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An investigation into the effect of pre-bending on the tube hydro-forging technology

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Abstract

Tube hydro-forging (THFG) combining with the pre-bending is an advanced method to manufacture the complex cross-sectional tubular component with curved axis. However, the effect of pre-bending on the subsequent THFG, especially on the critical internal pressure required to inhibit wrinkling, has not been clarified yet. Therefore, this paper makes a detailed study on it. At first, based on the energy method, the change rule between the critical internal pressure and the hoop strain was established when pre-bending was considered. Subsequently, the mechanics condition difference between single and double curvature differential segment during THFG was analyzed. Via the plastic theory, the distribution of hoop strain could be obtained. Mainly due to the uneven distribution of thickness and cold work-hardening caused by pre-bending, the maximum hoop strain at the outer straight-wall was greater than that at the inner straight-wall during THFG. Substituting the maximum hoop strain at the outer/inner straight-wall into the change rule, then their mathematical model of the critical internal pressure to restrain the wrinkling could be solved respectively. Finally, the critical internal pressure considering pre-bending was determined by that of outer straight-wall, and its value was always greater than the critical internal pressure without considering pre-bending under the same punch stroke. With the increase of bending radius, the critical internal pressure difference between considering and not considering pre-bending also increases. When the bending radius was 250 mm, the critical internal pressure difference was 33%, while it increased to 74% as the bending radius reduced to 100 mm, all of which were verified by experiment. The effect of friction coefficient on the critical internal pressure was also studied.

In conclusion, this work provided a new and more accurate prediction model of critical internal pressure to guide practical production for when existing the pre-bending.

Keywords: Tube hydro-forging; Pre-bending; Critical internal pressure; wrinkling; Tubular component

1 Introduction

Complex cross-sectional tubular component with curved axis is one of the important ways to realize the lightweight technology, so has been widely used in the automotive and aerospace industry. Especially by optimizing the axis, the assembly accuracy requirements can be satisfied while efficiently utilizing the bending and torsion resistance potential of the structure [1], as shown in Fig. 1. Usually, the initial tube needs a preforming operation like pre-bending to obtain the same or similar shape of the axis of the product, and then the cross-section shape is formed by the tube hydroforming technology. Therefore, the pre-bending step has important bearing on the final quality of the product. However, as we all know, the pre-bending will lead to a series of changes, such as the thinning of the outer layer and thickening of the inner layer, the cold work-hardening, the axis springback and the cross-sectional distortion [2]. Besides, the curved axis also will lead the difference in mechanics condition during subsequent process. For this reason, it is necessary to determine the influence of the above factors on the tube hydroforming process, which has become an urgent problem to be solved.

Fig. 1. Application of the complicated cross-sectional tubular components with curved axis.

In recent years, high pressure tube hydroforming (HPTH) has been the most popular technology to form the complicated cross-sectional tubular component [3]. By applying enough internal pressure and reasonable axial feeding, HPTH can make the materials expand to fit with the closed die, and then the ideal cross-sectional shape can be obtained. Actually, a

typical HPTH, like manufacturing exhaust manifolds, engine cradles and frames, involves pre-bending of tubes as the first step [4]. However, pre-bending will lead to the thinning of the outer layer. Consequently, compared with the straight tube, the outer layer of pre-bending tube is more easily to lead the further thinning and even cracking due to the expansion deformation during HPTH. In fact, because of curved axis, the material is difficult to achieve the feeding along the axial direction, which makes the cracking defect more obvious. Prabhu [5] pointed out that the process parameter of pre-bending has an important effect on HPTH, and found that the formability could be improved with the increase of bending radius. Gao [6] proposed that the reduction of friction coefficient can help lighten the degree of thickness thinning during pre-bending, which is also beneficial to alleviate the cracking defect after HPTH. Han [4] proposed a method to adjust the contact sequence of tube and die surface to solve the inhomogeneous deformation problem in hydroforming of a bending tube, effectively controlling the cracking defect. Moreover, there also found that the cross-sectional distortion during pre-bending will also aggravate the occurrence of cracking defect after HPTH [7]. Besides, a high internal pressure is needed in order to obtain the ideal cross-sectional shape during HPTH, and the pressure will be further improved because of the cold working-hardening caused by pre-bending.

In addition, Nikhare [8] proposed low pressure tube hydroforming (LPTH) to form the tubular components with complex cross-section. In contrast to HPTH, LPTH avoids the expansion deformation by introducing the action of die movement, so it can control the occurrence of cracking defect successfully. LPTH also can reduce the required internal pressure (is about 5% to 15% of HPTH) and the tonnage of the equipment greatly [9]. By this way, Hwang [10] studied the forming process for rectangular, square and triangular cross-section. The internal pressure is very small, thus there is no expansion deformation and the thickness uniformity is better. When manufacturing the complex cross-section through LPTH, Nikhare [11] found that the more complex the shape, the more difficult it is to fill the die cavity completely. Unfortunately, LPTH is only suitable to form the component with constant perimeter [12]. In particular, the cross-sectional distortion caused by pre-bending will lead to the unequal perimeter along the bending axis, which further increases the difficulty in forming complex cross-section. Besides, the bending moment is introduced during LPTH, hence the cross-sectional springback is inevitable [13].

By reviewing previous studies, it is found that the existing tube hydroforming technology has some inherent disadvantages when forming complex tubular components,

especially combining with the pre-bending process: 1. for HPTH, the pre-bending will lead to the thickness thinning, which aggravates the cracking defect; 2. the cross-sectional distortion caused by pre-bending restricts LPTH to form the complex cross-section, and so on. As a result, how to form the bending tubular components with complex cross-section and overcome the influence of pre-bending successfully has been an urgent problem. Chu [14] first proposed tube hydro-forging technology (THFG) to form bending tubular components with complex cross-section. In THFG, under sufficient internal pressure, the cross-section can be compressed by the upper die movement to obtain the required shape. Meanwhile, the cross-section with variable perimeter can be produced by setting different punch stroke along the axial direction. The material has always born the compression state in three directions, which can effectively avoid the occurrence of thickness thinning and cracking [15]. Moreover, this technology uses the clamping force as the driving force. In other words, the internal pressure only plays the function of supporting, thereby diminishing the demand for high-pressure equipment [16]. Most importantly, when THFG combines with the pre-bending, Lin [17] relied on hoop compression to eliminate the springback and cross-sectional distortion caused by pre-bending, and this further broadens the practicability of THFG.

THFG has many advantages in forming complex cross-sectional tubular component with curved axis, but because of the compression, the straight-wall is prone to wrinkling when the internal pressure is insufficient [18], which also exists in LPTH process [19]. Fortunately, Chu [14] pointed out that excessive hoop strain is the ultimate reason. Then through the energy method combining with finite element (FE) simulation, the critical internal pressure required to suppress the wrinkling has been obtained. However, the existing theoretical result has not considered the effect of pre-bending on THFG, especially on the critical internal pressure. Therefore, by energy method and plastic theory, this paper established a new mathematical model for the critical internal pressure when considering the pre-bending. At the same time, through the comparison with the straight tube, the influence of pre-bending on the critical internal pressure could be further revealed.

2 Tube hydro-forming

2.1 Process principle

Fig. 2. Schematic diagram of tube hydro-forming combining with pre-bending.

Fig. 2 shows the schematic diagram of tube hydro-forming combining with pre-bending. At first, the performing part with the desired curved axis is obtained through the pre-bending process. During THFG, the internal pressure plays a supporting role, and then the upper die moves Δ to compress the cross-section. If the internal pressure is enough, the material at the inner/outer straight-wall will be compressed and the perimeter of the cross-section will be reduced by 2Δ , but when the internal pressure is insufficient, the wrinkling will occur at the inner/outer straight-wall instead of stable compression deformation, as shown in Fig. 2. It is a typical compression instability defect caused by excessive hoop compression strain. The reasonable critical internal pressure p_{cri} and its loading path with the punch stroke Δ are the key to control wrinkling during THFG.

2.2 Critical internal pressure

As shown in Fig. 3, it is a traditional solution process of the critical internal pressure for straight tube, which is mainly composed of three steps [14]: in the step-1, the energy method is used to solve the change rule between the critical internal pressure p_{cri} and the hoop compression strain of THFG ε_{ϕ}^{th} ; in the step-2, the hoop strain distribution on the cross-section under the certain punch stroke Δ is determined according to FE method; in the step-3, by substituting the maximum hoop strain on the cross-section into the change rule, the critical internal pressure p_{cri} can be obtained under the certain punch stroke Δ at last.

Fig. 3. Traditional solution process of critical internal pressure.

For the straight tube, the initial state of each differential segment on the cross-section is the same, but for the pre-bending tube, the thickness and work-hardening degree of each differential segment is different. In addition, due to the existence of curved axis, the mechanics conditions of pre-bending tube must be different from that of straight tube during THFG. These changes not only affect the change rule, but also decide the specific distribution of the hoop strain on the cross-section. Therefore, the above factors inevitably lead to the critical internal pressure difference between considering and not considering pre-bending. Therefore, the following determines the state of each differential segment after pre-bending at first. Then, on the basis of pre-bending, through analyzing the mechanics conditions of pre-bending tube to solve the change rule and the hoop strain distribution. Substituting the maximum hoop strain into the change rule, the critical internal pressure considering pre-bending can be obtained finally.

3 Theoretical model

Before theoretical derivation, reasonable simplification can not only ensure the calculation accuracy, but also improve the calculation efficiency:

1. The Hollomon hardening law is taken to describe the stress-strain relationship of the material:

$$\bar{\sigma} = K \bar{\varepsilon}^n \quad (1)$$

Where K is the strength coefficient and n is the strain-hardening exponent, both can be obtained from the uniaxial tensile test.

2. According to the Von Mises yield criterion for isotropic material, the equivalent stress $\bar{\sigma}$ and the equivalent strain $\bar{\varepsilon}$ can be obtained:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_\varphi - \sigma_\theta)^2 + (\sigma_\theta - \sigma_t)^2 + (\sigma_t - \sigma_\varphi)^2} \quad (2)$$

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_\varphi - \varepsilon_\theta)^2 + (\varepsilon_\theta - \varepsilon_t)^2 + (\varepsilon_t - \varepsilon_\varphi)^2} \quad (3)$$

2. The pre-bending process is assumed to be plane strain and plane stress [20], that is, the normal stress σ_t^{bd} and the hoop strain $\varepsilon_\varphi^{\text{bd}}$ can be ignored, what can be expressed by Eq. (4):

$$\begin{cases} \sigma_t^{\text{bd}} = 0 \\ \varepsilon_\varphi^{\text{bd}} = 0 \end{cases} \quad (4)$$

3. The arbitrary cross-section of tube remains plane before and after pre-bending.

4. A plane strain state along the axial direction can be assumed during THFG:

$$\varepsilon_\theta^{\text{th}} = 0 \quad (5)$$

5. The distribution of stress-strain state and thickness at each differential segment are continuous after pre-bending and before THFG.

3.1 Results of pre-bending

As previously mentioned, the state of each differential segment after pre-bending should be solved firstly, and its specific solution process is shown in Appendix A. The distribution of thickness after pre-bending t^{bd} can be acquired, as shown in Eq. (6):

$$t^{\text{bd}} = \begin{cases} \frac{\rho_u t_0}{R} & \varphi = \frac{\pi}{2} \\ \frac{-(R + r \cos \varphi) + \sqrt{(R + r \cos \varphi)^2 + 4 \cos \varphi t_0 \rho_u}}{2 \cos \varphi} & \varphi \neq \frac{\pi}{2} \end{cases} \quad (6)$$

Simultaneously, due to the cold work-hardening, the distribution of flow stress $\bar{\sigma}^{\text{bd}}$ and equivalent strain $\bar{\varepsilon}^{\text{bd}}$ is also solved:

$$\bar{\sigma}^{\text{bd}} = K \left[\beta \ln \frac{R + (r + t^{\text{bd}}) \cos \varphi}{\rho_u} \right]^n \quad 0 \leq \varphi \leq \pi \quad (7)$$

$$\bar{\varepsilon}^{\text{bd}} = \beta \ln \frac{R + (r + t^{\text{bd}}) \cos \varphi}{\rho_u} \quad 0 \leq \varphi \leq \pi \quad (8)$$

Meanwhile, we can notice that the thickness and flow stress of each differential segment at the inner/outer straight-wall are the same. Next, based on the above pre-bending results, the mathematical model of the critical internal pressure will be established during THFG.

3.2 Change rule

Compared with straight tube, it is necessary to consider the cold work-hardening and thickness variation when calculating the change rule between the critical internal pressure p_{cri} and the hoop compression strain $\varepsilon_{\varphi}^{\text{th}}$ for pre-bending tube. Through the Appendix B., the change rule can be obtained:

$$p_{\text{cri}}^{\text{bd}} = \left[\left(\beta \varepsilon_{\varphi}^{\text{th}} + \varepsilon^{\text{bd}} \right)^{n+1} - \left(\varepsilon^{\text{bd}} \right)^{n+1} \right] \frac{3Kt^{\text{bd}}}{2\delta(n+1)} - \left[\left(\frac{\beta t^{\text{bd}}}{2} + \varepsilon^{\text{bd}} \left(\frac{2}{m^2\delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} - \left(\varepsilon^{\text{bd}} \left(\frac{2}{m^2\delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} \right] \frac{3Kt^{\text{bd}} \left(\frac{2}{m^2\delta} + \frac{t^{\text{bd}}}{2} \right)^{-n}}{L\delta(n+1)} \tan^{-1} \left(\sqrt{e^{-\varepsilon_{\varphi}^{\text{th}}} - 1} \right) \quad (9)$$

3.3 Hoop strain distribution

After obtaining the change rule, the following will solve the hoop strain $\varepsilon_{\varphi}^{\text{th}}$ distribution after THFG. It can be seen from Section 3.1 that the degree of work-hardening on each differential segment is different. According to the plastic theory, only when the equivalent stress of each differential segment reaches and exceeds its corresponding flow stress, the subsequent yield will occur, which results in the generation of hoop strain $\varepsilon_{\varphi}^{\text{th}}$ afterwards. In this case, whether the subsequent yield occurs can be judged by comparing the equivalent stress $\bar{\sigma}^{\text{th}}$ and its flow stress $\bar{\sigma}^{\text{bd}}$. Here, the equivalent stress $\bar{\sigma}^{\text{th}}$ can be obtained under the simultaneous action of a hoop stress $\sigma_{\varphi}^{\text{th}}$ and a normal stress σ_t^{th} :

$$\bar{\sigma}^{\text{th}} = \frac{1}{\sqrt{2}} \sqrt{(-\sigma_{\varphi}^{\text{th}} - 0)^2 + (0 - -\sigma_t^{\text{th}})^2 + (-\sigma_t^{\text{th}} - -\sigma_{\varphi}^{\text{th}})^2} \quad (10)$$

Therefore, if $\bar{\sigma}^{\text{th}} \geq \bar{\sigma}^{\text{bd}}$, the subsequent yield will occur, otherwise, the subsequent yield will not occur. The hoop stress $\sigma_{\varphi}^{\text{th}}$ is related to the hoop force F and the thickness t^{bd} :

$$\sigma_{\varphi}^{\text{th}} = \frac{F}{t^{\text{bd}}} \quad (11)$$

When the pre-bending radius R is fixed, the flow stress $\bar{\sigma}^{\text{bd}}$ and thickness t^{bd} is also

determined. Therefore, whether the subsequent yield occurs only depends on its hoop force F , so the distribution of hoop force during THFG will be analyzed below.

3.3.1 Hoop force distribution

Fig. 4. Static stress equilibrium model: (a) double-curvature differential segment, (b) single-curvature differential segment.

As shown in Fig. 4, due to the existence of curved axis, the differential segments on the cross-section can be divided into two categories: the double-curvature differential segment and the single-curvature differential segment. Due to the different mechanics condition, the distribution of hoop force at these two kinds of differential segment will be solved individually.

1 the double-curvature differential segment

The hoop force distribution is caused by friction force. When the friction coefficient μ is fixed, the friction force f is only determined by the contact pressure p' . According to Fig. 4(a), we first establish the static stress equilibrium model along the normal direction on the double-curvature differential segment:

$$\begin{aligned} & \sigma_{\varphi}^{\text{th}} [R + r \cos(\varphi)] d\theta \cdot t^{\text{bd}} \sin(d\varphi/2) + (\sigma_{\varphi}^{\text{th}} - d\sigma_{\varphi}^{\text{th}}) [R + r \cos(\varphi)] d\theta \cdot t^{\text{bd}} \sin(d\varphi/2) \\ & + 2\sigma_{\theta}^{\text{th}} rd\varphi \cdot t^{\text{bd}} \sin(d\theta/2) + prd\varphi \cdot (R + r \cos \varphi) d\theta = p' rd\varphi (R + r \cos \varphi) d\theta \end{aligned} \quad (12)$$

Among them, the $d\sigma_{\varphi}^{\text{th}}$ caused by friction can be got by further building the static stress equilibrium model along the hoop direction:

$$\begin{aligned} & \sigma_{\varphi}^{\text{th}} [R + r \cos(\varphi)] d\theta \cdot t^{\text{bd}} \cos(d\varphi/2) - (\sigma_{\varphi}^{\text{th}} - d\sigma_{\varphi}^{\text{th}}) [R + r \cos(\varphi)] d\theta \cdot t^{\text{bd}} \cos(d\varphi/2) \\ & = \mu p' [R + r \cos(\varphi)] d\theta \cdot rd\varphi \end{aligned} \quad (13)$$

Eq. (13) can be simplified:

$$d\sigma_{\varphi}^{\text{th}} = \frac{p' \mu r d\varphi}{t^{\text{bd}} \cos(d\varphi/2)} \quad (14)$$

Because of plane strain, when the subsequent yield occurs, the axial stress $\sigma_{\theta}^{\text{th}}$ meets the requirement: $\sigma_{\theta}^{\text{th}} = (\sigma_{\varphi}^{\text{th}} + \sigma_t^{\text{th}})/2$, and the normal stress σ_t^{th} can be expressed as: $\sigma_t^{\text{th}} = (p + p')/2$. Consequently, the relationship between the hoop stress $\sigma_{\varphi}^{\text{th}}$ and the axial stress $\sigma_{\theta}^{\text{th}}$ is obtained as follows:

$$\sigma_{\theta}^{\text{th}} = \frac{\sigma_{\varphi}^{\text{th}}}{2} + \frac{p}{4} + \frac{p'}{4} \quad (15)$$

Substituting Eq. (14) and Eq. (15) into Eq. (12), after simplification, we can get:

$$\sigma_{\varphi}^{\text{th}} t^{\text{bd}} \left[\frac{r}{2} + (R + r \cos \varphi) \right] + pr \left[\frac{t^{\text{bd}}}{4} + (R + r \cos \varphi) \right] = p' r \left[(R + r \cos \varphi) - \frac{t^{\text{bd}}}{4} \right] \quad (16)$$

Further arrangement, the contact stress p' can be expressed:

$$p' = \frac{\sigma_{\varphi}^{\text{th}} t^{\text{bd}} \left[\frac{r}{2} + (R + r \cos \varphi) \right] + pr(R + r \cos \varphi)}{r(R + r \cos \varphi)} \quad (17)$$

Assuming the differential segment along the curved axis is the unit length, so the contact pressure p' at the double-curvature differential segment can be described as:

$$p' = \frac{F \left[\frac{r}{2} + (R + r \cos \varphi) \right] + pr(R + r \cos \varphi)}{r(R + r \cos \varphi)} = \lambda F + p \quad (18)$$

Here, we can define $\lambda = [r/2 + (R + r \cos \varphi)]/[r(R + r \cos \varphi)]$, especially, which can be further simplified as: $\lambda \approx 1/r + 1/2R$ when $R > 2r$.

However, for the differential segment which does not enter into the subsequent yield, the axial stress $\sigma_{\theta}^{\text{th}}$ remains in the stress state after pre-bending:

$$\sigma_{\theta}^{\text{th}} = \sigma_{\theta}^{\text{bd}} \quad (19)$$

Similarly, when the subsequent yield does not occur, by substituting Eq. (19) into Eq. (12) and simplifying, we can get the following result:

$$p' = \frac{F}{r} + p + \frac{\sigma_{\theta}^{bd} t^{bd}}{R + r \cos \varphi} \approx \frac{F}{r} + p + \frac{\sigma_{\theta}^{bd} t^{bd}}{R} \quad (20)$$

Fig. 5. Hoop force distribution during tube hydro-forging.

After the contact pressure p' is obtained, the hoop force F distribution on the cross-section can be calculated. In this paper, BB' is the parting surface, so the direction of material flow during THFG is shown in Fig. 5. According to the flow law of material, the cross-section must have two fixed differential segments A (A') and D (D') which are relative to the die, hence the cross-section can be divided into two parts: ABCD segment and A'B'C'D' segment. When the punch stroke is Δ , the perimeter of ABCD and A'B'C'D' will be compressed by Δ simultaneously. For convenience, we can assume that the angle of AB segment is φ_{11} , the length of BC segment is l_{11} , the angle of CD segment is φ_{12} , likewise, the angle of A'B' segment is φ_{21} , the length of B'C' segment is l_{21} , and the angle of C'D' segment is φ_{22} . Meanwhile, according to the geometric relationship, there are following expressions:

$$\varphi_{11} + \varphi_{21} = \pi, \varphi_{12} + \varphi_{22} = \pi, l_{11} = l_{21}.$$

Therefore, according to Fig. 5, the force balance formula on the double-curvature differential segment can be established as shown in Eq. (21):

$$F_1 = F_0 - f = F_0 - \mu \frac{\varphi}{n} p' r \quad (21)$$

Substituting Eq. (18) into Eq. (21), we can get

$$F_1 = \left(1 - \mu \lambda \frac{\varphi}{n} r\right) F_0 - \mu \frac{\varphi}{n} p r \quad (22)$$

In the same way:

$$F_2 = \left(1 - \mu\lambda \frac{\varphi}{n} r\right)^2 F_0 - \mu \frac{\varphi}{n} pr \left[1 + \left(1 - \mu\lambda \frac{\varphi}{n} r\right)\right] \quad (23)$$

According to the analogy method, it can be concluded that:

$$F_n = \left(1 - \mu\lambda \frac{\varphi}{n} r\right)^n F_0 - \mu \frac{\varphi}{n} pr \left[1 + \left(1 - \mu\lambda \frac{\varphi}{n} r\right) + K + \left(1 - \mu\lambda \frac{\varphi}{n} r\right)^{n-1}\right] \quad (24)$$

When $n \rightarrow \infty$, the limit of F_n can be taken as:

$$F_n = e^{-\mu\lambda r\varphi} \left(F_0 + \frac{p}{\lambda}\right) - \frac{p}{\lambda} \quad (25)$$

Similarly, if the differential segment has not entered into the subsequent yield, by using Eq. (20), we can obtain its hoop force distribution:

$$F_n = e^{-\mu\varphi} \left(F_0 + pr + \frac{\sigma_{\theta}^{\text{bd}} t^{\text{bd}} r}{R}\right) - pr - \frac{\sigma_{\theta}^{\text{bd}} t^{\text{bd}} r}{R} \quad (26)$$

2 the single-curvature differential segment

Through the above analysis, the hoop force distribution on the double-curvature differential segment can be solved. Moreover, the hoop force distribution on the single-curvature differential segment also can be derived using the same method. As shown in Fig. 4(b), static stress equilibrium model on the single-curvature differential segment can be built at first:

$$2\sigma_{\theta}^{\text{th}} dl \cdot t^{\text{bd}} \sin\left(\frac{d\theta}{2}\right) + p dl \cdot R d\theta = p' dl \cdot R d\theta \quad (27)$$

By simplifying, the contact pressure p' on the single-curvature differential segment which has entered into the subsequent yield can be calculated as follows:

$$p' = \frac{F}{2R} + p \quad (28)$$

In the same measure, when $n \rightarrow \infty$, it can assume the limit of F_n :

$$F_n = e^{-\frac{\mu l}{2R}} (F_0 + 2Rp) - 2Rp \quad (29)$$

For the single-curvature differential segment that has not entered into the subsequent yield, substituting Eq. (19) into Eq. (27), and then we can get:

$$p' = \frac{\sigma_{\theta}^{\text{bd}} t^{\text{bd}}}{R} + p \quad (30)$$

Furthermore, for the differential segment without entering into subsequent yield, it can

be obtained by analogous method:

$$F_n = F_0 - \mu \left(\frac{\sigma_{\theta}^{bd} t^{bd}}{R} + p \right) l \quad (31)$$

3 hoop force distribution on the whole cross-section

Through the above efforts, the hoop force distribution on the whole cross-section can be obtained. For example, if all differential segments have entered into the subsequent yield, the hoop force distribution can be obtained in turn:

For the ABCD segment, if the hoop force at the differential segment A is F_A , we can get the hoop force F_B at the differential segment B:

$$F_B = e^{\mu \lambda r \varphi_{11}} \left(F_A + \frac{p}{\lambda} \right) - \frac{p}{\lambda} \quad (32)$$

The hoop force F_C at the differential segment C:

$$F_C = e^{-\frac{\mu r l_{11}}{2R}} (F_B + 2Rp) - 2Rp \quad (33)$$

The hoop force F_D at the differential segment D:

$$F_D = e^{-\mu \lambda \varphi_{12}} \left(F_C + \frac{p}{\lambda} \right) - \frac{p}{\lambda} \quad (34)$$

Similarly, for the A'B'C'D' segment, if the hoop force of the differential segment A' is $F_{A'}$, we can also get the hoop force $F_{B'}$ at the differential segment B':

$$F_{B'} = e^{\mu \lambda \varphi_{21}} \left(F_{A'} + \frac{p}{\lambda} \right) - \frac{p}{\lambda} \quad (35)$$

The hoop force $F_{C'}$ at the differential segment C':

$$F_{C'} = e^{-\frac{\mu l_{21}}{2R}} (F_{B'} + 2Rp) - 2Rp \quad (36)$$

The hoop force $F_{D'}$ at the differential segment D':

$$F_{D'} = e^{-\mu \lambda \varphi_{22}} \left(F_{C'} + \frac{p}{\lambda} \right) - \frac{p}{\lambda} \quad (37)$$

In which, $F_{A'}$ is equal to F_A , so it can be found that the hoop force at any differential segment on the cross-section can be solved by the hoop force F_A , such as:

$$F_C = e^{-\frac{\mu l_{11}}{2R}} \left[e^{\mu \lambda \varphi_{11}} \left(F_A + \frac{p}{\lambda} \right) - \frac{p}{\lambda} + 2Rp \right] - 2Rp \quad (38)$$

If the differential segment has not entered into the subsequent yield, it just needs to substitute Eq. (26) and Eq. (31) into the above analysis.

3.3.2 Stress-strain corresponding relationship

The hoop force distribution on the cross-section is obtained, and then by combining with

Eq. (11), the hoop stress distribution also can be acquired. In order to get the distribution of hoop strain on the cross-section, it is necessary to solve the stress-strain corresponding relationship.

Fig. 6. Stress/strain state at differential segment.

At first, when the hoop stress does not satisfy the subsequent yield condition, there is no plastic deformation on the corresponding differential segment:

$$\varepsilon_{\phi}^{\text{th}} = 0 \quad (39)$$

However, when the hoop stress $\sigma_{\phi}^{\text{th}}$ reaches the subsequent yield, the corresponding differential segment will undergo plastic deformation, and the hoop strain $\varepsilon_{\phi}^{\text{th}}$ will generate.

Considering that THFG satisfies the plane strain ($\varepsilon_{\theta}^{\text{th}} = 0$), so it can be obtained by combining the Eqs. (2) ~ (3):

$$\bar{\sigma}^{\text{th}} = \frac{\sqrt{3}}{2} |\sigma_{\phi}^{\text{th}} - \sigma_t^{\text{th}}| \quad (40)$$

$$\bar{\varepsilon}^{\text{th}} = \frac{2}{\sqrt{3}} |\varepsilon_{\phi}^{\text{th}}| \quad (41)$$

According to Appendix A., there is $\beta = 2 / \sqrt{3}$.

Substituting Eq. (40) and Eq. (41) into the Hollomon hardening law, it can be got:

$$\frac{1}{\beta} |\sigma_{\phi}^{\text{th}} - \sigma_t^{\text{th}}| = K |\beta \varepsilon_{\phi}^{\text{th}}|^n \quad (42)$$

The hoop stress $\sigma_{\phi}^{\text{th}}$ can be got by simplifying Eq. (42):

$$\sigma_{\phi}^{\text{th}} = K \beta^{n+1} \varepsilon_{\phi}^{\text{th}n} + \sigma_t^{\text{th}} \quad (43)$$

Consequently, the hoop force F can be described:

$$F = \sigma_{\varphi}^{\text{th}} t^{\text{th}} = \left(K \beta^{n+1} \varepsilon_{\varphi}^n + \sigma_t^{\text{th}} \right) t^{\text{th}} \quad (44)$$

Because of $\varepsilon_{\varphi}^{\text{th}} = -\varepsilon_t^{\text{th}}$, so the thickness after THFG t^{th} can be expressed by $\varepsilon_{\varphi}^{\text{th}}$:

$$t^{\text{th}} = t^{\text{bd}} \left(1 + \varepsilon_{\varphi}^{\text{th}} \right) \quad (45)$$

Combining Eqs. (44) ~ Eq. (45) and neglecting the high order item $\sigma_t^{\text{th}} \varepsilon_{\varphi}^{\text{th}}$, we get:

$$\frac{F - \sigma_t^{\text{th}} t^{\text{bd}}}{t^{\text{bd}} K \beta^{n+1}} = \varepsilon_{\varphi}^{\text{th}n} + \varepsilon_{\varphi}^{\text{th}n+1} \quad (46)$$

Therefore, we can establish the mathematical expression of hoop strain $\varepsilon_{\varphi}^{\text{th}}$:

$$\varepsilon_{\varphi}^{\text{th}} = \frac{1}{2} \sqrt{n^2 + 4n \left(\frac{F - \sigma_t^{\text{th}} t^{\text{bd}}}{K t^{\text{bd}} \beta^{n+1}} \right)^{1/n}} - \frac{n}{2} \quad (47)$$

Section 3.3.1 has obtained the hoop force distribution on the cross-section. Substituting it into Eq. (47), the hoop strain distribution on the cross-section can be obtained, and then the maximum hoop strain on the cross-section also can be got.

3.3.3 Relationship between maximum hoop strain and punch stroke

According to the analysis in 3.3.1 and 3.3.2, in order to establish the distribution model of hoop force and hoop strain when considering the pre-bending, three unknowns need to be solved, namely F_A , φ_{11} and φ_{21} . And for any punch stroke Δ_i , we can get the following three expressions:

(1) The integral of the whole hoop strain of ABCD segment is equal to the punch stroke Δ_i :

$$\int_0^{\varphi_{11}} \varepsilon_{\varphi}^{\text{th}} d\varphi + \int_0^{l_{11}} \varepsilon_{\varphi}^{\text{th}} dl + \int_0^{\varphi_{12}} \varepsilon_{\varphi}^{\text{th}} d\varphi = \Delta_i \quad (48)$$

(2) The integral of the whole hoop strain of A'B'C'D' segment is also equal to the punch stroke Δ_i :

$$\int_0^{\varphi_{21}} \varepsilon_{\varphi}^{\text{th}} d\varphi + \int_0^{l_{21}} \varepsilon_{\varphi}^{\text{th}} dl + \int_0^{\varphi_{22}} \varepsilon_{\varphi}^{\text{th}} d\varphi = \Delta_i \quad (49)$$

(3) The values of hoop force F_D and $F_{D'}$ are the same:

$$e^{-\mu\lambda\varphi_{12}} \left(F_C + \frac{p}{\lambda} \right) - \frac{p}{\lambda} = e^{-\mu\lambda\varphi_{22}} \left(F_{C'} + \frac{p}{\lambda} \right) - \frac{p}{\lambda} \quad (50)$$

By solving Eqs. (48) ~ (50), the three unknowns under any punch stroke Δ_i can be solved.

Substituting these three unknowns into the above theoretical derivation, the distribution of hoop force and hoop strain under any punch stroke Δ_i will be obtained, and then the maximum hoop strain under this punch stroke Δ_i also can be got.

3.4 Calculation process

Fig. 7. Specific calculation process.

Fig. 7 lists the specific calculation process for the critical internal pressure when considering the pre-bending. In order to solve the critical internal pressure under any punch stroke Δ_i , there needs a cycle calculation. At first, it is necessary to give an initial value of internal pressure p_0 . Subsequently, the corresponding maximum hoop strain on the cross-section can be solved. By substituting it into the change rule, we can get the temporary value of critical internal pressure $\bar{p}_{cri,i}$. If it meets the condition: $|p_0 / \bar{p}_{cri,i} - 1| \leq 1\%$, the critical internal pressure $p_{cri,i}^{bd}$ of pre-bending tube is $\bar{p}_{cri,i}$ ($p_{cri,i}^{bd} = \bar{p}_{cri,i}$). Otherwise, let $p_0 = \bar{p}_{cri,i}$ and re-substitute p_0 into the cycle calculation until the condition is met. Finally, by

iterating $\Delta_{i+1}=\Delta_i+d\Delta$, the loading path between the critical internal pressure and the punch stroke can be built for when considering pre-bending.

4 Experiment and FE simulation

4.1 Materials and experiment

Fig. 8. True stress-strain curve for DP800.

The material used in this paper was DP800 tube and its cross-sectional shape parameters were: $r = 14\text{mm}$, $l = 8.4\text{mm}$, $t_0 = 1.5\text{mm}$. The strip-shape specimens were cut from the tube along the axial direction. The true stress-strain curve of the material was obtained through the uniaxial tensile test, as shown in Fig. 8, and corresponding mechanical parameters were shown in Table 1.

Table 1 Material mechanical parameters for DP800.

Mechanical parameters	Value
Elastic modulus, E (GPa)	206
Poisson ratio, ν	0.3
Yield stress, σ_s (MPa)	710
Tensile strength, σ_b (MPa)	905
Strain hardening exponent, n	0.09113
Strength coefficient, K (MPa)	1064

Fig. 9. Experiment device: (a) rotary draw bending, (b) special hydro-forging machine.

In the practical production, there are many methods to realize the pre-bending process, such as rotary draw bending (RDB), press bending, pushing bending etc. Among them, RDB which is combined with computer numerical control (CNC) can obtain the required bending radius and angle stably. And through the application of mandrel, the bending wrinkling and the cross-sectional distortion both can be controlled. In addition, the cross-sectional characteristic of the pre-bending tube formed by RDB is more consistent along the axis, which is also conducive for the theoretical analysis. As shown in Fig. 9(a), the experiment device of RDB included pressure die, clamping die, mandrel and bending die. The radius of bending die was set as 100, 150, 200 and 250 mm respectively to study the influence of different bending radius R on the critical internal pressure.

THFG needs a special experimental device: hydro-forging machine which could accurately control the loading path between the internal pressure and the punch stroke. The machine was mainly composed of three parts: experiment setup, computer control system,

pressurization system, as shown in Fig. 9(b). Additionally, the experiment setup included upper die, lower die and sealing means. All the digital signals were collected into the computer control system, thus the experimental data could be output in real time. For the effect of friction coefficient, two kinds of experimental conditions were set up [21]: dry friction and MoS₂. Their corresponding friction coefficients are shown in Table 2. In order to reduce the experimental error, each group of experiment was repeated three times, and finally the average value was taken as the experimental result.

Table 2 Friction coefficients of different lubricants [21].

Conditions	Friction coefficient
Dry friction	0.09
MoS ₂	0.018

4.2 FE method

According to the experiment requirement, the RDB model and the THFG model were established respectively, as shown in Fig. 10. The RDB model also consisted of five parts: ① pressure die, ② clamp die, ③ mandrel, ④ tube, and ⑤ bending die. The dies were all defined as rigid body, while the tube was set as a deformed body with the isotropic material model. Meanwhile, the tube was discretized by the element type of C3D8R. In order to guarantee the accuracy of FE simulation, the element sizes were 2 mm and 5mm along the hoop and axial direction respectively, and the thickness direction was defined 3 elements. The radius of bending die was selected as 100, 150, 200 and 250 mm respectively. The THFG model adopted restart analysis, and the model mainly included three parts: ① upper die, ② tube (imported by pre-bending step), ③ lower die. Here, since the performing tube in this step was imported from the pre-bending step, its settings were the same as before. According to Table 2, the friction coefficient μ between the die and the tube was set to 0.09 and 0.018 respectively, and the Coulomb friction criterion was used in FE simulation.

Fig. 10. FE simulation: (a) rotary draw bending model: ① pressure die, ② clamp die, ③ mandrel, ④ tube, ⑤ bending die, (b) hydro-forming model: ① upper die, ② tube (imported from pre-bending step), ③ lower die.

5 Results and discussion

5.1 Experiment results

Fig. 11. Wrinkling situations of pre-bending tubes under different internal pressure.

Some of experiment results are shown in Fig. 11. When the punch stroke was 3 mm, different wrinkling situations could be obtained by applying different internal pressure. For the straight tube, according to the previous mathematical model deduced by Chu [14], its critical internal pressure p_{cri}^{str} needed to suppress wrinkling was 46MPa. However, for the

pre-bending tube, when the provided internal pressure p_{pro} is 46MPa, wrinkling only occurred at the outer straight-wall, but not at the inner straight-wall. When the p_{pro} reduced to 30MPa, wrinkling occurred at both inner and outer straight-wall. And the wrinkling could be restrained entirely until the p_{pro} expanded to 80MPa. From the experiment results, we can get that under the same punch stroke Δ , the difference of critical internal pressure not only exists in the pre-bending tube and the straight tube, but also in the inner and outer straight-wall of pre-bending tube.

5.2 Theoretical analysis of the pre-bending effect

The distribution of flow stress and thickness on the cross-section after pre-bending is studied at first, as shown in Fig. 12. When the $R = 150$ mm, the thickness reduction at the B'C' segment (outer straight-wall) is the most serious, and the thickness increase at the BC segment (inner straight-wall) is the most prominent. Similarly, due to the cold work-hardening, the flow stress at each differential segment all improve in different degrees. The flow stress distribution on the cross-section is approximately concave, and the cold work-hardening degree of BC and B'C' segment is the most serious, while the cold work-hardening degree of neutral layer is the least. Therefore, the distribution of thickness and flow stress will lead to the difference of plastic deformation between the pre-bending tube and the straight tube during the subsequent THFG. It can be found that the initial state of the differential segment at the inner/outer straight-wall is different due to the effect of pre-bending.

Fig. 12. Distribution of flow stress and thickness after pre-bending.

According to Section 3.2, we can obtain the change rule between the critical internal pressure and the hoop strain for the pre-bending tube. In Fig. 13, when pre-bending is considered, it can be clearly seen that the change rules on the inner/outer straight-wall are basically the same. When the R is 150 mm, for the inner/outer straight-wall, their thickness after pre-bending is 1.29 mm and 1.76 mm respectively, and their flow stress is 875 MPa and 878 MPa individually. A conclusion can be drawn by comparing the change rule of the inner/outer straight-wall: the difference in thickness has little effect on the change rule, while cold work-hardening has significant effect on that. Especially if the hoop strain is fixed, the critical internal pressure required for the inner/outer straight-wall is both greater than that required for the straight tube. With the increase of hoop strain, the change rule difference between pre-bending tube and straight tube also increases.

Fig. 13. Influence of pre-bending on the change rule.

After obtaining the change rule between critical internal pressure and hoop strain, it is necessary to analyze the hoop strain $\varepsilon_\theta^{\text{th}}$ distribution. For comparative purposes, the critical internal pressure for the straight tube $p_{\text{cri}}^{\text{str}}$ is applied to the pre-bending tube and the straight tube concurrently, that is, let $p_{\text{pro}} = p_{\text{cri}}^{\text{str}}$, as shown in Fig. 14, where the punch stroke Δ are taken as 0.5, 1, 2 and 3mm respectively. It can be seen from the figure that the fixed differential segments A (A') and D (D') on the pre-bending tube will move with the increase of Δ , but for straight tube, the above fixed differential segments always remain at the midpoint of the arc. The initial state of each differential segment is all different after pre-bending, which

leads to the asymmetry of plastic deformation at ABCD and A'B'C'D' segment during subsequent THFG.

More importantly, for ABCD and A'B'C'D' segment, the hoop strain $\varepsilon_{\phi}^{\text{th}}$ at the straight-wall is always greater than that at the arc under any punch stroke, whether the pre-bending is considered. This is because the differential segments B and B' are the parting surface, where the hoop force is the largest. And under the action of friction, the hoop force diminishes along both sides of the parting surface, and reaches its minimum value at fixed differential segments A (A') and D (D') respectively. According to Eq. (47), the hoop strain is proportional to the hoop force, so the hoop strain at the arc is always smaller than that at the straight-wall. In addition, it also can be found that when Δ is 0.5mm, the hoop strain does not occur at some differential segments. Due to the cold work-hardening caused by pre-bending, the hoop force F does not reach the subsequent yield condition of these differential segments.

Fig. 14. Hoop strain distribution under different punch stroke: (a) $\Delta=0.5\text{mm}$, (b) $\Delta=1\text{mm}$, (c) $\Delta=2\text{mm}$, (d) $\Delta=3\text{mm}$.

From the above analysis, it can be known that the hoop strain at the straight-wall is always greater than that at the arc, and reaches its maximum value at differential segments B

and B' respectively, whether pre-bending is considered. Wrinkling is most likely to occur at the maximum hoop strain, so the following will study the effect of pre-bending on the strain at differential segments B and B'. Obviously, under the same Δ and p_{pro} , the hoop strain at B' differential segment of pre-bending tube is larger than that at the same position of straight tube, while the hoop strain at differential segment B of pre-bending tube is smaller than that at the same position of straight tube, as shown in Fig. 14. This is because when the R is 150 mm, the thickness t^{bd} at B' differential segment reduces to 1.29 mm after pre-bending. On the one hand, the decrease of thickness results in the larger hoop stress under the same hoop force, which leads the differential segment B' is easier to enter into the subsequent yield firstly according to Eq. (10); On the other hand, based on Eq. (47), the thinner the thickness is, the greater the hoop strain will be under the same punch stroke at the differential segment B'. In addition, the thickness at differential segment B increases to 1.76 mm, which causes the opposite result compared with differential segment B'. The thickness of straight tube is between the above two differential segments, so its hoop strain is also between the above two differential segments. Meanwhile, according to the analysis of the change rule, although the provided internal pressure p_{pro} can inhibit the wrinkling of straight tube, it is not enough for pre-bending tube. Therefore, the wrinkling of the pre-bending tube will occur in this case.

Fig. 15. Hoop strain at the differential segments B and B' under the critical internal pressure.

Through the cycle calculation, the hoop strain $\varepsilon_{\phi}^{\text{th}}$ of the differential segments B and B' under the corresponding critical internal pressure can be obtained respectively, as shown in Fig. 15. When considering the effect of pre-bending, the hoop strain on the differential segment B' is always greater than that on the differential segment B under any punch stroke. However, for straight tube, the value of hoop strain on the differential segments B' and B is equal, and the value is just between that at the inner/outer straight-wall of pre-bending tube. The reason has been explained above. In addition, with the increase of punch stroke, the hoop strain difference between differential segments B' and B also increases.

Fig. 16. Loading path for outer/inner straight wall when the R is 150mm.

According to the specific calculation process shown in Fig. 7, the critical internal pressure required for the inner/outer straight-wall to suppress wrinkling can be obtained respectively, as shown in Fig. 16. When the different internal pressure is applied, there are three different results: 1. When the provided internal pressure p_{pro} is greater than the critical internal pressure $p_{\text{req}}^{\text{out}}$ of the outer straight-wall ($p_{\text{pro}} > p_{\text{cri}}^{\text{out}}$), the wrinkling can be completely inhibited; 2. However, when the provided internal pressure p_{pro} is smaller than the critical internal pressure $p_{\text{req}}^{\text{in}}$ of the inner straight-wall ($p_{\text{pro}} < p_{\text{cri}}^{\text{in}}$), wrinkling occurs on

the outer/inner straight-wall simultaneously; 3. Besides, when $p_{\text{cri}}^{\text{out}}$ f p_{pro} f $p_{\text{cri}}^{\text{in}}$ is applied, the wrinkling only occurs at the outer straight-wall, but not at the inner straight-wall. Consequently, the experimental results in Section 5.1 can be reasonably explained. The third result needs relatively high internal pressure, so its wrinkling height is less than that of the second result. Meanwhile, it can be seen that the critical internal pressure $p_{\text{cri}}^{\text{str}}$ of the straight tube is just between $p_{\text{cri}}^{\text{out}}$ and $p_{\text{cri}}^{\text{in}}$.

In conclusion, the critical internal pressure $p_{\text{cri}}^{\text{out}}$ of the outer straight-wall is the minimum internal pressure required to inhibit the wrinkling of pre-bending tube entirely. Therefore, the $p_{\text{req}}^{\text{out}}$ can be defined as the critical internal pressure $p_{\text{cri}}^{\text{bd}}$ of the pre-bending tube. And $p_{\text{cri}}^{\text{bd}}$ is always greater than $p_{\text{cri}}^{\text{str}}$ under the same Δ . Besides, with the increase of punch stroke, the effect of pre-bending is more significant. At first, the increase of punch stroke leads to amplify the hoop strain difference between the pre-bending tube and the straight tube; then, according to Fig. 13, the greater the hoop strain difference, the greater the change rule difference between the pre-bending tube and the straight tube. Therefore, the critical internal pressure enlarges with the increase of punch stroke.

5.3 Pre-bending radius

The following will mainly study the influence of the pre-bending radius R on the $p_{\text{cri}}^{\text{bd}}$. Here, the bending radius of 100, 150, 200 and 250mm are selected respectively.

Fig. 17. Influence of pre-bending radius on the change rule.

For the pre-bending tube, the $p_{\text{cri}}^{\text{bd}}$ is determined by its outer straight-wall. Fig. 17. shows the influence of different R values on the change rule between hoop strain and critical internal pressure for the outer straight-wall. Under the same hoop strain $\varepsilon_{\phi}^{\text{th}}$, the critical internal pressure $p_{\text{cri}}^{\text{bd}}$ increases slightly with the decrease of radius. According to the analysis in Section 5.2, the cold work-hardening caused by pre-bending is the main reason for the difference between pre-bending tube and straight tube. The smaller the R , the more severe the cold work-hardening on the outer straight-wall, which in turn leads to the greater the critical internal pressure required.

Fig. 18. Influence of pre-bending radius on hoop strain at differential segment B'.

It can be seen from the above that the maximum hoop strain of pre-bending tube is located at the differential segment B', and the effect of pre-bending radius on the maximum hoop strain is shown in Fig. 18. Apparently, when the punch stroke is fixed, the maximum hoop strain difference at the differential segment B' between pre-bending tube and straight tube decreases with the increase of pre-bending radius. The thickness reduction of outer straight-wall is the main cause of the hoop strain difference. According to Eqs. (6) ~ (7), as the bending radius increases, the effect of pre-bending on the thickness reduction and cold work-hardening is weakened, which leads to the decrease of maximum hoop strain difference.

Fig. 19. Effect of pre-bending radius on critical internal pressure p_{cri}^{bd} : (a) $\Delta=1$ mm, (b) $\Delta=2$ mm, (c) $\Delta=3$ mm, (d) $\Delta=4$ mm.

Then, by substituting the hoop strain at the differential segment B' into the change rule, the influence of the R on the critical internal pressure p_{cri}^{bd} can be obtained, as shown in Fig. 19. On the one hand, according to the change rule shown in Fig. 17., the smaller the R is, the higher the critical internal pressure p_{cri}^{bd} is under the fixed hoop strain; On the other hand, with the decrease of R , the maximum hoop strain difference between pre-bending tube and straight tube increases. Therefore, combining the above reasons, there is an important conclusion that with decrease of the R , the critical internal pressure difference between p_{cri}^{bd} and p_{cri}^{str} increases under the same punch stroke. For example, if the punch stroke Δ is 4mm (shown in Fig. 19(d)), the minimum value of the difference $\delta p_{min}^{\Delta=4}$ is 24.2MPa when the R is 250mm, and the difference reaches its maximum value $\delta p_{max}^{\Delta=4} = 52.9$ MPa when the R reduces to 100mm, which cannot be ignored in actual production. On the contrary, with the

decrease of punch stroke, the difference of critical internal pressure between p_{cri}^{bd} and p_{cri}^{str} also decreases. When the R is 100mm, if the punch stroke decreases to 2mm, the maximum value $\delta p_{max}^{\Delta=2}$ will reduce to 23.4MPa.

5.4 Friction coefficient

Fig. 20. Effect of friction coefficient on critical internal pressure: (a) $\Delta=3mm$, (b) $\Delta=4mm$.

Through the above theoretical analysis, it is found that friction is one of the important factors affecting the hoop force distribution. Therefore, in this paper, two lubrication conditions (dry friction and MoS₂) are selected to study the influence of friction on the critical internal pressure p_{cri}^{bd} , and the specific friction coefficient values are shown in Table 1. From Fig. 20, whether the pre-bending is considered, the increase of μ value will lead to the increase of the critical internal pressure under the same compression. The reason is that according to the theoretical analysis in Section 3.3, in order to produce the same punch stroke, the larger the μ value will increase the hoop force and the hoop strain required at differential segment B', and then the critical internal pressure required also will be enlarged. In addition, with the decrease of μ , the effect of pre-bending becomes more pronounced, that is, the critical internal pressure difference between p_{cri}^{bd} and p_{cri}^{str} increases with the reduction of friction coefficient. Therefore, the smaller the friction coefficient, the more important it is to consider the influence of pre-bending.

5.5 Practical application

Fig. 21. Automobile longeron formed by THFG combining with pre-bending.

The automobile longeron is a typical complex cross-sectional tubular component with curved axis, as shown in Fig. 21. There are two difficulties in forming this part: (1) because of 3D curved axis, the axis springback after pre-bending is large and difficult to control (springback was 10mm) (2) The cross-sectional shape is complex, and the perimeter difference between the maximum cross-section and the minimum cross-section can reach 15%. THFG combining with pre-bending is very suitable to manufacture this kind of components. According to the model of this paper, the critical internal pressure required for THFG when considering the pre-bending is 54MPa. Finally, not only the axis springback can be controlled (dropped almost to zero) [17], but also the variable cross-section along the axis can be obtained without wrinkling.

6 Conclusion

By combining energy method with mechanics analysis, the influence of pre-bending on the subsequent THFG, especially on the critical internal pressure required to inhibit wrinkling, was clarified in this research. The main following conclusions were obtained:

- (1) Through the energy method, the change rule of the critical internal pressure with the hoop strain can be obtained first for when existing the pre-bending. Because of cold work-hardening, the critical internal pressure of pre-bending tube is higher than that of straight tube under the same hoop strain, and their difference becomes more obvious with the decrease of bending radius. On the contrary, the variation of thickness caused by pre-bending has little effect on this change rule.
- (2) Based on the mechanics analysis, the hoop strain distribution and the maximum hoop

-
- strain considering pre-bending both can be determined. For pre-bending tube, mainly due to the thickness variation, the maximum hoop strain of the outer straight-wall is greater than that of the inner straight-wall. And the decrease of R value leads to the increase of the maximum hoop strain difference between pre-bending tube and straight tube.
- (3) After substituting the maximum hoop strain at the inner/outer straight-wall of pre-bending tube into the change rule, the mathematical model of the corresponding critical internal pressure p_{cri}^{out} and p_{cri}^{in} can be obtained respectively. When the provided internal pressure $p_{pro} \leq p_{cri}^{in}$, wrinkling occurs at both the outer/inner straight-wall, while $p_{cri}^{out} \leq p_{pro} \leq p_{cri}^{in}$, wrinkling only occurs at the outer straight-wall, and only under $p_{pro} \leq p_{cri}^{out}$, wrinkling can be suppressed entirely. Therefore, p_{cri}^{out} is defined as critical internal pressure p_{cri}^{bd} of the pre-bending tube.
- (4) The critical internal pressure p_{cri}^{bd} of pre-bending tube is always greater than that p_{cri}^{str} of straight tube under the same punch stroke. Moreover, with the decrease of bending radius, the difference between p_{cri}^{bd} and p_{cri}^{str} will increase. If the punch stroke is 4mm, the critical internal pressure difference is 33% when the R is 250mm, while the R decreases to 100mm, the critical internal pressure difference enlarges to 74%. Besides, the bigger the punch stroke, the more significant the pre-bending effect.
- (5) Whether pre-bending is considered, the smaller the friction coefficient is, the smaller the critical internal pressure requires. But the smaller the friction coefficient will lead the influence of pre-bending to be more prominent.

The above results were verified by experiment and FE simulation. This work provided a new prediction model of critical internal pressure on THFG, particularly when there is pre-bending. In conclusion, the effect of pre-bending on the THFG must be considered, so as to improve the reliability in practical application.

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1 Appendix A. Mathematical model for pre-bending

Fig. A-1. Parameter diagram of any cross-section during bending.

According to Eq. (4), the expression $\sigma_\phi^{bd} = \sigma_\theta^{bd}/2$ can be satisfied. Subsequently, by combining with Eq. (2) and Eq. (3), the equivalent stress $\bar{\sigma}^{bd}$ and the equivalent strain $\bar{\varepsilon}^{bd}$ after pre-bending can be reduced to:

$$\bar{\sigma}^{bd} = \frac{\sqrt{3}}{2} |\sigma_\theta^{bd}| \quad (A.1)$$

$$\bar{\varepsilon}^{bd} = \frac{2}{\sqrt{3}} |\varepsilon_\theta^{bd}| \quad (A.2)$$

Here, let $\beta = \frac{2}{\sqrt{3}}$.

On the basis of the previous study [22], the radius of neutral layer ρ_u after pre-bending can be determined as follows:

$$\rho_u = r \sqrt{\left(\frac{R}{r}\right)^2 - 1} \quad (A.3)$$

The normal strain ε_t^{bd} and the axial strain ε_θ^{bd} can be expressed as:

$$\varepsilon_t^{bd} = \ln \frac{t^{bd}}{t_0} \quad (A.4)$$

$$\varepsilon_{\theta}^{\text{bd}} = \ln \frac{R + (r + t^{\text{bd}}) \cos \varphi}{\rho_u} \quad (\text{A.5})$$

Here, t_0 is the initial thickness and t^{bd} is the thickness after pre-bending.

In case of $\varepsilon_t^{\text{bd}} = -\varepsilon_{\theta}^{\text{bd}}$, the distribution of thickness t^{bd} and equivalent strain $\bar{\varepsilon}^{\text{bd}}$ after pre-bending can be achieved by combining Eq. (A.4) and Eq. (A.5):

$$t^{\text{bd}} = \begin{cases} \frac{\rho_u t_0}{R} & \varphi = \frac{\pi}{2} \\ \frac{-(R + r \cos \varphi) + \sqrt{(R + r \cos \varphi)^2 + 4 \cos \varphi t_0 \rho_u}}{2 \cos \varphi} & \varphi \neq \frac{\pi}{2} \end{cases} \quad (\text{A.6})$$

$$\bar{\varepsilon}^{\text{bd}} = \beta \ln \frac{R + (r + t^{\text{bd}}) \cos \varphi}{\rho_u} \quad 0 \leq \varphi \leq \pi \quad (\text{A.7})$$

By substituting the Eq. (A.7) into the Hollomon hardening law, the flow stress distribution $\bar{\sigma}^{\text{bd}}$ after pre-bending can be obtained:

$$\bar{\sigma}^{\text{bd}} = K \left[\beta \ln \frac{R + (r + t^{\text{bd}}) \cos \varphi}{\rho_u} \right]^n \quad 0 \leq \varphi \leq \pi \quad (\text{A.8})$$

Appendix B. Solution of the critical internal pressure by energy method

Fig. B-1. Schematic of the wrinkling plate being suppressed: (a) perfect plate, (b) wrinkling plate.

As shown in Fig. B-1, if there is a small edge displacement $u_{\varphi}/2$ and the internal pressure p is enough, the wrinkling will not produce and still remain the perfect plate, which needs the plastic strain energy in a perfect plate E_0 . However, when the internal pressure p is insufficient, wrinkling will occur, and the bending energy in a wrinkling plate is E_w . According to the energy method [23], if the wrinkling can be suppressed by internal pressure p exactly, the difference between E_0 and E_w is the minimum external work W_p which is executed by the

internal pressure p and can be expressed as:

$$W_p = E_0 - E_w \quad (\text{B.1})$$

First, we can assume that the mode shape of the wrinkling plate is a cosine wave:

$$y = \frac{\delta}{2}(1 - \cos mx) \quad (\text{B.2})$$

In which, L denotes the wrinkling wave length and δ is the maximum wrinkling amplitude. Besides, m is the frequency of the cosine mode, which can be expressed as:

$$m = \frac{2\pi}{L - u_\varphi} \quad (\text{B.3})$$

For a perfect plate, if a certain edge displacement $u_\varphi/2$ is applied, the hoop compression strain $\varepsilon_\varphi^{\text{th}}$ can be determined as:

$$\varepsilon_\varphi^{\text{th}} = \ln \left(1 - \frac{u_\varphi}{L} \right) \quad (\text{B.4})$$

Therefore, the m also can be expressed as:

$$m = \frac{2\pi}{Le^{\varepsilon_\varphi^{\text{th}}}}$$

Assuming that the length of the plate before and after wrinkling is equal, both are L . Therefore, according to the arc length integral criterion, we can get:

$$L = \int_0^{L-u_\varphi} \sqrt{1 + y'^2} d\varphi \approx \int_0^{L-u_\varphi} 1 + \frac{1}{2} y'^2 d\varphi \quad (\text{B.5})$$

Through Taylor expansion, the maximum wrinkling amplitude δ can be calculated by the hoop compression strain $\varepsilon_\varphi^{\text{th}}$:

$$\delta = \frac{2}{\pi} \sqrt{u_\varphi (L - u_\varphi)} = \frac{2L}{\pi} \sqrt{e^{\varepsilon_\varphi^{\text{th}}} - e^{2\varepsilon_\varphi^{\text{th}}}} \quad (\text{B.6})$$

In addition, Cao [23] takes the wrinkling in a deep drawing operation as the research object and assumes that the external force F satisfies $F = F_{\max} - F_{\max} (y / \delta - 1)^2$, so the critical internal pressure p_{cri} can be described:

$$p_{\text{cri}} = \frac{3(E_0 - E_w)}{2\delta L} \quad (\text{B.7})$$

According to Eq. (5), a plane strain state along the axial direction is assumed, thus the equivalent strain can be expressed as:

$$\bar{\varepsilon}^{\text{th}} = \beta \varepsilon_{\varphi}^{\text{th}} \quad (\text{B.8})$$

Substituting Eq. (B.8) into the Hollomon hardening law to solve the equivalent stress:

$$\bar{\sigma}^{\text{th}} = K \left(\beta \varepsilon_{\varphi}^{\text{th}} \right)^n \quad (\text{B.9})$$

For pre-bending tube, compared with straight tube, it is necessary to consider the effect of thickness variation and work-hardening when calculating E_0 and E_w . Therefore, based on the result in Section 3.1, E_0 and E_w can be expressed as follows respectively [23]:

$$E_0 = \iint \bar{\sigma}^{\text{th}} d\bar{\varepsilon}^{\text{th}} dV = \frac{KLt^{\text{bd}}}{n+1} \left[\left(\beta \varepsilon_{\varphi}^{\text{th}} + \bar{\varepsilon}^{\text{bd}} \right)^{n+1} - \left(\bar{\varepsilon}^{\text{bd}} \right)^{n+1} \right] \quad (\text{B.10})$$

$$E_w = \frac{2Kt^{\text{bd}}}{n+1} \left[\left(\frac{\beta t^{\text{bd}}}{2} + \bar{\varepsilon}^{\text{bd}} \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} - \left(\bar{\varepsilon}^{\text{bd}} \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} \right] \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right)^{-n} \tan^{-1} \left(\frac{m\delta}{2} \right) \quad (\text{B.11})$$

By substituting Eq. (B.10) and Eq. (B.11) into Eq. (B.7), the change rule between the critical internal pressure and the hoop compression strain under pre-bending condition can be obtained:

$$p_{cr} = \left[\left(\beta \varepsilon_{\varphi}^{\text{th}} + \bar{\varepsilon}^{\text{bd}} \right)^{n+1} - \left(\bar{\varepsilon}^{\text{bd}} \right)^{n+1} \right] \frac{3Kt^{\text{bd}}}{2\delta(n+1)} - \left[\left(\frac{\beta t^{\text{bd}}}{2} + \bar{\varepsilon}^{\text{bd}} \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} - \left(\bar{\varepsilon}^{\text{bd}} \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right) \right)^{n+1} \right] \frac{3Kt^{\text{bd}} \left(\frac{2}{m^2 \delta} + \frac{t^{\text{bd}}}{2} \right)^{-n}}{L\delta(n+1)} \tan^{-1} \left(\sqrt{e^{-\varepsilon_{\varphi}^{\text{th}}} - 1} \right) \quad (\text{B.12})$$

According to the Cao's mathematical model [24], for two certain wrinkling lengths L_1 and L_2 , there is a corresponding transition strain $\varepsilon_{\varphi}^{\text{th}}$, thus the following relationship can be built as:

$$p_1(L_1, \varepsilon_{\varphi}^{\text{th}}) = p_2(L_2, \varepsilon_{\varphi}^{\text{th}}) \quad (\text{B.13})$$

Eventually, we can use computer software to obtain the $\varepsilon_{\varphi}^{\text{th}}$ with different wrinkling lengths L , and then substitute it into Eq. (B.12) to build the change rule between the hoop compression strain $\varepsilon_{\varphi}^{\text{th}}$ and the critical internal pressure p_{cri} :

1 $p_{\text{cri}} = f(\varepsilon_{\varphi}^{\text{th}})$ (B.14)

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Statements & Declarations

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Author Contributions

Caiyuan Lin performed the Data curation and Writing - original draft;

Guannan Chu contributed significantly to the Conceptualization, Methodology and Writing - review & editing;

Lei Sun performed the Writing - review & editing.