average metal transportation haul distance (km)	0.5
Average slag transportation haul distance (km)	0.5
Additional Costs	
Other costs including general administration cost (percentage % of the total operating cost)	25
Rates	
Selling price of metal (₹ / tonne)	500000
Tax rate (%)	34
Discount rate (%)	11
Grade Interval Precision	
Grade Interval increment (%)	0.01

4.3 Result-outputs

The data has been loaded into the program and analyzed by the *COG-MDP* model, and the final optimized output for each component was obtained and shown in Table 4.

Table 4 Summary of the results output of all the eight modules.

Abbreviation used for the output variables:								
OCOG = Optimum mill <i>COG</i> % ; T (T_o) (t)= Total tonnage of ore production (tonnes); S_{α,β} = Overall Stripping ratio; C_n (t/yr) = Tonnage of concentrate produced per year; T_l (t/yr) = Tonnage of mill tailing produced per yr; T(C_n) (t) = Total tonnage of concentrate produced ; T(T_l) (t) = Total tonnage of mill tailing produced; S_g (t/ yr) = Tonnage of slag produced per year ; T(Q_r) (t) = Total tonnage of metal produced; ḡ =Average head grade % T(S_g) (t) = Total tonnage of slag produced; T(CF_o) (₹) = Total Investment ; D_p (₹/ yr) = Depreciation; R_v (₹/ yr) = Revenue generated per yr; AOMC (₹/ yr) = Annual operating and maintenance cost ; GP (₹/ yr) = Gross profit per yr; N_p (₹) = Net profit per yr; T(N_p) (₹) = Total Net profit; CF_y (₹) = Annual cash-flow; AC (₹/ yr) = Annual other costs including GAC & fixed cost; T(R_v) (₹) = Total Revenue generated								
Modules	M-1/4/6	M-2	M-3	M-5	M-7	M-8A	M-8B	M-8C
Particulars	values	values	values	values	values	values	values	values
Mining (production in tonnes (t))								
Q_m (t/ yr)	2132818	2540000	3788250	2538177	2132818	4191642	5680398	26547068
OCOG (%)	0.33	0.23	0.52	0.26	0.33	0.27	0.22	0.24
Life (yrs)	38.00	32.00	22.00	32.00	38.00	19.00	15.00	3.00
T (T_o) (t)	66168302	76532073	49194329	73422942	66168302	72386565	77568451	75495696
T (T_w) (t)	13472903	3109131	30446876	6218263	13472903	7254640	2072754	4145509
S_{α,β}	0.20	0.04	0.62	0.08	0.20	0.10	0.03	0.05
Mill (production in tonnes (t))								
Q_c (t/ yr)	1772009	2440840	2340000	2340000	1772009	3809819	5532559	25165232
ḡ (%)	1.133	1.017	1.380	1.050	1.133	1.061	1.006	1.028
C_n (t/yr)	71397	88320	114884	87393	71397	143809	198108	920194
T_l (t/yr)	1700613	2352521	2225116	2252607	1700613	3666010	5334452	24245038
T(C_n) (t)	2713071	2826231	2527440	2796564	2713071	2732376	2971619	2760583
T(T_l) (t)	64623286	75280661	48952560	72083436	64623286	69654189	80016774	72735114
Smelter/ Refinery (production in tonnes (t))								
Q_r (t/ yr)	18500	22885	29768	22645	18500	37263	51333	238437
S_g (t/ yr)	52897	65435	85115	64748	52897	106546	146775	681757
T(Q_r) (t)	703000	732322	654900	724635	703000	708002	769994	715311
T(S_g) (t)	2010071	2093909	1872540	2071930	2010071	2024374	2201625	2045271
Profits (₹ in million)								
T(CI_o) (₹)	616	784	972	768	616	1261	1764	8142
D_p (₹/ yr)	16	24	44	24	16	66	118	2714
R_v (₹/ yr)	9250	11443	14884	11322	9250	18632	25666	119219
AOMC (₹/ yr)	4232	5250	6935	5185	4232	8530	11783	54664
AC (₹/ yr)	2848	3538	4651	3493	2848	5745	7943	36835
GP (₹/ yr)	2154	2630	3254	2621	2154	4290	5823	25006
Tax (₹/ yr)	732	894	1106	891	732	1458	1980	8502
N_p (₹/ yr)	1422	1736	2148	1730	1422	2831	3843	16504
CF_y (₹/ yr)	1438	1760	2192	1754	1438	2898	3961	19218
NPV (₹)	12208	14653	16948	14609	12208	21454	26717	38821
T(R_v) (₹)	351500	366161	327450	362317	351500	354001	384997	357656
T(N_p) (₹)	54026	55552	47248	55350	54026	53792	57647	49511

In this model, the input values (operational and economical) have been taken irrespective of the time. Therefore, the values and results will behave accordingly as per the user's input values. Thus, the mine planners or experts

will run their module as per the current situation and get their results to optimize accordingly. If there will be a significant increase in the pricing dynamics in the coming years, the planning experts will run through the program and optimize accordingly. Therefore, there is no need to introduce the price forecasting technique in this model because there are a host of variables where the cost-escalation factor will be applicable, including the entire mine, mill, and smelter and refinery stages. Therefore, considering all the variables will make things more complex and may lead to an indefinite solution.

4.4 Graphical interface-result module

The graphical user interface of the result module for the above-described outputs is represented in various graphs through Figures 10, 11, 12, and 13, respectively.

Fig. 10. (a) Tonnage versus Grade distribution of deposit **(b)** Cumulative tonnage, Average grade versus COG

Fig. 11. Showing all the NPV values versus COG(%) related with the eight modules.

Fig. 12. Showing the 3-D line graph of NPVs versus $COG(\%)$ versus life of mine of all the eight modules.

Fig. 13. Showing the 3-D line graph of NPVs versus $COG(\%)$ versus mine production (Q_m) of all the eight modules.

5. Discussions of results

The results after the simulation from all the eight modules have been presented through tables and figures. The observations according to the given grade-tonnage and techno-economic variations are discussed below in Table 5.

Table 5 Conclusive summary sheet of the output variables of all the eight modules for the open-pit metalliferous deposit (MCP).

Sl. No.	Optimum mill <i>COG</i> (%)	<i>NPV</i> (in Millions ₹)	Conditions (<i>L</i> -Limited; <i>UL</i> -unlimited)	Observations (Mine: <i>M</i> ; Concentrator: <i>C</i> ; Refinery: <i>R</i>)
1.	0.33	12208	Module - 1: <i>M, C, R</i> (<i>L, L, L</i>) Module - 4: <i>M, C, R</i> (<i>UL, UL, L</i>) Module-6: <i>M, C, R</i> (<i>L, UL, L</i>) Module - 7: <i>M, C, R</i> (<i>UL, L, L</i>)	The optimum <i>COG</i> value for the Module 1, 2, 4 and 7 is same. In all the above four modules, there are different limiting conditions, and out of these three limiting conditions, <i>R</i> becomes the final limiting function, and hence it decided the final <i>C</i> and <i>R</i> values. Therefore, in these scenarios, all four modules are getting the same results as the final (<i>Q_m</i> , <i>Q_c</i> , and <i>Q_r</i>) values. In these four modules, same optimal <i>COG</i> (%) of 0.33 with the maximum <i>NPV</i> of ₹ 12208 million have been found.
2.	0.23	14653	Module - 2: <i>M, C, R</i> (<i>L, UL, UL</i>)	In <i>Module 2</i> , <i>M</i> is the limiting function. It has decided the ultimate <i>C</i> and <i>R</i> values, and correspondingly, the limiting logical conditions on <i>M</i> , <i>C</i> , and <i>R</i> values contributed to the final (<i>Q_m</i> , <i>Q_c</i> , and <i>Q_r</i>) results. This module has given the optimal <i>COG</i> (%) at 0.23 with the maximum <i>NPV</i> of ₹ 14653 million.
3.	0.52	16948	Module - 3: <i>M, C, R</i> (<i>UL, L, UL</i>)	In <i>Module 3</i> , <i>C</i> is the limiting function. It has decided the ultimate <i>M</i> and <i>R</i> values, and correspondingly the limiting logical conditions on <i>M</i> , <i>C</i> , and <i>R</i> values contributed to the final (<i>Q_m</i> , <i>Q_c</i> , and <i>Q_r</i>) results. This module has given the optimal <i>COG</i> (%) at 0.52 with the maximum <i>NPV</i> of ₹ 16947 million.
4.	0.26	14609	Module - 5: <i>M, C, R</i> (<i>L, L, UL</i>)	In <i>Module 5</i> , <i>M</i> and <i>C</i> are the limiting function, whereas <i>R</i> has a scope of unlimited capacity. So, limiting conditions of <i>M</i> and <i>C</i> have decided the ultimate values, and correspondingly the limiting logical conditions on <i>M</i> , <i>C</i> , and <i>R</i> values contributed to the final (<i>Q_m</i> , <i>Q_c</i> , and <i>Q_r</i>) values. This module has given the optimal <i>COG</i> (%) at 0.26 with the maximum <i>NPV</i> of ₹ 14609 million.

				Module 8(A)/ 8(B)/ 8(C)
5(a).	0.27	21454	Module – 8 (A): Taylor (Taylor, 1986) M, C, R (UL, UL, UL)	When all the capacities (mining, concentrator, and smelter and refinery) are unlimited, the optimal $COG(\%)$ and the corresponding maximum NPV , mine production and mine life have been calculated according to Taylor's (Taylor, 1986) mine life rule, by Long's (Long, 2009) rule and by the DP based algorithm (the presented $COG-MDP$ model). It has been found that in all the three cases, mining capacity becomes the limiting function. Therefore, the logic sequentially advances and decides the final C and R values, and correspondingly the limiting logical conditions on Q_c and Q_r values will contribute to the final results. It may be observed that among the three methods for the unlimited conditions, the DP based algorithm gives the maximum NPV of ₹ 388201 million at $COG(\%)$ of 0.24. A major advantage of the DP based algorithm is that it optimizes all the four variables – $COG\%$, NPV , mine production and mine life simultaneously.
MM	0.22	26717	Module – 8 (B): Long (Long, 2009) M, C, R (UL, UL, UL)	
5(c).	0.24	38821	Module – 8 (C): DP based algorithm ($COG-MDP$ model described in the paper) M, C, R (UL, UL, UL)	

It may be observed that the results obtained from all the different modules using the real life data behave dynamically as per the existing grade-tonnage, techno-economic criteria, and other limiting conditions.

6. Conclusion and recommendations

The significance of developing a new evaluation technique, such as the one proposed in this study, lies in providing a pragmatic analytical approach to the mining industry that allows the mine planner or analyst to obtain a more realistic estimate of an open-pit project value accounts for the given techno-economic conditions. The three primary stages utilized in COG optimization depend upon the limiting conditions of mining, milling, and smelting and refining. In each of these sectors, several smaller divisions may be established. By individually optimizing mine and COG in planning stages, a guarantee of obtaining the highest NPV is generally lost.

This necessitates developing a suitable approach for finding the optimum COG of ore, and hence a multi-value dynamic programming has been developed and used to compute COG for a metalliferous deposit. In this model, discrete values have been utilized for Dynamic Programming (DP), a more deterministic approach than stochastic one. The developed model is based on the concept of dynamic programming and thus finds the solution much more straightforward than other optimization techniques. The fundamental framework and dynamics guide this model rather than tailoring it to time series. This increases the model's generic validation, but it takes a long time to design and parameterize. Its key strength is that it optimizes several policies simultaneously, that the results are connected

and mutually reliable, and effectively validates against facts. As in this model, *COG* optimization has been carried out after finalizing the Ultimate Pit, and the pit optimization follows the objective function of maximizing profits irrespective of time. Therefore, whenever one does mining, it will create wealth, and the economics will behave as per the existing scenario. As a result, mine planners must re-run the *COG-MDP* optimizer model depending on market conditions and modify their intermediate stages of excavation following demand and economics. In general, the mine planners can choose their optimization in three stages.

- First stage by initially excavating higher-grade ore to recover capital, operating, and running costs and reduce the payback period.
- Second stage, by recovering the depreciation and tax amounts, can mine with the average-grade or lower grade.
- Third stage, when the machines or plants are fully depreciated, they can go for mining the lower grade ore and sustain the working of mines.

This work has the novelty of dealing with eight different optimization possibilities. So far, only six potential outcomes have been dealt with as on date by Lane (Lane, 1964). The computer can produce the result in seconds from the complex system of mine, mill, and smelter and refinery comprising of eight different possibilities. Thus, it will be a useful industry-oriented tool to plan a mining project for a given deposit considering all the possible dynamic conditions. An alternate way to Lane's algorithm and other current methodologies, based on mathematical programming, is described here as a practical and generic method for determining optimal *COG*, production rate, mine-life and the global *NPV*.

The Malanjkhand copper complex, for example, faces a variety of production restrictions. Besides, these operations will generally have a well-defined operating range, which may vary in actual situations. The model ignores the volatility and variations in economic characteristics, such as commodity pricing, stockpiling, dilution, multi-mineral aspects, and mine-restoration. The stockpiling option has not been included as that may increase the complexities enormously and lead to an incomplete solution. The objective of this model is to introduce a universal approach for resolving the optimality scenario related to *COG* of open-pit metalliferous deposits.

This optimization framework methodology can handle technically more acceptable relations, functions, and multi-dynamic aspects. However, the software provides enough flexibility to include or exclude the parameter(s) by changing the design's code; therefore, there is ample room for the expansion and modification of the model to cover additional scenarios. The intention here is to introduce a general method for the solution with a deterministic mathematical approach.

However, there is a scope to modify the program by including the stochastic behavior of the variables (commodity price, operating and maintenance expenses, fixed cost, discount rate etc.) to make it more applicable but at the cost of increasing the complexities. Though, in the future, if there is a significant increase in the price dynamics, the planning experts will run through the program and optimize accordingly. However, based on mining conditions

and economics, the material categorized as waste could become economically viable to process in the future, necessitating a shift in policy. So, the utility and complexity of stockpiling is a more extensive area to explore.

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